Examining Mathematical Representation to Solve Problems in Trends in Mathematics and Science Study: Voices from Indonesian Secondary School Students

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Article Info	Abstract
	This study describes the mathematical processes in TIMSS (Trends in Mathematics and Science Study) to reflect issues. A descriptive qualitative methodology was used in this analysis as the research approach. The research allocated the respondent to the 6th grade students of Muhammadiyah 6 in the sub-district of Dau, Malang regency, East Java, Indonesia. Using a TIMSS 2011 test and interview, data was obtained. The results of the study showed that visual representation took place in both classes of high, medium, and low capacity. Students of high and low capacity conducted the symbolic representation process at the comprehension level, while students of medium capacity performed the symbolic representation process at the problem-solving stage. In addition, as they wrote topics, the writing of the conclusions was done by students who had high skill. Students who had medium potential when they wrote questions completed the writing of the conclusions. The students with a low level of skill wrote what was understood and asked about the issues.

Introduction

Over the last decade, the achievement of Indonesian students in mathematical lessons has not been fulfilled. A study of TIMSS scores between 1999 and 2004 by the OECD revealed that scores were significantly low. We have to improve students' mathematical learning processes so that they can meet the goal (Skoumios & Skoumpourdi, 2021). TIMSS is a math and science examination for students 15 years of age and older. The purpose of this program is to gather international assessments of the abilities of students in math and science (Mullis et al., 2016).

The TIMSS needs a high level thinking ability for evaluation because the assessment includes cognitive and content dimensions. The content dimension influences the cognitive process of evaluating information. The content dimension for grades eight in Singapore is arithmetic, algebra, geometry, data and opportunity (Ebersole & Kanahele-Mossman, 2020). The high thinking ability level in mathematics would be useful in problem solving, reasoning, proof, and talking about mathematics (NCTM, 2000). Representation is one of the process of

constructing and abstracting mathematical knowledge considered important in developing the understanding and optimizing students' thinking skills, especially in learning mathematics.

Representation is a method one uses to clarify what one is saying and how one is saying it (NCTM, 2000; Rangkuti, 2014; Widakdo, 2017). Mathematical representation is an essential skill a student should develop when learning mathematics (NCTM, 2000). It allows for mathematics to be translated into language that makes sense to the reader (NCTM, 2000; Tripathi, 2008). The most important function of mathematical representation is to visualize mathematical ideas from one's own thoughts using a configuration of symbols (Golding & Kaput, 1996; Yuniawatika, 2015; Polat, 2020).

A graph or a picture represents or symbolizes aspects of concepts. Representation in mathematics is merely a supplement to solve mathematical problems and they have a very narrow range of representation abilities (Utomo, 2021). The result of interviews made by Hudiono (2005) with one of the teachers showed that display methods such as tables and pictures have no impact on students' knowledge acquisition. The 2012 released results from the TIMSS tests also reveal that Indonesian students are not doing very well with the skills of representing math ideas and concepts (Armadan, Somakin & Indrayanti, 2017; Mullis, Martin, Foy & Arora, 2012).

Based on the above rational explanation, if students were in grade four and eight, they should possess skills in mathematical representation in order to be able to compete in TIMSS. According to Piaget, students at the age of 11 years or older enter into a formal operation stage where they start being able to think abstractly. During this stage, one of the key characteristics that begins to emerge is the students' ability to think about problems in a daily life context. Hence, it was decided that the subject of the research that had the advanced study stage (Grade 8) and whose performance in the TIMSS Advanced Studies test was better than that of the grade VIII subjects in Korea was chosen because these subjects are at the formal operation stage and they might make a variety of representation so that it is necessary to know how they consider mathematical problem solving (Munalikatasari & Rosyidi, 2016; Mullis et al., 2016).

