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Abstract

Education reforms in the United States and abroad have increased efforts to improve student interest and capacity in STEM (science, technology, engineering, and mathematics). Despite these attempts, students still have little opportunity to engage in STEM learning in K-12 education. This qualitative case study was designed to investigate how incorporating STEM into teacher education can promote STEM teaching as well as enhance math instruction. The study took place in the fourth year of an undergraduate teacher education program spanning a secondary math and science Methods course and student teaching. Guided by the framework of sensemaking, individual interviews, teaching artifacts, and written reflections for four teachers were analyzed to identify moments of dissonance that pushed participants to reach new understandings about the learning and teaching of math. Findings indicated that learning to teach math through the lens of STEM shifted pre-service teachers" instructional emphasis in two ways: 1) figuring out math vs. learning about math; and 2) teaching math through authentic STEM contexts as opposed to focusing purely on mathematics. However, experiences in student teaching can either enhance or stifle these gains. Findings suggest the role of teacher education in promoting STEM education by shifting prospective teachers" mindsets about mathematics and teaching.

Introduction

Given a global increase in complex economic, health, and environmental challenges, attention has turned to the improvement of science, technology, engineering, and mathematics (STEM) education in the United States (NSF, 2016; NAE & NRC, 2014; U.S. Congress Joint Economic Committee, 2012) and abroad (English, 2016; Marginson, Tytler, Freeman, & Roberts, 2013; Thomas & Watters, 2014). Yet research indicates student interest and motivation for STEM is in decline, particularly in the West (Thomas & Watters, 2014). The status of STEM education in the United States is particularly alarming. Despite three decades of reform efforts intended to improve STEM education (e.g. AAAS, 1989; Sanders, 2009), a recent national landscape study found typical K-12 math and science instruction rarely provides students with opportunities to engage in STEM learning, with the fewest interdisciplinary connections occurring in high school coursework (Banilower et al., 2018).

In the United States, the latest educational reforms in math, Common Core Standards for Mathematics (CCSM) (NGA & CCSSO, 2010), and science, the Next Generation Science Standards (NGSS) (NGSS Lead States, 2013), "advocate for purposefully integrating STEM by providing deeper connections among the STEM domains" (Kelley & Knowles, 2016, p. 2). CCSM and NGSS elevate the practices of mathematicians, scientists, and engineers as learning outcomes in tandem with content knowledge. The practices represent the skills, mindsets, and social norms for interaction and developing knowledge within the discipline (Cobb & Jackson, 2011). With a practice-based approach, the instructional goals shift to students *figuring out* rather than *learning about* the content (Duschl & Grandy, 2013). The disciplinary practices include intentional synergies, emphasizing shared mindsets and approaches in mathematics, science, and engineering (Stage, Asturias, Cheuk, Daro, & Hampton, 2013). Thus, the disciplinary practices provide a lever to not only transform math and science education, but also to advance STEM education (Kelley & Knowles, 2016; Stohlmann, Moore, & Roehrig, 2012).

However, research on the implementation of practice-based teaching tends to focus on each discipline in isolation, and typically focuses on in-service rather than pre-service teachers (Allen & Penuel, 2015; Loper, McNeill, González-Howard, & Marco-Bujosa, 2019; Berland et al., 2016; Marco-Bujosa, McNeill, González-Howard, & Loper, 2017; McNeill, Marco-Bujosa, González-Howard, & Loper, 2018; Selling, 2016). Moreover, research indicates that the shift to practice-based instruction is difficult for teachers, particularly for mathematics (Abel, Search, & Salinas, 2020; Selling, 2016). Specifically, teacher beliefs about the nature of mathematics (Holm & Kajander, 2012; Nolan, 2012; Selling, 2016) and prior experiences learning math, generally through didactic pedagogies, have a greater influence on instruction than teacher education (Bolden, Harries, & Newton, 2010; Nolan, 2012). More research is needed about how the disciplinary practices can promote STEM education (Kelley & Knowles, 2016) while enhancing teaching and learning within each discipline. The present study was designed to address this gap in the literature. This study used the lens of sensemaking to examine how prospective secondary teachers" beliefs about teaching math were shaped by a STEM practice-based instructional framework introduced in a secondary science and math methods course. Specifically, this paper addressed the following research questions:

1. What learning experiences in the Methods course facilitated pre-service secondary math teacher sensemaking of the STEM practices?
2. How did these learning experiences shape pre-service teacher sensemaking about how to teach mathematics?
3. How did pre-service teachers incorporate STEM practices into their mathematics instruction as student teachers?

Related Research

Conceptualizing STEM Education

STEM has been vaguely defined in education (Breiner, Johnson, Harkness, & Koehler, 2012). There are competing conceptualizations about how STEM integration can be accomplished, underscoring tensions between the relative emphasis on disciplinary content, the contexts that frame learning, and disciplinary practices. For example, while Moore and colleagues (2014) argued STEM instruction "is based on connections between the subjects and real-world problems" (p. 38) to address one content area, Kelley and Knowles (2016) argued STEM instruction must integrate "STEM content of two or more STEM domains, bound by STEM practices within an authentic context for the purpose of connecting these subjects to enhance student learning" (p. 3). The present study integrates these perspectives: STEM teaching utilizes disciplinary practices within authentic STEM contexts to support student learning of one content area, mathematics.

Disciplinary practices provide an opportunity to productively integrate STEM disciplines in math instruction. The synergies between the math practices (MP) and science and engineering practices (SEP) in CCSM and NGSS were purposeful, including, for example, connections between mathematical thinking and reasoning (SEP5); explanation and argumentation (MP3 & SEP7); and modeling (MP4 & SEP2) (Stage et al., 2013) (see Figure 1).

