Concrete Materials in Primary Classrooms: Teachers' Beliefs and Practices about How and Why they are Used

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A study was conducted to explore the beliefs and practices of 49 New South Wales (NSW) primary school teachers regarding their beliefs and practices concerning the use of concrete materials in the learning and teaching of Number and Algebra. This paper reports on elements of the study regarding why and how teachers use concrete materials. Not only do teacher beliefs influence their classroom practice, Buehl and Beck (2014) propose that teacher practices may impact teacher beliefs suggesting they are interrelated. This paper sought to situate teacher beliefs and practices upon a conceptualisation of this interrelationship based on aspects of classroom practice involving concrete materials. The study employed survey methodology utilising a questionnaire and semi-structured interviews to gather data that were analysed to identify key themes regarding teachers' prevailing beliefs about the use of concrete materials. These themes included a belief in a social constructivist approach to learning and teaching mathematics, a cognitive dimension and engagement. On the basis of these findings an overview is suggested that positions teacher beliefs and practices relating to the use of concrete materials and how it might be used to shift inservice and preservice teachers' beliefs towards a more constructivist teaching approach.

Keywords: concrete materials · teacher beliefs · primary school teachers

Introduction

Along with other factors, including knowledge and attitudes, teachers base decisions about their choice of classroom strategies on their beliefs (van der Sandt, 2007). Furthermore, beliefs play an important role in the decisions preservice teachers make regarding their choice of teaching experiences (Valcke et al., 2010). These decisions inservice and preservice teachers make about their practice are influenced by their beliefs about models of teaching mathematics (Ernest, 1989). Along with this, Buehl and Beck (2014) have suggested that not only do teacher beliefs influence their practice, but that teacher practice may influence their beliefs.

Teacher practice involves finding strategies to assist students understand the underlying concepts of abstract mathematical ideas. This can be a challenge for some primary teachers, and they are encouraged to employ a variety of strategies to enhance the learning of their students (Carpenter et al., 1996). One particular strategy recognised as being of potential benefit to students' understanding in mathematics is the use of concrete materials to represent mathematical ideas (Moyer, 2001). Using concrete materials as a way of representing mathematical ideas is recommended by the National Council of Teachers of Mathematics (NCTM) in their *Principles and Standards for School Mathematics* (2000). Moreover, although there have been some equivocal and negative results, various studies have demonstrated that the use of concrete materials in the classroom may be more successful than using abstract symbols alone, can aid memory, and may assist in developing a deeper understanding of mathematical concepts, particularly when used in conjunction with appropriate classroom strategies (Carbonneau et al., 2013; Sarama & Clements, 2016; Thompson, 1994).

Research has gathered evidence about the reasons teachers give for using a range of concrete materials in mathematics (eg. Perry & Howard, 1997; Swan & Marshall, 2010) and how they might affect student achievement (eg. Carbonneau et al., 2013). This study focussed on the beliefs of NSW primary school teachers' regarding their use of concrete materials in the strand of Number and Algebra (NSW Education Standards Authority (NESA), 2019). The learning and teaching of Number and Algebra concepts involves abstract thinking. This may make the use of concrete materials to model these concepts of particular benefit to students, especially when combined with effective instruction (Carbonneau et al., 2013).

Literature Review

Students' learning is impacted by the pedagogical choices made by their teachers. These choices are influenced by belief systems concerning the learning and teaching of mathematics (Ernest, 1989; Wilkins, 2008). Moreover, just as a teacher's beliefs may be influenced by their practice, teacher practice may impact teacher beliefs (Buehl & Beck, 2014). A better understanding of this relationship may enable strategies to be developed that could lead to improved learning outcomes for students along with helping to shift inservice and preservice teachers' beliefs regarding a constructivist approach to mathematics teaching and learning. This study considered this issue in the context of the use of concrete materials in primary classrooms.

Concrete Materials

The term 'concrete materials' is used synonymously with the term mathematical manipulatives throughout mathematics education literature. Discussions on mathematical manipulatives describe pictorial representations (Sowell, 1989) and can include both physical and virtual manipulatives. This study focusses on physical manipulatives. They can be specifically designed for use in mathematics classrooms, such as counters, base 10 blocks or Cuisenaire rods. Whereas a broader definition of concrete materials may include objects such as toys or dolls (McNeil & Jarvin, 2007), Moyer (2001) defines concrete objects as those used in order for students to conceptualise an abstract mathematical idea. Moreover, Perry and Howard (1997) define them as objects students use to explore visually, often in conjunction with a hands-on activity. Students should be able to touch and hold concrete materials and move them around (Moyer, 2001) and use them to "experiment and explore" (Demetriou, 2015, p. 1912). They can be structured such as pattern blocks or unstructured such as popsicle sticks (Marshall & Swan, 2005). Perry and Howard's (1997) definition is used in this study to define concrete materials with an emphasis on the structured form, while incorporating Moyer's (2001) notion, they are used to make abstract mathematical concepts tangible.

The teaching of mathematics using concrete materials has been espoused by educational theorists from at least the time of Pestalozzi in the early 1800s (Sowell, 1989). Montessori's work early last century was built around the idea of learning through exploration of manipulatives (Mix, 2010). Zoltan Dienes advocated the use of base ten blocks during the 1960s (Booker et al., 2010) while Cuisenaire rods were widely promoted during the 1960s and early 1970s (Perry & Howard, 1997).