The primary objective of this study was to discover how students created TIMSS problems. Research has generally been more focused on mathematical abilities and there are few studies that use TIMSS problems in the students' process of mathematical representation. The previous research conducted by Yumiati & Noviyanti (2017) was focused on the differences in abilities in mathematical representation among Junior High School (SMP) students taught using inquiry and conventional methods. A research was made by Minarni & Napitupulu (2017) that was focused on developing educational materials to improve students' mathematical reasoning ability. A research performed by Widakdo (2017) showed how students can improve mathematical representations using project-based learning. Agasi & Rudhito's (2014) study focused on the problem solving abilities of eighth graders. Cahyati and Kriswandani (2017) examined students' understanding of mathematical concepts in solving TRMS problems. Just as Witri, Putra & Gustina (2014) demonstrated, students' mathematical abilities were also measured by TIMSS.

The study has found a correlation between mathematics representation and learning models. In the type of representation, we selected TIMSS mathematical problems and asked students to address problems in a problem-oriented way. Students' process of representation may inform the teachers about the forms and processes of representations used by students to solve TIMSS problems, which might be used by the teachers to prepare TIMSS problems. Representation develops the understanding of students but also serves as a communication tool for the teachers. Teachers should witness their pupils' understanding of the material they have studied.

Research Question

The question we were trying to answer is described as: "What is the mathematical representation process carried out by students of Junior High School with high, medium and low ability in solving TIMSS issues?"

Literature Review

Mathematical Representation

It is another means to understand the importance of law (Asmara, 2014; NCTM, 2014). Modelling mathematics helps students grasp the meaning of mathematical concepts better. Some forms of mathematical representation are diagrams, pictures, numbers, graphics, symbols, tables, expressions, and mathematical notations (Rahmawati et al., 2017; Yudhanegara & Lestari, 2015). Representation is more than a method used as a way to learn and teach mathematics, according to Antonio Soares et al. (2016), but it also important factor for students to understand mathematical knowledge and skill, and a criterion for teachers to evaluate their students' mathematics abilities and performance. The teachers may also instruct their students to use forms of analogy, visual images, or other representations to clarify mathematical ideas (Fennell & Rowan, 2015). In different ways, mathematical concepts can be interpreted. Mathematical representations create the basis of how one can perceive and apply their ideas. Representation aids students in illustrating maths or concepts (Minarni & Napitupulu, 2017).

NCTM (2014) expresses that: a) A process of representation involves the translations of problems or ideas into new forms in which the translation involves the use of diagrams or physical models into symbols and phrases that represent the ideas. b) A process of representation also includes changes of diagrams or physical models into words and phrases that represent the ideas. And c) The process of representation may also be used by translators as a vehicle to analyze verbal problems and make their meanings clear. In Fadillah (2011) and Budiharso & Tarman (2020), there are two reasons why representation is required. Firstly, fluency in translating between different representations is the fundamental skill of students to grow, which will have a significant impact on students learning mathematics. Secondly, students should be prepared to develop their own representation in order to have good skill and understanding and versatile ideas that can be used to solve problems.

Goldin and Koput (1996) say that representation is either indirect or direct. Internal representation is an integral

part of the development of the architecture of algebraic thoughts (Minarni & Napitupulu, 2017; Pape & Tchoshanov, 2001; Solikhah & Budiharso, 2020). Another example that mathematical concepts require internal (mental) representation to refer to Configurations in internal representation can not be observed directly. External representation is one's interior reflection or one's visual system (Debrenti, 2013; Goldin & Kaput, 1996; Minarni & Napitupulu, 2017; Pape & Tchoshanov, 2001; Solikhah & Budiharso, 2019). The degree to which students' ideas are expressed into verbal, visual, and symbolic forms is a representation of the solutions students find to problems they encounter.