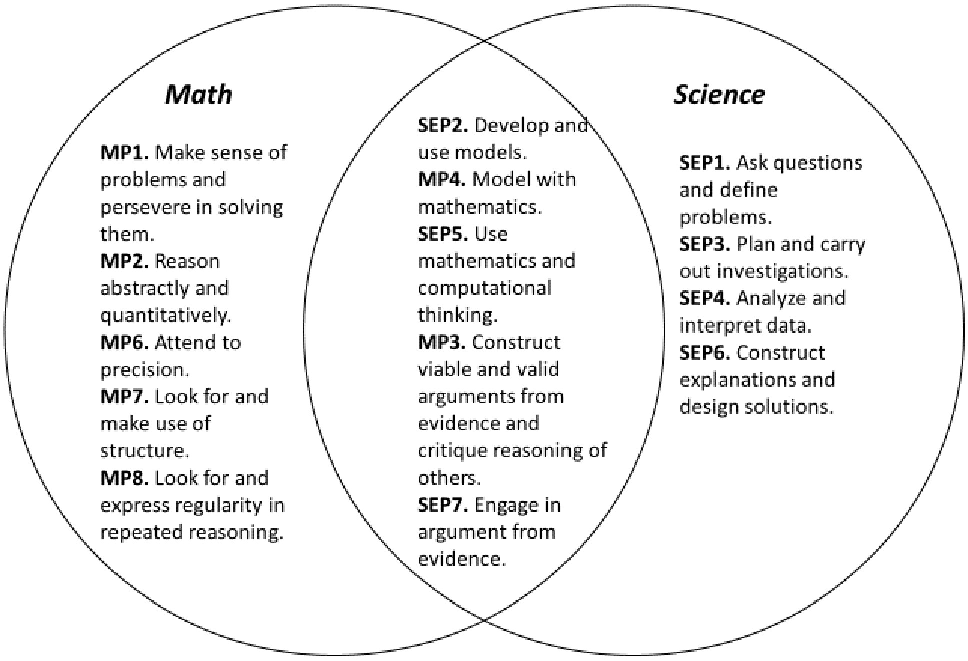

Figure 1. Science and Math Practices (adapted from Stage et al., 2013)

Research indicates practice-based instruction can enhance student learning of content by providing greater focus to the curriculum and developing key mathematical and scientific concepts and skills (Commission on Mathematics and Science Education, 2009; National Research Council, 2012). For mathematics, engaging students in the math practices has been found to promote deeper conceptual understanding (Boaler & Staples, 2008; Kazemi & Stipek, 2002) and improved self-efficacy (Boaler & Greeno, 2000; Cobb, Gresalfi, Hodge, & Hodge, 2009) as compared to traditional instruction. However, research examining the skills, beliefs, knowledge, and experiences necessary to effectively implement STEM instruction is limited (Frykholm & Glasson, 2005; Stohlmann et al., 2012), and little research has examined teachers" experiences teaching STEM through the disciplinary practices.

Math Teacher Education

Historically, mathematics teacher education has struggled with specifying the type of knowledge that is necessary to effectively teach math. A greater emphasis has been placed on prospective teachers taking high-level mathematics courses, largely based on the assumption that more content knowledge leads to better teaching (Aubrey, 2007). Despite many efforts, research has not linked mathematical expertise to student learning (e.g. Eisenberg, 1977; Fennema & Franke, 1992). Thus, research in math education has shifted to identify and operationalize the nature of mathematical knowledge necessary for teaching math (Grossman, Wilson, & Shulman 1989; Hill, Ball, & Schilling, 2008). For example, Hill, Ball, and Schilling (2008) distinguished between three forms of mathematical understandings for teaching: 1) specialized knowledge of mathematical ideas, explanations, and procedures; 2) common content knowledge or "knowledge that is used in the work of teaching in ways in common with how it is used in many other professions or occupations" (p. 377); and "knowledge at the mathematical horizon" (p. 378) which refers to a teacher‟s ability to make connections between different mathematical concepts and topics (Guberman & Gorev, 2015).

Research indicates that prospective teachers are introduced to specialized mathematical knowledge, often the canonized, pure knowledge of mathematics, in coursework. This knowledge is primarily conveyed through lecture (Baxter & Williams, 2010; Lobato, Clarke, Ellis, Lobato, & Ellis, 2005; Singleton, 2015). Thus, through taking advanced mathematics, prospective teachers are socialized into the discipline of mathematics (Nolan, 2012). But, essential domains of mathematical knowledge for teaching are missing from their content preparation due to the emphasis on specialized, pure mathematics, specifically *common knowledge* and *knowledge at the horizon* (Hill et al., 2008). Also, most teachers, including prospective teachers, have little experience engaging with the mathematical practices or applied mathematics (Guberman & Gorev, 2015; Selling, 2016) called for in the CCSM. Thus, mathematics teacher education inadvertently perpetuates the "apprenticeship of observation" (Lortie, 1975, p. 61) which reinforces more traditional, didactic approaches and prioritizing specialized and abstract mathematical knowledge. Therefore, transforming math instruction must focus on not only *how* math is taught; *what* is taught must also be considered. This study is designed to explore how an emphasis on the intersections between STEM disciplines (Stage et al, 2013) in a secondary math and science teaching methods course created productive dissonance for prospective teachers to support interdisciplinary STEM thinking by introducing alternatives for both the content and pedagogy employed in high school math classes.

