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Teaching Place Value to Students With Learning Disabilities in Mathematics

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Abstract

According to National Council of Teachers of Mathematics (NCTM), students must develop a conceptual understanding of place value by second grade to understand that mathematics processes are more than a set of procedural steps. Yet, students with learning disabilities in mathematics typically struggle to understand place value in a deeper, conceptual sense, which can have a negative impact on their future performance where application of place value concepts are foundational. Therefore, early interventions targeting place value instruction are critical for building a solid foundation in mathematics for students with learning disabilities. This article provides teachers with an overview of five place-value principles, which are critical for student success; and evidence-based practices for teaching those principles.

Keywords

mathematics education, place value, learning disabilities

When students enter school, mathematics instruction focuses on forming a foundational understanding of numbers. In the primary years, teaching centers upon counting, cardinality, number relationships, and *subitizing*, which is the ability to immediately identify the number of objects in a group (Witzel & Little, 2016). However, as students progress through school, instruction shifts from teaching students to count by individual units to counting by making sets of 10, also known as *unitizing*. Unitizing sets of 10 is where the formal instruction of place value begins.

Place value is the concept that a digit's value is dependent upon its position within a number (Reys et al., 2014). Place value instruction should begin by introducing sets of 10 because the place value system is a base-10 system. According to Reys et al. (2014), the "base-10 system means a collection in which the value 10 determines a new collection and is represented by the number 10 and consists of 10 digits, 0 through 9" (p. 149).

Many state standards place heavy emphasis on learning place value because it is a predictor for learning later skills, such as operations with whole and rational numbers (Moeller et al., 2011). The National Council of Teachers of Mathematics (NCTM, 2000) stated, "it is absolutely essential that students develop a solid understanding of the base-10 numeration system and place value concepts by the end of grade 2" (p. 81). However, students with a learning disabilities (LD) in mathematics often struggle to develop a deep conceptual understanding of the mathematical ideas that encompass place value (Landerl & Kolle, 2009). Therefore, it is critical to provide them with early interventions through evidence-based practices, such as the use of precise vocabulary, explicit instruction, and multiple representations, to help them develop the place value skills that are required as mathematics becomes more complex (Bryant et al., 2011).

According to Van de Walle et al. (2019), there are five overarching principles that contribute to a student's deeper understanding of place value. These five principles include (a) understanding sets of 10, (b) positions of digits in numbers, (c) patterns in numbers, (d) composing and decomposing

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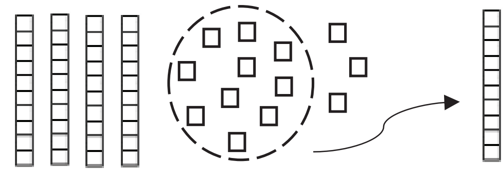
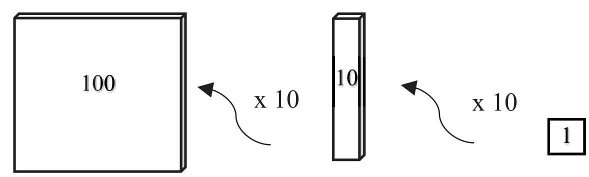
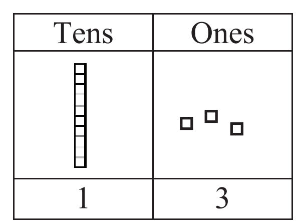
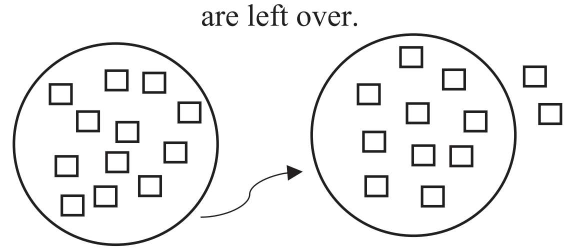
Essential Term	Example
Decompose The ability to represent a given number in *x* amount of hundreds, tens, and ones.	347 can be represented in the following ways: ▪ 3 hundreds, 4 tens, and 7 ones ▪ 3 hundreds, 3 tens, and 17 ones ▪ 2 hundreds, 14 tens, and 7 ones
Exchange Principle The understanding that 10 of any place value can be exchanged for one of the next place value.	Fifty-three is the same as 5 groups of ten and 3 ones. 
Multiplicative Pattern of Place Value The notion that each successive place value is 10 times the value as the previous one.	 This model shows that 10 ones make 1 ten and 10 tens make 1 hundred. This pattern continues infinitely.
Positionality The value of a digit is determined by its position within the number, such as the hundreds, tens, and ones place.	 The value of the 1 is 10 because it is in the tens place.
Unitizing The act of making a set, such as a set of 10, out of a given group of objects.	If a student is given 12 ungrouped objects, they can partition them into one set of ten and tell how many are left over. 

