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Eighth Grade Students' Misconceptions and Errors in Mathematics Learning in Nepal

Mukunda Prakash Kshetree
Tribhuvan University, NEPAL

Bed Raj Acharya
Tribhuvan University, NEPAL

Bishnu Khanal
Tribhuvan University, NEPAL

Ram Krishna Panthi
Tribhuvan University, NEPAL

Shashidhar Belbase*
United Arab Emirates University, UNITED ARAB EMIRATES

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Abstract: This paper explores misconceptions and errors (M/Es) of eighth-grade students in Nepal with a quasi-experimental design with nonequivalent control and experimental groups. The treatment was implemented with teaching episodes based on different remedial strategies of addressing students' M/Es. Students of control groups were taught under conventional teaching-learning method, whereas experimental groups were treated with a guided method to treat with misconceptions and errors. The effectiveness of treatment was tested at the end of the intervention. The results showed that the new guided treatment approach was found to be significant to address students' M/Es. Consequently, the students of experimental groups made significant progress in dealing with M/Es in mathematical problem-solving at conceptual, procedural, and application levels.

Keywords: *Mathematical conceptions, misconceptions in mathematics, students' errors in mathematics, Nepal.*

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Introduction

The misconception seems a common phenomenon among students in different disciplines along with mathematics (Ojose, 2015). Misconceptions and errors in mathematics learning have been a concern for researchers, scholars, and mathematics teachers. There are numerous studies on students' misconceptions and errors in mathematics (e.g., Aliustaoglu et al., 2018; Burgoon et al., 2017; Mohyuddin & Khalil, 2016). These misconceptions and errors could be due to various reasons, for example, student disposition to mathematics (Kusmaryono et al., 2019), teaching framework (Skott, 2019), teaching skills (Organization of Education Economic Cooperation and Development [OECD], 2019), students' preconceptions (Diyanahesa et al., 2017), limited understanding (Saputri & Widyaningrum, 2016), lack of appropriate modeling (Blazar & Kraft, 2017), and lack of higher-order thinking skills (Kusmaryono et al., 2020), to name a few. Mathematical misconceptions seem to be associated with inaccurate ideas that students develop in mathematics due to a lack of clarity in concept learning. Such misconceptions may have a root to their prior knowledge, which they generalized inappropriately (Im & Jitendra, 2020), and they consider either that what they are doing is correct or are not sure what they are doing (Neidorf et al., 2020; Rushton, 2014). An error may occur due to incompetence or lack of awareness to check the answers given (Hansen et al., 2014). The persistent misconceptions may interfere with students' ability to understand mathematical concepts and may cause a frequent repetition of the errors (Im & Jitendra, 2020). Such error may lead to low performance, causing anxiety toward the subject leading to negative attitudes and poor images of mathematics (Belbase, 2013).

Hansen (2006) described 'misconception' as a person's perception of a concept when it is meant to conflict with the believed meaning and understanding in mathematics. Students' prior inappropriate learning experiences might result in misconceptions (Mc Neil & Alibali, 2005). Therefore, misconceptions are consequences of students' attempts to generate their knowledge (Brodie, 2014; Olivier, 1989). These misconceptions can be intelligent attempts based on inaccurate or partial prior experience that may add to constructivist theories to view errors as part of misconceptions

* **Corresponding author:**
Shashidhar Belbase, Department of Curriculum and Instruction, UAE University, Al Ain, United Arab Emirates. ✉ sbelbase@uaeu.ac.ae

or conceptual structures because of underlying misconceptions (Vermeulen & Meyer, 2017). In this sense, the misconception may produce errors, and consequently, misconceptions and errors (M/Es) can be viewed as an interrelated construct of misunderstanding instead of two separate entities (Bush, 2011; Tendere & Mutambara, 2020). If errors are corrected immediately and superficially without analyzing their root causes, they may repeatedly resurface due to naïve knowledge (Chi & Roscoe, 2002). Students' M/Es in mathematics can help teachers decide on suitable remediation strategies by playing a constructive role in classroom discussions (Hiebert et al., 1997). Therefore, the analysis and treatment of students' mathematical M/Es should be a fundamental concern of mathematics teachers to address the variety and complexity of students' mathematical M/Es; mathematics teachers require specific knowledge and skills to analyze and address them (Skott, 2019).

Many researchers have focused on students' mathematical misconceptions and errors' from different perspectives (Cline et al., 2020; Muzangwa & Chifamba, 2012). To set out the functional role of M/Es, teachers need to build a healthy classroom environment where students should be free to take risks and try out ideas without being ridiculed (Nesher, 1987). Teachers need to set a balanced tone to build such a classroom culture (Hiebert et al., 1997). To reduce M/Es on time, the teachers should treat them as early as possible before it passes on to the students as habits in the long run (Egodawatte, 2011). Teachers should focus on three significant areas of concern, such as *categories* of misconceptions and errors, *reasons* for misconceptions and errors, and *measures* to overcome the misconceptions and errors (Sisman & Aksu, 2015). These misconceptions and errors may result in any stage during concept formation through interiorization of a concept, condensation of the newly learned concepts with older concepts, and reification of the concept (Sfard, 1991; Vermeulen & Meyer, 2017).

Despite the abundance of studies on students' M/Es in international contexts, there is very limited evidence of studies in this area in Nepal. There is a persistent low achievement in mathematics in the primary, middle and high school national examinations (Education Review Office, 2015; Mathema & Bista, 2006). There might be several factors accountable for such low achievement in mathematics, for example, socio-economic factors, teacher and school factors, and curriculum and assessment (Pangeni, 2014; Rijal et al., 2017). However, there is almost no focus on the students' M/Es in mathematics as one of the factors in their low achievement. In this context, the current study aimed to assess students' M/Es in classroom practices and apply different remedial strategies to treat them in Nepal. The research question that guided this study was-- How does a guided teaching approach remedy students' misconceptions and errors? The subsidiary research questions were:

R1: Was there a significant impact of a guided approach to teaching mathematics to address students' M/Es on students' achievement?

R2: Was there a significant impact of a guided approach to teaching mathematics to address students' M/Es on conceptual understanding in mathematics?

R3: Was there a significant impact of a guided approach to teaching mathematics to address students' M/Es on procedural understanding in mathematics?

R4: Was there a significant impact of a guided approach to teaching mathematics to address students' M/Es on application level understanding in mathematics?

The research question can be justified from a theoretical base—*learning difficulties* that may lead to mathematical misconceptions and errors, and a practical base related to *addressing misconceptions and errors*. The difficulties in learning mathematics attributed to students' inability to process information at the rate of the instructional pace, lack of timely response and feedback from teachers, worry about mathematics, and difficulties in learning to process and then being detached from it (Brown & Burton, 1978), or even due to 'students' carelessness or overloading of working memory' (Lemaire et al., 1996). Consequently, a clear understanding is needed to remedy the M/Es so that students will be successful in learning mathematics (National Council of Teachers of Mathematics [NCTM], 2014). Even fundamental algebraic concepts or operations, like addition and subtraction of like and unlike terms, may involve too complicated cognitive processes. Nonetheless, teachers may superficially mark those processes as obvious and think that it is not necessary to deal with them rigorously and ignore or underestimate students' difficulties (Campbell, 2009; Schoenfeld, 1985). Then it misleads the students and teachers (Stanic, 1986). Further, misconceptions in one domain of contents may cause misconceptions in another related domain.

Teachers need to understand student's logic behind his/her thinking that lays errors, which can be done by using thinking aloud protocols and adapting diagnostic interview procedures (Mulungye, 2016). If the misconceptions behind errors were identified, they would help teachers to design remedial strategies to treat them (Mulungye, 2016). After locating students' M/Es, they should be treated through different remedial strategies, which is crucial when there are poor learning outcomes from traditional mainstreaming pedagogy (Osei, 2007). Remedial course and process, theoretically, is preferred instead of repeating the same material with the assumption that it does not motivate learners (Osei, 2007). The misconceptions and errors are part of the learning process and can be remediated with explanation and regular practice (Bush, 2011). Mainly, teachers may use two ways of remediating M/Es-- incidental correction applied while practicing mathematics and systematic corrective work as a supplementary treatment to address the concerned problems (Luitel, 2005). Experienced teachers can predict students' M/Es, and prediction of the errors goes well means manifestation of teachers' awareness towards students' M/Es (Mulungye, 2016). The systematic remedy process starts with identifying students' M/Es and then moves along with implementing teaching episodes prepared by incorporating their M/Es (Mulungye et al., 2016).