Conceptual Framework: Sensemaking

This study is framed by the sociological concept of sensemaking to explore changes in pre-service teachers‟ beliefs about the teaching and learning of mathematics through STEM. Sensemaking describes how people "structure the unknown" (Waterman, 1990, p. 41). Applied to learning, new knowledge is actively constructed by individuals based on how they notice or select information, make sense of this information, and take action based upon their interpretations (Weick, 1995). Often, sensemaking occurs when individuals encounter situations that interrupts their beliefs, triggering cognitive dissonance (Weick 1995). These interruptions lead to teachers questioning their abilities and knowledge, thus generating opportunities for professional growth (Grossman & Stodolsky, 1995)

The sensemaking framework has been utilized in research in math and science education to elucidate teachers" interpretations of new educational reforms, including CCSM and NGSS. Collectively, findings indicate a shift in teacher beliefs and instruction is dependent upon their ability to contrast what is new with their prior experiences (e.g. Abel, Search, & Salinas, 2020; Allen & Penuel, 2015; Marco-Bujosa, McNeill, et al, 2017). Furthermore, sensemaking is contextually dependent (Weick, 1995). Thus, school values, instructional norms, and routines have been found to shape teacher interpretation of new pedagogies (Spillane, 1998). For example, Marco-Bujosa and colleagues (2017) found in-service science teachers who actively engaged in contrasting their prior instruction with the practice-based goals of a new curriculum implemented instruction that was more closely aligned with the scientific practice of argumentation. While comparably less research in math education has utilized sensemaking, Abel, Search, and Salinas (2020) developed a guide how educators interpret and scaffold student engagement in the mathematical practices based upon the sensemaking framework. Thus, sensemaking is an appropriate analytic tool to interpret math teacher uptake of STEM integration. In the present study, sensemaking was utilized as a lens to explore how these experiences shifted pre-service teachers" understandings of how to teach math.

Method

This article employed a holistic multiple-case study methodology (Creswell, 2007) to explore how pre-service secondary math teachers engaged in sensemaking about teaching math through the lens of STEM. Case studies enable the researcher to emphasize the real-world context surrounding the case, making it appropriate to examine the complexity of sensemaking (Yin, 2013). The researcher selected four cases bounded within one academic year to illustrate the ways in which STEM learning experiences disrupted pre-service math teacher beliefs and pedagogical decisions (Creswell, 2007). Study participants included one cohort of four secondary teachers in an undergraduate teacher education program. Data were collected during the 4th year of the program in which participants enrolled in the Methods course (Fall) for secondary math and science teaching and the full practicum for student teaching (Spring). The focus of this study is on analyzing representations of teacher sensemaking across these settings, notably a retrospective interview, instructional artifacts, and written reflections.

Research Context and Participants

The focal undergraduate teacher education program for this study was in a mid-sized private Catholic university in the mid-Atlantic region of the United States. All undergraduate and graduate students intending to teach math or science were required to take a course entitled "Methods of Teaching Mathematics and Science in Secondary School" (i.e. "Methods"). The math content courses were taken separately from education courses. Eleven math content courses were required for a major in secondary math education. A double major in mathematics and education required an additional four courses in upper-level mathematics and analysis, for a total of 15 content courses. The program also required ten education courses, which advanced the knowledge, skills, and dispositions necessary to effectively teach diverse students. As a component of the undergraduate courses, prospective teachers engaged in educational settings as after school tutors, instructors, facilitators, coaches, and other roles involving interaction with youth. However, student teaching in the senior year of the program was their first formal engagement with teaching math in a high school.

The Methods course met weekly in the Fall semester of the senior year. Methods was designed to be an active, thought-provoking instructional environment for prospective high school math and science teachers. Readings, classroom activities, and assignments were purposefully designed to disrupt pre-service teachers‟ preconceived notions about the teaching and learning of mathematics. Given that the course prepared both math and science teachers, it focused on the connections between the disciplines to guide instructional design. These connections were emphasized in the course in two ways: utilizing shared disciplinary *practices* for mathematics and science (Stage et al., 2013) and authentic *STEM contexts* to motivate and engage student learning (e.g. Moore, Glancy, Tank, Kersten, & Smith, 2014) to teach the *content* identified in national and state frameworks.

The course focused on the intersections between the math and science/engineering practices (Stage et al., 2013) (see Figure 1). Students engaged in these practices in a variety of ways, including readings, videos, reviewing sample lesson plans, and, most importantly, in class "student hat" activities. These "student hat" activities engaged pre-service teachers in the discipline from the perspective of the learner (Gibbons & Cobb, 2017). These activities intentionally disrupted pre-service teacher understanding of disciplinary concepts, instructional goals, and understanding of how students would react to instruction (Lowell & McNeill, 2020) and allowed teachers to practice new methods (Lampert et al., 2013). Examples of these activities included a "Card Sort," promoting engagement in explanation and argumentation (SEP7 & MP3), "the Mobius Strip," a hands-on investigation addressing the practices of asking questions and looking for patterns (SEP1/MP8), and "Puff Cars," an engineering design challenge. The lead researcher also served as the instructor of the course. Tables 1 and 2 offer a more detailed overview of the topics, goals, activities, and assignments in the Methods course. Table 3 offers a more detailed description of one student hat activity, the Card Sort, to illustrate how these activities were framed, structured, and debriefed in class time.

Participants began their student teaching assignments in the Fall, concurrently with Methods. Pre-service teachers were paired with an experienced high school math teacher who served as a mentor and instructional coach. They observed their cooperating teacher‟s instruction twice a week. Full-time student teaching took place during the Spring semester, from January to early May. As student teachers, they gradually assumed the full schedule of teaching responsibilities. A university supervisor observed five lessons over the semester to provide support and evaluate their teaching quality and professional growth.

This study focused on the 4 undergraduate mathematics education majors enrolled in the Methods course. All consented to participate in this study. (A total of 7 students enrolled in the course, including 3 graduate students who were not included in this study.) All four study participants identified as white females. All taught high school math for their student teaching placement. The researcher was the university supervisor for 3 participants (with the exception of Grace) (see Table 4 for more information each participant and their student teaching placements.)