Figure 1. Essential place value terms.
Note. The terms presented in the figure are discussed throughout the article and are essential for the teaching and understanding of place value concepts.

numbers in flexible ways, and (e) conceptualization of larger numbers. This article provides strategies for teaching these five principles to students with LD in mathematics through intervention activities and evidence-based practices. Essential place value terms that are discussed within the article are presented in Figure 1.

Understanding Sets of 10

Before students receive formal place value instruction, they count using a count-by-ones approach. The count-by-ones approach is considered a pre–place value stage because students have not yet formed an understanding of base-10. The National Research Council (NRC) has suggested that "to begin to understand the base-10 place value system, children must be able to view 10 ones as forming a single unit of 10" (NRC, 2009, p. 45). The progression from a pre–place value stage to understanding sets of 10 is as follows: (a) using the count-by-ones approach, (b) making groups of 10 with ungrouped models, to then (c) counting using a standard base-10 approach.

As early as Pre-K and kindergarten, students begin counting into the double digits. However, it is likely that when young students count, they are using a count-by-ones approach, meaning they have memorized the sequence of counting into the double digits but lack the understanding that the digit in the 10s place represents groups of 10 (McGuire & Kinzie, 2013). For instance, students may count 16 teddy bears one-by-one and write the number with the digits 1 and 6. Nevertheless, without formal place value instruction, students with LD in mathematics may not understand that the 1 in 16 represents a group of 10 and the 6 represents 6 ones. Teachers can determine whether students are in a pre–place value level of understanding through an observational assessment. If students count one-by-one and continue to do so as numbers get larger, rather than making groups of 10 to make counting easier, then they are in a pre–place value level of understanding and will require formal instruction to begin unitizing groups of 10.

To introduce unitizing to students with LD in mathematics, teachers can have students count ungrouped models, such as ungrouped beans, connecting cubes, and other manipulatives, by first making sets of 10 (see Figure 2). Utilizing ungrouped objects for this type of activity allows for students to explore the *exchange principle*, which is an understanding that a group of 10 in any place value can be traded for one in the successive place value. Unitizing is beneficial because it allows for students to count the sets of 10 first and then count the ones that are left over (e.g., two groups of 10 and three left over is equal to 23). As students gain experience counting by 10s, they begin to see the value in unitizing compared to the counting-by-ones approach and are ready to progress to standard base-10 groupings.

After students understand the exchange principle, teachers can introduce the standard base-10 approach to representing numbers. In the standard base-10 approach, a set of 10 is made for every opportunity possible, resulting in the least amount of place value blocks needed to represent a number (see Figure 3). Utilizing concrete (e.g., base-10) or semi-concrete (e.g., pictorial representations) models can aid students in developing the concept of standard base-10 groupings (Mix et al., 2017). Specifically, pre-grouped base-10 models are beneficial when teaching the standard base-10 approach because the models are already grouped by ones, 10s, or 100s, making it easier to make a trade. For instance, when counting by 10s to reach a value of 100, a trade can be made for a pre-grouped 100s model. When base-10 models are introduced, the vocabulary referring to the models must be consistent throughout instruction and across grade levels (Townsend et al., 2012). Thus, teachers should refer to concrete representations as a unit or one, a 10 rather than a rod or stick, and a 100 instead of a flat.

