String Tension in Pendulum and Circular Motions: Forgotten Contributions of Huygens in Today Teaching And Learning

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Abstract
Many authors suggest that is necessary to include the most important episodes from physics history in teaching and learning in order to give students some ideas about the nature of science. The pendulum-related aspects are considered as very adequate for that purpose. Unfortunately, when some physics textbook authors present historical information, it is frequently incomplete and even false. In the first part of this article, we present and comment important results of Huygens' analysis of two problems related to pendular and circular motions. These results were published in his work about centrifugal force "*De vi centrifuga*". In the second part, we consider the destiny and different treatments of his results in today physics teaching and learning, noting that Huygens' contribution is never mentioned. These treatments go from (1) testing students (and teachers') ideas about tension force in pendular motion to (2) mathematical derivation of Huygens' results or posing his problems to students. Finally, we briefly suggest how original results might be used to give students an idea about development of mathematical tools in physics and to explore students' creative experimental thinking.
Key words: History of physics, tension in pendulum, conceptual understanding of pendulum physics, measurement by force sensor.

INTRODUCTION

History of physics and science and their use in teaching were once quite polemic and controversial (Hanson, 1955; Brush, 1974), being an issue calling for critical considerations (Russel, 1981; Jung, 1994). Especial attention was given to teachers' knowledge and attitudes (King, 1991). Nevertheless, many authors agreed and suggested that was necessary and useful to include the most important episodes from physics history into teaching and learning in order to give students some initial ideas about the nature of science. With that aim on mind, that theme was treated in international meetings and the results were published in proceedings (Brush & King, 1972; Bevilcqua & Kennedy, 1983; Finley et all., 1995). The articles about pedagogical usage of physics and science history appeared also in Spanish (Navaro, 1983; Sánchez Ron, 1988; Gil Pérez, 1993)

A decisive positive impulse to that movement was given by Professor Matthews by his ground-breaking book "*Science Teaching: The Role of History and Philosophy of Science*" (Matthews, 1994) and its revised and expanded edition (Matthews, 2014). He eloquently describes how history and philosophy of science contributes to the resolution of persistent theoretical, curricular, and pedagogical issues in science education, giving arguments why it is essential for science teachers to know and appreciate the history and philosophy of the subject they teach and how this knowledge can enrich science instruction and enthuse students in the subject. Through many historical examples, Matthews reveals to students, teachers, and researchers the foundations of scientific knowledge and its connection to philosophy, metaphysics, mathematics, and broader social influences.

Considering the important role physics textbooks play in shaping teaching and learning, a considerable research attention was also paid to treatments the textbooks give to historical materials. Leite (2002) developed a theoretically grounded checklist for analyzing historical content of five physics textbooks. Her conclusion was that those textbooks were hardly able to give students an adequate image of science and scientists' work. Other researchers came to similar conclusions (Thao-Do & Yuenyong, 2013; Wei, Li & Chen, 2013). It was also found that physics textbook presentations of some historical episodes were incomplete (Rodriguez & Nias, 2004) or even false (Slisko & Hadzibegovic, 2017; Leone, Robotti & Verna, 2018).

Pendulum As Important Curriculum Element In Science And Physics Education

Far for being just a device of pure physics, the pendulum is fascinating because of its intriguing history and the range of its technical applications that span many fields, from engineering to entertainment (Baker & Blackburn, 2005; Pook, 2011). As such, study of different aspects of pendulum physics and its history can be an ideal context for cultivating science literacy (Matthews, 2000).

The study and utilization of different aspects and features of pendulum motion caused an important scientific, cultural, horological, philosophical, and educational impact. The International Pendulum Project (IPP) (Matthews, Gauld & Stinner, 2005) was a collaborative research effort whose objectives were to examine this impact, and to demonstrate how historical studies of pendulum motion might help teachers improve science education by developing enriched curricular material, and by showing important connections between pendulum studies and other parts of the school curricula (mathematics, social studies and music).