Table 1. Methods Course Overview Classes 1-7

Class/Topics	Guiding Questions	Activities	Select Readings	Assignments
Class 1: Making student thinking visible	What does it mean to know math/science? How do you know what your students know?	1. Icebreaker: Drawing your classroom. 2. Instructional Strategies Brainstorm	How do my students think? Diagnosing student thinking	
Class 2: Math and Science Standards	How can math/science teaching better address the needs of the learner while covering essential content?	Activity: Math Classroom Video Analysis	1. How People Learn (Ch 2) 2. Finding overlap in math, ELA, and science standards (Stage et al., 2013)	Teaching Statement
Class 3: Instructional Design	How can I teach for understanding and transfer/application of knowledge?	1. Student Hat Activity: Mini-Lessons 2. Collaboration: Unit Planning	1. How People Learn (Ch 3) 2. Making mathematical practices explicit (Selling, 2016)	Mini-lesson #1
Class 4: Instructional Design	How can I teach for understanding and transfer/application of knowledge?	1. Video: Critique a Math Lesson 2. Critique your Mini-Lesson	1. From Common Core Standards to Curriculum: Five Big Ideas 2. Successfully teaching math (Little, 2003)	Mini-lesson #1 Reflection
Class 5: Reasoning Abstractly and Quantitatively	How can I design math/science instruction within authentic STEM contexts? How can I engage students in reasoning?	1. Video: Identify Math in a Science Lesson 2. "Student Hat" Activity: Walking Sticks, Graphing	How People Learn (Ch 4, How Children Learn; Ch 5, Mind Brain)	Teaching Toolkit #1
Class 6: Asking Questions	How can I design math/science instruction within authentic STEM	1. "Student Hat" Activity: Mobius Strip	1. How People Learn (Ch 6) 2. Discrepant events	Design One Lesson Plan

	contexts? How can I engage students in questioning?	2. Collaboration: Unit Planning	(Wright & Govindarajan, 1995, p. 24-28)	
Class 7: Explanation and Argumentation	How can I design math/science instruction within authentic STEM contexts? How can I engage students in explanation and argumentation?	1. "Student Hat" Activity: Fossil Card Sort 2. Collaboration: Unit Planning	1. For ELLs: Vocabulary beyond the definitions (Roberts et al., 2013) 2. Bridging the language barrier in mathematics (Winsor, 2007)	Teaching Toolkit #2

Table 2. Methods Course Overview Classes 8-14

Class/Topics	Guiding Questions	Activities	Select Readings	Assignments
Class 7: Modeling	How can I design math/science instruction within authentic STEM contexts? How can I engage students in modeling?	"Student Hat" Activity: Modeling Tanker Implosion	1. NCTM Mathematical Practices to Actions Toolkit 2. Scaffolding student participation in mathematical practices (Moschkovich, 2015)	
Class 8: Assessment	How can I design assessments that provide useful feedback to students about their learning and to myself about my teaching?	1. Activity: Analyze Standardized Assessment Items 2. Collaboration: Design Assessment Items	1. Importance of everyday assessment (Black, 2003, p. 1-11) 2. Developing assessment items: How-to guide (Henriques et al., 2006, p. 15-30)	Field Reflections #1 & #2 due
Class 9: Assessment	How can I design authentic performance assessments for disciplinary practices?	Video: Evaluating student participation in the practices in a math lesson	1. Project 2061, Aligning Assessment with Learning Goals 2. Chapter 4: Rubrics	Teaching Toolkit #3

			and scoring guides (Enger & Yager, 2001, p. 62-88)	
Class 10: Problem-Based Learning	How does problem-based learning support student learning of key concepts and skills?	Video: Evaluating student participation in the practices in a STEM project	1. Problem-based learning (Hmelo-Silva, 2004) 2. Geometry and PBL (Schettino, 2012)	Teaching Toolkit #4
Class 11: Engineering	How does engineering design integrate STEM in the classroom?	"Student Hat" Activity: Puff Cars	NGSS and the landscape of engineering (Moore, et al., 2017)	Field Reflections #3 & #4 due
Class 12: Engineering	How can I incorporate engineering design in my teaching? (in my unit?)	1. "Student Hat" Activity: Puff Cars Redesign 2. Student Hat Activity: Mini-Lessons	*How People Learn* (Ch 7, Effective Teaching)	Mini-lesson #2 (hands-on)
Class 13: Diversity and Equity	How are issues of social justice relevant to the math and science? What does it look like teach math and science for social justice?	1. Diversity survey 2. Social justice in math case study 3. Collaboration: Critical Friends Peer Feedback	It doesn't add up African American students' achievement. (Ladson-Billings, 1997)	Mini-lesson #2 Reflection
Class 14: Reflection	How have my beliefs about teaching changed? How does my mini unit reflect course goals?	Reflection: Comparing Your Classroom Drawings	How People Learn (Ch 8, Teacher Learning)	Mini-Unit Plan

Table 3. Description of a Sample Student Hat Activity, Card Sort†

Practices	Extrapolation on Math Practice††	Task Description	Experience Debrief	Reflective Practice
MP3: Construct viable arguments	• Understand and use assumptions, definitions, and results to	Background: Groups are shown a picture of a "mystery fossil" and	1. What did you talk about when you were discussing the	1. What is the typical pattern of conversation you have observed in student teaching?

and critique the reasoning of others. SEP7: Engage in argument from evidence.	construct arguments; • Justify conclusions, communicate to others, and respond to the arguments of others. • Reason inductively about data, make plausible arguments considering the context. • Compare the effectiveness of two arguments, distinguish correct reasoning, and explain flaws.	receive an envelope containing “evidence” cards.†††The Task: • Work in pairs or small groups to categorize evidence cards as supporting the following claims: 1. The fossil tooth came from a prehistoric mountain lion. 2. The fossil tooth came from a prehistoric shark. 3. Other •	evidence? 2. How were your discussions similar and different between the first and second sort? 3. How did you and your group make connections between the evidence and the claim when discussing evidence in the card sorts?	2. What do you consider “productive” discourse in the science or math classroom? 3. How could you engage students in explanation/argumentation in your unit? a. What would work well? What would be challenging? (for you and students) b. How could you create a learning environment that would promote productive student to student discourse?

