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THE EXAMINATION OF ARGUMENTATION BASED PROBLEM SOLVING PROCESSES OF 10TH GRADE STUDENTS IN THE CONTEXT OF QUADRATIC EQUATIONS

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Abstract: The aim of this study is to examine the argumentation-based problem solving processes of 10th grade students in the context of quadratic equations. The study was carried out in a public high school in Istanbul. The study group consists of three male students selected by the method of "homogeneous sampling". In the research, special case study, one of the qualitative research models, was used. The data in the application process were collected with two problem-solving activities related to quadratic equations. The students tried to solve the problems together by doing group work with one week interval for both problems. The problem solving processes of the students were recorded with the video recorder and the discussions between the students were analyzed according to Toulmin's argumentation model. According to the findings of the research, it was determined that the students used the argument (65), least rebuttal (3) and qualitative (3) components among the argumentation components.

Students produced less arguments in the first problem solutions than the second problem. For this reason, differences were observed in students' ability to produce arguments according to the qualifications of the questions. In addition, after the argumentation study, the students stated that the problem solving process contributed in the context of understanding the problem for themselves, expressing their thoughts, persuading their opinions by defending their ideas, and communicating together and they want to be found again in similar working environments.

Key words: Argumentation, Toulmin model, problem solving, quadratic equations.

1. Introduction

With the rapid change in science and technology, the needs of the individual and society have also changed. As a result, it is aimed to raise problem-solving individuals who produce knowledge, are persistent, have a strong communication and empathy, can look critically at events, contribute to the society and culture they live in (MoNE, 2018). National Council of Teachers of Mathematics [NCTM] (2000) states that in order to provide effective mathematics teaching, students should be encouraged to prepare learning environments where they will establish a cause-and-effect relationship and to explain their thoughts together with their reasons.

In our country, mathematics education was carried out with traditional methods of behavioral approach until 2005, and in 2005, the teaching of mathematics was started with a constructivist approach, and in 2013, the curriculum was narrowed, and the spiral programming approach was started by giving up the linear programming approach (Aydın, Laçin & Keskin, 2018). Finally, the 2013 Secondary School Mathematics Program was revised, and a constructivist curriculum was created within the framework of values, skills and competencies, using meta-cognitive skills, providing meaningful and permanent learning, associated with preliminary cognitive knowledge, and integrated into different disciplines (MoNE, 2018).

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The constructivist approach aims to develop students' cognitive and affective skills. One of the topics studied on the scope of these skills and what they should include is “Mind Habits”. When the skills in the studies on mental habits are taken into consideration in the context of mathematics, they are named as “Mathematical Mind Habits”. Cuoco, Goldenberg and Mark (1996) define “Mathematical Mind Habits” as learning to approach mathematical problems and to think about mathematical concepts that are similar to the ways used by mathematicians.

Mathematical mind habits include cognitive (problem solving, reasoning, relating, communicating) and affective (perseverance, curiosity, criticism-criticism, questioning) skills (Körükcü, 2015). The skills of criticizing and questioning mathematical mind habits skills are also at the center of the constructivist learning theory. It can be said that it is important to create inquiry-based learning environments that can bring these skills to students. Llewellyn (2002) inquiry-based learning; stated that the student was actively involved in the learning process, tried to solve and create his own questions and learned new information with the help of his old knowledge, as a process in which the learner takes place. The aim of the inquiry-based learning is to enable the student to acquire the skill of structuring the information himself.

Having a learning environment based on inquiry contributes to the increase of students' interest and adaptation to the lesson and the development of researcher aspects. The student who makes inquiries in the discussion environment supports the formation of new information by sharing their knowledge and experience (Shunk, 1996). At the same time, it was observed that the students in the group discussion environments gained a habit of working together, increased communication among students, gained experience in listening, acting together, taking responsibility within the group and making decisions together, increased self-confidence and developed a positive attitude towards mathematics (Dereli, 2008). In this context, it can be said that inquiry-based learning environments will be useful in training students who have acquired mind habits and acquired intellectual thinking skills.

In the literature, it is seen that there are many studies on science related to research and inquiry based learning. When the studies conducted in this field are analyzed, Duran and Dökme (2017) stated that the lessons taught with this approach positively affect students' academic success and attitudes towards the course. Kaya and Yılmaz (2016) determined that students showed improvement in both contexts in their researches where they investigated the effect of inquiry-based learning on students' achievements and scientific process skills. Similarly, Wu and Krajcik (2006) observed that in their studies with 7th grade students, there were improvements in the process skills and abilities of students in inquiry-based learning environments. In his study, Stohr-Hunt (1996) found that as the frequency of interrogation activities of students increases, their success in science increases.

In the light of the above, it can be said that inquiry-based learning approach is important. One of these approaches is Argumentation Based Learning (ABL) approach. Argumentation is the process of providing information to convince people about the validity of a particular claim and proving it with supportive and refuting ideas (Toulmin, 2000). According to Thoron and Myers (2012), argumentation is that students can produce alternative results with their reasons using the data presented to them to prove their claims.

The first studies on the ABL approach were carried out by Toulmin in 1958. Toulmin's (1958) ABL approach consists of six different components: claim, data, rationale, supportive, qualitative, and refuting. The first three of these components are the main framework of the ABL approach of Toulmin and the others are auxiliary components. The claim from the components of this approach; Opinions that are advocated verbally or in writing are the ideas put forward to convince the person. Data; information on which the claim is based, supports the claims and helps to reach the claim. The warrant is statements that explain the relationship between claim and data (Driver, Newton & Osborne, 2000). At this stage, justifications are put forward to bridge the gap between the alleged argument and the data supporting it. Auxiliary components of the ATO approach are backing and rebuttal. Backing are explanations that reinforce the validity of the warrant. The rebuttal are statements that weaken the effect of the warrant or emphasize that it is invalid. The qualifier is expressions that show the degree

of the data's effect on the claim. The six-component argumentation model of Toulmin is presented in Figure 1.