Indicator of Mathematical Representation

In 2000, the National Council of Teachers of Mathematics (NCTM) said that mathematics programs in preschool to college-level students should allow students to develop and use representation to organize, document and communicate mathematical ideas. They should identify, choose, and apply a mathematical representation, to solve a problem, and use representation to model and interpret physical, social and computer concepts. Elsewhere, Suryana (2012) tells us that the following mathematical representation should be used. Indicators of visual representation include the representation of diagrams, graphics, or tables to aid problem solving, and the use of visual representation to make things clearer. Indicators of symbolic representation consist of making equations or mathematical models from other kinds of representation; making statements or conjectures from numeric representations; including mathematical expressions in solving problems. The indicators of verbal representation comprise of making questions based on data or representations; and answering problems with verbal representation.

The research will be classified as visual, mathematical and verbal (words or written texts). Table 1 shows mathematically represent the objective of his research.

Table 1. Mathematical Representation Indicator

Representation	Indicator
Visual Representation	Students present problems into the forms of pictures to solve them.
Symbolic Representation	Students use mathematical expressions using symbols or numbers to solve problems.
Verbal Representation	Students write interpretations of problems presented.

Solutions of TIMSS Problems

TIMSS does a study of math and science high school graduates (Mullis et al., 2016). TIMSS is like international test which is made by the International Association for the Evaluation of Educational Achievement (IEEA). Integrated Environmental Assessment is a certain organization that takes care of global environmental issues. NRC was established in 1958 (Mullis & Martin, 2013). In TIMSS studies of mathematics and science, they are divided into two domains: contents and cognitive (Mullis et al., 2013, 2008, 2004, 2000). Mathematics has four subjects: number, algebra, geometry and opportunity. The number domain contains all integer numbers, even

numbers, fraction numbers, ratios, decimal numbers, percentages and proportions. Algebra is concerned with equations and inequalities, algebraic operations and expressions and/or relations and functions. Geometry measurements, forms, and positions and transfers are part of geometry content. Besides, LUCC will cover interpreting data, data characteristics and opportunities.

TIMSS problems, which are an international level that measure student success in the field of mathematics and science of all over the worlds, are used to determine student's ability. Solutions to the problems in this report can be described as a lengthy process of chasing the largest possible understanding, afterwards working on possible/important planning, followed by a long process of implementation and then short reviews on results (NCTM, 2014). One way to look at this question is to think of a hypothetical situation like someone who understands a problem if they can know what is known and asked by the problem. After everyone has already discussed the solution to the problem and everyone has agreed on the approach to taking, then it can be called the stage of planning. At the start of a project, a student might think of a method to use or method concept (concept of a way to do something) that has been thought before such as, "I'll choose the stone method." At the stage of reviewing, a student retests the result of the answer she/he has got and observes the steps used in solving it.

Methods

Design

A case study design and qualitative methodology were used in this recent study (Yin, 2003). A case study was chosen because it focuses on a small group of students and allows for in-depth examination of a specific case. A descriptive qualitative analysis is a survey or study that focuses on identifying, exploring, assessing, and observing the subject's situation based on the available information. The aim of this recent study was to see how students performed during the teaching process in terms of mathematical representation. The argument for this analysis was compelling because the researchers were able to delve deeper into the phenomena rather than merely generalizing results (Yin, 2009). A case may be interesting because of its unique characteristics or because it is mundane. This study used qualitative methods to explain themes based on evidence identified by the researchers in terms, phrases, sentences, and paragraphs (Cresswell & Miller, 2000).

Participants

Participants in this study were 6 (six) students in the 4F class of Muhammadiyah School in the Dau Subdistrict, Malang Regency, East Java Province. They had average, medium, and low IQ, or they had above average to subpar, intelligence for their age. The participants of the study were picked from a group of middle schoolers in an equal amount of math classes. We decided to use the sampling approach, in which we used an act of convenience to include six participants (e.g., Creswell & Miller, 2000). There are preferred here. It turned out that optimizing the efficiency of searching for emerging themes and evaluating their relative importance according to contextual conditions provided an/do better answers (Erlandson et al. 1993, p. 82). The team of researchers was worried that the participants they could not represent the middle school mathematics teachers in

the district well by himself and him only serving as examples. We defined the participants in groups of the first, second, third, fourth, and fifth in this instance. The participants in this experiment were T5 and T6 girls.