† Activity based on The Argumentation Toolkit, www.argumentationtoolkit.org (González-Howard, Marco-Bujosa, McNeill, & Loper, 2018)
†† NGA & CSSO, 2010 (http://www.corestandards.org/Math/Practice/MP3)
††† Evidence cards included a variety of statements featuring data (e.g. “the fossil tooth is 5 centimeters long”) and science ideas (e.g. “the fossil tooth was found in sandstone, which is sedimentary rock made of layers”)

Table 4. Participant Information

Teacher	Gender	Race/ Ethnicity	Undergraduate Majors	Student Teaching School Type	Student Teaching Courses Taught
Amy	Female	White	Education + Mathematics	Catholic HS	Algebra 2 Algebra 3
Dawn	Female	White	Education + Mathematics	Public HS Low performing	Honors Geometry Computer Science
Grace	Female	White	Education + Mathematics	Public HS High performing	Honors Algebra 2 College Prep Geometry
Sabrina	Female	White	Education + Mathematics	Public HS High performing	AP Statistics AP Computer Science College Prep Algebra 2

Data Collection

This study utilized data sources that provided evidence of teacher sensemaking about how to teach math. The primary data sources included individual interviews, teaching artifacts (e.g., teacher-developed lessons, instructional resources, and assessments) and written reflections. Supplemental data, such as field reflections, teaching statements, and field notes from observations, were utilized to triangulate findings. Data collection spanned the academic year (August to June) from the Methods course and student teaching. Data sources are described in further detail below.

The individual interview took place in-person at the end of the academic year following student teaching in May or June. The protocol consisted of semi-structured and retrospective prompts. Interviews ranged between 38 minutes to 1 hour and 6 minutes in duration. Questions aligned with the sensemaking framework and addressed how participants" beliefs about math teaching were challenged, changed, and enacted throughout the year. For example, one question prompted participants to describe a critical moment that forced them to confront their beliefs about math education, thereby providing insight into their sensemaking. Interviews were audio-recorded and transcribed for analysis.

Instructional artifacts from Methods included the two mini-lessons and the final unit (Table 2). These were selected because all four participants identified these assignments in the interview as having an impact on their beliefs and instructional practices. The mini-lessons were 10-minute lessons addressing content of their choice. The assignments required a formal lesson plan, enacting the lesson during Methods, and written reflection. The first mini-lesson had no requirements, but the second lesson required a hands-on investigation. The final unit was a ten-day instructional plan. The unit required an introduction, a justification, scope and sequence, lesson plans and student materials, and a written reflection. Participants also discussed the teaching artifacts and observed lessons in the retrospective interviews, elaborating on their sensemaking from the written reflections. From student teaching, lesson plans and written reflections from the five formal observations conducted by a university supervisor were also analyzed for evidence of sensemaking.

Data Analysis

Guided by the sensemaking framework, analysis focused first on documenting shifts in participant beliefs and instructional practices (Weick, 1995). Two research assistants and the lead researcher engaged in separate readings of the individual interviews to jointly develop a preliminary emic coding list (Corbin & Strauss, 2008). The researchers continued this separate and collaborative coding for a second reading of all interviews. We then used the same coding framework to code the teaching artifacts and written reflections in chronological order. The lead researcher created data displays and matrices to search for patterns and make comparisons within and across cases (Miles, Huberman, & Saldaña, 2014). These comparisons were used to develop detailed case reports for each teacher that summarized their sensemaking about how to teach math across their experiences in the Methods course and student teaching.

The study was designed to address four aspects of trustworthiness: credibility, transferability, dependability, and confirmability (Lincoln & Guba, 1985). Credibility was assured in data collection and the research design of the study. Given that the researcher also served as the instructor of Methods and was a university supervisor, participants were invited after all coursework was completed and student teaching evaluations had been submitted. The study was designed in this retrospective manner in order to mitigate potential biases in participant responses. Additionally, the data was independently coded by three members of the research team, with weekly debriefing conversations to discuss coding and provide opportunities for alternative interpretations. Furthermore, the case summaries provided thick description and rich detailed data about the sensemaking processes of the participants, which also increased the credibility of this research. The case summaries were also utilized as participant member checks, in which the participants evaluated the degree to which the research interpretations aligned with their experiences. Finally, the summaries were also reviewed by an external research auditor with expertise in math education to confirm, refute, or provide alternative explanations. To enhance transferability, detailed descriptions of the participants, the context in which this study took place, and the research methods are provided in this manuscript to enable "a reader to be able to decide whether the prevailing environment is similar to another situation with which he or she is familiar and whether the findings can justifiably be applied to the other setting" (Shenton, 2004, p. 63). Dependability and confirmability were ensured in several ways, notably the use of multiple data sources (interview, instructional observations, lesson plans, and written reflections) to triangulate findings (Lincoln & Guba, 1985).

With a small sample of four teachers from just one university, broad generalizations are limited. In addition, aligning with the framework of sensemaking (Weick, 1995), the data sources were, by design, self-reported. Furthermore, it is not possible to assure that the participants‟ views and experiences are typical of other pre-service secondary math teachers (Maxwell, 2005). Regardless, the findings elucidate the role teacher education can play to enhance STEM education and to transform mathematics education.