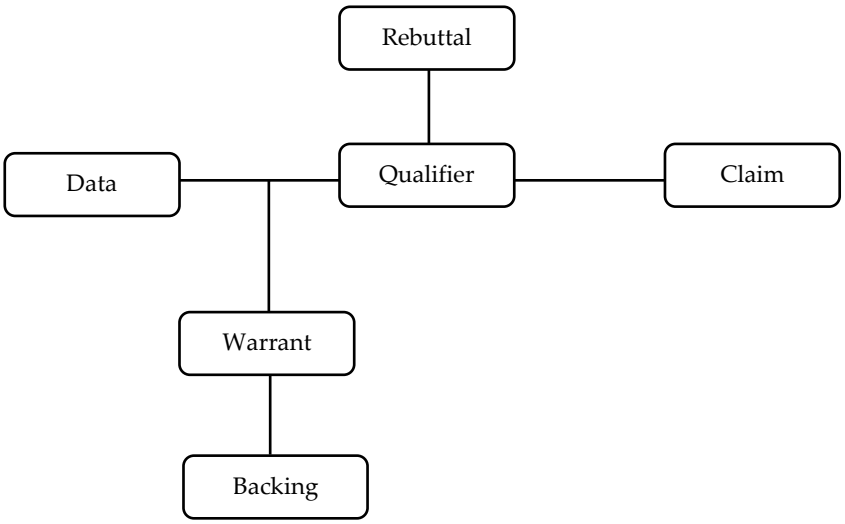

Figure 1. *Six-component structure of the Toulmin model.*

The ABL approach advocates that providing information from a single source, depending on the teacher or the book, negatively affects mental development and creativity, and that what matters is generating ideas rather than knowledge, and supports processes that attach importance to thinking (Fisher, 2004). In this approach, which is based on research-inquiry strategies, students have the opportunity to discuss scientific studies with their friends (Hohenshell, 2004). While the ABL approach is used in learning environments, students are expected to make claims based on data and to defend their claims using justifications, promoters and qualifiers. In this process, students with the same idea support the allegations, while students with opposing opinions create new ideas using rebuttal. The role of the teacher is to guide students both in creating arguments and proving their claims with justifications and supporters (Can, İşleyen & Küçük Demir, 2017).

When the literature is examined, studies on argumentation are generally in the field of science and there are a limited number of studies in mathematics (Brown & Redmod, 2007; Brown & Reeves, 2009; Can, İşleyen & Küçük Demir, 2015; Dinçer, 2011; Doruk, 2016; Duran, Doruk & Kaplan, 2017; Krummheuer, 2007; Küçük Demir, 2014; Mercan, 2015; Mueller, 2009; Mueller & Yankelewitz, 2014). When these studies are examined, it can be seen that the ABL approach has positive effects on both cognitive and affective components. In terms of cognitive components, after the lessons taught with the ABL approach, it was observed that there was an increase in students' mathematics achievement, their ability to use mathematical operations while solving problems and to develop new approaches. In terms of cognitive components, it was observed that after the lessons taught with the ABL approach, students' mathematics achievement increased their ability to use mathematical operations while solving problems and develop new approaches (Brown & Redmod, 2007; Brown & Reeves, 2009; Küçük Demir, 2014; Mercan, 2015). In the context of affective components, Baydaş, Yeşildağ, Hasançebi and Kilis (2018) affirm that ABL positively affects university students' desire for discussion; Brown and Redmod (2007), Martin and Hand (2009) and Kaya, Erduran and Çetin (2010) stated that the ABL increased students' desire to participate actively in the learning process; Kıngır, Geban and Günel (2011) stated that students think that the argumentation-based learning environment gives them more opportunities to learn.

On the other hand, in the study conducted by Can, İşleyen and Küçük Demir (2015), it was observed that the success rates of the two groups who were taught with the ABL approach and the teacher-centered approach increased after the teaching, but there was no significant difference in academic

success between the two groups. In their study, Duran, Doruk and Kaplan (2017) determined that the ABL approach positively affects students 'mathematics achievement, while it does not affect students' anxiety. In the studies of Duran, Doruk and Kaplan (2018), it was observed that the ABL approach was not effective in the context of students' gaining metacognitive awareness, but contributed to the development of quality arguments and reasoning skills.

In line with the above, it can be said that the argumentation process is effective in the development of critical thinking and communication skills. Considering the importance of these skills in mathematics education, it is foreseen to be used in the process of algebraic thinking, which plays a role especially in the transition from concrete to abstract, which students have difficulty in understanding (Chazan, 1996; Sfard, 1991; Kieran, 1992).

When looking at the historical development of algebra, it is seen that it is always taught through problem solving, but these problems are rather artificial problems that remain rather abstract from life (Katz, 1997). Therefore, students think that algebra is not included in daily life, cannot understand the importance of algebra and do not believe its necessity. However, as Katz (1997) stated, the subject of algebra and especially quadratic equations has been used as a measurement tool in determining people who may become managers in the future and have leadership qualities. Understanding the subject of quadratic equations, which is one of the important topics of algebra, is necessary for advanced studies in both mathematics and other disciplines (Lopez, Robles & Planell, 2015).