Literature Review

Problem analysis for understanding M/Es

There are several studies on students' misconceptions and errors in mathematics, their sources, and teacher efforts to minimize their misconceptions and errors through interventions or modifications in teaching-learning of mathematics (e.g., Fumador & Agyei, 2018; Mulungye's, 2016; Vermeulen & Meyer, 2017; Walick, 2015). Walick (2015) conducted a study on the *problem-analysis model* to propose a model for analyzing and solving algebraic problems. The study has shown a significant relationship between the model and students' skills required for learning algebra. It helped to examine students' deficit skills and identify their misconceptions through analyzed algebraic problems and their solutions. Likewise, Mulungye's (2016) study exposed the *nature and origin of students' errors and misconceptions* in algebra and examined the influence of remediation of M/Es through classroom practices among secondary-level students. The study has been able to find out students' M/Es patterns, teachers' knowledge of students' M/Es, and develop remedial actions for M/Es. It recommended that more focus should be given on students' conceptual understanding rather than procedural parts and teacher-students interaction to identify and remedy students' M/Es (Bush & Karp, 2013). This study provided enough ideas and synopses to identify students' M/Es, prepare, and implement remedial strategies to address those M/Es.

Teacher knowledge of students' M/Es

Vermeulen and Meyer (2017) studied *teachers' knowledge and students' misconceptions* about using equal signs on 57 students of sixth grade and five mathematics teachers. They used a questionnaire for all students, a focus group interview for teachers, and individual interviews with six students. They reported that most students were not able to comprehend the meaning of the equal sign correctly. This result corroborated teachers' knowledge and skill to teach the concept related to the equal sign, and they could prevent students' misconceptions about the use of the equal sign. Fumador and Agyei (2018) researched *students' algebraic M/Es* to explore the diagnostic conflict approach's impact to identify and remedy students' M/Es. The study, conducted among 114 students of high school level, has analyzed an impact on the remediation of M/Es, followed by an examination of teachers' knowledge and skills to treat students' M/Es. The study showed that the diagnostic conflict teaching approach was significantly effective than the conventional method to remedy students' M/Es and enhance their achievements. Ung et al. (2019) studied identifying different *algebraic errors and misconceptions* behind them using an explanatory research design. They used an assignment and two tests of algebra among twenty-six students of the school. They identified five types of common errors: conjoin errors, sign errors, and errors due to misapplication of rules, misinterpretation of cancellation, and misuse of the distributive property.

Mishra (2020) conducted a study entitled *conception and misconception in teaching arithmetic at the primary level* to identify and analyze primary level students' conceptions, misconceptions, and alternative conceptions on arithmetic. He also identified the teachers' difficulty level and problems in teaching arithmetic. For this study, he selected 160 primary school teachers (one from each school) and 320 students (two from each school). The study has discussed students' typical perceptions of conceptions, misconceptions, and alternative conceptions. The study has suggested teachers' skills to treat students' misconceptions and reform pedagogical content and knowledge. Im and Jitendra (2020) studied *students' misconceptions in proportional reasoning*. They conducted this study on a sample of 338 students with mathematical learning disabilities. The study result demonstrated a positive effect of schema-based instruction to enhance students' learning in the experimental group. This study showed that mathematical misconceptions, such as wrong ideas and faulty understanding of mathematics concepts, could be reduced by applying a specific teaching strategy like schema-based instruction.

Constructivism and students' M/Es in mathematics

Students' M/Es in mathematics can be a natural part of everyday experience of learning and teaching (Hansson, 2020). In a constructivist view, misconceptions may play a crucial role in the teaching-learning process because they are part of perturbations and productive struggles to construct new mathematical conceptions. The traditional teaching-learning approach applies drill and practice (Thomas, 2017), but constructivist approach focuses on the re-organization of the learners' experience by reconstructing conceptual connections (Mohammed & Kinyo, 2020). According to Von Glasersfeld (1995), constructivist teachers tend to explore how students perceive the problem and why their path towards a solution seems promising (Sergei et al., 2019). According to Piaget (1970), children may learn mathematics not by internalizing the formula and rules enforced by an external authority. Still, they learn mathematics by constructing meaning from the inside on their natural thinking abilities. Therefore, if errors are committed, they arise because the children are thinking and not because they are careless. It is part of a learning environment of engaging students to correct errors with reasons instead of correcting them mechanically by the social process (Ernest, 1991). Constructivist teaching may help students negotiate goals and objectives (Qarareh, 2016), pose problems of emerging relevance to students (Soysal & Radmard, 2018), emphasize hands-on and real-world experiences (Polman et al., 2020), seek and value students' points of view (Vintere, 2018), see the social context of the content and create new understanding, and test with task and use errors to inform students about their progress (Ernest & Albert, 2018). With these views and assumptions, the phenomenon of construction can be used while identifying students' M/Es and remediating them by preparing and using lesson plans in this study as M/Es are considered context-dependent (Neidorf et al., 2020).

There are three ways to deal with M/Es such as (i) making students and teachers aware of possible M/Es that is prevention is better than cure (Neidorf et al., 2020), (ii) using the didactic method that is a conventional method with a detailed explanation and re-teaching the concepts from the beginning (Hennessey et al., 2012), and (iii) using M/Es in a diagnostic (cognitive) conflict teaching method in a cooperative approach (De la Torre & Minchen, 2014; Swan, 2001). This study used the last approach, classroom practice, and group interaction to identify, diagnose, and remedy students' M/Es. Students' learning difficulties are attributed to their underdevelopment of logical thinking (Piaget, 1970). According to Bruner (1990), there are two central themes around its idea, which include: (i) knowledge acquisition is an active process, and (ii) knowledge is actively constructed by relating new information to a previously acquired internal model. Teachers may use cognitive conflict in thinking process which is inevitable for learning, which is the way students try to equilibrate their cognitive tension (positive mental disturbance) due to the conflict (perturbation) (Hackenberg, 2010). In this context, learning can be seen as increasing enculturation into practice by contextualizing the content and process in the classrooms amid dealing with M/Es as a natural phenomenon of education (Wenger, 1998).

Methodology

This study adopted a quasi-experimental research design with two nonequivalent groups (control and experimental). Therefore, it was conducted in intact classes by ensuring daily classes running, usually followed by forming small peer groups. The study was conducted with pre- and post-tests among the students of both the non-randomized control and experimental groups (Creswell, 2012). The pre-test was subjected to three objectives: finding the level of students in control and experimental groups, benchmarking students' achievements, and identifying their M/Es. However, the objective of the post-test was to measure the effectiveness of the remedial treatment. The control group was taught with conventional methods, whereas the experimental group was instructed with teaching episodes involving three remedial strategies. Therefore, the teaching through a new treatment approach with remedial strategies was an independent variable, whereas students' achievement score was the dependent variable. Some controlling efforts were applied through a purposive sampling method while selecting similar schools and teachers and determining the level of students.

Experimental Design

The population of this study consisted of all students in the eighth grade of the public schools of Kathmandu valley. Four groups of eighty students studying in the eighth grade of two public schools were purposively selected. Two groups were randomly selected into experimental groups with thirty-seven students. These groups were treated through a new remedial approach of the teaching-learning mathematics, whereas the other two groups having forty-three students were under the conventional teaching-learning method.

Four pre-service secondary mathematics teachers from two public schools in Kathmandu were trained about the origin and patterns of algebraic conceptions, misconceptions, and errors, followed by remedial strategies to treat students' M/Es. The remedial strategy was focused on practice, cognitive conflict, and communicational approach. A weeklong training (3 hours each day) was provided to prepare teaching episodes based on students' common algebraic M/Es and remedial strategies. After orienting teachers about theoretical perspectives of M/Es, the researcher (the first author) demonstrated two model teaching episodes (one from algebraic expressions and another from word problems), focusing on incorporating students' M/Es and implementing those remedial strategies to treat their M/Es. During the training, each teacher prepared two teaching episodes and demonstrated one turn by turn. The researcher visited selected schools, observed their classes, and guided them for two months to achieve the objectives of this study.

The students' errors were explored and triangulated by examining pre-test answer sheets, class works, in-class concept tests, and observation of the individual and group works. An in-depth interview was administered with twenty-four students (six from each group) to understand their misconceptions in each algebraic error. Further, a focus group discussion consisting of 6-8 students was conducted among the students. The peer discussions and interactions during the teaching and learning in those classes were also taken into consideration. The qualitative findings from these discussions have not been reported in this study as they were commissioned in other publications. The identified M/Es committed by students were shared with their teachers as well. This study organized those errors into seven categories related to different kinds of algebraic misconceptions as per the design suggested by Perso (1991).