Accepting established validity of pendulum as a topic in physics curricula (Matthews 2000), Galili and Sela (2004) studied the place of pendulum motion in the physics curriculum of the high school in Israel. The data were gathered through presenting results of the nationwide matriculation examination in its units of Mechanics and Research Laboratory for the Advanced Placement program (several thousands of students). Although the assessment questions and problems mainly tested students' performance, and less their understanding of the subject, the study, by discussing these problems and questions, allowed a perception of the extent to which the pendulum topic is addressed in High School physics instruction. Their results could be useful in a discussion on the nature of the requirements and values encouraged by the particular educational system and the strengths and weaknesses of the adopted educational policies.

HUYGENS' PROPOSITIONS ON PENDULUM AND CIRCULAR MOTIONS

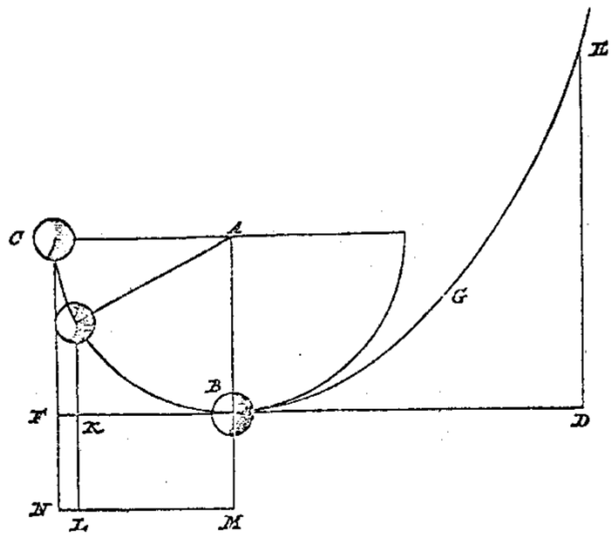
Dutch mathematician, physicist, and astronomer, Christiaan Huygens (Figure 1), was one of the most important protagonists of the Scientific Revolution. He discovered the rings of Saturn, the moon Titan, invented the pendulum clock, measured gravitational acceleration and carried out groundbreaking studies on optics and centrifugal forces (Yoder, 2004; Andriesse, 2005; Bell, 2012).

Figure 1. *Christian Huygens (1629 – 1695)*

Our focus in this article is Huygens' treatise on centrifugal force (*De vi centrifuga*). Although it was written in 1659, it was published eight years after his death in 1703. In the treatise, Huygens resolved surprisingly well two problems, both related to quantification of tension forces (https://www.princeton.edu/~hos/mike/texts/huygens/centriforce/huyforce.htm). Here come two propositions from that treatise:

Proposition XVI

> "If a simple pendulum is set in motion with the maximum lateral oscillation, i.e. if it descends through the whole quadrant of the circle, when it reaches the lowest point of the circumference it will pull its string with a force three times as great as if it were simply suspended from it.