In more recent times, the use of concrete materials has been advanced by government education authorities both within Australia as well as internationally. For example, the United States of America's *Common Core Standards for Mathematics* (Common Core State Standards Initiative, 2020) promotes their use for modelling and in problem solving. In Singapore's *Mathematics Syllabus: Primary One to Six (*Ministry of Education Singapore, 2012) it is suggested that teachers use concrete materials to promote the discovery and understanding of abstract mathematical concepts. The Australian Curriculum, Assessment and Reporting Authority's (ACARA) *Australian Curriculum: Mathematics* (2020) suggests students should use them to build patterns and models, while in the *NSW Syllabus for the Australian Curriculum: Mathematics K–10 Syllabus* (NESA, 2019) the incorporation of concrete materials in learning and teaching is mentioned as a means of modelling mathematical concepts from Early Stage 1 (Foundation/Kindergarten) through to Stage 4 (Grades 7 and 8). Moreover, support documents, such as *Key Characteristics of Effective Numeracy Teaching P–6* (Department of Education and Early Childhood Development (State of Victoria), 2010) advocate the incorporation of concrete materials in learning and teaching experiences.

Further support for the incorporation of concrete materials can be found in constructivism. Constructivism is a complex entity. The ideas underpinning constructivism have not coalesced to form a single theory, rather constructivism is a fusion of many theories that advance that "learning is a process of constructing meaning" (Amineh & Asl, 2015, p. 9). Moreover, constructivism proposes students should be allowed to actively construct their own meaning (Bobis, Mulligan, & Lowrie, 2009) building on previously acquired knowledge and ideas through cogitation and sense making (Amineh & Asl, 2015; Reys et al., 2020). Concrete materials allow students the opportunity to reflect on the representations of mathematical ideas which they have built which helps to deepen their understanding of new concepts (Hiebert et al., 1997). Furthermore, as part of Vygotsky's sociocultural theory of learning (Rieber & Robinson, 2004), social interactions can be accommodated through group work with learning focused around the use of the concrete materials accommodating a social constructivist approach in the classroom.

A learning theory involving a progression from concrete to abstract has been proposed by both Piaget and Bruner. Piaget's stages of cognitive development link learning to a timeline (Snowman & Biehler, 2006). It describes a progression through age-related stages from concrete through to abstract understanding, necessitating the use of concrete materials at appropriate stages of learning. However, more recent theories question Piaget's stages arguing that even young children may be capable of abstract thought while adults could benefit from concrete modelling when learning something new (Mix, 2010).

The learning sequence described by Bruner (2006) begins with concrete representations, moving to images followed by the development of abstract thought. He "saw learning as a developing process that could be influenced by teaching" (Bruner, 1986, as cited in Bobis et al., 2009, p. 6). The teacher's choice to incorporate concrete materials into mathematics learning experiences influences students' learning and their opportunity to develop an understanding of new concepts.

Another important aspect of students' learning is their thinking. It is key that a teacher understands their students' thinking in order to design appropriate learning experiences for them (Carpenter et al., 1996). One way of doing this is through the use of concrete materials as they can allow teachers to see what and how children are thinking (Skemp, 1989).

Research into whether concrete materials are of use in mathematics classrooms, has produced equivocal results regarding their effectiveness (McNeil & Jarvin, 2007; Sowell, 1989). Some studies involving student testing have had positive results (eg. Carbonneau et al., 2013; Liggett, 2017; Sowell, 1989). Sowell (1989) carried out a meta-analysis on data collected from 60 studies. She found there was a significant positive difference between achievement of students who were taught using concrete materials compared to those given abstract instruction, this difference being most significant when the students received instruction using concrete materials for a year or more. Similar results were found by Liggett (2017). His study involved 43 Grade 2 students randomly assigned to a treatment group (n=22) and a control group (n=21). Students completed pre-tests and post-tests. Students who received the intervention, availability of concrete materials (unifix cubes) once before and during post-test, performed better on the post-test than the control group students.

A meta-analysis by Carbonneau et al. (2013) drew on 55 studies, from kindergarten to college level, where instruction using concrete manipulatives was compared to that using only abstract symbols. They found the results, regarding improved student achievement, were statistically significant with effect sizes ranging from small to moderate. While effects on retention ranged from moderate to large, they found problem solving, transfer and justification produced small effects. They noted, however, a few factors played a role in student achievement including the choice of manipulatives as well as how much guidance was given by the teachers. Overall, students in studies utilising manipulatives that were detailed or more real (eg. plastic pizza) did not achieve as well as those in studies using plain, uninteresting ones (eg. wooden blocks). Moreover, teacher guidance in the learning activities indicated the more guidance the higher the achievement.

Other studies refute positive findings. Uttal et al. (1997) claim “several intensive, longitudinal studies of the use of manipulatives in individual classrooms have shown that children do not readily acquire new mathematical concepts from using manipulatives” (p. 38). They added there does not appear to be a clear advantage in using concrete materials compared to “more traditional methods of instruction” (p. 38). These findings highlight the complex nature of the relationship between the use of concrete materials and student learning. It is not simply the inclusion of concrete materials in classroom activities that will enhance student learning but the manner in which teachers employ them to illustrate or model mathematical concepts. To use them effectively, the teacher needs to understand the concept and more importantly have the necessary pedagogical content knowledge (Carpenter et al., 1996) in order to maximise the likelihood of students understanding the new concept.

Research to date confirms concrete materials are only beneficial when they are used ‘properly’ by teachers who understand how to use them to help students construct their own knowledge (McNeil & Jarvin, 2007). Moreover, Perry and Howard (1997) highlight “there are concerns apparent in the literature with regard to teachers over enthusiasm to use manipulatives as a conduit to students’ learning of mathematics” (p. 27). Just because students use concrete materials, does not necessarily imply they are learning. However, while some research is inconclusive about the benefits of the use of concrete materials (Stanley et al., 2008) more recent meta-analysis by Carbonneau et al. (2013) indicates there is some evidence the use of concrete materials may be beneficial to student achievement. So while many studies have explored the effect of the use of concrete materials on student achievement and a number of studies have looked at the type of concrete materials used and how frequently they are used (e.g. Figueira-Sampaio et al., 2013; Perry and Howard, 1997; Swan et al., 2007)) fewer studies have investigated how and why concrete materials are used.