Positions of Digits

The second principle, positions of digits, involves the understanding that the value of a digit is determined by its position within a number, also known as *positionality* (Witzel & Little, 2016). Teaching this principle can be achieved through (a) using a place value chart to emphasize positionality, (b) teaching the use of the zero in the place value system, and (c) representing numbers in the expanded form.

Students with LD in mathematics who have not mastered the concept of positionality might incorrectly represent two 10s and three ones as 32 or 203. Teachers can teach the concept of positionality by representing numbers with base-10 blocks and having students write the number in a place value chart. A place value chart serves as a scaffold for students and ensures that the digits are written in their corresponding place values. To model correct positionality for the number 23, the teacher can place two 10s blocks and three ones blocks on the place value chart in their correct columns. The student then can write the *standard form* of the given number underneath the blocks. The standard form of 23 will be written with a 2 in the 10s column and a 3 in the ones column.

In addition, students need to be exposed to numbers that contain zeros so that they can form an understanding of how zero is used within our place value system. To teach the value of zero within a number, teachers can display base-10 blocks for numbers such as 105 and 230. Teachers can model how to place the zero in a place value position for which there are no displayed representations. However, it is important to abstain from calling these zeros "placeholders" to avoid the misconception that they have no value. Teachers should be mindful of two common errors that are typical for students with LD in mathematics in relation to the zero (Reys et al., 2014). First, students record the place value blocks as they are seen, such as writing 105 as 15, omitting the zero that is needed in the 10s place. Second, students write numbers in standard form as they are spoken orally, such as writing 305 for 35. To address this error, teachers can display both 35 and 305 using place value models and