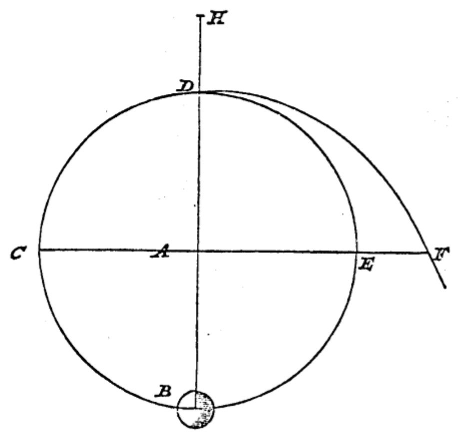


If ball C attached to A by string AC descends through a quadrant of circumference CB, when it arrives at B it will pull string AB with a force three times as great as if it were hung by its weight alone. For, first, the velocity at which it would continue to move along the straight-line BD, if the string were released at B, is the same as that which it would have at point F, if it were to fall perpendicularly through CF. But, in that case, it would acquire just enough speed [*tantem celeritatem*] to traverse twice the distance of this CF with a uniform motion at the same time in which it fell from C to F. Therefore, at B the ball has the tendency to traverse the line BD, which is twice AB, at the same time in which it would fall from A to B (not considering, that is, the force of its gravity, by which it would in the meantime also have descended and described some parabola). Let BGE be a parabola, of which B is the vertex and AB one-half the latus rectum. Since, therefore, the recession of ball B from circumference BC while it is moved along BD with a uniform motion may, at the beginning, close to point B, be taken as the same as the recessions from parabola BGE, it holds that the centrifugal force that the ball has at B from revolution alone is a tendency to recede from center A, or from circumference BC, with a motion accelerated according to the numbers 1, 3, 5, 7, etc., and consequently is similar to that tendency by which the body tends to fall, which we call gravity. But this tendency in ball B is as much as would be in a body equal to it that would traverse with accelerated motion the distance DE at the same time in which it would traverse distance BD with a uniform motion, i.e. in a time equal to that in which the ball would fall from A to B with a likewise accelerated motion. Therefore, because DE is twice BA, the centrifugal tendency of the ball at B is twice its gravity. But another tendency is added by gravity here, by which ball B (at the same time in which it would fall from A to B) now also tends to traverse the same amount of distance with a naturally accelerated motion downward. Therefore, with both tendencies taken together, it tends to traverse, with a motion accelerated according to 1, 3, 5, 7, a distance equal to both DE and AB, i.e. three times this AB. Wherefore, the force with which the body descending from C pulls at point B is three times that which arises from the simple weight of ball B hanging freely. *Which also agrees exactly with experience*."

It can be seen that Huygens has shown, using proportional and geometrical reasoning of Galilean type, that the centrifugal force on a pendulum string, after running from horizontal to vertical position, is two times bigger than the pendulum weight. In that way, the tension force of the string in the vertical position is three times bigger than the weight of the pendulum bob at rest. Huygens did not provide details of how that theoretical result was verified experimentally.

Proposition XVII

> A globe hung on a string from the center of a circle perpendicular to the horizon cannot be revolved around the circumference of this circle unless the string can support six times the weight hung [on it].



> Let BCDE be a circle standing perpendicularly to the horizon, and from the center A let ball B be suspended. I say that, in order that this ball be able to revolve along the circumference BCDE, it is necessary that the string be able to support six times the suspended weight B. For, in order that the string remain extended when the ball passes through point D and descends through arc DE, the velocity of the ball there must be such that, if it were released, it would describe parabola DF of which AD is one-half the latus rectum. Whence, it must have as great [a velocity] as a body falling from height HD, one-half of this DA, would have at D. Therefore, in order that [the ball] ascending from B through semicircle BCD have the said velocity left over at D, the speed at B must be so much as to enable it to ascend perpendicularly to point H. For, having this speed at B, by whatever path it reaches height D, it will always retain so much speed as to enable it further to ascend perpendicularly, or by any other path, to H; that is, as much speed will be left to it as it would acquire falling from height HD, which we said it needed at point D.

This result, in modern view, means that the tension of the string with the ball in position B is six time bigger than the tension of the string when the ball passes through position D.

Over many years, the concept of centrifugal force, crucial part of Huygens derivations, was discussed extensively in physics education journals (Hagenow, 1935;

Lenzen, 1939; Blackwood, 1944; Bauman, 1980) and even in a mathematical one (Jagger & Lord, 1995). It is very strange that the authors never mentioned Huygens approach to this controversial and, in fact, today forbidden concept!