How teachers use concrete materials has been researched employing a number of methodologies. Some involved self-reported use of concrete materials (e.g. Perry & Howard, 1997; Marshall & Swan, 2005) while others have been conducted using classroom observation (e.g. Moyer, 2001).

Intensive instruction and support were provided by Moyer (2001) to ten middle school teachers in the use of concrete materials and they were provided with a class kit of resources. Throughout one year, she collected extensive data using observations, interviews and a self-report questionnaire. Comparing concrete material use to student achievement, Moyer found the way in which the concrete materials were being used by the teachers did not help students gain a deeper understanding of the mathematics they were being taught, despite the intensive instruction and support. She discovered teachers viewed the use of these manipulatives as a 'reward'. They believed that when students were using them, they were not doing 'real math' (p. 185) thus the concrete materials were used for a change of pace or to have a break.

In contrast to Moyer's findings, Howard et al.'s (1997) research data indicated concrete materials were used in ways that might facilitate more 'serious' learning. Through conducting a survey, they found primary teachers (n=603) used concrete materials for demonstration (83%), with student choice (71%) and with teachers and students agreeing how they are used (56%). Furthermore, their research revealed concrete materials were used for students to verify their work (45%) and as a support for students having difficulties (65%).

To explore why concrete materials were used, Marshall and Swan's (2008) study employed a questionnaire and they supplemented the questionnaire data with volunteer interviews, to deepen their understanding of responses. Their research involved 820 teachers, from 250 primary and middle schools. The researchers found over 95% of respondents believed student learning was enhanced by the use of concrete materials. This is in line with Howard et al.'s (1997) data showing 94% of primary teachers surveyed said they believed the use of concrete materials supported the learning of students. However, while 64% of respondents from Howard et al.'s (1997) study indicated student enjoyment was a reason for their use, Marshall and Swan found only 23% of participants' responses mentioned, collectively, engagement, enjoyment, fun, motivation or heightened interest. Other reasons given for why concrete materials were used found in Marshall and Swan's study included to aid visualisation (18%), provide opportunities for hands on learning (16%) and improve understanding (15%).

The study reported here further explores some of the ideas studied by Howard et al. (1997) and Marshall and Swan (2008). Moreover, it builds on these ideas, looking for themes, and identifies more ways teachers use concrete materials and further reasons why primary teachers choose to use them.

Teachers' Beliefs

Teachers' beliefs and practices significantly impact the choices they make about how concrete materials are used in the classroom. The type of activities and learning experiences a teacher chooses to implement is strongly influenced by their beliefs about the learning and teaching of mathematics (Beswick, 2011; Buehl & Beck, 2014; Clark & Peterson, 1986; Ernest, 1989; Francis, 2015; Lui & Bonner, 2016; Richardson et al., 1991; Thompson, 1992; Vacc & Bright, 1999). Beliefs about the teaching of mathematics cover aspects such as how a teacher chooses to teach, including the form of instruction and the choice of activities and resources utilised. The classroom experience designed by a teacher is influenced by how they believe a student learns. Although there are many beliefs concerning mathematics education, beliefs as utilised in this study broadly encompass a constructivist philosophy, where students build their understanding of mathematical concepts through exploration and student-focused activities, versus the instructionist approach which sees students as receptors of knowledge (Ernest, 1989). A teacher is more likely to embrace the use of concrete materials, if a teacher's beliefs about the learning and teaching of mathematics are founded in constructivism as constructivism is about the student constructing their knowledge and reflecting on their learning and concrete materials can potentially be utilised to support this endeavour (Hiebert et al., 1997).

Research supports the idea beliefs play an important role in explaining the actions of teachers (Beswick, 2011; Buehl & Beck, 2014; Calderhead, 1996; Ertmer, 2005; Kane et al., 2002; Pajares, 1992; Wilkins, 2008). "Few would argue that the beliefs teachers hold influence their perceptions and judgments, which, in turn, affect their behaviour in the classroom" (Pajares, 1992, p. 307). Although society, education systems and schools set boundaries and have expectations about how a teacher should teach, a teacher to a large extent has the freedom to choose their own instructional strategies (Dexter et al., 1999; Fullan, 1991). It is a teacher's beliefs that are "stronger predictors of behaviour" (Ertmer, 2005, p. 28) than teacher knowledge. Compared to knowledge, beliefs play a far greater role in directing the choices teachers make about their classroom practices.

Nevertheless, teachers' practices may reciprocally influence their beliefs. Buehl and Beck (2014) conducted a meta-analysis of 257 articles concerning the relationship between teachers' beliefs and practices. Research included how beliefs influence practice, how practice influences beliefs, the disconnection between practice and beliefs along with how they influence each other. Their analysis concluded that rather than restricting the relationship to simply teacher beliefs influence teacher practice, a better way of viewing the relationship between beliefs and practices is one of interdependence. Buehl and Beck's conceptualisation of the relationship between teacher beliefs and teacher practices has them centrally located joined by a double-headed arrow, implying they influence each other. Moreover, their model overlays teacher beliefs and practices upon a complex arrangement of internal and external factors, recognising many other influences play a part in their relationship.

So while much research has explored teachers' beliefs and practices and, in particular, in relation to mathematics, mathematics teaching and mathematics learning, fewer studies have explored primary teachers' beliefs, more specifically, related to use of concrete materials, particularly with regards to how they are used.