Findings

This section begins summary of the learning experiences in Methods that had the greatest impact on pre-service teacher sensemaking and the degree to which STEM was reflected in instructional artifacts. These findings are elaborated upon in individual cases for the four teachers that offer insight into their sensemaking and implementation of STEM in their math instruction.

The Methods course engaged pre-service teachers in numerous activities intended to promote teacher sensemaking and uptake of practice-based and interdisciplinary STEM approaches to teaching math. According to the pre-service teachers in the retrospective interview, the learning experiences that had the greatest impact on their teaching included instructional design activities which prompted participants to adopt the mindset of the teacher and the student hat activities (see Table 5). With respect to instructional design, the second mini-lesson and the final unit required different elements of the course goals for the STEM practices, notably including a hands-on investigation (mini-lesson #2) and emphasize one or more disciplinary practices (final unit).

Table 5. Learning Experiences Highlighted by Pre-Service Teachers

Participant	Teacher Perspective			Student Perspective
	Mini-Lessons	*Final Unit*	*Peer Collaboration*	*"Student Hat" Activities*
Amy	X	X	X	
Dawn	X	X	X	X
Grace	X	X		
Sabrina	X	X		X

Pre-service teachers integrated practice-based teaching strategies into their math instruction during the Methods course to varying degrees (see Table 6). In the lesson plans for two mini-lessons and the final unit, pre-service teachers more often incorporated the practices implicitly through the goals and the nature of the mathematical tasks, thereby emphasizing students figuring out the math rather than explicitly listing a mathematical practice. The first mini-lesson served as a baseline for the instructional preferences of pre-service teachers; three participants (with the exception of Amy) designed lessons with direct instruction reflecting the traditional didactic pedagogies prevalent in math education (see Table 6).

Table 6. Instructional Artifacts Reflecting Practices and STEM Contexts

Participant	Methods			Student Teaching				
	Mini-Lesson 1†	*Mini-Lesson 2*	*Final Unit††*	*Obs 1*	*Obs 2*	*Obs 3*	*Obs 4*	*Obs 5*
Amy	F	F	MP(STEM)	N	N	N	F(STEM)	F
Dawn	N	F	MP(STEM)	N	N	F	F	N
Grace	N	MP	MP(STEM)	N	F	N	F	N
Sabrina	N	F	MP(STEM)	F	MP	F(STEM)	F	F

† "MP" indicates explicit practice listed and addressed; "F" indicates students engaged in "figuring out" without explicitly stating the practice; "N" indicates practices not addressed. †† (STEM) indicates context in which math content is connected to science, technology, or engineering.

In contrast, for the second mini-lesson, which required a hands-on investigation, all lessons engaged students in the practices, either implicitly (Amy, Dawn, and Sabrina) or explicitly (Grace). For the final unit, all four explicitly featured mathematical practices as learning goals and designed appropriate activities to engage students in the practices to learn math content. All developed final units that not only emphasized the practices but also provided appropriate STEM connections throughout the unit and for the final assessment. However, teacher inclusion of practice-based instruction and STEM contexts in student teaching varied greatly. Only Sabrina consistently incorporated practices into every observed lesson, and only Sabrina and Amy made STEM connections.

Overall, these findings indicate there is variation in how pre-service teachers experienced, interpreted, and enacted mathematics instruction. The following cases are intended to offer rich descriptive and contextual information necessary to offer insight into the learning experiences that promoted the most pedagogical growth (RQ #1), their sensemaking processes (RQ #2) and implementation of elements of STEM instruction as a student teacher (RQ #3). Each case presented to address three main assertions regarding their sensemaking and implementation: 1) pre-service teachers interpreted the STEM practices as promoting students" abilities to *figure out* rather than *learn about* math; 2) pre-service teachers provided authentic STEM contexts for learning math compared to emphasizing pure mathematics; and 3) experiences in student teaching either promoted or inhibited pre-service teacher continued implementation and growth with respect to the STEM practices. Cases are presented in order, from the strongest (Sabrina) to the weakest (Grace) to provide contrasts between the cases with respect to their sensemaking and experiences as student teachers.

Sabrina

Figuring out vs. Learning about

Sabrina emphasized that engaging in practice-based “student hat” activities was critical in shifting her beliefs about how to teach math (Table 5). Adopting the perspective of the learner allowed her to reconsider what it means to learn mathematics. Sabrina noted the activities that positioned teachers as learners, “were really good for dispositional growth and rethinking what we would do in the classroom and putting us in the position of the students to see what we would do in the lesson." Sabrina identified particular activities as influencing her approach to teaching, notably the “Mobius Strip,” the “Card Sort,” “Puff Cars,” and participating as “students” for peer enactment of mini-lessons.

Sabrina self-identified that these experiential practice-based activities helped her reconsider the value of more open-ended explorations to figure out the math for themselves. Sabrina contrasted this approach to teaching mathematics with her own experiences learning math:

> *[I]n high school I didn't do that. It was lecture and take notes and do your homework and come in the next day and do it again. It worked for me, so I didn't second guess it until I got into the Methods class and saw all of these other options than just doing a lecture every day.*

Thus, the activities Sabrina participated in as a learner in the Methods course provided an essential counterpoint to the more traditional, lecture-based approaches to teaching mathematics. Moreover, in the retrospective interview, Sabrina highlighted the impact of engaging in these activities while learning how to design lessons and units:

> *I had never written a lesson plan before, and by the end of Methods, obviously it wasn't totally easy, but it was easier because I had to create lessons before and make them in a way that wasn't just lecture to engage them more.*

This shift to emphasizing student conceptual exploration of mathematics was evident in the instructional artifacts she designed throughout the Methods course. For example, in the second mini-lesson, Sabrina designed and implemented an activity called “M&M Statistics.” The activity she designed was strikingly different from her first mini-lesson, which consisted of more traditional pedagogical approaches, including lecture and practice problems (see Table 6). Sabrina designed an introductory lesson for statistics that utilized a hands-on activity to facilitate student exploration of basic statistical elements. Specifically, students were given a random sample of M&Ms and were given the task to apply their prior knowledge of proportions to predict the ratio of each color of M&Ms in a box. Through this activity, students engaged in the practices of mathematical modeling (e.g. creating graphs and making predictions) and argumentation to justify their conclusions. In her written reflection on this lesson, Sabrina commented, "I think I did a good job of emphasizing how data collection could help us to form conclusions for interesting questions, and the lesson got students actively participating in the math practices rather than just learning about them."