When the renewed 2018 secondary education mathematics curriculum is examined, it is seen that this process cannot be fully reflected in learning environments, although these subjects are taught with daily life problems. However, it can be said that questioning the subject of quadratic equations in a rich learning environment by using daily life problems will strengthen students' conceptual structures. According to Osborne (2005), the ABL approach contributes to students' constructing their own conceptual understanding by making predictions, proving their beliefs by stating their reasons and evaluating the opposed (opposite) arguments presented. Therefore, it was necessary to plan the research considering the contribution of the ABL approach to creating the desired learning environment and the limitations of the studies in this field. In this context, in this study, 10th grade students' argumentation-based problem solving processes in the context of quadratic equations are examined. In accordance with this purpose;

- What argumentation components do students use in problem solving processes?
- How often are students using argumentation components in problem solving processes?
- What are the students' views on the problem solving process?

answers to questions were sought.

2. Method

2. 1. Research Design

Special case study, one of the qualitative research models, was used in the research. Special case study is a qualitative research method in which the researcher examines and defines a particular situation in depth using data collection tools such as observation, interview, documents, reports (Creswell, 2007). In this study, this method was chosen because it was a study carried out by observing and discussing the determined students after the subject of the quadratic equations and no generalization anxiety was experienced.

2. 2. Study Group

The study group was selected from the purposive sampling methods by analogy sampling method. The participants of the study consist of 3 students studying in a tenth grade in a state vocational and technical anatolian high school in Istanbul in 2018-2019 academic year. These students were selected on the basis of volunteering, among the students who can express their feelings and thoughts well, have high communication power, get the highest grades in the exams held during the semester, and

stand out in class participation and make comments. In order to observe the components of the ABL approach on the chosen topic, students who have hypothesis, who can explain the reasons, who can comment, and who have a mathematical basis are preferred. All three of the selected students are men. In the study, the real names of the students were kept and pseudonyms such as Burak, Ahmet and Enes were used.

2. 3. Data Collection Tools

As a data collection tool; two problem solving activities, student feedback form, video and voice recorder were used. The problems used in the research are "Water Container Problem" and "Sapling Planting Problem" and are given in Appendix 1 and Appendix 2. The student opinion form consists of 7 items (See Appendix 3).

While determining the problem solving activities, especially the recently revised secondary school mathematics textbooks were examined and a pool of verbal problems consisting of 5 questions on the subject of quadratic equations was created. These questions are presented to the views of three mathematics teachers and two academics working in the field of mathematics education, whether they are understandable, appropriate for the level of students and the targeted objectives. As a result of the feedbacks, 2 problems were determined by making the necessary arrangements. At this stage, the compliance percentage of the experts was determined as 90 with the formula of Reconciliation Percentage = [Consensus / (Consensus + Disagreement)] x 100 specified by Miles and Huberman (1994).

2. 4. Data Collection

Firstly, a suitable working environment was prepared to enable students to solve problems in the discussion environment and to observe this process. Then, students were seated around a table and each of them was given worksheets with the problem written. Students were asked to try to solve the problem together by first reading the problem individually and then thinking together, expressing the places where they listened to each other's ideas and agreeing. This process was carried out in two different time periods. The students solved the "Water Container Problem" in the first week and the "Sapling Problem" problem in the second week. The application process of the first problem took 11.58 minutes and the application process of the second problem took 22.16 minutes. This process was recorded by the researcher, the second author of the study. In order to ensure the continuity of the discussion environment and the active participation of all three students, the researcher asked the students, "Have you evaluated all the data regarding the problem?" , "Why do you think so?", "Do you agree with your friends' opinion, why?" questions such as. In addition, a student opinion form consisting of 7 questions was applied in order to get the opinions of the students about the process after the application (See Appendix 3).

2. 4. Data Analysis

The video recordings of the problem solving activities with the students were transferred to the computer environment and then converted into written text. In this text, students' speeches and explanations about the solution of the problem are examined according to the Toulmin model. In addition to the model of Toulmin, the question component was used in the study. While creating explanations and guiding examples of these components, the views of Toulmin (1958) were adhered to. The data of students' problem solving processes were analyzed in accordance with the explanations in Table 1.

Table 1. *Thematic Framework and Guiding Examples Used to Analyze the Data in the Problem Solving Process*

Data analysis components	Guiding examples
Claim: It is the ideas or the result found for the solution of the problem.	• … becomes 20. • … there is missing data in the problem.
Data: They are real statements that support claims.	• The area of the rectangular region is found by multiplying the short edge by the long edge.

	• The perimeter of the rectangular region is found by adding up all the edge lengths.
Warrant: Statements that reveal the relationship between data and claims.	• … I multiplied 4 to 7 because we multiplied the short edge and the long edge to find the area. • … Because we gathered two short, two long sides to find the perimeter, I collected two 4's and two 7's.
Rebuttal: Statements that explain situations where the warrants are not valid. Opposing examples are also addressed in the rebuttal.	• … If he asked for his surroundings, we would collect the edges. • … If we had asked about the area, we would have hit the edges. • … If he asked for his surroundings in cm instead of m, we would add zero next to him.
Backing: It is the explanations that support the warrant and make it stronger.	• …If I divide the rectangular region consisting of 4 units and 7 units into 4 rows and 7 columns, there will be 28 unit squares. Therefore, its area is 28 square units.
Qualifier: They are statements that indicate the degree of certainty of the claim.	• … We always get the result when we multiply the short and the long edge to find the area of the rectangular region.
Question: Expressions that students and teachers point to each other in the process that require answers	• … Why do you think so? • … Do you agree with your friend's opinion, Emir? • … Why did you add those numbers?