The students' M/Es can be illustrated with seven examples. First, students' M/Es were related to *elementary level of understanding*, for example, fundamental meanings of algebraic symbols and variables, e.g., square root of (a^2+b^2) = a+b. Another example was *mishandling and misinterpreting* algebraic symbols, e.g. $(x+y)^3 = x^3+3xy+y^3$. The second category of M/Es, were associated with *manipulative activities* with unfinished answers. For example, $a^{m-n} = a^{-mn}$ which shows misconception with an equal sign (considering it as a step making sign) and detaching variables and terms, e. g., $3x^2/x = 3^2$. The third kind of M/Es were related to *application level* with overgeneralization of mathematical rules and concepts. For example $(x+y)^n = x^n + y^n$ shows an ignorance of indices and parentheses in the simplification of expressions such as x - (y + z) = (x - y) + z. The fourth kind of M/Es were observed in problems by using patterns with structural confusion in algebraic expressions, such as $1+1/x^3$ and $(1+1/x)^3$. The fifth type of ME/s were related to *the translation of word problems* with replacement of successive key words by algebraic symbols while transforming into algebraic equations, e.g., substitution of x=1 and y=2 into xy =12. The sixth kind of M/Es were related to *the analysis and generalization* of the problem due to lack of identification of relationship among the variables dependent on simple arithmetical calculations. In addition, the seventh type of M/Es were observed in solving equations followed mechanical drill and practice, and any way trying to reach the final step and solution (Kshetree, 2020). Some examples of students' works that demonstrate such M/Es have been presented in Figure 1.

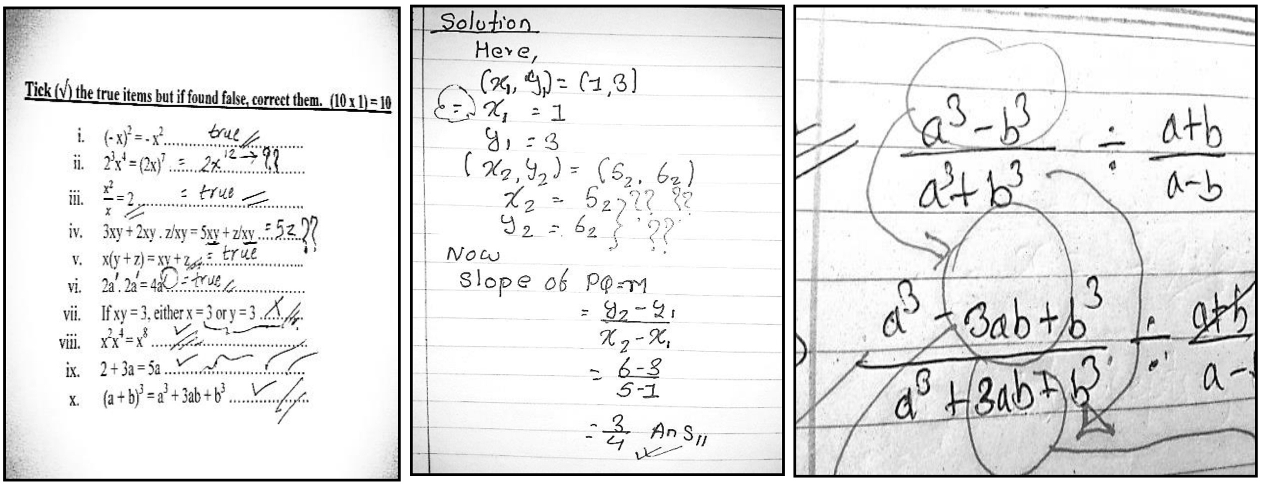

Figure 1. Some examples of M/Es in students' understanding, simplifying, and problem-solving in algebra

The teaching episodes were intervened through three classroom activities such as task intervention strategy, peer group activities, and reflection with a conclusion. These activities were implemented in a strategic way to overcome students' M/Es. In the first strategy, the teachers introduced the concepts of teaching algebraic expression through the relevant open-ended questions (related to variables), story-telling (about variables and their relations), examples related to real life situation, any news (to set a context), puzzles (e.g., number puzzles), teaching and learning materials and models. Further, the teachers used to conduct a discussion over the students' M/Es with required remedial strategies. In the meantime, they also involved the potential M/Es in the discussion as the students' likely to commit them. Sometimes, they used to ask students a few of the tricky questions related to algebraic operations in order to test students' knowledge and skills, including their M/Es. Actually, the questions contained conceptual obstacles, which were prepared from already identified and analyzed common M/Es. It was done in either whole class or in peer groups (depending upon the nature of M/Es). In this process, teachers also adopted a special c*ommunicational approach* in which they used varieties of teaching and learning materials, algebraic models, graphs, charts, wooden and paper materials in peer or group works. With this approach, the different learning paces, abilities, interests, and strategies of the students were addressed and treated or remediated their M/Es in their group works.

The second strategy was peer or group activities with more time for activities that followed engagement, exploration, explanation, and elaboration as per the constructivist approach. Each group leader presented their group work in the whole class turn by turn, where the peers and teacher provided feedback. This helped in dealing with students' M/Es through exposing any misconceptions identifying by the groups. The other group members would pinpoint the M/Es and counter them with their views to resolve such M/Es. The teacher would provide remedial teaching or short explanation with examples to further treat the M/Es if any existed even after group discussions. In addition to *practicing problems* in groups and giving them praising words, the teacher used to model (in whole class) the solutions of the questions they attempted correctly. However, no one student was discouraged even when they did wrong. The wrong answers became a source for learning and treating students' M/Es.

The third strategy implanted in the experimental classrooms was reflection and conclusion**.** The students were asked to reflect on any problems they solved but had M/Es as pointed out by other students or the teacher. They were asked about their thoughts and understanding of the concepts, procedures, and applications of the algebraic problems and how they developed new understanding after further discussion with other group members and explanation by the teacher. In the reflective discussions, the students were asked to think of how they found the causes or reasons behind their M/Es and share them with their group or the whole class. In the meantime, teachers facilitated resolving students' conflicts and then helped them to come up with a common conclusion. Moreover, the students were given an opportunity to consolidate newly learned concepts by providing additional questions too. The feedback and comments made them understand the ideas well, and they became aware of their M/Es.

The teachers used three strategies – practice, cognitive conflict, and communication as re-enforcement for students to overcome their M/Es. Under the 'practicing strategy' of reducing students' M/Es, students were encouraged to learn from positive reinforcement on their expected correct behaviors, whereas the unwanted behaviors were gradually minimized (Campbell, 2009). Therefore, the teachers' behavior needed to be concentrated on the creation of such a classroom environment in which students would not only repeat the required behaviors and change the unwanted behaviors, but they also developed such a habit of learning as well. Regarding the use of 'cognitive conflict' with the explanation method, the teachers used to select the wrong answers and related questions from the students' works and then developed similar problems by focusing the identified errors to distribute them for additional group works. The questions were designed so that they could create cognitive conflict in students while solving them. After discussing those questions in peer groups, each peer group leader presented their views and solutions to the whole class. Then the teacher noted down those answers on the whiteboard and explained the misconceptions behind identified errors concerning the related definition with similar or counterexamples.

In the communicational approach, the researcher and teachers developed and used different mediating communication tools such as meta-cards, flashcards, mathematical models, graphs, and charts to provide them substantial knowledge as and when needed. These strategies were applied to dislodge students' M/Es and develop the required concepts. While implementing those remedial strategies, the five steps proposed by Swan (2001) were also considered such as -- start with examining the conceptual framework of the students, share concepts in the classroom, call for conflicting discussion, resolve conflicting ideas and develop new concepts through discussion, and consolidate learning through problem-solving.

A pre-test tool was designed from eighth-grade algebra contents with conceptual (7), procedural (7), and application-level (7) questions of easy, medium, and difficult types based on the specification grid designed by the Curriculum Development Center (CDC, 2015). The pre-test was administered to the students of both control and experimental groups. The intervention of teaching algebra by guided approach to address M/Es continued for two months in the experimental group. The teaching approach was the same as usual (mostly traditional chalk-and-talk teacher lead lectures) without focus on M/Es in the control group. Then, at the end of the intervention on the experimental group after two months, a post-test was administered to examine the effect of the intervention on the experimental group compared to the control group. The post-test items also were parallel to the pre-test but of different contents in algebra as the coursework progressed. It also included items of conceptual (7), procedural (7) and, application-level (7) in algebra contents for grade eight.

Validity and Reliability

The validity and reliability of the experimental process could be addressed with content and construct validity of the classroom process, test-items construction, and implementation. The reliability of the tests (pre- and post-tests) was examined with the split-half method.

The test papers were prepared from the prescribed curriculum, textbook, and specification grid as prescribed by the Curriculum Development Center (CDC, 2015). The study was conducted for a short period, and no event occurred during this period, which could change the behavior of students. The threat in students' biological and psychological state within the subject and treatment effect over the period of experiment was controlled by taking the students of the same age bracket of the same class for a short period. The subject matter was controlled by teaching same content having same characteristics to the students of both control and experimental groups even by using same teaching aids. The test items were also same for both the groups. In this period, there was no change in sampled students in both the groups (control and experimental). The students were not pre-informed for their tests. Further, the observation of peer group activities of the classroom teaching and learning practices and interviewing students were also done by the researcher himself. Therefore, there was no interaction effect of testing implemented among the students. The researcher himself oriented the prospective teachers for the implementation of independent variables. By following these norms of selection, the possible interactive effect of biasness was controlled. Similarly, the experiment was conducted in natural classrooms, though there was a slight change in students' sitting arrangement for the group works when needed. Students came to be acquainted with such a practice, and later on, it was changed into their habit of the learning process.