Sabrina‟s instructional design choices and written reflections provided further evidence of this mental shift toward practice-based teaching, highlighting the benefits of engaging students in figuring out the mathematics for themselves. For example, in her unit reflection, she noted, "[t]his unit addresses the main goals of high school math teaching that we have covered in our course because it stresses the mathematical practices, specifically the use of modeling in the classroom. It encourages active participation from students, and student inquiry and exploration." Additionally, Sabrina explained that she chose to focus on quadratics to reimagine how this topic could engage students in thinking mathematically rather than simply memorizing specific formulas:

> *Throughout this semester, I came to realize that maybe if the topic was taught in a different manner, it could have been more appealing and easier to grasp…. There is a good variety of teaching methods in this unit including hands-on experimentation, student inquiry activities, direct instruction, and group collaboration, and I think the topic lends itself well to incorporating so many different teaching styles and strategies.*

In the unit reflection, Sabrina intentionally contrasted the more didactic, teacher-centered pedagogies she had experienced as a student, with the student-centered, practice-based goals of the unit she developed. She provided evidence of her use of a variety of teaching strategies as a strength of the unit. For example, the second lesson in the unit utilized Desmos, an online learning platform, to engage students in exploring different graphs through modeling. She explained, the purpose of this activity was for students to "discover certain properties of parabolas when graphing them, and how the graphs could be written as equations of the parent function $y=x^2$" "rather than teaching [transformations] directly." Therefore, rather than a traditional approach to mathematics in which students memorize the equations and rules, she intentionally designed an introductory activity to conceptually engage students collaboratively exploring and discovering the rules. Overall, Sabrina‟s case illustrates the power in the practices to shift the instructional emphasis to students figuring out the mathematics, specifically placing the emphasis on students‟ mathematical thinking and conceptual understanding as opposed to content coverage. Sabrina prioritized open-ended challenges in which students interacted with the content by using the disciplinary practices.

STEM Contexts vs. Pure Mathematics

Throughout the semester, Sabrina also began to emphasize real-world STEM contexts to frame student learning of mathematics. For example, in the final interview, she commented that she appreciated the variety of learning activities she was introduced to in Methods that involved STEM applications, notably the Mobius Strip and Puff Cars. Her instructional design goal was to consider, "how to incorporate math into activities like that." Her unit featured math laboratory investigations and simulations that connected the math content to other disciplines. Her unit was designed for a 9th grade honors algebra class studying quadratic functions. In her rationale for the unit, she explained connecting the content to the real-world was a central goal:

> *The main learning goal will be for students to acquire knowledge on how to represent and analyze quadratic functions through modes such as graphs, factored form, standard form, etc. This knowledge could then be extended to applications in the real-world to help students understand the benefit of modeling real-life phenomena through tools in mathematics.*

Her instruction featured activities in which students utilized math as a tool for explaining natural phenomena. Two specific learning activities included in the unit illustrate these real-world STEM applications for quadratics. For the first, students created a mathematical model of the projectile motion of a tennis ball tossed to a partner. Sabrina stated the purpose of this activity was, "engaging in the math practices from a first-hand view" and to "develop the skill set of analyzing phenomena in the real-world through the model of a quadratic equation." Thus, students engaged in mathematical modeling through a scientific phenomenon typically introduced in a physical science class, motion. In addition, Sabrina designed a performance assessment in which students were tasked with finding a real-world example of a parabola. Students would take a "selfie" with the parabola, "because I do not want you to just find a picture from Google. That takes all the fun out of this project!" Students would upload this photo into Demos and apply their knowledge of quadratics to identify key features and determine the equation. Students then presented their parabola to the class, "because it is important for students to justify and explain their ideas to fully demonstrate their understanding." With this performance assessment, Sabrina illustrated her emphasis on students understanding math as a tool to explore and understand mathematical phenomena in the natural and engineered world. By encouraging students to see the math around them and to apply their mathematical knowledge in creative ways, students developed a deeper conceptual understanding of quadratics. Therefore, Sabrina‟s math instruction emphasized real-world mathematics and mathematical practices over the products that are typically valued in mathematics education.

In her written reflection on the unit, Sabrina explained she purposefully designed this unit to shift the way quadratics are typically taught, intentionally contrasting her pedagogical approach with didactic methods:

> *I noticed that quadratics were a key component of high school mathematics because they are addressed all the way from Algebra I to Calculus, and I did not feel there was a lot of real-world application or active learning in most units on quadratics that I have seen or experienced.*

This sentiment emphasized Sabrina‟s shift in teaching mathematics from emphasizing procedural knowledge to math serving as a tool for students to connect math to other disciplines.

Student Teaching

Sabrina completed her student teaching in a suburban high school in an upper-middle class community (Table

4). She worked closely with two cooperating teachers in two math courses: Probability and Statistics, an honors class of seniors, and on-level Algebra 2, mostly sophomores. Her cooperating teachers were supportive of her STEM teaching goals and allowed her freedom in the classroom to design and implement her own lessons. Three example activities provide evidence of how Sabrina integrated STEM practices and real-world applications into her instruction: Graph of the Day, Math Investigations, and Card Sorts.