Data Collection Technique

With the use of various approaches, this qualitative analysis focused on numerous sources of evidence in pursuit of convergence and corroboration (Yin, 1994). This research examined data derived from data gathering and instructional techniques. Several students with a portion of their video lessons recorded, and all of their lesson plans were made available to the principals, as well (Co-PIs). One of the PI's obligations was asking for access to data (in this case, the video lessons and lesson plans) to be made available (so, these could be used) This study relied on a total of 10 videos of 6 students, of the highest quality, and every video was recorded by 6 different students.

Video observations were our primary data source because videos allowed us to pause and rewind for extensive analyses of lessons. The tests were taken, and the answers provided, the results were analyzed, and the questions were asked. Based on the national curriculum, the tests in this study were tweaked. specially from the content on the previous version of the program of the Program for International Student Assessment (PISA) Tasks such as these are used to obtain a representation of students' mathematical understanding. Lecturer of math: The problems for the experiment were discovered through experience in the course before data collection began. For a better understanding of the process of using mathematics in TIMSS problems, we conducted interviews. They also recorded everything that was said and will be incorporated into future discussions. In a series of interviews between students and teachers, the questions were based on the answers provided by the students.

Observation

As far as research is concerned, gathering information and observations is concerned; this is the major factor (Merriam & Tisdell, 2016). The use of filmed examples of the mathematics lessons was a key to the success of this study, for those were viewed as significant variables. The two videos that were recorded verbatim by the study's first author were observed and filmed verbiage was used for the transcripts. it was estimated that the duration of each video was between 45 and 50 minutes

Documents

Artifacts such as bone and antlers provided the second possible study data. The university professors recorded their lessons and uploaded and shared the information about lesson plans and other teaching materials with the students who took part in the video. Examination of all related documents (e.g., lesson plan, handouts) during the data analysis was performed to guarantee the research object's credibility. Students spend their time investigating the environment (investigating the classroom), the people who teach there (specifically) and what they are investigating (specifically), and that which aspect is being examined (intently) (Merriam & Tisdell, 2016). Students received lesson plans and handouts to aid them in their understanding of the various objectives

of the class as well as narrative information about what the teacher was hoping to accomplish during class time. Artifacts developed by these teachers were of their own volition and were not made to satisfy test design requirements. Artifacts, however, were invaluable for triangulation of data.

Field Notes

Each video observation was documented with field notes, which included a general overview of the lesson (e.g., the instructor spent 5 minutes on engagement) as well as reflective notes detailing the researchers' viewpoints and observations of the lesson (e.g., teacher spent more time in phase of exploration). in their classrooms. True, though all of the information had been gathered from a videotape of the lesson, there were still some additional details that could not be found out by looking at the clips.

Technique of Data Analysis

As descriptive as possible, qualitative data on subjects' observations and/commentaries were used in this study, this research utilized subjects' creative observations and commentary. When the quantitative data has been first obtained, it is sifted through numerous examples to arrive at the final answer. The collective efforts of everyone on the team dramatically reduced the available data. The skat effect is where, after each three points is made, one roll of the dice is followed by two consecutive points in the direction of bank, which is ideal in this case, followed by three points in the opposite direction to the bank (and results in a new points being added in bank) as long as the pattern holds. This research was done by showing the results of the interviews and experiments as well as the results in a written format. The last stage of the TIMSS mathematical representations occurs following the completion of the student mathematical student responses and interviews.

Finally, the researchers advised that students complete a sequence of tasks that include interacting with a new concept, doing more practice, and examining progress in a new environment, or environment. Data analysis looked at this pattern. to ensure the credibility of the data The researcher recorded and edited each video using the 5E coding rubric During the rubric grading process, each lesson received a score of 1 (present) or 0 (absent). each lesson earned an overall grade, which was equal to the total of the numbers of their descriptor scores However, video-recorded evidence demonstrated why each descriptor's score was correct. a carefully selected and random-sampled video lessons were pre-coded to control for observer inference while maintaining semantic precision to determine inter-reliability.