The student achievement reliability was examined using the split-half method, the correlation between the two forms was 0.731, and the Spearman-Brown coefficient was 0.844 (for equal length). These coefficients showed that the test items in the pre- and post-tests were reliable.

Data Collection, Analysis, and Interpretation

The pre-test and post-test were administered in order to study and compare the progress made by experimental and control groups of students. Therefore, the data collected for this research were mainly the test scores of the students of experimental and control groups. The three levels—conceptual, procedural, and application -- were categorically considered and balanced while preparing test items. Further, the observational notes were also made based on classroom practices. The data collected from pre-test and post-test were analyzed and compared to measure the treatment effect in the students.

The distribution of achievement of students in the pre-test and post-test for the control and experimental groups were examined for normality. Based on the test of normality of the distribution of the achievements in the pre- and post-tests, a non-parametric test (e.g., Mann-Whitney U Test) was applied to compare the results of pre- and post-tests of both experimental and control groups. The effect size of the intervention was assessed by using Cohen's formula ($r = Z/\sqrt{N}$), where Z is standardized test statistics, and N is a total number of samples. According to Cohen's criteria, the effect size of 0.1 was considered low, 0.3 was considered medium, and 0.5 was considered high (Fritz et al., 2012). The scores obtained by the students of control and experimental groups were further categorically analyzed and interpreted as per their three levels -- conceptual, procedural, and application. The effect size of the intervention was also assessed by using Rank Biserial Correlation at 0.01 level of significance to examine the impact of the intervention on the post-test scores compared to the pre-test, besides Cohen's r. The scores obtained by the students of control and experimental groups were further categorically analyzed and interpreted as per their three levels -- conceptual, procedural, and application.

Results

Normality Tests for Scores of Control Group versus Experimental Group

The control group included 43 student participants, whereas the experimental group comprised of 37 students. The scores from both control and experimental for pre- and post-tests were examined for normality to decide whether to apply parametric or non-parametric tests for the comparison of groups (experimental and control groups) (Table 1).

Table 1. Results of normality test for the scores of control and experimental groups in both pre and post-tests

Control Vs Experiment		Kolmogorov-Smirnov			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Pre-Test	Control	0.164	43	0.005	0.942	43	0.030
	Experiment	0.131	37	0.109	0.972	37	0.470
Post-Test	Control	0.106	43	0.200	0.959	43	0.131
	Experiment	0.189	37	0.002	0.916	37	0.008

Both Kolmogorov-Smirnov and Shapiro-Wilk tests for normality of students' scores from control and experimental groups during pre- and post-test showed that null hypothesis for normality was rejected for pre-test control group scores and post-test experimental group ($p < 0.05$). At the same time, the null hypothesis for normality could not be rejected for the pre-test scores from the experimental group and post-test scores from the control group ($p > 0.05$). For this mixed results for normality tests, non-parametric test, such as Mann-Whitney U Test, was applied on the data to compare the mean rank scores between the control and experimental groups for pre- and post-tests (Table 2).

Table 2. Results of Mann-Whitney Test for ranks of students' scores in control and experimental groups for pre- and post-test

Tests	Control vs Experimental	N	Mean Rank	Sum of Ranks
Pre-Test	Control	43	40.43	1738.50
	Experiment	37	40.58	1501.50
	Total	80		
Post-Test	Control	43	31.23	1343.00
	Experiment	37	51.27	1897.00
	Total	80		

The results in the Table 2 showed that the mean ranks of scores of control and experimental groups were almost similar to the pre-test, whereas the mean ranks were higher for the experimental group than the control group in the post-test. The result showed that there was no statistical significantly difference between the experimental and control groups for pre-test (Control: Mean Rank = 40.43, n = 43; Experimental: Mean Rank = 40.58, n = 37), U = 798.500, Z = 0.029, and p = 0.977 > 0.05 two-tailed (Table 3). Therefore, there was no support for the alternative hypothesis for the pre-test results. However, the there was a statistically significance difference between the control and experimental groups for post-test results (Control: Mean Rank = 31.23, n = 43; Experimental: Mean Rank = 51.27, n = 37), U = 1194.00, Z = 3.859, and p = 0.000 < 0.05, two-tailed. In this case, the alternative hypothesis was supported and hence rejected the null hypothesis, which indicated a significant gain in the post-test in the experimental compared to the control group. The effect size was high moderate (0.3 < r < 0.5) in the post-test whereas it was very low in the pre-test (Table 3).

Table 3. Results for Mann-Whitney Test for comparison of mean ranks for the control group and experimental in pre- and post-tests

Groups	Pre-test	Post-test
Total N (Experimental + Control Groups)	80	80
Mann-Whitney U	798.500	1194.000
Wilcoxon W	1501.500	1897.000
Test Statistic	798.500	1194.000
Standard Error	102.815	103.255
Standardized Test Statistic (Z)	0.029	3.859
Asymptotic Sig. (2-sided test)	0.977	0.000
Effect Size ($r = Z/\sqrt{N}$)	0.003	0.431

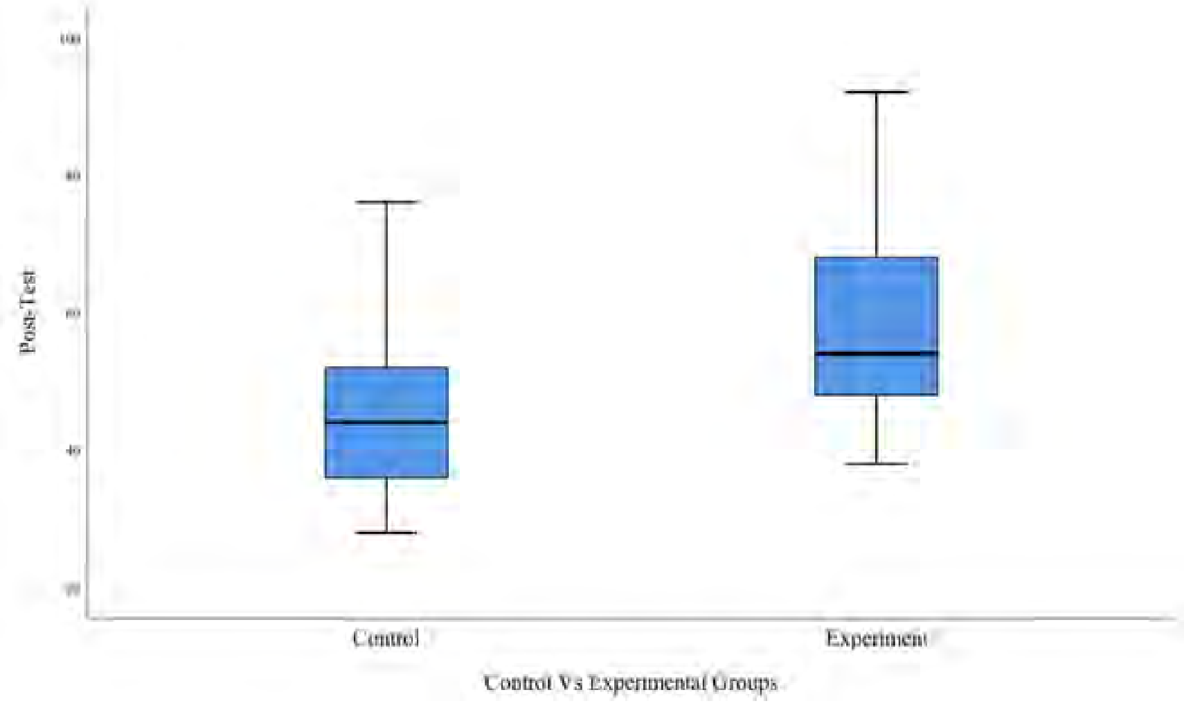

Figure 2:Post-test scores of experimental and control groups that showed a gain in all the three quartiles and minimum-maximum scores from the control group to the experimental group