Teachers' Beliefs about concrete materials

A review of the literature from the past 20 years, involving studies which mention they have explored teachers' beliefs specifically related to the use of concrete materials, produced limited results. Some more relevant studies are outlined here. These studies include how beliefs could be used to predict how frequently concrete materials were used (Uribe-Flórez & Wilkins, 2010), beliefs related to enabling and disabling factors regarding their use (Golafshani, 2013), beliefs about effective mathematics teaching and learning (Perry, 2007), beliefs about the impact of their use on both students and teachers (Akkan, 2012) and beliefs about how the use of concrete materials might influence classroom practice (Tran, 2015).

A survey conducted by Uribe-Flórez and Wilkins (2010) involved 503 primary teachers from the USA. They found that the grade level being taught and teachers' beliefs about concrete materials could be used to predict how frequently teachers used concrete materials in their teaching.

An investigation into beliefs about the use of physical and virtual manipulatives and their links to classroom practice by Golafshani (2013) involved four Canadian Grade 9 teachers. The project ran over 21 weeks, utilised a questionnaire and classroom observation and involved professional learning. The teachers designed lessons which were piloted and were developed as 'model' lessons for Grade 9 classes in the local district. Golafshani found some teachers viewed manipulatives as an option while others saw them as a necessity. Furthermore, the teachers offered possible enabling and disabling factors concerning the use of manipulatives. While the teachers noticed the incorporation of manipulatives had some positive effect on students who find mathematics challenging, the main finding from the study was that the secondary teachers believed their use enhanced student engagement.

Like Golafshani (2013), Akkan's (2012) research in Turkey involved beliefs about physical and virtual manipulatives. In this study, Akkan surveyed 148 teachers and 228 preservice teachers from preschool, lower and upper primary and secondary levels. Results from primary teachers (n=86) related to physical manipulatives will be presented here. The study found teachers agreed impacts on students from using concrete materials included that physical manipulatives support individual needs, they improve students' mathematical and problem solving skills, have a positive effect on students' attitude to mathematics, allow mathematical reasoning and enhance understanding. The teachers disagreed with the idea that physical manipulatives prompted memorisation.

In the context of exploring effective mathematics teaching and learning, Perry (2007) interviewed 13 Australian primary school teachers regarding their beliefs about mathematics, mathematics teaching and mathematics learning. Concrete materials were discussed in relation to teachers' beliefs about student learning. While all teachers believed concrete materials had a role to play in primary mathematics learning, eight believed they should be made available to students whenever they wanted them. Similar to Akkan's (2012) study it was noted concrete materials support understanding. In addition, teachers mentioned visualisation, the hands-on aspect and connections made between classroom and real-life mathematics and between concrete and abstract ideas, as being reasons for using concrete materials. In contrast to Akkan's findings, a teacher mentioned that by "touching it, talking about it [and] listening to others share ideas" (Perry, 2007, p. 277) about concrete materials, they provide students with the opportunity to build memory through the ability to draw on a range of senses.

Research by Tran (2015) explored teacher beliefs about teaching mathematics using manipulatives and how this might influence classroom practice. The study involved interviews with three teachers who teach Grades 4-8 in Canada. The teachers reported they used concrete materials with the whole class, to model mathematical ideas, to promote active discovery and exploration and as an assessment tool. Numerous reasons were given for why they used concrete materials. As with Akkan (2012) and Perry's (2007) studies, teachers believed their use promoted understanding while, in line with Golafshani's (2013) findings, they mentioned engagement. Furthermore, like Perry, Tran noted teachers mentioned the use of concrete materials as a visual aid and to connect the abstract with the concrete, whilst their use to support the development of mathematical skills, a similar finding to Akkan, was highlighted. Further reasons given were to help English Language Learners, promote active learning and as a comfort, to reduce maths anxiety.

So, while there is some research from the past 20 years specifically concerning teachers' beliefs regarding the use of concrete materials, it is not extensive. More of the research relates to why teachers use concrete materials and beliefs related to the impact of their use on student learning. This area encompasses a wide range of possible aspects and will be further explored in this study. Less research appears to have investigated teacher beliefs around how they are used, warranting its focus in this study.

Research Questions

This paper presents an investigation into the use of concrete materials by primary school teachers in NSW. It explored their beliefs about why and how they are used in the classroom.

While acknowledging the numerous internal and external factors which impact teacher beliefs and practices, Buehl and Beck's (2014) conceptualisation of the interdependent relationship between teacher beliefs and teacher practices underpins this study. Teachers' beliefs and professed practices regarding the use of concrete materials were positioned within this conceptualisation and, in particular, possible influences of teacher practice on teacher belief were examined. The research in this paper was guided by the following research questions:

1. Why do teachers believe they should use concrete materials?
2. What classroom practices are employed when using concrete materials?

These questions were explored with a focus on the Number and Algebra strand of the NSW Mathematics syllabus. This content strand includes the topics Whole Numbers, Addition and Subtraction, Multiplication and Division, Fractions, Decimals and Percentages and Patterns and Algebra (NESA, 2019).

Methods

Primary school teachers' use of concrete materials in the classroom was investigated for this study. A survey methodological approach was taken, utilising a questionnaire and semi-structured interviews, in order to capture rich data. Forty-nine teachers, participants at a NSW primary mathematics conference, completed the questionnaire and four of these teachers were later interviewed. This paper focuses on the aspects of the study regarding why and how teachers use concrete materials.