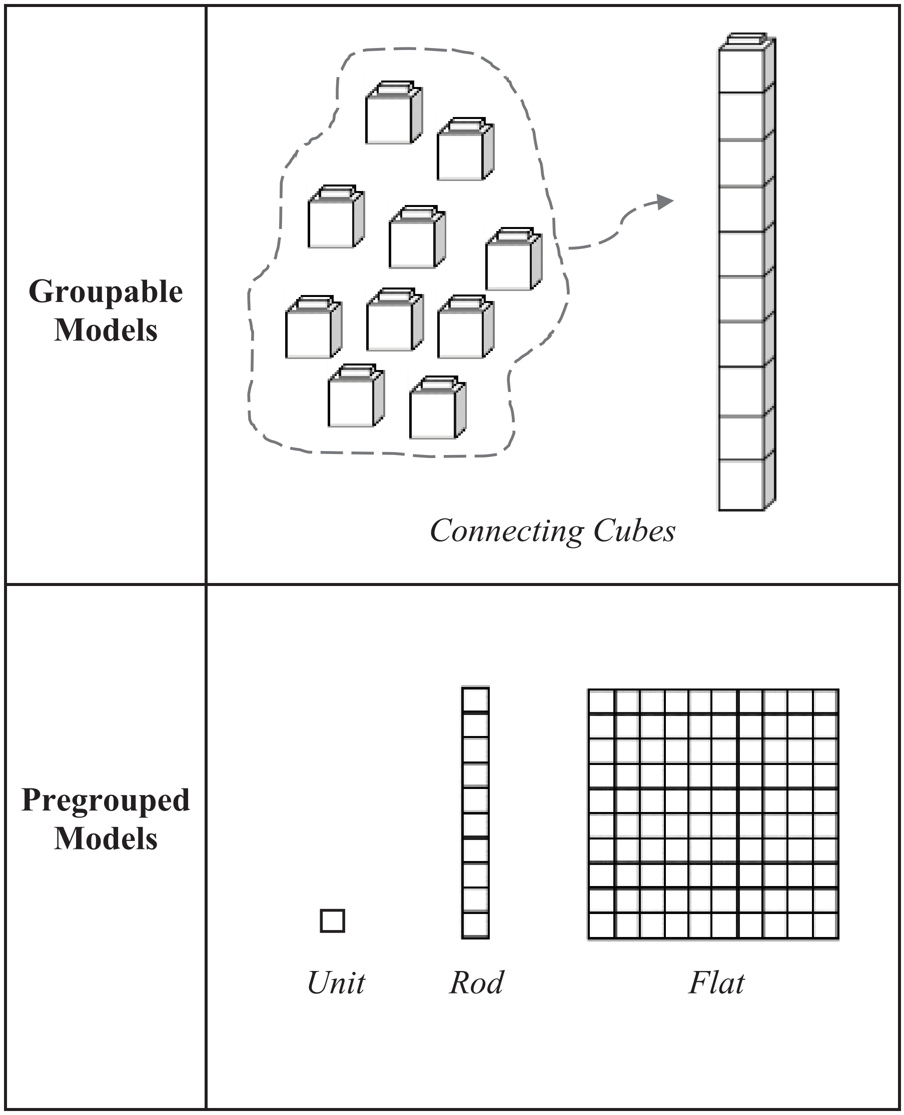


Figure 2. Groupable versus pre-grouped models.
Note. Groupable models, such as interlocking cubes, can easily be taken apart. Pre-grouped models, such as units, rods, and flats, are best for trading activities as they cannot be taken apart.

have the students write the numbers in a place value chart. The teacher then can lead a discussion on the differences between the two displays.

Next, teachers should introduce the *expanded form* to continue developing a students' understanding of positionality. The expanded form is a way of representing a number that shows the value of each digit as a string of addends (e.g., 235 is equal to 200 + 30 + 5). Students with LD in mathematics often have the misconception that the largest digit, regardless of its position, has the greatest value. However, writing numbers in the expanded form will increase students' conceptual understanding of digits and their values. For example, expanding the number 235 demonstrates that the 2 has a greater value than the 5 because the 2 is in the 100s place and has a value of 200, while the 5 is in the ones place with a value of five. To further develop this concept, teachers can have students practice writing the expanded form of different numbers that contain the same digits (e.g., 352, 523, 253). This practice will reinforce how the digit values change as their position changes.

The intervention activity presented in Figure 4 shows how to teach the second principle, positions of digits. For this activity, the teacher can write the standard form of a

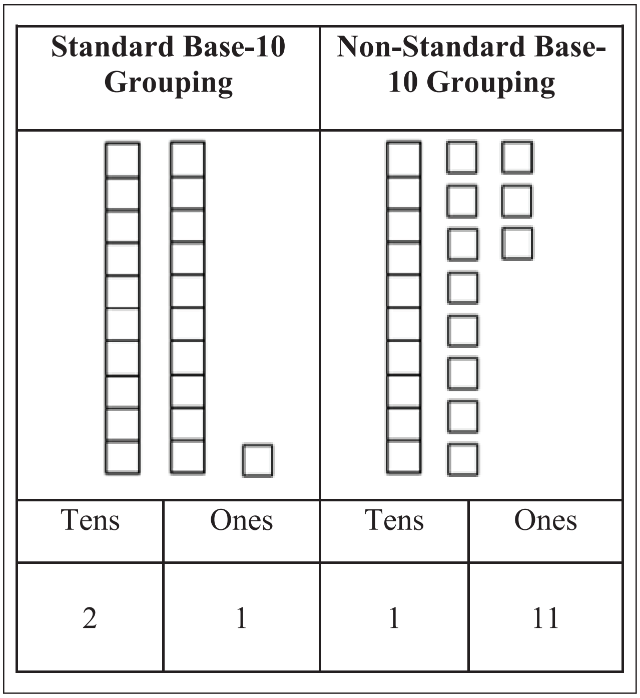


Figure 3. Standard and non-standard base-10 groupings.
Note. The standard base-10 grouping represents the number with the least base-10 blocks, whereas the non-standard base-10 grouping represents the number in different, yet equivalent way.

number, such as 2,315, in a place value chart and have students represent the number with either base-10 blocks or a pictorial representation. The teacher then can demonstrate how to represent that number of base-10 blocks in the expanded form using explicit instruction. For example, the teacher can say, "The number written in the place value chart is 2,315. I am going to expand this number by writing out the value of each digit." The teacher should model how to determine the value of each digit and write those values beneath the corresponding blocks and then say, "Next, I will write a plus sign between each value to show that the number 2,315 is the sum of the values of those digits. Two thousand plus 300 plus 10 plus five is equal to 2,315." Alternatively, teachers can provide students with the expanded form and have them fill in the place value chart with the standard form and pictorial representation.