The sound recordings obtained from the interviews with the students were transferred to the computer and then converted into written text exactly. These data were analyzed with descriptive analysis approach in four stages given below, as defined by Yıldırım and Şimşek (2011, p.256).

i) Creating a framework for descriptive analysis: The framework that Şengül and Rabbit (2019) used in their studies was used to examine students' views on the process. The thematic framework created is shown in Table 2.

Table 2. *Thematic Framework and Guiding Examples Used to Analyze the Data During the Interview Process*

Themes	Guiding Examples
Cognitive Experience	• If I encounter a similar problem in the future, I can. • It was easy to solve the problem with the equation. • It was difficult to create equations.
Emotional Experience	• The process was interesting and fun. • I'm bored / I was not bored. • I felt bad.
Linguistic Experience	• It is easier to explain my opinion linguistically.
Social Sharing Experience	• We solved the problem quickly with solidarity. • Solving the problem as a group was easier than solving it alone.

ii) Processing of data: Depending on the frame given in Table 2, the data is read and organized under the heading of the themes. Data not related to themes were left out.

iii) Description of the findings: The edited data are presented descriptively with direct quotations where necessary.

iv) Interpretation of the findings: The findings were explained and related to each other.

While making thematic coding, opinions of two faculty members working on argumentation were consulted. The consensus in expert evaluations was calculated with the formula specified by Miles and Huberman (1994). As a result of this calculation, the compromise percentage was found as 95 and it was concluded that the determined categories were consistent. Questions that could not be reconciled were reexamined and discussed until a full classification consensus was reached. For the reliability of the study, the research data were recorded to be open to the opinions of other researchers.

3. Findings

In this section, the argumentation components of the students and their views on the process are presented.

Argumentation Components of the Students in the Process

In this section, the data obtained from the problem solving processes of the working group are presented according to the components in Table 3. The data obtained in the context of "Water Container Problem" and then "Sapling Problem" were included.

Table 3. *Understanding Process of the "Water Container Problem"*

1	**Enes:** If the container is x, the other is y and the total container number is z, then x + y = z	**Claim**
2	**Ahmet:** No. The distance between is x.	**Claim**
3	**Burak:** There is a relationship between the number of container and the distance. If we say the total container x, the distance must be x	**Rebuttal, Claim**
4	**Enes:** He said the distance between the container at the beginning and at the end is 110m. If we say z at the beginning, z at the end, 110 m.	**Data, Claim**

When Table 3 is examined, in the context of understanding the problem, Enes made a claim expressing the total number of vessels using variables (1). Ahmet claimed that the distance should be x (2). Burak stated that there is a relationship between the number of vessels and the distance by using digesters, and that the total number of vessels and the distance should be expressed with the same variable (3). Enes made a new claim using a data given in the question (4).

Table 4. *Planning Process of the "Water Container Problem"*

1	**Burak:** The distance between the original and the last container is 110 m. We have to find x. x is equal to the total number of containers and the distance	**Data, Claim, Claim**
2	**Burak:** If the distance is 10, if the number of containers is 11, it provides each other and the distance is 110.	**Claim**
3	**Ahmet:** So how will its equation be?	**Question**
4	**Researcher:** Do it if you want. Maybe then you can also set up the equation	**Claim**
5	**Burak:** Get 11 containers. It becomes 11 at the distance between two containers. What kind of equation can we establish? Can we say x. (x + 1) ?	**Claim, Question, Quesstion**
6	**Burak:** If we say a is the number of containers, will the equation be a (a + 1) ?	**Claim, Question**
7	**Ahmet:** The same happens.	**Claim**
8	**Burak:** Yes, nothing has changed.	**Backing, Claim**

When Table 4 is analyzed, Burak argued that using the data in the context of planning, the total number of vessels and the distance between them should be expressed as x and should find x (1). Afterwards, Burak claimed that if the distance was 10, and the number of vessels was 11, the distance would be 110 and verify each other (2). Thereupon, Ahmet asked the question of what equation the

solution proposed by Burak should be (3). The researcher also made a claim to help them establish equations (4). Burak said that if the container number is 11, the distance between the two containers will be 11, and can the equation be x. (x + 1)? asked the question (5). Then Burak, if we take the number of cups a, will equation a. (a + 1) be? asked the question (6). Ahmet said that nothing has changed (7). Burak supported him and stated that nothing has changed (8).

Table 5. *Process of Implementing the Plan of the "Water Container Problem"*

1	**Enes:** We said 11 cups. 10.11 =110	**Claim**
2	**Researcher:** You are very close to the solution	**Claim**
3	**Burak:** We think something is missing. Since the number is small, we found it logically. We wouldn't have found it if it weren't small. Equation is a must!	**Claim, Backing**
4	**Enes:** Let's make a-1. 11.10 i.e. a. (a-1)	**Claim**
5	**Burak:** But the a-1 is the distance between the top and bottom cup. We need to find the number of containers. Equation must be equal to x. We said a container number. I think it's unreasonable.	**Rebuttal, Claim, Claim, Backing**
6	**Researcher:** Enes, why did you think a-1?	**Question**
7	**Enes:** There are 11 containers. 11 meters, distance between two vessels. The product of the beginning and the end should be 110.	**Claim**
8	**Burak:** If we say a to the number of containers. How many containers, 11. If we count the distance, it will be 2,4,6,8, 10. a. (x-1). In other words, if the distance between two vessels is a, the total number of vessels becomes a. Then a. (a-1) = 110	**Backing, Claim**

When Table 5 is examined, Enes made a claim in the context of plan implementation (1). The researcher stated that they are very close to the solution (2). Burak said they thought something was missing. He stated that the equation is necessary by using promoters (3). Enes said that the equation should be a.(a-1) (4). However, Burak said that Enes' claim was unreasonable by using a rebuttal (5). The researcher asked Enes why he thought it was a-1 (6). Enes made a claim to answer the researcher's question (7). Finally, using the supporter, Burak said that the equation should be a.(a-1) = 110 (8).