Questionnaire

The questionnaire had two components. The first part asked teachers for background information including their teaching experience, enjoyment of and confidence in teaching mathematics and other demographic information. The second part included questions about their practices and beliefs regarding the use of concrete materials. To enhance the reliability of the questionnaire, several items were adapted from one used by Marshall and Swan (2008) in their research on the use of concrete materials. Including questions that have been used previously means they have already been tested (Hyman et al., 2006). Other influences on the design of the questions came from research by Carpenter et al. (1996), Moyer (2001) and Perry and Howard (1997).

Respondents were asked to rate their agreement to statements on a 5-point Likert scale – strongly disagree, disagree, uncertain, agree, strongly agree. This made the analysis of the results straightforward and allowed for calculation of frequencies. In numerous questions respondents did not choose the statements strongly agree and strongly disagree, therefore any responses which included these were combined with agree and disagree, respectively (see Tables 1 and 2). Cross tabulations were produced. Data from free response questions were used to clarify other responses and summarised. Further exploration of the free response data sort to identify concepts related to the themes which emerged from the interview data.

Interviews

At the end of the questionnaire, teachers were asked to indicate their willingness to participate in semi-structured interviews. This approach can provide greater depth of information for the researcher and is relatively straightforward to analyse (Burns, 2000). This method was employed to collect elaborations of questions from the questionnaire and to gain a deeper understanding of teachers' reasons for using concrete materials. Teachers were asked to respond to questions and encouraged to add any further comments they believed clarified their views or ideas. Moreover, teachers were asked to describe some of their teaching experiences using concrete materials.

To collect data from the interviews they were audio-recorded and later transcribed and analysed using open, axial and selective coding as described by Neuman (2011). Open coding was initially conducted looking for potential themes. This was followed by axial coding, organising the data into categories that clustered related concepts together within the themes. Lastly, selective coding was utilised to look at all the data to gather illustrations of the final themes. All coding was completed by one researcher to ensure consistency.

Results and Analysis

Questionnaire

The 49 participants had a wide range of teaching experience and teachers of all primary grade levels were represented. Approximately one quarter were in their first five years of teaching while almost one third had been teaching for more than 25 years.

Eighty percent of those surveyed said they enjoyed teaching mathematics while 94% indicated they enjoyed using concrete materials when teaching mathematics. Several respondents commented their enjoyment of teaching mathematics was due to the 'hands on' learning experiences they were able to employ to engage their students.

While examining reasons primary teachers identified for using concrete materials in the learning and teaching of mathematics (see Table 1), results indicated all respondents agreed the use of concrete materials enhances student engagement. Moreover, all those who responded to the question on whether their use enhances understanding, agreed it did.

There was substantial agreement that the use of concrete materials aids visual (96%) and kinaesthetic (100%) learners. While 86% of teachers surveyed agreed the use of concrete materials is 'essential to help students' understanding of mathematical concepts', only 80% of respondents agreed with the statement 'it is difficult to help students understand mathematical concepts without them'. This could indicate a conflict in beliefs of a few respondents as they are suggesting they believe concrete materials are essential and yet, they believe it is not difficult to help students to understand concepts without them. The importance of concrete materials to allow students to represent their thinking was strongly supported by 94% of respondents.

Table 1
Possible Reasons for the Use of Concrete Materials in the Teaching and Learning of Mathematics (N = 49)

Statements	Disagree	Uncertain	Agree	No response
Enhance student engagement	0	0	49 (100%)	0
Enhance understanding	0	0	48 (98%)	1 (2%)

Aid visual learners	2 (4%)	0	47 (96%)	0
Aid kinaesthetic learners	0	0	49 (100%)	0
It is difficult to help students understand mathematical concepts without them	4 (8%)	5 (10%)	39 (80%)	1 (2%)
Essential to help students' understanding of mathematical concepts	1 (2%)	4 (8%)	42 (86%)	2 (4%)
Allow students to represent their thinking about mathematical concepts	0	1 (2%)	46 (94%)	2 (4%)

When asked what concrete materials they actually used, the teachers indicated they used them in a variety of ways (see Table 2). Most respondents (90%) indicated they used concrete materials to demonstrate an idea or concept when teaching Number or Algebra, while 94% believed it was important for students to use the concrete materials themselves. The results show eight respondents, who are teachers from across all stages, either agree or are uncertain that concrete materials are usually used for fun activities and games rather than for 'real' learning. Of these eight respondents, four had less than five years teaching experience and two had 35 years of experience. This could indicate teachers in their early years of teaching have not yet developed an understanding of how concrete materials can be used beyond their use for fun activities and games and yet the same can be said about the two very experienced teachers.

For effective use of concrete materials, 88% of respondents agreed it is important to ask good questions that challenge students to think. A further comment about how concrete materials are used in the classroom was:

> They are great for assessment with children who work at higher levels and won't engage, by asking them to use the concrete materials to teach another student a skill or process. You can then see their processes. (Experienced female Year 1 teacher)

Responses to the statement 'drawing diagrams is just as effective as using concrete materials' resulted in some interesting data. Just over 30% of respondents were uncertain about this while 26% agreed with the statement and 37% disagreed. This spread of responses may indicate teachers are unsure about the role of diagrams in effective teacher practice and whether or not or how diagrams might be utilised instead of or alongside the use of concrete materials. This may be an area for further study.

Table 2

Classroom Practices Related to How Concrete Materials Are Used in the Teaching and Learning of Mathematics (N = 49)Statements	Disagree	Uncertain	Agree	No response
Demonstrate an idea or concept when teaching Number or Algebra	2 (4%)	1 (2%)	44 (90%)	2 (4%)
Important for students to be able to use them themselves	2 (4%)	0	46 (94%)	1 (2%)
Usually used for fun activities and games but not for 'real' learning	41 (83%)	3 (6%)	5 (10%)	0

To use them effectively, it is important to ask good questions that challenge students' thinking	2 (4%)	2 (4%)	43 (88%)	2 (4%)
Drawing diagrams is just as effective as using concrete materials	18 (37%)	15 (31%)	13 (26%)	3 (6%)

In summary, the questionnaire data provided an overview of teachers' beliefs and practices related to their use of concrete materials. Some of these ideas are further explored through the analysis of the interview data.