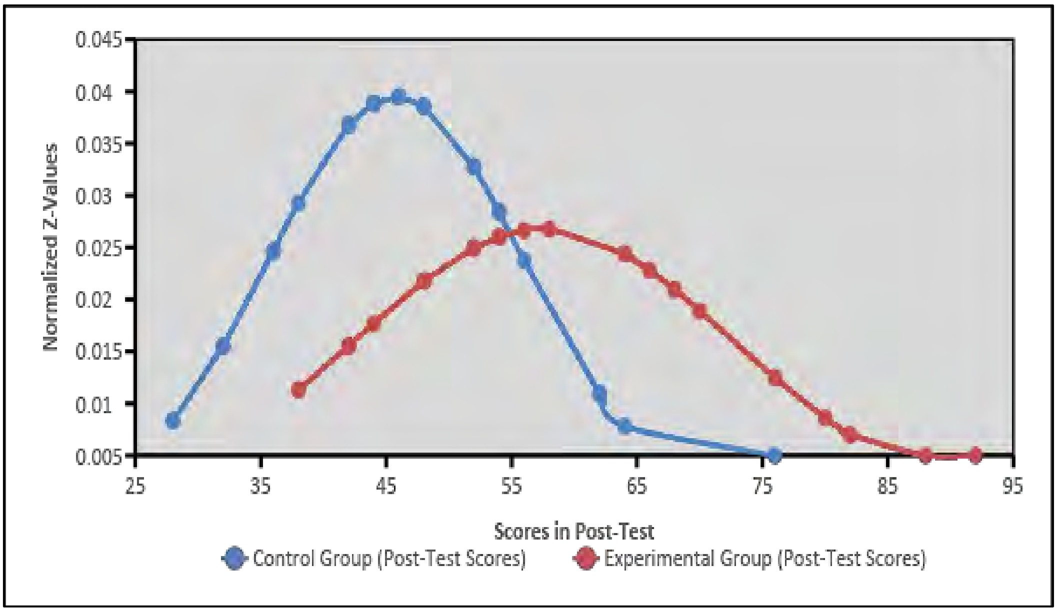

Figure 3: Normalized post-test scores of experimental and control groups

The pre- and post-test results in Figure 2 showed a gain in the experimental group's test scores compared to the control group in all three quartiles, including shifts of minimum and maximum scores. The graph in Figure 3 showed a considerably increasing trend of the scores of the students of the control group up from 28 to median score of 46, and then it was steeply fallen to be ended at around 68. Whereas the scores of experimental groups were found to be started at 38, reached the median score of 56, and then it touched off the ceiling score that was 87. In this way, it showed a clear distinction between the experimental and control groups' scores, where the normalized z-score curve for the experimental group was flattened and shifted to the right.

Mann-Whitney U Test was conducted to examine the differences between pre- and post-test scores in both experimental and control groups (Table 4). The purpose of this test in the experimental and control group was to see the post-test scores were significantly different from the pre-test scores due to the approach to teaching mathematics in the context of M/Es.

Table 4. Results for Mann-Whitney Test for comparison of mean ranks for the control group and experimental groups in pre- and post-tests

Groups	**Control**	**Experimental**
Total N	86	74
Mann-Whitney U	1515.000	1297.500
Wilcoxon W	12461.000	2000.500
Test Statistic	1515.000	1297.500
Mean Rank (Pre-Test)	29.77	20.93
Mean Rank (Post-Test)	57.23	54.07
Standard Error	115.135	92.319
Standardized Test Statistic (Z)	5.129	6.640
Asymptotic Sig. (2-tailed)	0.000	0.000
Effect Size ($r = Z/\sqrt{N}$)	0.553	0.772

The results in the Table 4 showed that the mean ranks of scores of pre- and post-test scores of control groups were significant (Pre-Test: Mean Rank = 29.77, n = 43; Post-Test: Mean Rank = 57.23, n = 43), U = 1515.000, Z = 5.129, and $p = 0.000 < 0.05$ two-tailed (Table 4). Therefore, there was a significant improvement in students' performance in the post-test control group compared to the pre-test. Similarly, there was a statistically significance difference between the pre- and post-test scores of the experimental group (Pre-Test: Mean Rank = 20.93, n = 37; Post-Test: Mean Rank = 54.07, n = 37), U = 1297.500, Z = 6.640, and $p = 0.000 < 0.05$, two-tailed. In this case, the alternative hypothesis was supported and hence rejected the null hypothesis, which indicated a significant gain in the post-test in the experimental compared to the pre-test score in the same group (Table 4).

Since both the control group and experimental group had a significant gain in the post-tests compared to the pre-tests in the respective groups, this challenged the notion of effectiveness of the intervention of the guided method of teaching to overcome M/Es. Therefore, it was necessary to examine whether the intervention had a significant impact on the experimental group compared to the control group after the two months of the study period. The effect size was high ($r > 0.5$) in both control and experimental group, however, it was higher for the experimental group than control group ($r_{control} = 0.553$ and $r_{experimental} = 0.772$) (Table 4). To further conform the effect of intervention in the experimental group, a Rank Biserial Correlations between a continuous variable (student scores) and dichotomous variable (pre and post-test) were computed to examine the effect of instructional practice in both control and experimental groups (Table 5).

Table 5. Rank Biserial Correlation (Spearman ρ) between pre- and post-test scores in the control and experimental groups

			Pre- and Post-Test	Overall Scores
Spearman's rho (Experimental Group)		Corr. Coeff.	1.000	0.777*
		Sig. (2-tailed)	-	0.000
		N	74	74
Spearman's rho (Control Group)	Pre- and Post-Test	Corr. Coeff.	1.000	0.556*
		Sig. (2-tailed)	-	0.000
		N	86	86

*Correlation is significant at the 0.01 level (2-tailed)

The results of the Rank Biserial Correlations in Table 5 showed that the correlations between the students' scores in pre- and post-tests were significant ($p < 0.01$). The experimental group had a greater effect size ($r = 0.777$, $N = 37$, and $p < 0.01$) than that of the control group ($r = 0.556$, $N = 43$, and $p < 0.01$), indicating that intervention in the experimental group had a greater effect than the traditional teaching-learning approach in the control group.

Student Achievement in Procedural, Conceptual, and Application Levels

In order to find out whether the scores of different tests found in favor of the experimental group were proportionately distributed in three levels of the cognitive domain (conceptual, procedural, and application), it was designed for their comparative study under each level separately. Indeed, it was a plan to test the effect of the implemented treatment, so the test items were prepared accordingly based on three levels of the cognitive domain. Thus, the scores of each test were divided as per these three levels.

The scores from both control and experimental for pre- and post-tests were examined for normality to decide whether to apply parametric or non-parametric tests to compare groups (experimental and control groups) for all three levels (Table 6). Both Kolmogorov-Smirnov and Shapiro-Wilk tests for normality of students' scores for conceptual, procedural, and application levels from control and experimental groups during pre- and post-test showed that null hypothesis for normality could not be rejected only for pre-test scores in application level for the experimental group, post-test scores in application level for both control and experimental groups ($p > 0.05$). For the rest of the scores, at least one of the tests (either Kolmogorov-Smirnov or Shapiro-Wilk or both) rejected the null hypothesis. Because of this mixed results for normality tests, non-parametric test, such as Mann-Whitney U Test, was applied on the data to compare the mean rank scores between the control and experimental groups for pre- and post-tests in the conceptual, procedural, and application levels.

Table 6. Tests of normality for the distribution of student scores in conceptual, procedural, and application levels (Pre- and Post-Test)

Control vs Experiment		Kolmogorov-Smirnov			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Pre-Test Conceptual	Control	.147	43	.021	.948	43	.050
	Experiment	.165	37	.013	.945	37	.065
Post-Test Conceptual	Control	.171	43	.003	.950	43	.061
	Experiment	.161	37	.016	.938	37	.040
Pre-Test Procedural	Control	.193	43	.000	.910	43	.003
	Experiment	.153	37	.028	.954	37	.129
Post-Test Procedural	Control	.133	43	.055	.948	43	.051
	Experiment	.231	37	.000	.929	37	.021
Pre-Test Application	Control	.188	43	.001	.941	43	.028
	Experiment	.132	37	.103	.969	37	.387
Post-Test Application	Control	.111	43	.200	.975	43	.471
	Experiment	.135	37	.084	.952	37	.110

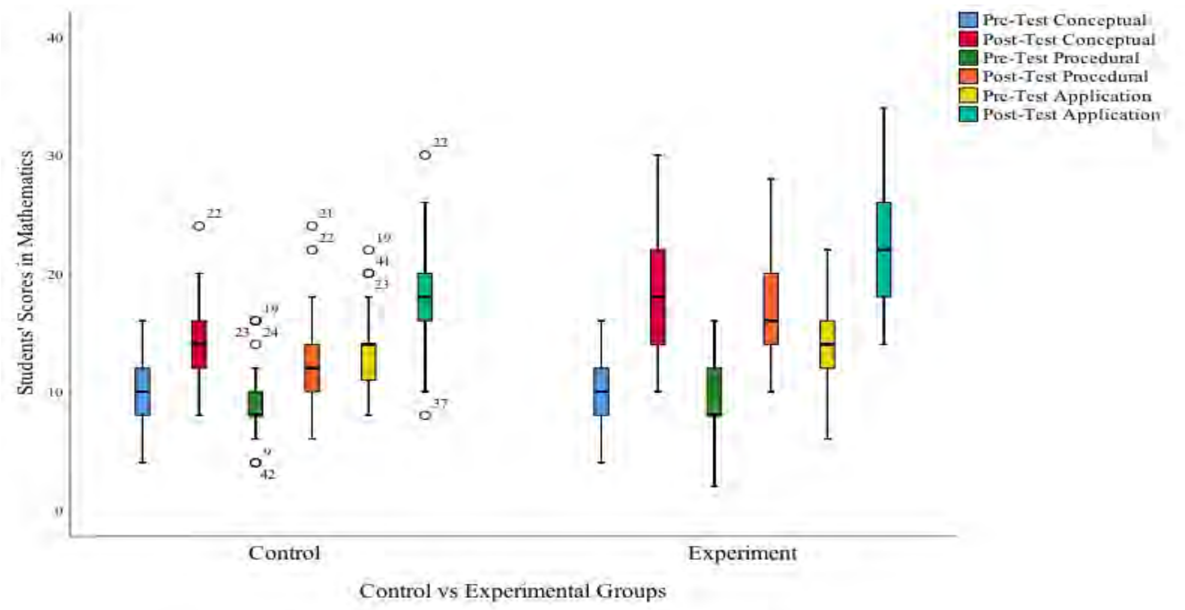

Figure 4. Distribution of students' mathematics achievement scores in pre- and post-tests for control and experimental groups