Results

Process of Representation Made by the Student with High Ability

The student with high ability in the process of representation and communication is ST (the abbreviation of the name of the student studied). The researcher conducted an in-depth analysis of the topic, introducing contrasting arguments to satisfy the need for scientific representation.

Based on the student's preceding work done in Picture 1, the student undertook the process of visual representation of this problem on the basis of a straight line provided by ert and positioning at the point B with the distances so that the sign is that ert the midpoint A is between the point A and B, with the distance of the midpoint B is between B and C. This shows that the student was able to visualize a picture and therefore solve the math problem given.
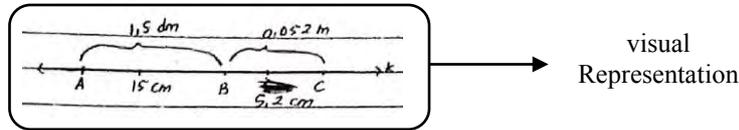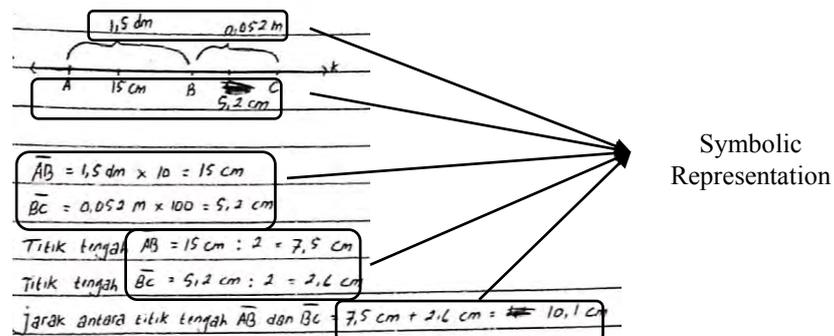

Picture 1. Process of ST's Visual Representation

The student's interpretations about the interview during the semester are as follows: The student views those materials as representation of a problem solving tools. In the above interview, ST reports about that he has many symptoms.

P	:	*"What do you know about this problem?"*
ST	:	*"There is a line k with points of A, B, C and the distance between the point A the point B is $1,5\ dm$ and the distance the point B to the point C is $0,052\ m$"*
P	:	*"Can you understand the question of the problem?"*
ST	:	*"Yes mam, I can, here it is instructed to look for the midpoint between the points $\overline{AB}$ and $\overline{BC}$ "*
P	:	*"What did you do after understanding the problem?"*
ST	:	*"I drew the line, mam."*
P	:	*"Why did you draw the line first before solving this problem?"*
ST	:	*"Because by drawing the line it is easier to solve the roblem given especially in putting the points A, B, C in the line".*

The symbolic representation made by ST is given in Picture 2.


Picture 2. Process of ST's Symbolic Representation

In Picture 2, the student undertook symbolic representation, at the stage of solving the problem, by writing a symbol for the unit of length in the straight line with the distance between the point to the point of then that

from the point to the point of and writing the symbol for the name of the line with. ST also wrote the symbols for the distance between two points and, and, and the change of units from to, to, and from to. The cause of the writing of symbols was that he was used to writing symbols at the process of solving problems and he understood the symbol he wrote.