Interviews

Four female primary school teachers were interviewed to gain more in-depth information about beliefs and practices related to the use of concrete materials. Sally[1] was in her third year of teaching and a grade leader for Year 2. Kathryn had been teaching for 27 years. She was an Assistant Principal and was teaching Kindergarten. Jane was an Acting Principal and was teaching Year 4. She had 31 years teaching experience. Barbara was the principal in a one teacher school, teaching all primary year groups, with 28 years of teaching experience.

On analysing the data, some key themes emerged. These are presented here along with other data related to the research questions. The areas covered in the interview included, understanding, thinking and reasoning, other benefits to students, other reasons and practices.

Understanding

The interviewees all reiterated a major reason for using concrete materials was to enhance understanding. Sally clarified this by adding their use builds the foundational understanding that allows a student to move from concrete to abstract thinking. Early in her teaching career she was confronted with Year 5 students who did not have a conceptual understanding of fractions. On utilising concrete materials to illustrate the concept she said she soon realised how beneficial they were in helping students understand.

Barbara related her early experience of learning mathematics, believing she was mainly given an instrumental understanding of mathematical concepts at school. As an adult she has developed a relational understanding of concepts and believed it was important to teach using strategies that promote the development of a relational understanding of concepts.

By engaging both weak and more able students, Jane suggested concrete materials allow students to process their understanding in different ways. She added they provide opportunities for students to use their understanding and skills in a variety of contexts.

The interviewees all mentioned the importance of the visual and kinaesthetic elements involved in employing concrete materials and how being able to see, touch, feel and hold the materials was vital to gain a deep understanding of the mathematics concepts. Kathryn commented that by using concrete materials teachers are giving students a perceptual understanding of concepts. Barbara believed it was this imagery that builds understanding, "if you can picture something in your head you can see how it works".

A variety of reasons given to justify the use of concrete materials were linked to promoting understanding, but Kathryn highlighted another idea. She believed concrete materials not only

[1] All names are pseudonyms.

allow students to demonstrate their understanding, but they can show a student's lack of understanding, illustrating the misconceptions they may hold.

Thinking and reasoning

Jane suggested concrete materials provide students with an avenue to accurately record their thinking believing there can be times when students may not have the skills to do this with pen and paper. She added tasks designed around concrete materials can give students a need to talk and articulate their thinking and believed there is "power in having to explain your model to someone else". However, she believed "that the concrete materials' use is still only as good as the substantive conversation that's generated from it". Jane provided tasks to her students which included extra or insufficient resources. She required students to analyse the problem, choose appropriate materials and give reasons for their conclusions.

Kathryn maintained that concrete materials allow students to make connections between different representations. She teaches the metalanguage of mathematics and encouraged her students to use this while reflecting in their learning logs about the tasks they had completed, including activities that have involved the use of concrete materials.

Sally mentioned the benefits of concrete materials to students' thinking. She commented they helped students process the steps of their thinking.

Other benefits

When using concrete materials, Jane believed students are in charge of their learning. Using them, she suggested, provides a supported learning environment where students feel confident in taking risks as their product does not need to be a permanent record, as pen and paper may be perceived. They can simply pull apart what they have created and remodel it. In conjunction with this Jane found students concentrate for longer periods, extending their 'on task' time.

Both Barbara and Jane mentioned the opportunities for choice and creativity that concrete materials afford. They both exemplified this with patterning in algebra.

Sally and Barbara both discussed the benefits of using concrete materials to aid in differentiating tasks. Jane and Barbara believe if students model concepts with concrete materials it helps them to remember.

Practices

All teachers reported using a wide variety of teaching strategies when incorporating concrete materials. They usually utilised concrete materials on a daily basis and they regularly incorporated them in daily number work to support fluency and automaticity.

Kathryn mentioned it was easier to demonstrate a concept using concrete materials than to try to explain it while Jane liked to employ them when students engaged in peer-to-peer teaching. She liked to use them for games as she believed this helped build social skills.

Barbara outlined her use of concrete materials for a place value activity where students used dice to generate numbers and then build a representation of them using the base ten blocks.

Information gathered from the interviews complemented the survey data. The interviews provided further information regarding the use of concrete materials to enhance understanding and thinking and exemplified some teacher practices. They introduced ideas which had not been explored in the questionnaire around reasoning and other benefits to students. These findings will be further examined in the discussion.

Discussion

There are many reasons teachers choose to incorporate the use of concrete materials in their classrooms. The data presented a number of these. They include improving students' understanding, engagement, thinking, memory, social interactions, learning style, and fluency and automaticity.

For discussion purposes, these reasons along with the choice of strategies teachers employ when using concrete materials are presented under four broad groupings. These are: belief in a social constructivist philosophy of teaching and learning mathematics; other teacher beliefs; belief in a cognitive dimension and beliefs about engagement/fun. Each of these is briefly discussed here with reference to the data and the literature. It should be noted however these groupings are not discrete and there are elements common to more than one perspective.