Figure 4 showed that there was a shift in the students' performance from the control group to the experimental group at all three levels. These shifts in the scores in pre- and post-tests have been discussed under separate sub-headings.

Conceptual, procedural, and application-level comparison (Pre-Test)

A non-parametric test (Mann-Whitney U Test) was administered for scores achieved by the students of both the groups in the pre-test at three levels – conceptual, procedural, and application (Table 7 and Table 8).

Table 7. Results of Mann-Whitney U Test for mean ranks of scores in the conceptual, procedural, and application levels in the control and experimental groups (Pre-Test)

	Control vs Experiment	**N**	**Mean Rank**	**Sum of Ranks**
Conceptual	Control	43	40.99	1762.50
	Experiment	37	39.93	1477.50
	Total	80		
Procedural	Control	43	40.47	1740.00
	Experiment	37	40.54	1500.00
	Total	80		
Application	Control	43	38.86	1671.00
	Experiment	37	42.41	1569.00
	Total	80		

The result showed that there was no statistical significantly difference between the experimental and control groups for pre-test in conceptual knowledge in mathematics (Control: Mean Rank = 40.99, n = 43; Experimental: Mean Rank = 39.93, n = 37), U = 774.500, Z = - 0.207, and p = 0.836 > 0.05 two-tailed (Table 7 and Table 8). Therefore, there was no support for the alternative hypothesis for the pre-test results in the area of students' conceptual knowledge in mathematics. In the similar way, there was no statistical significantly difference between the experimental and control groups for pre-test in procedural knowledge in mathematics (Control: Mean Rank = 40.47, n = 43; Experimental: Mean Rank = 40.54, n = 37), U = 797.000, Z = 0.015, and p = 0.988 > 0.05 two-tailed (Table 7 and Table 8). Therefore, there was no support for the alternative hypothesis for the pre-test results in the area of students' procedural knowledge in mathematics. Similarly, there was no statistical significantly difference between the experimental and control groups for pre-test in application knowledge in mathematics (Control: Mean Rank = 38.86, n = 43; Experimental: Mean Rank = 42.41, n = 37), U = 866.000, Z = 0.692, and p = 0.489 > 0.05 two-tailed (Table 7 and Table 8). Therefore, there was no support for the alternative hypothesis for the pre-test results in the area of students' conceptual knowledge in mathematics.

Table 8. Independent-Samples Mann-Whitney U Test for comparison of achievements in the control and experimental groups (Pre-Test)

Criteria	Conceptual	Procedural	Application
Total N	80	80	80
Mann-Whitney U	774.500	797.000	866.000
Wilcoxon W	1477.500	1500.000	1569.000
Test Statistic	774.500	797.000	866.000
Standard Error	101.512	101.278	101.871
Standardized Test Statistic (Z)	-0.207	0.015	0.692
Asymptotic Sig. (2-sided test)	0.836	0.988	0.489

Conceptual, procedural and application-level comparison (Post-Test)

A non-parametric test (Mann-Whitney U Test) was administered for scores achieved by the students of both the groups in post-test at three levels – conceptual, procedural, and application (Table 9 and Table 10).

The result showed that there was a statistical significantly difference between the experimental and control groups for post-test in conceptual knowledge in mathematics (Control: Mean Rank = 32.33, n = 43; Experimental: Mean Rank = 50.00, n = 37), U = 1147.000, Z = 3.427, and p = 0.001 < 0.05 two-tailed (Table 9 and Table 10). Therefore, there this supported for the alternative hypothesis for the post-test results in the area of students' conceptual knowledge in mathematics. In the similar way, there was a statistical significantly difference between the experimental and control groups for post-test in procedural knowledge in mathematics (Control: Mean Rank = 30.74, n = 43; Experimental: Mean Rank = 51.84, n = 37), U = 1215.000, Z = 4.093, and p = 0.00 < 0.05 two-tailed (Table 9 and Table 10). Therefore, this supported the alternative hypothesis for the post-test results in the area of students' procedural knowledge in mathematics. Similarly, there was a statistical significantly difference between the experimental and control groups for post-test scores in application-level knowledge in mathematics (Control: Mean Rank = 31.84, n = 43; Experimental: Mean Rank = 50.57, n = 37), U = 866.000, Z = 3.620, and p = 0.000 < 0.05 two-tailed (Table 9 and Table 10). Therefore, this supported the alternative hypothesis for the post-test results in the area of students' application-level knowledge in mathematics. The effect size due to intervention (guided teaching method) to overcome the M/Es were 0.383, 0.458, and 0.405. The effect size was large (almost 0.5) for procedural level knowledge compared to conceptual and application levels.

Table 9. Results of Mann-Whitney U Test for mean ranks of scores in the conceptual, procedural, and application levels in the control and experimental groups (Post-Test)

	Control vs Experiment	N	Mean Rank	Sum of Ranks
Conceptual	Control	43	32.33	1390.00
	Experiment	37	50.00	1850.00
	Total	80		
Procedural	Control	43	30.74	1322.00
	Experiment	37	51.84	1918.00
	Total	80		
Application	Control	43	31.84	1369.00
	Experiment	37	50.57	1871.00
	Total	80		

Table 10. Independent-Samples Mann-Whitney U Test for comparison of achievements in the control and experimental groups (Post-Test)

Criteria	Conceptual	Procedural	Application
Total N	80	80	80
Mann-Whitney U	1147.000	1215.000	1168.000
Wilcoxon W	1850.000	1918.000	1871.000
Test Statistic	1147.000	1215.000	1168.000
Standard Error	102.579	102.499	102.895
Standardized Test Statistic (Z)	3.427	4.093	3.620
Asymptotic Sig. (2-sided test)	0.001	0.000	0.000
Effect Size ($r = Z/\sqrt{N}$)	0.383	0.458	0.405

The effect of treatment was found significantly useful in the experimental group students compared to those of the control group regarding the conceptual level of understanding. The graphical presentation of the post-test scores of both the groups was as given in Figure 5.

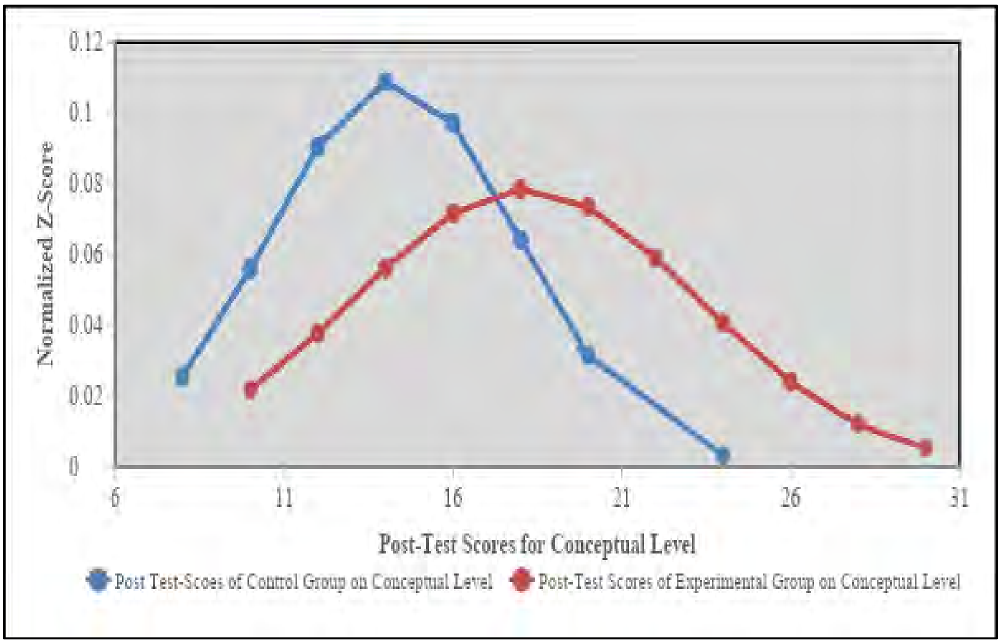

Figure 5: Normalized conceptual level scores of post-test of experimental and control groups

The graph in Figure 5 showed that the control group students' scores ranged from 8 to 24, whereas that of the experimental group between 10 and 30. The control and experimental groups' median scores were 14 and 18, respectively, demonstrating that the experimental group's normalized post-test score shifted to the right. It was an evidence of having a better conceptual understanding of the students of the experimental group.