Table 6. *Evaluation Process of "Water Container Problem"*

1	**Burak:** Let's try. We said a = 11, the distance between them is a-1, so it becomes 10. 11.10 = 110. a. (a-1) = 110.	**Warrant, Claim**
2	**Researcher:** If you want, analyze the equation you find	**Proposal**
3	**Ahmet:** a^2-a= 110 .	**Claim**
4	**Burak:** a^2-a- 110=0. Let's factor it out.	**Claim, Claim**
5	**Burak:** (a-11).(a+10)=0 and a=11 ve a=-10 .	**Claim, Claim**
6	**Enes:** But the number of containers cannot be negative. So a = 11 should be.	**Warrant, Claim**
7	**Ahmet:** Yes, it cannot be negative. Answer 11.	**Qualifier, Claim**

When Table 6 is examined, it is seen that Burak verified his claim by using justification during the evaluation process (1). The researcher made a guide to the students and suggested that they analyze the equation (2). Ahmet said that the equation should be a2-a = 110 (3). Burak claimed that the equation should be taken as a^2-*a- 110* = 0 and it should be factored (4). Afterwards, Burak said that the expression of the equation in multipliers would be (a-11). (a+ 10) = 0, and there would be a = 11 and

a = -10 (5). Enes argued that the number of vessels should not be negative, so the answer would be 11 (6). Using the qualifier, Ahmet confirmed that the answer was 11 (7).

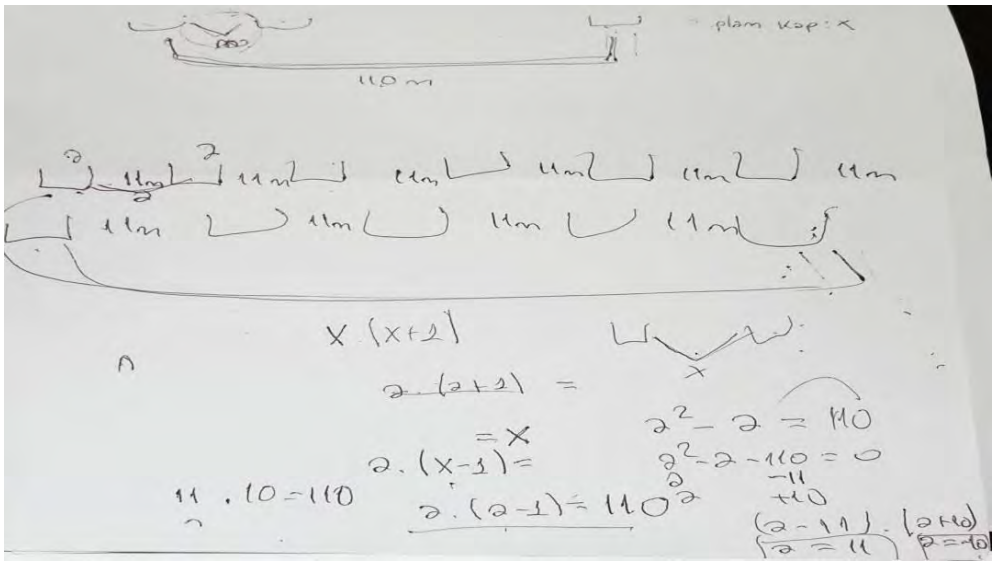

Figure 2. *Students' solutions to the "Water Container Problem"*

Table 7. *Understanding Process of "Planting Sapling Problem"*

1	**Burak**: We should start with the first item	**Claim**
2	**Burak**: Let's solve the first two items, then move on to the third item?	**Question**
3	**Enes:** Yes, we should	**Claim**

When Table 7 is analyzed, it is seen that Burak claims to start solving the problem by using the data in the first article (1). Then, to consult his friends, Burak asked, "Let's solve the first two items and then move on to the third item? " (2). Enes endorsed Burak's thought and claimed that this should be started (3).

Table 8. *Planning Process of the "Planting Sapling Problem "*

1	**Burak:** If we say x the number of saplings planted by Hüseyin, the number of seedlings planted by Taner becomes 2x-10.	**Claim, Warrant**
2	**Ahmet:** If we say x the number of seedlings planted by Taner, the number of seedlings of Gülsen becomes x / 2 +20.	**Warrant, Claim**
3	**Burak:** If we say Hüseyin to x, Taner's 10 minus of 2 times x is 2x-10 and both must be equal, so it's 10.	**Claim, Data,Warrant**
4	**Researcher:** Why should they be equal to each other?	**Question**
5	**Researcher:** For example; If my age is less than 10 times your age, do we have to be equal to each other?	**Question**
6	**Ahmet:** No, we are not.	**Claim**
7	**Researcher:** No other data? You should evaluate all the data	**Question**
8	**Burak:** We can start with item 3.	**Claim**

When Table 8 is analyzed, it was seen that Burak made a claim by making a justification in the context of creating a plan (1). Similarly, Ahmet made a claim by justifying (2). Burak reiterated his previous claim and made a new claim by connecting with a data (3). The researcher then asked a question (4). Then, the researcher repeated the question by sampling (5). Ahmet responded to this question with the

claim (6). The researcher once again posed a question (7). Burak answered this question with a claim (8).