Derivation And Measurement Of String Tension Of A Pendulum

Today the derivation of the value of tension for pendulum string, for the case considered by Huygens, is carried out by using the concept of net (centripetal) force. In the vertical position of the pendulum with string length L, the net force (the difference between the tension T and the bob's weight W = mg) is, according the second Newton law, equal to the product of the mass m and centripetal acceleration:

$$T - W = \frac{mv^2}{L}$$

$$T = mg + \frac{mv^2}{L}$$

Using the ley of conservation of mechanical energy, where "zero" level of potential energy correspond to the lowest position of the bob, one has:

$$mgL = \frac{mv^2}{2},$$

where L the length of the pendulum and the radio of the circular trajectory and v is instantaneous speed of the bob when the pendulum is in the vertical position, after moving one quadrant. Interchanging the positions of the terms L y 2, one gets:

$$2mg = \frac{mv^2}{L}.$$

Centripetal force is two times bigger than the weight of the pendulum bob.

Inserting this equality in the second equation, the expression for the tension becomes:

$$T = 3\,mg.$$

The tension of the string is three times bigger than the weight of the pendulum bob. Contemporary mathematical tools used in physics aren't only advantage when compared with those in Huygens' time. Measurement tools had a similar progress, and it is possible to verify experimentally the above result tension force, something that Huygens claimed but didn't provide details. We made measurement of the tension force, for the pendulum motion described by Huygens, using a commercial force sensor.

The pendulum, shown in the Figure 2, was made of a wooden sphere with a diameter of 5.5 cm (having the weight of 0.427±0.003 N and a spring whose length is 0.46 m. The data were recording by using a force sensor, model CI-6537, connected to interphase Science Workshop 750 of Pasco. The frequency of capture was 1,000 Hz.

In the initial position, the string of the pendulum is hold in the horizontal position. Consequently, the tension of the spring is zero.

Figure 2. *The pendulum and the launching system in the horizontal position.*

The sphere was hold in its adequate position by a vertical string (Figure 3).

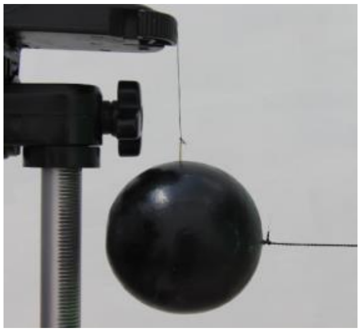


Figure 3*. The launching system.*

The sphere is set into motion by burning that vertical string. In that way, the launching process is well controlled. It means that initial launching effects on string tension were eliminated and could not influence posterior tension measurements. The Figure 3 show the graphics of four pendulum launching. The behavior of the pendulum was acceptable, and it was possible to determine the value of tension and its error. The average tension value of four launching was T=1.29±0.01 N. This result is in a good agreement with theoretical prediction of 1.28 ±0.01 N.

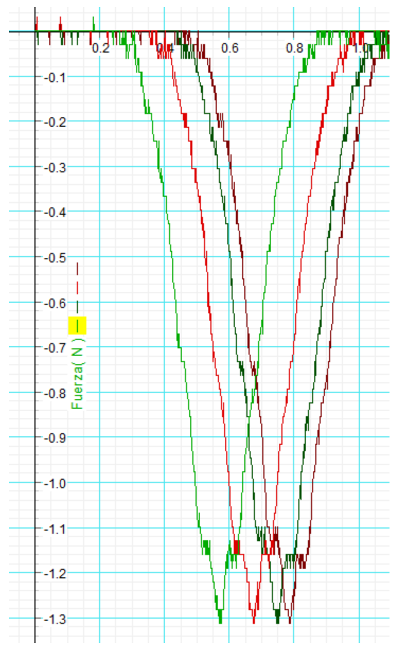


Figure 4. *The changes of tension force in four launching.*

Students' And Prospective Teachers' Conceptual Difficulties And A Possible Improvement

Knowing how difficult for first class scientists was to understand physics of pendulum and circular motion, it is not surprising that today students face conceptual difficulties when facing that type of motions.

The first formal research of students' conceptual difficulties related to pendulum string tension was carried out by Viennot (1985). She designed a "pendulum test" and gave it to 60 students in their first year in a scientific curriculum at the University of Paris, after a two-month introductory mechanics' course. The third question asked students to draw the net force that acts on the pendulum ball while moving through the vertical position (Figure 5).