Patterns of Numbers

The next principle, patterns of numbers, introduces the concept that our place value system follows a multiplicative base-10 pattern that continues infinitely. That is, each successive place value is 10 times the value as the previous one (e.g., 222 is equal to 200 + 20 + 2), which may be difficult for students to understand if they have not been introduced to multiplication (Van de Walle et al., 2019). Understanding the *multiplicative pattern* of place value will help students with LD in mathematics begin to understand larger numbers and facilitate their learning as they advance into decimals. Proportional models, which maintain a physical base-10 proportion, are ideal for teaching the multiplicative pattern of place value (see Figure 5). Non-proportional models, such as place value chips, differ in that they do not physically maintain a base-10 proportion.

One way teachers can help students construct the concept of the multiplicative pattern is by displaying a number with the same digit in multiple place values (e.g., 333 or 555) and representing that number with both base-10 blocks and the expanded form. This will help students see that the value of each digit increases by multiples of 10. Teachers can begin by writing 333 in a place value chart (see Figure 6) and then have students build the number with proportional place value models and write the expanded form beneath the model (300 + 30 + 3). The teacher should discuss how the digits in 333 are the same but have different values because of their positions. Each three increases in value by a power of 10 as you move to the left. The expression [(3x100) +(3x10) +(3x1)] can be utilized to show how the values of the digits are determined. Placing the expression beneath the model and the expanded form will help students make connections between the multiple representations displayed.

Composing and Decomposing in Flexible Ways

The fourth principle of learning place value is the ability to *compose* and *decompose* numbers in flexible ways, which is a prerequisite for understanding regrouping with multi-digit operations. Students with LD in mathematics tend to make more regrouping errors with multi-digit operations than their typically achieving peers, which could stem from their inability to flexibly compose and decompose numbers (Lambert & Moeller, 2019). Therefore, it is important for students to practice representing numbers with a non-standard base-10 approach (e.g., representing 243 as one 100, 14 tens, and three ones) before beginning multi-digit operations.

When students are able to compose and decompose flexibly, they can show that a number can be represented in different ways. For example, the number 347 can be composed of three 100s, four 10s, and 7 ones or three 100s, three 10s, and 17 ones. However, the equivalency between the two representations may be difficult for students to understand. Teachers can reinforce this concept with groupable base-10 models that can be taken apart and put together, such as connecting cubes or interlocking base-10 blocks (Van de Walle et al., 2019).

Place Value Chart						
Place value	Hundred Thousands	Ten Thousands	One Thousands	Hundreds	Tens	Ones
Pictorial representation						
Standard form			2,	3	1	5
Expanded form			2,000 +	300 +	10 +	5

Figure 4. Intervention activity: Positions of digits.
Note. The place value chart allows students to see multiple representations of the same number in a single graphic organizer.