Table 9. *The Plan Implementation Process of the "Planting Sapling Problem"*

1	**Enes:** We will multiply Taner and Gülsen, we will equal 75 times Hüseyin.	**Claim, Claim**
2	**Burak:** x.(x/2 +20) = 75x	**Claim**
3	**Ahmet:** Taner was minus 10 than 2 times of Hüseyin. We said Taner x. We wrote Gülsen connected to x, it was wrong.	**Data, Warrant,Claim**
4	**Burak:** We are doing something wrong. From the first item, Taner became 2x-10. But in item 2, we got x again.	**Claim, Rebuttal**
5	**Enes:** Then, in Article 2, we will take Taner Bey as 2x-10.	**Claim**
6	**Ahmet:** Yes, we should get 2x-10 there too.	**Claim**
7	**Burak:** 75x= ((2x-10)/2+ 20). (2x-10) .	**Claim**
8	**Burak:** It'll be complicated if we multiply.	**Claim**
9	**Researcher:** Can be simplified?	**Question**
10	**Burak:** 75x=(2x+30)(x-5).	**Claim**
11	**Enes:** $2x^2 - 55x - 150 = 0$ equation occurs.	**Claim**

When Table 9 is examined, Enes made two claims in the context of plan implementation (1). Burak, on the other hand, put forward a claim that expresses Enes' claim transactionally (2). Ahmet made a new claim using the rationale and stated that the solution made by his friends was wrong (3). Using the rebuttal, Burak stated that his previous claims were false (4). Enes made a new claim (5), and Ahmet confirmed it (6). Burak expressed the equation required for the solution (7) and claimed that the product is complex (8). The researcher then asked a question (9). Burak and Enes made claims about the solution (10) and (11).

Table 10. *Evaluation Process of "Planting Sapling Problem"*

1	**Burak:** This equation is not solved. The factors of 150 are 50 and 3 and the middle term is not.	**Claim, Warrant**
2	**Researcher:** Could you have an operating error?	**Question**
3	**Burak:** Is Taner and Hüseyin equal?	**Question**
4	**Researcher:** Where did you write equality?	**Question**
5	**Burak:** We wrote in item 3. I think we made a mistake there.	**Claim, Claim**
6	**Researcher:** Check your operations there.	**Claim**
7	**Researcher:** Did you reach the same equation? Why do you think it's wrong?	**Question, Question**
8	**Burak:** There is a mistake. I want to factor it out, but it doesn't. From 150 = 50.3 comes 53x.	**Claim, Warrant, Data**
9	**Ahmet:** But there are $2x^2$ at the beginning .	**Data**
10	**Burak:** Yes, I didn't think of it. Let's get 30 and 5,is it ok ?	**Question**
11	**Ahmet:** One must be negative. Must be 30 and -5.	**Claim, Claim**
12	**Burak:** Let's equal the factors to zero. There are 30 and -5/2.	**Claim, Claim**
13	**Burak:** Let's try. We need to take whichever equation provides.	**Claim, Warrant**

	When we put the roots in the equation, if it equals 0, it does.	
14	**Researcher:** Doesn't roots need to provide the equation because we solve it by equaling to 0?	**Question**
15	**Ahmet:** Yes it should.	**Claim**
16	**Researcher:** What is -5/2 as unknown?	**Question**
17	**Enes:** Number of saplings	**Claim**
18	**Ahmet:** The number of saplings is not -5/2.	**Claim**
19	**Burak:** Yes. -5/2 saplings can not be. The number of saplings cannot be negative. In this case, the number of saplings should be 30.	**Claim, Warrant, Claim**
20	**Enes:** Yes. It should definitely be 30.	**Claim, Qualifier**

When Table 10 is examined, in the context of the evaluation process, Burak made a claim that the equation cannot be solved by providing a reason (1). The researcher then asked a question (2). Burak also asked a question (3). The researcher once again posed a question (4) and Burak made claims in response to this question (5). The researcher claimed that they should check the procedures (6) and posed two questions (7). Burak made a claim by using data and providing justification (9). Ahmet expressed a data (10) and Burak posed a question accordingly (11). Then, Burak stated that it is necessary to equal the factors to zero and the resulting values (12). Burak suggested that the solution would be reached by trying the values found by justifying (13).Thereupon, the researcher asked a question (14). Ahmet made a claim in response to this question (15). The researcher asked another question (16). Enes and Ahmet made a claim in response to the question (17) and (18). Burak put forward the claim by giving reasons (19). Enes repeated Burak's claim using a qualifier (20). Afterwards, Burak made claims by giving reasons and continued his claim using qualifiers (21).