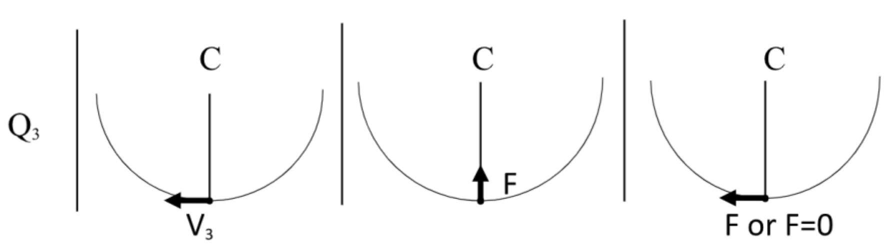


Figure 5. *The diagram for the third question in the pendulum test (with information about velocity vector) and more frequent students' conceptions.*

Almost 60 % of students were able to indicate that net force is directed toward C. Nevertheless, 30 % of students thought that the net force acted in the direction of motion.

That important students' alternative conception of "motion force" became widely known that and it is sometimes described without citing the research of Viennot. For example, Ryder (2001) wrote: Take a group of first-year physics undergraduates. Present them with a diagram of a pendulum passing through equilibrium position (Figure 6). Then ask them to draw an arrow showing the direction of the overall force on the bob as it passes through the position drawn. What would you expect the different responses from the students to be?

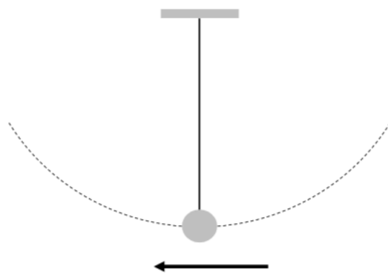


Figure 6. Diagram used by Ryder.

In answers to the pendulum question, students often draw an arrow in the direction of motion, rather than towards the hanging point. Other students' alternative conceptions related to the string tension came out, too. Chudková and Musilová (2000) applied to 50 secondary students two tests, one related to conical pendulum and the other to simple pendulum. In the second test one task was:

"Find the magnitude of the tension force of the string at the equilibrium position."

Not a single student gave a correct answer. Almost 70 % of the students meant that the tension of the string had the magnitude mg.

Gaisman (2006) found, in a research carried out with 24 engineering students who were taking a differential equations course, that students had various conceptual and procedural difficulties related to pendulum physics. One group had an idea that gravity force acts in the direction of motion and the other group didn't consider the spring tension. Those who had correct ideas about gravity and tension forces faced difficulties while trying to decompose them.

It is important to know that even some textbook authors make errors when dealing with forces acting on pendulum in motion (Santos-Benito & Gras-Marti, 2005).

Simple pendulum test, designed by Czudkova and Musilova (2000), was administered to the cohort of 29 preservice physics teachers undergoing the postgraduate teacher certification program at one of the teacher education institutions in the UK (Dandare, 2018).

The results were unsatisfactory: only 3 respondents out of 29 answered scientifically accurately the whole questionnaire related to simple pendulum. In addition, 10 respondents claimed that the weight of the bob and the tension in the string would balance each other resulting in no net force and, consequently, no acceleration of the bob while passing through equilibrium (vertical) position.

One possibility to improve prospective teachers' conceptual knowledge is to practice with them a representational approach (Waldrip & Rusdiana, 2012).

This pedagogical intervention was implemented with 24 prospective physics teacher students at State University of Malang, enrolling Selected Topic of School Physics course. Using mixed-methods design, this study concluded that (1) students' mean score of conceptual test increased significantly; (2) students became able to use operational definition of acceleration to analyze multi-flash motion diagram, and (3) students remediated their misconceptions of acceleration. However, few students stuck in their misconception that acceleration of a shot-up object is decreasing with its elevation, and the tension in the rope of a swinging pendulum is equal to the weight of its bob.