Then, the process of the verbal representation conducted by ST is given in Picture 3.
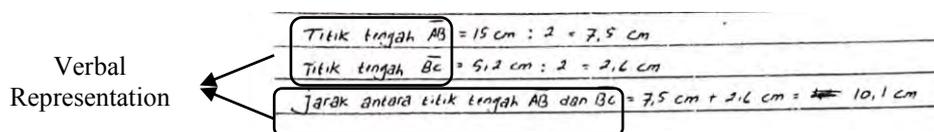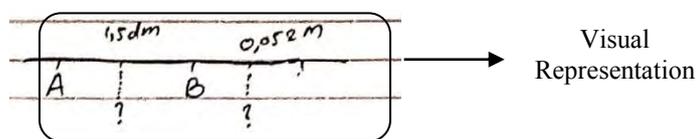

Picture 3. Process of ST's Verbal Representation

At the stage of writing a statement in finding the distance between the midpoint O and O as shown in Figure 3. Furthermore, ST also wrote the final result of his research. This is supported by the statement ST wrote in order to clarify what he/she had done in order to solve the problem. The following is ST's results from the interview.

> P : *"What did you write the statement about the distance between the midpoint*
> $\overline{AB}$ *and* $\overline{BC}$ in the process of the solution to the problem*?"*
>
> ST : *"To make it more clearly mam if I found the distance between the midpoint*
> $\overline{AB}$ *and* $\overline{BC}$ in line with the question given in the problem*"*

Process of the Representation in the Student with Medium Ability

The subject with an average of medium ability in the process of representation is Susan. See the group of things, not made by hand, which produce a variety of relations. The process of visual representation used by SS to solve the problem is made by making a straight line and determining the points where the point is located between the points and as shown in Picture 4.
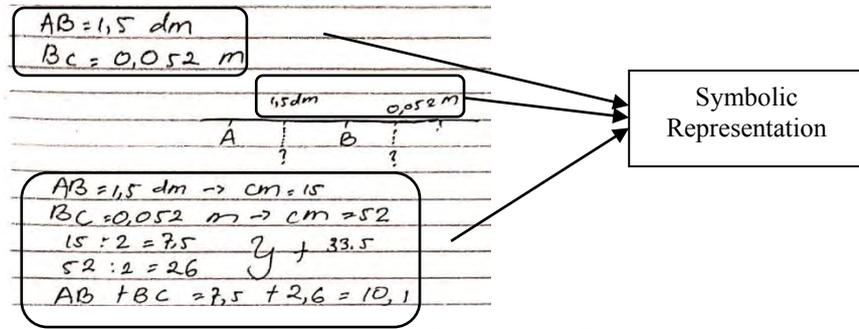

Picture 4. Process of SS's Visual Representation

In the figure, SS determined the distance between the point A to B and between the point A and B by measuring the distance from the point A to B, then he decided which midpoint would be between the points A and B, and determined that the point between A and B should be the midpoint. The midpoint of the points is intended to know the position of the point asked in the problem. In SS's visualization, A shows how to solve the problem. This information is derived from the interview. Interview with SS is aimed to get information from the answer to the written questions.

> P : *"Why did you draw the line?"*

SS : *"To help me do the problem, mam."*

P : *"Do you always draw first before doing a problem?"*

SS : *"Yes mam, because by drawing, I understand what is the form like"*

P : *"Do you know how to draw a straight line?"*

SS : *"Yes mam, like my picture"*

The process of symbolic representation conducted by SS is given in Picture 5.

Picture 5. Process of SS's Symbolic Representation

It was made by Jean Benac when he understood the problem, and drawing a straight line when he solved the problem. When he wrote out the symbols he understood the problem in terms of a line with a unit of length. The symbol for the point A of the line (k) was drawn near the straight line (k). The post, which is centered at point B, was written with the symbol for the point overhead, and the symbol for the length of the object between the point B and the C as showing. In the response, SS wanted to symbolize the length and unit of the line segment in terms of another unit and for the question/problem, he used another symbol and he would need to surround the other symbols. The spelling of the symbol for the unit of length could be seen in the distance between the midpoint of and where he merely wrote as shown in Picture 4.5. There is a need to have a "x" in the middle of the results in the interview because the writer thinks it is not necessary.