Belief in a social constructivist philosophy

Fundamental to the effective use of concrete materials is the belief in a social constructivist approach to the learning and teaching of mathematics. Belief in a social constructivist approach means teachers see learning as student construction of knowledge, including the use of and reflection on physical actions (Reys et al., 2020). To facilitate this, a teacher with these beliefs would likely embrace the use of concrete materials. In referring to their use, it has been suggested that “children should be encouraged to incorporate them into their mathematics learning in their own ways” (Perry & Howard, 1997, p. 30). The current research found 94% of respondents believed it was important for students to use concrete materials themselves. As soon as the students have the concrete materials in their own hands, it is likely the teacher is no longer the focus of the lesson. As Jane, a very experienced Year 4 teacher, mentioned, “it puts the students in charge of their learning”.

Jane believed tasks designed around concrete materials can give students an opportunity to talk. A strength lies in the need to discuss and explain what they have constructed to others and the conversation generated from their use. Jane added that they provide a setting for students to process their understanding in different ways.

Other teacher beliefs

All interviewees reported incorporating concrete materials in a variety of tasks involving an array of teaching and learning experiences. These included exploration, individual, group and pair work. The data suggested there was extensive use of concrete materials to demonstrate ideas or concepts. Such findings are consistent with earlier research (Perry & Howard, 1997; Swan & Marshall, 2010). Kathryn, an experienced teacher currently teaching Kindergarten, gave an example where concrete materials were used as part of a more holistic experience which included the teaching of mathematical vocabulary and student reflection in a journal.

Cognitive dimension

Summarising the data, the support for thinking provided by the use of concrete materials was reported to be an important reason for incorporating them in class activities. Marshall and Swan (2005) propose the use of concrete materials “may be justified in a setting where the teacher uses the manipulatives as a catalyst to encourage thinking” (p. 145). In reference to their use, participants' beliefs about the benefits to students' learning included: articulating their thinking, helping to process the steps of thinking, seeing the processes (students are using), accurately recording their thinking, analysing and giving reasons, and making connections. This supports Skemp's (1989) theory that concrete materials allow teachers to see what and how students are thinking. Moreover, 94% of survey respondents believed concrete materials allowed students to represent their thinking while 88% agreed it was important to ask good questions to challenge students to think when using concrete materials.

Both Jane and Barbara proposed if students model concepts with concrete materials it helps them to remember. All the interviewees related they usually utilised concrete materials on a daily basis and regularly incorporated them in daily number work to support fluency and automaticity.

Some of the uses for concrete materials related to thinking appear to be related to metacognitive activities. The benefits of the support of concrete materials to metacognition could represent an area for further investigation as there is support for there being a positive relationship between metacognition and achievement in mathematics (Özsoy, 2011).

Engagement, fun and games

Marshall and Swan (2008) reported many teachers believed concrete materials "helped to engage students, or provided them with enjoyment or were 'fun'" (p. 342). Although some may see these as similar ideas, enjoyment and fun can be quite different to engagement. Engagement implies an involvement in learning (Attard, 2009) that does not necessarily follow from finding fun or enjoyment in an activity. It is not clear whether participants in this research drew a distinction between engagement and, enjoyment and fun.

Similarly to Golafshani's (2013) findings, all respondents agreed concrete materials enhance student engagement. This is not exactly the same term used in the research of Perry and Howard (1997) who reported on enjoyment or Swan and Marshall (2010) who grouped "heighten interest, helped engage students, enjoyment, fun and provided motivation" (p. 16). The distinction between engagement and enjoyment may be an important one. Engagement may support learning while enjoyment may not. Given the belief by so many respondents that concrete materials enhance engagement, it would be beneficial to know whether they understand the difference between engagement and enjoyment and explore whether there is a link between these and concrete materials, and whether the link results in improved learning.

Rather than being used for 'real' learning, a study by Moyer (2001) revealed a number of teachers believed concrete materials were used for fun. In the current study, the findings do not appear to be as strong as Moyer's, showing 16% of respondents, who are teachers from across all stages, either agree or are uncertain as to whether concrete materials are usually used for fun activities and games rather than for real learning. This could be because the participants in this study were attending a conference and could be more informed about the use of concrete materials. Of those respondents who agreed or were uncertain as to whether concrete materials are usually used for fun activities and games rather than for real learning, half had less than 5 years teaching experience and a quarter had 35 years of experience. A possible reason for this finding could be that these early career teachers may have mainly experienced the use of concrete materials for fun. While, it is possible the teachers mentioned here who had many years teaching experience are not familiar with more current theories on mathematics teaching and learning. As Barbara mentioned, in regard to more experienced teachers, "they are set in their ways."

Concrete materials are used by Jane for games as she believed this helps build social skills. Research suggests the use of games in mathematics learning increases 'time-on-task' (Bragg, 2012) and so perhaps there is scope for further study of the links between games and the use of concrete materials. In general, Jane found students' time-on-task increased with the use of concrete materials, believing students are able to concentrate for longer. Furthermore, Jane believed games could benefit students by reinforcing understanding and improving memory.

Teachers appear to use concrete materials because they believe they benefit students' understanding of mathematical concepts and engage students in their learning. However, it was not clear whether teachers have a deep understanding of what effective use of concrete materials looks like or whether they realise engagement does not necessarily equate to learning. These could be areas for further research. Moreover, exploring the role of concrete materials in student thinking and the connection between concrete materials and engagement might warrant further investigation.

Much evidence supports the idea teachers' beliefs influence their practice. The findings here concur with this idea although additionally they elucidated possible factors that might influence the reciprocal relationship of practices influencing beliefs. In Figure 1 elements from the current study regarding teacher practices and teacher beliefs are presented. Through implementing classroom practices that are influenced by their beliefs, teachers were able to observe a range of impacts that appeared to benefit student learning. These impacts in turn may have confirmed teacher beliefs related to the use of concrete materials. However, further research is necessary to explore if and how teacher observations of impacts on student learning when using concrete materials might influence teacher beliefs.