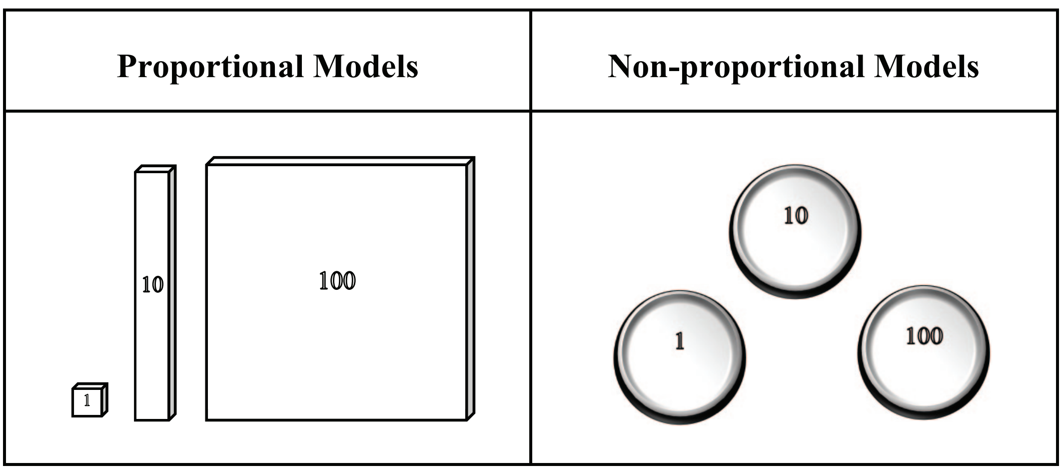


Figure 5. Proportional versus non-proportional place value models.
Note. Proportional models maintain the proportion of their value (e.g., the model for a ten is one-tenth the size of the hundred model). Non-proportional models, such as place value disks, do not maintain a base-10 proportion.

The intervention activity in Figure 7, *equivalent representations*, demonstrates how to teach students to decompose numbers in flexible ways. To teach this concept, teachers can draw a table with three columns and label the headings of the columns hundreds, tens, and ones. Then, display the number 204 using the least number of groupable base-10 blocks. The teacher should write a two in the 100s column, a zero in the 10s column, and a four in the ones column. Next, the teacher can demonstrate how to represent 204 in a different but equivalent way by decomposing the two 100s blocks into one 100 and ten 10s. The teacher can explain the processes above by saying, "I am going to represent the number 204 in a different but equivalent way. To do this, I will decompose one of the hundreds into ten 10s." Now, the teacher should have one 100, ten 10s, and 4 ones displayed. Tell the students, "Notice that although I decomposed a 100 into 10s, I still have the same number of units. The value of this number has not changed." Then, underneath 204, the teacher can write a 1 in the 100s place, a 10 in the 10s place, and a 4 in the ones place. Teachers should continue guiding students through decomposing the same number in different ways using the steps above. Remember to use consistent and accurate vocabulary by asking students to

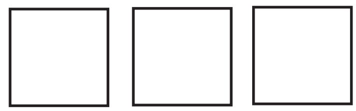
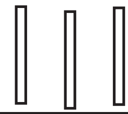
Place Value Chart			
Place value	Hundreds	Tens	Ones
Pictorial representation			
Standard form	3	3	3
Expanded form	300 +	30 +	3
Equation	(3 x 100) +	(3 x 10) +	(3 x 1)

Figure 6. Intervention activity: The multiplicative pattern of place value.
Note. Using a number that has the same digits in this activity allows for students to understand how each digit will increase in value by a multiple of 10, reinforcing the multiplicative pattern that is inherent to the base-10 place value number system.

compose or *decompose* rather than *make* or *break apart* (Townsend et al., 2012).

Conceptualization of Larger Numbers

Within many state standards, students are expected to read and write numbers up to 1,000 in multiple ways by second grade. By fourth grade, that number typically increases to the millions. However, students tend to have a difficult time conceptualizing numbers greater than 100 (Byrge et al., 2014). To help students understand the fifth principle, conceptualization of larger numbers, teachers can (a) reinforce the multiplicative structure of place value, (b) practice previously taught place value principles with larger numbers, (c) explicitly teach students when to add a place value, and (d) utilize real world examples.

One contributing factor to the difficulty of understanding the value of larger numbers is that students do not have enough practice with concrete and semi-concrete manipulatives that represent larger values. As students begin to practice modeling larger numbers with base-10 blocks, they are unable to represent larger place values with proportional models (e.g., representing the millions place with base-10 blocks). Since the larger place values won't be physically represented, the process may become too abstract. Teachers can continue to reinforce the notion that as numbers get larger, each place value increases in magnitude by 10, reinforcing the multiplicative pattern of place value. Teachers can use a "10 of this makes one of those" demonstration to reinforce the concept of numbers getting proportionally larger. For this demonstration, use a unit base-10 model to represent the ones place. Discuss how 10 ones make a 10 and display the model for a 10. Next, show how ten 10s make a 100 and display the model for a 100. Continue this process until there are no longer base-10 blocks available to represent the next value. Ask, "How large would the next model be if we had one to represent 10 of these?" and "how large would the model after that be?"