The process of the verbal representation made by SS is given in Picture 6.
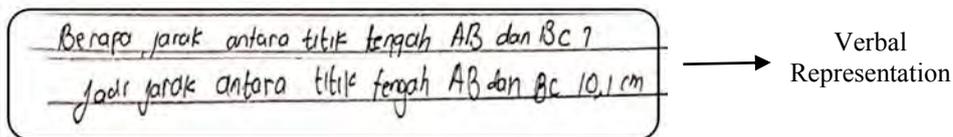
Picture 6. Process of SS's Verbal Representation

Based on Figure 6, SS wrote a question about the location of P(2,2), and then he constructed a conclusion in the form of a statement about the value of the distance between P(2,2) and Q(2,2). The interview with SS revealed that he wrote a conclusion to explain the end result of all the processes used in solving the problem.

Process of Representation Made by the Student with Low Ability

The significant topic of the student's work in the process of creating the portrait is SR. The blink of an eye in

time represents the process of representation. Picture 7 shows that SR's plan worked because he drew a straight line k and determined the location of the points namely the point B between the point A and the point C. It can be seen that the distance between point A and point B is farther than the distance between point B and point C, therefore the answer is yes. From results of the interview, SR determined that is longer than; therefore, is higher in decimal numbers.
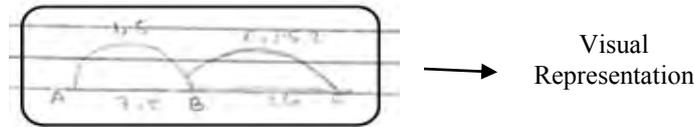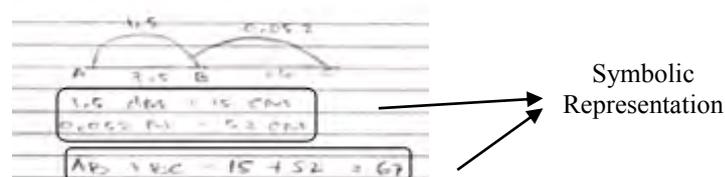

Picture 7. Process of SR's Visual Representation

The results of the interview are reinforced by the answer. The interview with SR is below.

> P   :   *"Why did you draw the line k?"*
>
> SR   :   *"To make easy to determine the positions of the points mam"*
>
> P   :   *"Does the distance between the point A and the point B is equivalent with the distance between the point B and the point A?"*
>
> SR   :   *"Different, mam"*
>
> P   :   *"Which one is longer, the distance from the point A to the point B or the point B to the point C?"*
>
> SR   :   *"The distance from the point B to the point C"*
>
> P   :   *"Why is the distance between the point A and the point B is shorter than that of the point B to the point C?"*
>
> SR   :   *"Because $1{,}5\ dm$ is smaller than $0{,}052\ m$"*

SR then conducted the process of the following symbolic representation. Picture 8 shows the problem was solved in the form of symbols. The symbols were for a unit of length and a unit of length. However, there is a symbol for a unit of length written with a capital letter such as the symbol for meter (written with and the symbol for the line segment was written with instead of). It seems SR is not able to understand mathematical symbols completely. He missed the significance of symbols in the solution process because he did not recognize them for what they were.

Picture 8. Process of SR's Symbolic Representation

In SR, there is a process for step wise representation.
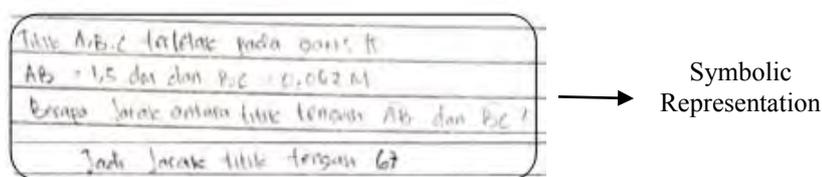
Picture 9. Process of SR's Verbal Representation

In picture 9, SR did the verbal processing which was to understand the problem, rewriting the problem, and generating what is asked. SR wrote about his conclusion regarding the result of his work. In order to clarify the cause of the illness, the patient informed the doctor about her family background. This response came from a male participant whom I had interviewed.