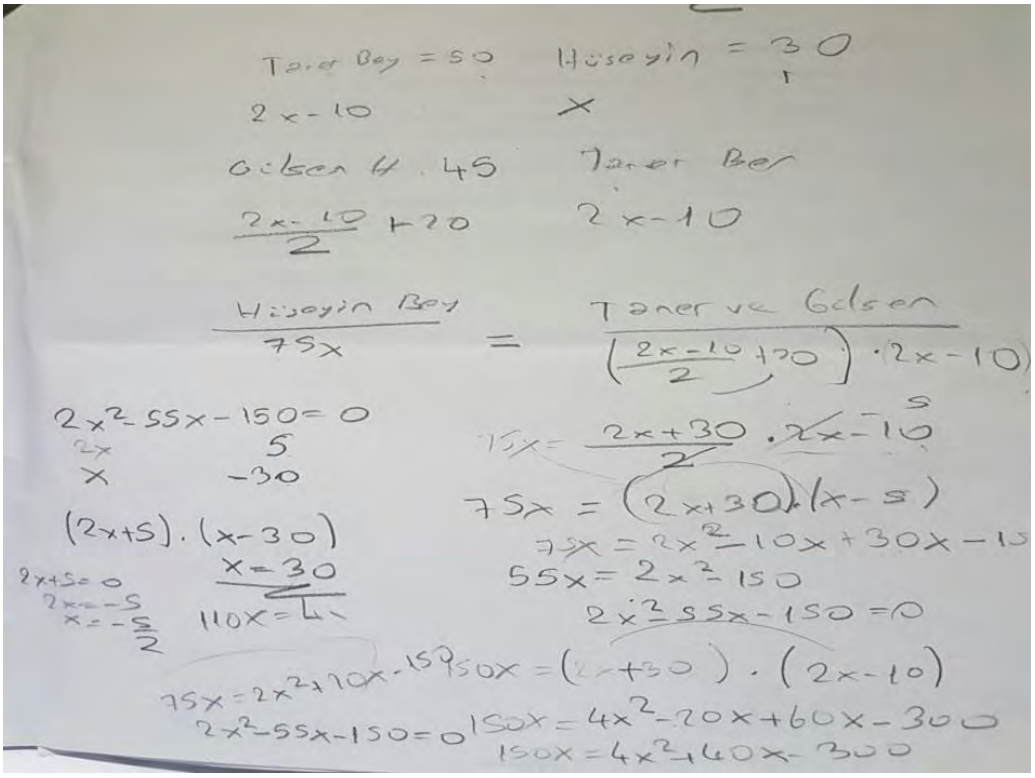

Figure 3. *Students' solutions to the "Planting Sapling Problem"*

The findings regarding how often the students use each of the data, claim, warrant, backing, rebuttal and qualifier components in the two problem solving processes are shown in Table 11.

Table 11. *Argumentation Components Used by Students in Problems and Number of Usage of These Components*

	Question	Data	Claim	Warrant	Backing	Rebuttal	Qualifier	Argument
Water Container Problem	5	2	28	2	4	2	1	44
Sapling Problem	13	3	37	12	0	1	2	68
TOTAL	**18**	**5**	**65**	**14**	**4**	**3**	**3**	**112**

When Table 11 is examined, we can observe 5 questions, 2 data, 28 claims, 2 warrants, 4 backings, 2 rebuttals and 1 qualifier in the solution process of the students' Water Container Problem; 13 questions, 3 data, 37 claims, 12 reasons, 1 rebuttal and 2 qualifiers in the solution process of the "Sapling Problem". When the two problem processes are considered together, it was determined that the students used a total of 18 questions, 5 data, 65 claims, 14 reasons, 4 supplements, 3 rebuttals and 3 qualifiers.

Students' Views on the Process

The forms containing the opinions of the students after the problem solving process are examined by the content analysis method and presented below.

It was seen that the students' opinions about how problem solving as a group affects them positively or negatively in reaching the solution with the argumentation method. The students expressed the positive effects as follows: Enes said, "My friends think about something I cannot see and we find a solution for the question," Ahmet said, "So many ideas come out." On the other hand, Burak commented, "When working with the group, one can think that the other cannot think, and that is how he taught group work." Students agree that the problem solving as a group with the argumentation method does not affect them negatively in reaching the solution.

The question: "What are the easy aspects of problem solving with the argumentation method?" Students answered that , "It helped us to progress systematically", "It enabled us to reach the solution faster and easier", "I could think better".

It was observed that the students could not clearly express the difficult sides of problem solving with the argumentation method. Burak said, "Sometimes, there can be confusion when questioning, but on the contrary, we reach an easier solution by questioning". While Ahmet replied, "If you cannot do the correct questioning, you are moving away from the answer, but you are as close as possible", Enes stated that there is nothing difficult.

The students were asked to express their deficiencies in the argumentation process, which they considered good and incomplete. It was observed that the students answered this question sincerely. Burak stated that he is good at constructing and processing the problem in his mind, but he made attention mistakes while solving the question. Ahmet stated that not trying to come up with a solution-oriented idea is a good thing, but trying to solve the problem in my own mind and then sharing it with his friends is a shortcoming.

Each of the students stated that they were satisfied with their group friends and problem solving process and stated that they want to take part in such a study again. Burak answered, "I want to take part because I think I have improved myself." It was very useful. I understood the issue I did not understand by solving a question. " Ahmet said, "I want because my point of view is changing. I saw that I could reach the answer if I didn't give up any question. He also showed the importance of friendship and how well mathematics works in everyday life. "

As a result, students stated that working in groups and making inquiries facilitated the problem solving process, the process was useful and they wanted to be in a similar environment.

3. Discussion, Conclusion and Implications

In the study where 10th grade students' argumentation processes in the context of mathematical problems were examined, it was observed that students used the most claims (65), least refutation (3) and qualitative (3) components of the argumentation components. This was followed by the backing (5) and the data (5). When looking at the other components, it was determined that students used the warrant component 13 times and the question component 18 times during the entire application process. When similar studies are analyzed, Şengül and Tavşan (2019) determined that students used the most claim component during the application, while they did not include the backing component at all and included very few warrant and rebuttal. Doruk and Kaplan (2017) came to the conclusion that students use the claim component the most and include very few backings and rebuttals. Çinici, Özden, Akgün, Herdem, Karabiber and Deniz (2014) found that students mostly use the claim component, while they cannot use the rebuttal component. When the findings of the research were evaluated separately for each problem, it was found that the students produced a total of 44 arguments during the first application period and a total of 68 arguments in the second order (Table 11). In addition, in the first problem-solving process, students were able to produce 28 claims and present only 2 reasons, while in the second process they produced 37 claims and presented 12 reasons. It is seen that there is an increase in both the number of arguments and the level of presenting claims and justifications while passing from the first application to the second. It can be said that these results are obtained as a result of students gaining more experience and better understanding of problem solving with argumentation method. However, it was determined that students' rebuttal production levels decreased during the first to second applications. In fact, it is clear that in both applications, students cannot produce rebuttals at the desired level. This result is thought to be due to the fact that the students were inexperienced in the first application and that the students mostly put forward the claims with their reasons. It can also be said that another reason for generating more arguments in the second application may have resulted from the second problem involving more data and encouraging students to question more. Considering similar studies in the literature, it is stated that as the number of applications increases, the number of arguments produced by students increases. (Nussbaum & Edwards, 2011; Kuhn & Moore, 2015; Duran, Doruk & Kaplan, 2017; Şengül & Tavşan, 2019). This finding of the study is in parallel with the studies mentioned. The study also coincides with the findings of Zohar and Nemet's (2002) research with high school students that the number and quality of argument production increases as the process progresses.