The question related to tension in the pendulum string while moving through equilibrium position was:

> "X and Z mark the highest and Y the lowest positions of a 50.0 kg bob swinging as illustrated in the diagram. What is the tension in the rope at point Y if we neglect the mass of the rope and air resistance?
>
> (A) 200 N (B) 300 N (C) 500 N (D) 625 N (E) 700 N."
>
> The diagram presented is given in the Figure 7.

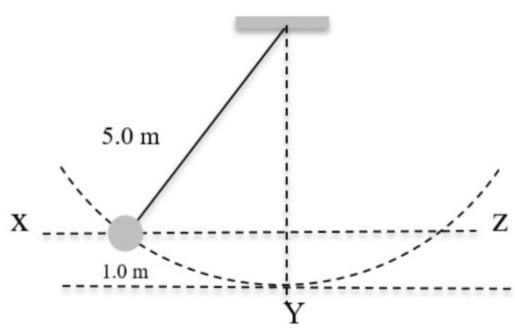


Figure 7. *The diagram that was part of the question related to the string tension.*

The results of the pretest were: B (8 %); C (67 %) and "No answer" (25 %). It can be seen that not a single prospective teacher gave a correct answer (E)!

The performance improved greatly at the posttest: B (4 %); C (34%); E (54%) and "No answer" (8 %).

Huygens' Problems In Textbooks And Manuals For Exams

Huygens' first problem is today used in study guides, without mentioning that Huygens was the first to formulate and solve it.

Here comes one example:

"**Question 4**

A simple pendulum, initially with the string taut and horizontal, is released from the rest… Find the expression for the tension T in the spring when the pendulum reaches its vertical position. Hence, deduce if T depends on the length of the spring.

Answer

Conservation of (mechanical) energy

KE at vertical position = PE at horizontal position

$$\frac{1}{2}mv^2 = mgl \quad \text{(in the usual notations)}$$

$$v^2 = 2gl.$$

Force = mass x acceleration at vertical position

$$T - mg = \frac{mv^2}{l} = \frac{m(2gl)}{l} = 2mg$$

$$T = 3\ mg.$$

T does not depend on length l of string." (Toh, 2016, p. 102).

A more challenging is the tension problem when the initial position of the pendulum is not horizontal. Such is an objective question with four offered answers:

"When the bob of a simple pendulum passes through the position of equilibrium, the string is subjected to a tension equal to twice the weight of the bob. Through what maximum angle from the vertical can the pendulum be deflected?

(a) 30^0 (b) 45^0 (c) 60^0 (d) 75^0" (Kumar & Juneja, 2006, p. 194)

Second Huygens' tension problem is also used in physics textbooks without giving deserved credit. Here come two examples:

"A ball is tied to a cord and set-in rotation in a vertical circle. Prove that the tension in the cord at the lowest point exceeds that at the highest point by six times the weight of the ball." (Sears, Zemansky & Young, 1963, Problem 6-33, p. 140).

"A ball whirls around in a vertical circle at the end of a string. The other end of the string is fixed at the center of the circle. Assuming the total energy of the ball–Earth system remains constant, show that the tension in the string at the bottom is greater than the tension at the top by six times the weight of the ball." (Serway & Jewett, 2008, Problem 63, p. 225).

CONCLUSION AND RECOMMENDATIONS FOR TEACHING

Although Huygens gave important contributions to understanding physics of pendulum and circular motions, he is not mentioned in physics textbooks. How his contributions might be used with today students?

One possibility is to give students reading task to try to understand Huygens' proportional and geometrical reasonings and to compare them with today's mathematical tools. In that way, they will sense better the progress physics has made since Huygens' time.

The other task might be to ask them how Huygens might have verified experimentally his claim that the tension force in the vertical position is three times bigger than the weight of the pendulum bob. Such type of task are creative challenges rarely given to students. On many occasions we, as physics teachers, were surprised how creative our students could be!

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