P   :   *"Why did you rewrite the problem and the question of the problem?"*

SS   :   *"I don't know, mam"*

P   :   *"Why did you write a conclusion in the form of a statement "the distance between the midpoint is* 67*?"*

SS   :   *"To make the teacher if the result I obtained is* 67*"*

Students of the TIMS student can represent themselves, however they are not in control of solving the EPT problems themselves. Instead, teachers and staff can devise a method for solving a particular thing in particular ways.

Table 2. Process of Students' Representation Based on Ability

Representation Ability	Visual	Symbolic	Verbal
High	Presenting a picture at planning the problem solving.	Writing a symbol completely at the stage of problem solving.	Writing a conclusion from the result of the calculation obtained.
Medium	Presenting a picture at planning the problem.	Writing a symbol when understanding the problem and solving the problem.	Writing a question at understanding the problem and writing a conclusion from the result of the calculation obtained.
Low	Presenting a picture at planning the problem solving.	Writing a symbol at the problem solving.	Rewriting the problem and question of understanding the problem.

Discussion

Our findings show that the students' mathematics abilities demonstrated in this study are high, medium, and low performances in solving the TIMSS problems. Students with high intellect do visual representation in their heads. According to Arcavi (2003), visualization makes an important part in constructing mind, understanding and concretize abstracts thoughts to solve math problems. At the end of the class, students are making symbolic representation by writing a complete symbolic representation showing the solution to the problems as a process of verbal representation. According to Mahardiningrum and Ratu (2018), the students with high abilities in mathematics may be able to propose their ideas well for solving problems.

In the process of there being helpful questions given by the students with a medium level of ability, then they would represent and solve problems by writing symbols as a symbolic representation process. On the down side, their writing of symbols is missing of completeness. Students who are good readers may be influenced to be more proficient in the way they interpret TIMSS issues are created as a comparison of the outcomes of the students' responses and interviews, and analyze textual material. Students are asked what is wrong with a specific situation and they would have to make a conclusion by using example sentences from the problem. Before actually solving the problem, they try to visualize each piece of the puzzle so that each fits into the whole.

Students with low ability might act out the process of solving the problem by writing symbols such as math symbols and sentence symbols on the board. When students are doing their homework, they often make a visual representation by visualizing a picture. We need to work on keeping students' visual skills intact so they can produce good visualization (Bal, 2014). As they were asked to take a problem and rewrite it as an embodiment of their understanding, they can substantially increase their problem-solving crafts. Those students who have difficulty in making representation, or can't represent a number of different values, will experience difficulty in solving difficult problems since representation is one of the things that is necessary in solving problems (Aryanti, Zubaidah & Nursangaji, 2013; Montague, 2004).

Conclusion

As a result of the TIMSS testing, it has been determined that students' ability to perform in class is greatly improved, and along with this the use of laptops, smartboards, and e-notebooks is also said to greatly improve student's performance in class. When exemplifying a visual representation task, students of varying ability needed to use their strengths, while students with low ability used their weaknesses, and students with high ability used their strengths to solve the problem. Processing is completed at two separate points: once at the level of comprehension, and once at the problem solving stage. Students with high and low skills are automatically identified at the stage of comprehension during the symbolic representation process, while students with intermediate skills perform the symbolic representation process at the solving stage. First, when they are reading the problems themselves as well as reading the answers and questions, the answers are presented in the form of these final written products. Second, when students read the problems, questions, and

answers, they easily learn to convert these into verbal speech by themselves and make conclusions with the help of this process. When people's utterances are transformed into written characters by them and read by others, they can easily understand these. This method has the most advantage for learning to understand the problems and the answers to them, but it is difficult for people with low ability of verbal expression, who have reading problems that make it difficult for them to understand the written characters and write the answers or write questions.

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