That is, the effect of treatment was found significantly fruitful in the students of the experimental group for the procedural level. The comparative study has been made easier with the help of a graphical presentation provided in Figure 6.

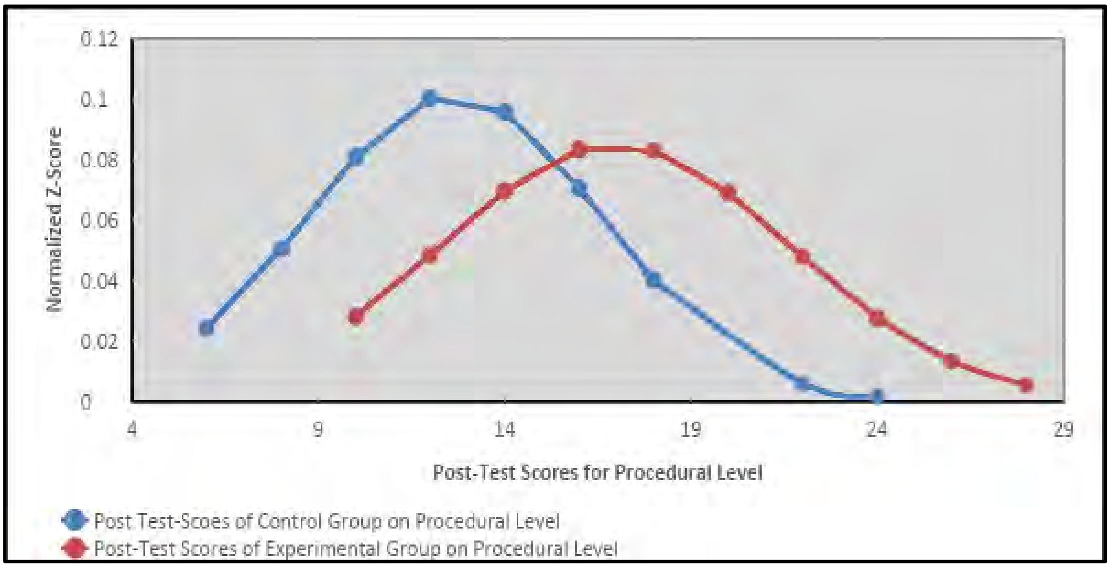

Figure 6: Normalized procedural level scores of post-test of experimental and control groups

The graph in Figure 6 showed that the control group scores ranged from 6 to 24, and the range for the experimental group was from 10 to 28. The control group's median score was 12, and that for the experimental group was 16 showing a shift of the normal curve for the experimental group to the right. In this way, the treatment effect was found to be substantial in achieving procedural skills.

These statistical facts conclude that the treatment effect on the application level of the students of experimental groups was found significantly more effective. The two groups' achievement scores have been depicted in the graphical presentation, as given below in Figure 7. The graph visualized that the experimental group's achievement was higher than the control group of students in the skill of application-level of cognitive development after the intervention of new treatment, as portrayed in the graph in Figure 7.

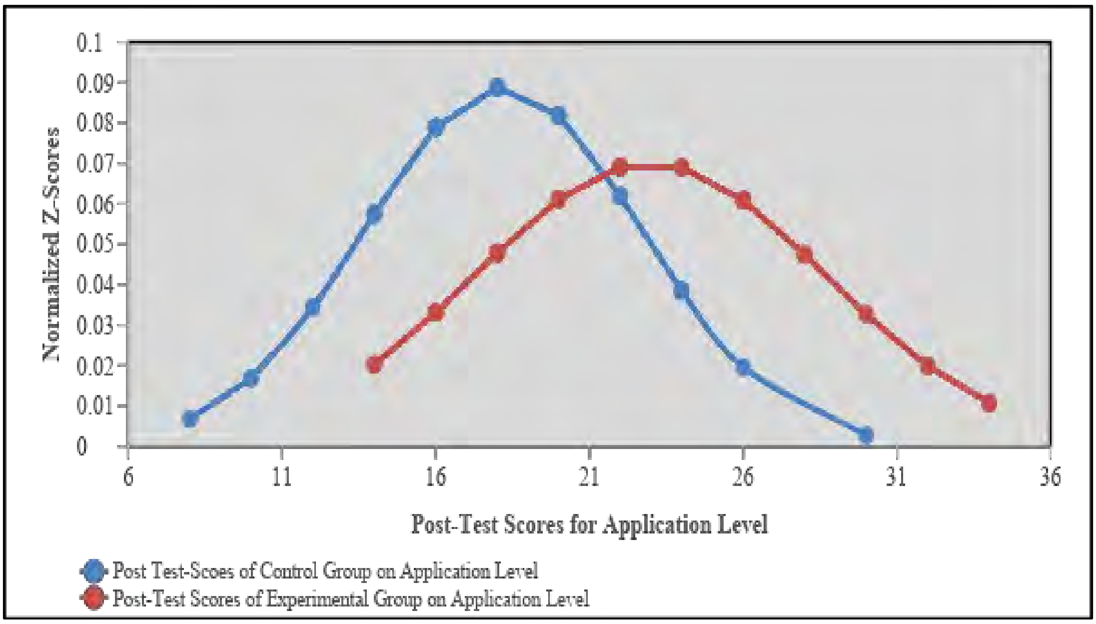

Figure 7: Application-level scores of post-test of experimental and control groups

The graph in Figure 7 showed that the control group scores ranged from 8 to 30 and that for the experimental group from 14 to 34. The control and experimental groups' median scores were 18 and 22, respectively, showing a shift of the experimental group's normal curve to the right. Therefore, the experimental group's achievement was higher than the control group in the application-level of cognitive development after the intervention.

Discussion

The results of the analysis of students' mathematics achievement with Mann-Whitney U Test for pre-test showed that both the control group and experimental groups were equivalent as there was no significant difference in the mean ranks of scores between the two groups at 0.05 level of significance. However, the post-test data showed a significant difference between the control and experimental groups' mean ranks of scores for overall achievement and achievements at the three levels – conceptual, procedural, and application ($p < 0.05$). The effect size due to guided teaching (intervention) was moderate to high as Cohen’s r for the control and experimental groups for overall achievement were $r_{control} = 0.553$ and $r_{experimental} = 0.772$. Biserial rank correlation coefficient between groups (experimental and control) and scores in tests were medium to high ($\rho_{control} = 0.556$ and $\rho_{experimental} = 0.777$). Comparing the effect sizes for the three levels, the procedural level had the effect size nearly high (near 0.5), demonstrating that the treatment was more effective for improvement in students' procedural knowledge and helped to overcome such M/Es related to computational procedures. However, there was a considerable impact on the student achievement in conceptual and application levels too ($0.3 < r < 0.5$).

The experimental group achieved more than the control group in the post-test in the conceptual level of mathematics intervention. The conceptual change with a gain in students’ learning of mathematics could be attributed to the effects of *conceptual assignments* in the classroom activities in the experimental group (Eryilmaz, 2002; Mutambara et al., 2020). The significant achievement of the students of the treatment group was not just as an effect of the new intervention. Instead, it became possible because of purposefully designed and implemented remedial teaching approach to treat their identified M/Es. These approaches might have supported students in conceptual representation of the problems to visualize and relate the variables (Scheuermann & van Garderen, 2010) and help them understand the meaning (Ada & Kurtuluş, 2010). Then students might have grasped the conceptual meaning of the problems presented and associated variables (Yilmaz et al., 2018). Swan (2001) argued that such an approach significantly reduced students' M/Es. It supported students to actively construct and reconstruct their knowledge through peer interaction, discussion and sharing their ideas for possible solutions, settling conflicting ideas with feedback, and valuing alternative methods to solve problems. Further, the remedial teaching strategies supported students to increase their critical view, creative thinking, and positive attitude towards inquiry of algebraic relations during the study period (Fumador & Agyei, 2018).