According to the findings obtained from the interviews with students after the problem solving process with argumentation, it was determined that this process had positive effects on students in terms of cognitive, affective, linguistic and social sharing components. In the context of social sharing and affective components, students expressed their opinion that working and questioning in groups facilitates the problem solving process, they are satisfied with their group friends and problem solving process and are very eager to take part in such a study again. For example, a student named Ahmet said, “I would like to take part because I think I have improved myself. At the same time, I understood the importance of friendship and how much mathematics worked in everyday life”and Enes said,“I want it. It was very useful. I understood the issue I did not understand by solving a question”. In line with these opinions, the thoughts of the students about both the application and the process aroused curiosity about learning in themselves, and it can be said that the affective mind habit skills are also positively affected. This finding of the study supports the findings of the study by Körükcü (2015).

Ahmet, one of the students who gave opinions about the process, also stated that taking part in this study changed his viewpoint to the questions and that he can reach the answer if he does not give up any question. This suggests that the argumentation process also contributes to the component of persistence, especially from affective mind habits. When similar studies are examined, it is stated that the argumentation method arouses interest towards the lessons, makes problem solving enjoyable and easier, and that students improve their communication skills with each other (Karataş, 2008; Şengül & Tavşan, 2019; Özgen & Pesen, 2008 ve Duran, Doruk & Kaplan, 2017).

When the students' views about the process are evaluated in a cognitive dimension, Ahmet commented, "When solving problems together, there are too many ideas, three people think instead of one person, your friend thinks you cannot think, accelerating and facilitating progress to this result".

Enes said, “My friends think something that I cannot see and find a solution for the question from there” and Burak said, “Although I sometimes confused while questioning, I actually found that I could find a solution easier and improved myself.” This result of the study shows that the process of argumentation and problem solving helps students acquire problem solving, reasoning, associating and communicating skills from the components of cognitive mind habits. Research results of Şengül and Rabbit (2019) with secondary school students also support the study. Similarly, Duran, Doruk & Kaplan (2017) and Küçük-Demir (2014) determined that the argumentation-based learning approach contributed to the development of students' mathematical reasoning and creative thinking skills while Kwon, Rasmussen and Allen (2010) found that teaching through the inquiry-based learning method positively influences students' ability to model and solve the problem.

Considering the above, Toulmin's ABL model can be said to support the theory of mental habits put forward by Cuoco, Goldenberg and Mark (1996). In addition, Toulmin's ABL approach provides students with the opportunity to experience their own ancestors' scientific processes used in the past, to experience their own problem solving processes and contribute to their intellectual thinking skills at an early age. In this context, it is thought that it would be beneficial to encourage teachers to use this method while teaching in their classrooms by providing them with information about ABL.

In this study, since it was considered as a limitation of the study that the study was carried out with three 10th grade students with high mathematics achievement, it may be suggested to conduct studies with students at each grade level and at different success levels. In researches about the ATO approach, it can be suggested to examine the effect of this approach on students' inquiry, reasoning and logical thinking skills In addition to these, in researches about the ABL approach, it can be suggested to examine the effect of this approach on students'

- questioning, reasoning and logical thinking skills
- the levels of enriching metacognitive information and concept images
- critical thinking level and communication skills

in different learning areas of mathematics.

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Appendıx 1: Water Container Problem

The residents of Güzelyalı Neighborhood placed water containers at equal intervals on the pavement of a linear road passing through their neighborhoods for street animals to drink. The distance between two consecutive cups is equal to the total number of cups. If the distance between the first and last container is 110 meters, find the total number of containers used.

Appendix 2: Planting Sapling Problem

Three friends, a teacher of mathematics, Taner, Gülsen and Hüseyin decide to create a "Mathematics Memorial Forest" in Ankara and planting the saplings they provide on Sunday. These friends meet Emrah on their way back at the end of the day. Emrah asks questions about the planted saplings and learns the following information:

- The number of saplings planted by Taner is 10 minus 2 times the number of saplings planted by Hüseyin.
- The number of saplings planted by Gülsen, 20 more than half of the number of saplings planted by Taner.
- 75 times the number of saplings planted by Hüseyin is equal to the multiplication of the number of saplings planted by Taner and Gülsen.

According to this information learned by Emrah, how many saplings were planted in total?

Appendix 3: Student Interview Form

1) How do you think that progress by questioning and discussing in a group environment positively affected the problem solving process?
2) In your opinion, how has progress negatively affected the problem solving process by questioning and discussing in a group setting?
3) What do you think are the easy parts of problem solving by questioning?
4) What do you think are the difficult parts of problem solving by questioning?
5) When you evaluate yourself, what do you think are the behaviors that you are good at during the application process and the behaviors you are missing?
6) Are you satisfied with the process and group friends? Please explain
7) Would you like to take part in such a study again? Why?