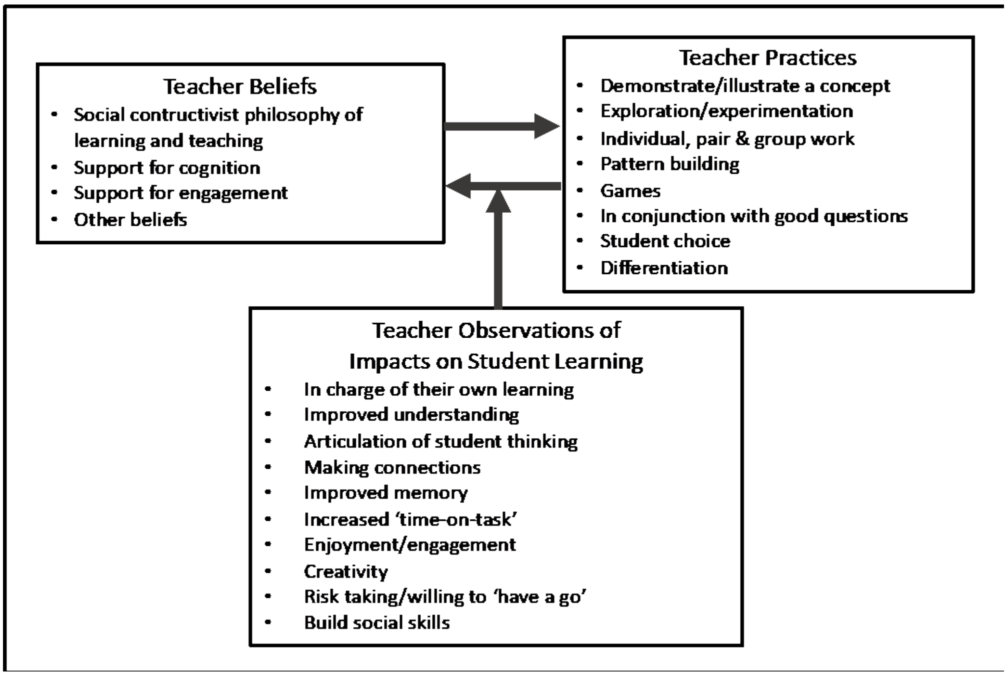

Figure 1. An overview proposing possible links between teacher beliefs, practices and observations in the context of the use of concrete materials

Preservice teachers come to their educational studies with deep-rooted, existing beliefs about mathematics learning and teaching which are often founded in their own more traditional, teacher-centred experiences of mathematics education (Valcke et al., 2010). Teacher educators aim to shift preservice teachers' beliefs to encompass a more constructivist approach. To do this teacher educators introduce preservice teachers to learning and teaching strategies which support the development of more a student-centred classroom, which they may then implement when in the classroom, however, this may be insufficient to have any impact on their beliefs (Valcke et al., 2010). Both Swars et al. (2006) and Valcke et al. (2010) suggest that preservice teachers might need to be provided with opportunities to reflect on their experiences in order to shift their beliefs. To facilitate this reflection, teacher educators could utilise the observations of practising teachers in the classroom that indicated to the teachers the impacts on student learning from the use of concrete materials. These observations may provide a starting point for guiding preservice teachers' reflection on their practice as, if they notice the effectiveness of strategies based in constructivism, then they may be more likely to embrace this approach (Sarws et al., 2006). This might be an area for further research.

Conclusion

There were numerous limitations of this study affected by its duration, scope and resources. Concrete materials were defined using a narrow scope. The focus was on the structured form, utilised for making concrete, abstract mathematical concepts. However, concrete materials come in a variety of shapes and sizes and including a wider range may have enriched this study.

The size of the sample, although adequate to gain small insights, would have benefited from being larger. A random sample may have given more reliable and generalisable results. The survey was self-reporting and what respondents say they do may not be the same as what they do in practice. Classroom observations would be an effective and more reliable way of getting a true representation of what is happening inside classrooms and to discover whether there are consistencies between professed beliefs and classroom practices.

The aim of this research was to explore teachers' beliefs and practices regarding the use of concrete materials in primary classrooms. The results from this study support the use of Buehl and Beck's (2014) conceptualisation of the interrelation between beliefs and practices. Teachers' beliefs and practices significantly impact why concrete materials are used in the mathematics classroom. There are many reasons why concrete materials might be utilised in a mathematics classroom. They could be used to help children think, make a concept visible, engage learners, help them to move from concrete to abstract thinking, focus conversation and articulate ideas or for reinforcement and consolidation.

Furthermore, teachers' beliefs and practices considerably effect how concrete materials are used in the mathematics classroom. They may be used to demonstrate or explore a concept, build patterns or play games.

While some findings in this study give support to previous findings, little articulation of the use of concrete materials for differentiation of tasks had been previously reported nor mention of their use in conjunction with the use of good questions. Their use to increase concentration, promote creativity and to support risk taking are additions to the many and varied reasons teachers give for why they use concrete materials. This study identified some overarching ideas related to the use of concrete materials amongst these a belief in a social constructivist approach to classroom practice, a cognitive dimension to their use which included allowing students to articulate their thinking and engagement.

Moreover, this study has enriched our understanding of teacher beliefs and how they may be influenced by teacher practices and has furthered our knowledge on their relationship in the context of the use of concrete materials. Possible links have been proposed between teacher beliefs, practices and observations of impacts on student learning. While it is well established that teacher beliefs influence their practice, the role these observations may play in influencing or shifting inservice and preservice teachers' beliefs towards a more constructivist approach to learning and teaching warrants further investigation and may help us learn whether these observations play a part in teacher practices influencing teacher beliefs.

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