The main causes of improvement in the post-test score could be the strategies that help reconstruct mathematical knowledge and skills through cognitive conflict led by self-initiation or teacher, including positive reinforcement through peers' practice, teachers' appropriate communicational approach, and conceptual change strategy. In this strategy, inconsistencies were presented to the students where they could see, think, and make changes in their incorrect mental structures into correct ones (Campbell, 2009). In fact, there were several steps required to get a solution to the problem, and any of these steps could introduce errors. The students might not break down a solution into multi-steps; then, they might be piled up after the first step (Campbell, 2009). Those situations were addressed and made them comfortable to reach the last step correctly with the cooperation of peers and the teacher's hands-on support. Through this approach, teaching and learning allowed students to discuss M/Es friendly in peer groups (Golub, 1988). The higher-order cognitive talk took place, which promoted a higher level of understanding, conceptualizing, and application. The discussion over students' M/Es provided opportunities for meaning-making and mutual feedback system, which resulted in a better understanding of concepts for all. In this way, the findings regarding the effectiveness of the treatment were found consistent with the theoretical understandings as conceptualized in this study.

Since the teaching episodes were prepared based on students' M/Es and guided by principles of constructivism, it was found most effective and consistent as it showed a powerful influence on the students' motivation and academic achievement. Students' progress and achievement were significantly affected by their misconceptions (Sarwadi & Shahrill, 2014). The rational choice theory was applied for choosing problem, peers and group activities, and early functional theory for their self-esteem (Doise, 1990), which could have positively influenced students to commit errors freely without fear and then master new approach which contributed in learning. As Palincsar and Brown (1984) claimed, students' improvement was possible with some constructive activities as observed in the classroom were as taking turn, listening more, reasoning, respecting and being responsible, using teaching-learning materials, discussing to relate the problem in empirical ways, finding the mathematics patterns, discussing M/Es, reflecting and sharing, and describing in small groups of like-minded friends. In order to avoid errors, teachers and students should skillfully select mathematical tasks (Hansen et al., 2014). Thus, it showed that the students' higher achievement was as expected and consistent through mastery of mathematical tasks. The treatment produced students' intellectual synergy with the social stimulation of mutual engagement in a common endeavor.

In the post-test, the experimental group students performed better than those of the control group in the procedural level by reducing errors significantly. This improvement in procedural fluency is attributed to the focused attention to the critical steps in problem-solving episodes in remedial teaching (Diaz et al., 2020; National Research Council, 2001). The students who had misconceptions in procedural computations and simplification in algebraic and numerical problems seemed to benefit from the intervention to minimize M/Es (Makonye, 2011). Moreover, the early exchange theory of learning (Haralambos & Heald, 2006) says that students feel comfortable exchanging their every idea among their entrusted peers. Along with this idea, each student performed his or her own task as it was guided by individual accountability, where each one was clear for his/her role of action. They learned the required concepts at their own pace and strategy. Students also added that debating in the peer group and verbalizing their ideas helped them develop explicit concepts towards encountered M/Es. Learning by verbalization was consistent with Vygotsky's (1978) concept of egocentric speech, where he claimed its significant role in learning. Further, Vygotsky (1978) added that it happens by providing opportunities such as interacting, arguing, conceptualizing a problem, solving them, and discussing alternative solutions. They were also challenged with higher-cognitive level questions in-group work; thus, the students extended their zone of proximal development (Vygotsky, 1978) and achieved better scores in the experimental groups.

In the post-test, the experimental group had scored more than the control group in the application level of intervention to reduce misconceptions and errors in mathematics problem-solving. This improvement could be attributed to the problem-solving episodes designed to contextualize the problems to fit into the students' daily life activities (Agustyaningrum et al., 2018; Diaz et al., 2020). The reduction of M/Es in the application level could be because of treatment (intervention) that attributed to improvement in conceptual understanding, procedural fluency, strategic problem-solving competencies, and adaptive reasoning in the context of the problems (Makonye & Fakude, 2016; Schoenfeld & Kilpatrick, 2008). As an outcome of the intervention of the remedial teaching-learning approach in the students of the same standard, the experimental groups' students were comparatively better regarding immediate learning achievement. Further, the classroom observations showed that the peer work activities with specified learning objectives, managing the student-friendly environment, and teaching based on students' prior knowledge and ability were useful for treating students' M/Es. Thus, the remedial strategies implemented in the classroom teaching-learning practice were found useful for the remedy of students' M/Es related to all three learning levels: conceptual understanding, procedural knowledge, and implicational skills (Baidoo, 2019).

Students' active participation was necessary to construct their mathematical knowledge, and the study showed that the same phenomenon was applied to deconstruct their misconceptions and errors. It was found that the misconceptions and errors were deeply rooted in students' minds; thus, they could not be easily dislodged, and few of them were resurfaced as well. Therefore, teachers had a significant role, though not at all, who could estimate and project those possible M/Es in their lesson plans. However, the study's findings may not support fully the constructivist approach as the only way to improve M/Es as the students learned more by practicing rules, formulas, and problem-solving and careful implementation of step-by-step procedure even after having a conceptual understanding. In this sense, the higher-level performance in the experimental group could be not only due to the constructivist approach to engagement, exploration, explanation, and elaboration in the classroom approach for guiding students, but it could be due to focused and guided problem solving with repeated discussion and practice with step-by-step procedural and conceptually guided methods (Kirschner et al., 2006).

Conclusion

This study is evidence that learning mathematics without identifying and treating students' M/Es cannot be meaningful. As a result, the M/Es are not iterated only; moreover, they keep on fitting new concepts into prior misconceptions that pass the entire learning process into the channel of M/Es. This study showed that students' M/Es could be identified and treated if the teachers have the required knowledge and skills. However, teachers' pedagogical content knowledge (PCK) has not incorporated identifying and treating students' M/Es in Nepal. Therefore, teachers need such PCK to prevent students' M/Es by identifying sources and employing appropriate measures to deal with such issues during teaching-learning mathematics. Mathematics teacher education programs at the universities and teacher training packages or modules by the Ministry of Education should focus on the crucial measures that deal with students' M/Es in mathematics. Further study is recommended to explore different categories of students' M/Es in cognitive, metacognitive, and dispositional aspects from understanding to synthesis levels.

Recommendation

The remedial strategies discussed and implemented in this study played a vital role in making learning mathematics useful and efficient. The positive reinforcement under practice in the guided approach to deal with M/Es encouraged students to correct their potential confusions and mistakes. The cognitive conflict mode supported students to fit in new and correct concepts. It was made possible by identifying and overwriting their old misconceptions and errors. The mathematical concepts were not only learned meaningfully through the communicational approach, but they were also made closer to a real-life situation by using various learning tools. This study showed that the deteriorating situation of teaching-learning mathematics could be improved in a significant way if teachers are trained about the development of teaching episodes by incorporating different remedial strategies to discuss and address students' mathematical misconceptions and errors. In this way, the results of this study inform primarily the school teachers and secondly curriculum planners and practitioners, textbook writers, teaching-learning material developers, and other stakeholders to broaden their understanding of how M/Es in mathematics can be identified and thoughtfully engaged in treating them through different remedial strategies. In this way, this study's findings can be implemented in real classroom practices to help students address their M/Es and make the teaching-learning process meaningful and joyful. As claimed by Hansen et al. (2014), teachers have to plan their teaching effectively to expose and discuss M/Es so that students can think critically, reflect upon their own experiences, ask questions to teachers and listen to teachers' explanations carefully.

Limitations

The study was limited only to identifying and treating students' common patterns of M/Es. Therefore, it did not cover individual types of typical M/Es. It was limited to specific course and topics in algebra for grade eight. The study was conducted on four groups of students in two community/public schools of Kathmandu valley. These schools were selected by purposive sampling method. Therefore, it was limited to a small sample of schools and students. The results of the study have a limited generalizability. The comparison of control and experimental groups has been limited to pre-test and post-test only. The causes of M/Es might be because of teacher's characteristics, curriculum, and learning environment separately or jointly. Similarly, it did not include the impact of students' affective factors related to M/Es in mathematics.

There has been almost no practice to analyze students' M/Es in Nepal. There are very limited attempts of designing and implementing teaching episodes to analyze students' M/Es in mathematics, although some studies in English had such attempts (e.g., Maharjan, 2009). Even the implemented authorized textbooks and materials may not be equally appropriate for all types of students who are not considered in the implementation of courses. The teaching-learning methods have not been appropriately designed based on the nature of the content and course of mathematics (Mathema & Bista, 2006). The most general educational methods seem ineffective for mathematics learning. Further, they are not enough to address students' diversified backgrounds and learning strategies and pace. As a result, students' achievement in mathematics has not reached the expected level (Education Review Office, 2015). Therefore, the limitations can be attributed to theories, empirical studies, and classroom practices in the area of M/Es in students' mathematics in Nepal.

Authorship Contribution Statement

Kshetree: Concept design for the study, data collection and analysis. Acharya: Literature review, critical revision of manuscript. Khanal: Theoretical framework, critical revision of manuscript. Panthi: Discussion, critical revision of manuscript. Belbase: Data analysis, interpretation, discussion, final approval of manuscript.

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