However, as teachers introduce larger numbers, it should not be expected that generalization of previously learned place value concepts will transfer. Time should be spent teaching students how to apply the concepts of place value that were taught with smaller numbers to larger numbers, such as teaching students how to represent four and five digit numbers using the expanded form. In addition, students with LD in mathematics commonly encounter difficulties counting across decades, centuries, and millennia (e.g., 99 to 100; 999 to 1,000). Typically, students will feel comfortable trading 10 ones for a 10, however, having ten 10s will result in the need for another trade and the addition of a new place value, the 100s place. Therefore, teachers should explicitly teach students when to add a new place value. Teachers can teach this concept by having students act out the trading of base-10 blocks from 99 to 100. Teachers can begin with a display of 90 using nine 10s and prompt students to continue counting by ones. When students get to 99, the teacher may ask, "What comes next?" Students will need to trade the 9 ones for a 10 and then the

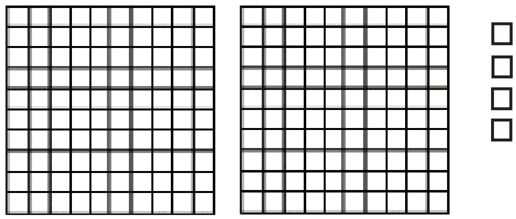
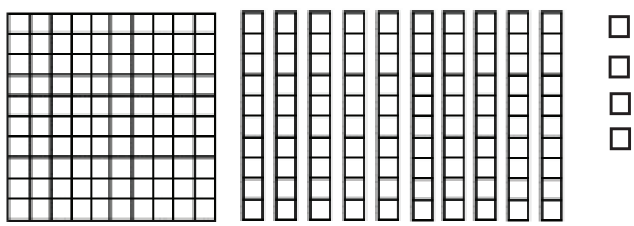
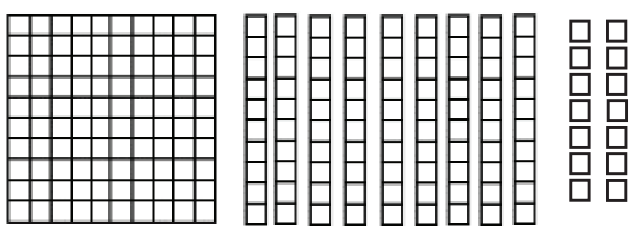
Base-10 Blocks	Hundreds	Tens	Ones
	2	0	4
	1	10	4
	1	9	14

Figure 7. Intervention activity: Equivalent representations.
Note. The number 204 can be composed and decomposed in different ways. It can be represented using a standard base-10 approach or using an equivalent non-standard base-10 approach. The ability to flexibly compose and decompose numbers lays the foundation for multi-digit operations, such as subtraction, that requires regrouping.

ten 10s for a 100. This activity can be repeated for the trading of base-10 blocks from 999 to 1,000. Teachers can reinforce the patterns of place value by reviewing that a group of 10 in any position adds one digit to the left of a number.

Finally, teachers should provide real-world connections to large numbers (e.g., 1,000, 1,000,000, etc.), to help students relate to numbers that may seem too abstract. To help students understand big numbers, teachers can place students in small groups to answer questions such as, “How many minutes do you spend in school each day?” or “How old is a 10 year old in days?” Teachers can expose students to pictures with large numbers such as a baseball field filled to capacity. Children’s books that have been written to help students understand the difficult concept of extremely large numbers and beyond, can be read aloud to students. Two examples of such books are *If You Made a Million* and *How Much is a Million?* by D.M. Shwartz.

Conclusion

Understanding the principles of place value outlined in this article are foundational for deeper mathematics learning: understanding sets of 10, positions of digits in numbers, patterns in numbers, composing and decomposing numbers in flexible ways, and the conceptualization of larger numbers. Place value is not a skill expected to be mastered in a single day, unit, or year. Thus, the progression through these principles should begin very early in schooling and continue for several academic years. Due to the unique challenges faced by students with LD in mathematics throughout their academic years, explicit and systematic instruction of place value at an early age and purposeful recursive review can be helpful in remediating future difficulties, such as those experienced with whole number multi-digit operations and decimal understanding.

Author's Note

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