Each day, Sabrina began the Probability and Statistics class with a real-life graph addressing current events, advertisements, or media articles. Sabrina identified the “Graph of the Day” as her greatest success in student teaching. According to Sabrina, the purpose of this activity was two-fold; students had the opportunity to apply the knowledge and skills they were learning in class and the real-world graphs made explicit connections between course content and statistical knowledge and skills students would use in other fields. She stated the goal of this activity “was to emphasize the importance of statistics in the world around us, and to find a fun and relevant way to get students thinking more critically when encountering graphs in their everyday lives.” Several examples of graphs included: “Global Temperature and Number of Pirates,” which emphasized drawing reasonable conclusions, and “Comparisons of Average Winter Temperature” to focus on global temperature trends over time and identify outliers.

Sabrina also implemented two distinct types of instructional activities that emphasized the mathematical practices: math investigations and card sorts. Sabrina cited that her experiences in the “student activities” in Methods were inspirational for her to adapt and extend her thinking to teaching math. In the retrospective interview, she commented that in student teaching, “I used a lot of the ideas from methods class… I don't think I used any specific lesson plans in Methods or mini-lessons, but I used those as a basis for.... other lesson plans.” One observed math investigation was implemented in the Algebra II class, called “Bouncing Balls.” Students engaged in a laboratory that would be expected in a science class. Drawing upon the lesson she developed in her mini-unit about modeling projectile motion, students worked in groups to predict, collect data, and ultimately develop a mathematical model (MP4) describing and predicting the bounciness of balls. Sabrina reflected that witnessing student engagement further convinced her of the power of practice-based instruction, “I was extremely impressed by how well the Bouncing Balls activity was received by my students, and it made me realize that I can trust them to participate in more fun and engaging activities while still learning the material of the course,” and “I enjoyed implementing a more hands-on and explorative activity in this Algebra 2 class, and it opened my eyes to some more practices that I could bring into the classroom.” Her written reflection on this lesson also emphasized her attention to developing student conceptual understanding. “By asking them about their graphs and their predictions, I was able to guide them in the direction of considering the connection between the activity and the exponential situations we had been studying previously.” These comments indicated that Sabrina believed investigations were not just for science; investigations are opportunities to engage students in meaningful, applied mathematics and provided the teacher with a formative assessment of student understanding.

Sabrina also utilized Card Sorts in student teaching to promote the practices of explanation and argumentation (MP3). In one observed lesson in Probability and Statistics, Sabrina implemented a card sort in which students worked in groups to correctly sort nine scatterplot graphs in order of the strength of the correlation. Students were given the task to discuss and justify their decisions in an oral argument to the teacher. In her reflection, Sabrina noted:

> *Students were successfully able to describe the direction, strength, and form of a scatter plot, but they struggled more when faced with the task of using that information and applying it in context.*
>
> *My questioning style has developed throughout the course of the semester, and from the start, it has always been important for me to make sure I know why students are arriving at the answers they have, and for them to have the ability to justify and rationalize mathematically.*

These reflections illustrate how Sabrina was attending to student conceptual understanding through practice-based instruction. She focused on her own growth as a teacher, specifically as a facilitator of student thinking by asking targeted questions to gauge and develop student reasoning skills.

In the retrospective interview, Sabrina commented that teaching math through STEM improved her instruction. She reflected that her greatest success in student teaching was "[i]ncorporating those activities that were more creative and less traditional math" because she "saw the connections for my students." For Sabrina, these comments illustrated that this STEM approach to mathematics instruction allowed her to shift both what she taught and how she taught math, with observed learning benefits for her students. The combination of learning experiences in Methods introducing and reinforcing STEM pedagogy, as well as the supports and successes in student teaching, served to shift her mindset away from traditional mathematics instruction toward integrated STEM to support math learning.

Amy

Figuring out vs. Learning about

Amy emphasized active student engagement in the practices, specifically those associated with reasoning, explanation, and communication. Her interpretation of practice-based teaching for mathematics emphasized the processes of learning as opposed to more traditional products (e.g. getting the right answer). Her sensemaking was strongly impacted by the instructional design activities and the opportunity to collaborate with peers during class to share ideas and reflect on instructional design (Table 5). These experiences promoted her sensemaking about math teaching by valuing and revealing learning processes, shifting the teacher"s role to be a facilitator of student learning. Through the instructional design activities in Methods, Amy was able to integrate and apply what she had learned in the Methods course, her math content courses, and other education courses to achieve these instructional goals. Interestingly, Amy was the only participant who did not employ traditional didactic methods for her first mini-lesson (Table 6). She designed a hands-on activity in which teams of students collaboratively explored the criteria for triangle congruence. Her lesson plan emphasized students figuring out the criteria by observing patterns:

> *Students will be given a bag with a piece or pieces of a triangle to represent one of the criteria for proving congruent triangles…They will work together to see if the given pieces can or cannot produce more than one triangle…*

She centered the lesson around students figuring out the criteria based upon her own experiences reaching new conceptual understandings in a college geometry course, while she was enrolled in Methods:

> *The way my college level geometry class is being taught is by assuming we remember all the rules and theorems we learned in high school. At the most basic level, we have to know how to prove triangles congruent. In my class, we quickly skimmed over the criteria to prove two triangles congruent, and something in that moment clicked for me in a way it never had before. I made the connection that the reason why the criteria works is through utilizing transformations.*

In her reflection, she noted that she intended to shift her geometry course from memorization to understanding mathematical patterns. This shift was prompted by her own experiences as a learner in her math class. Thus, similar to Sabrina, having the opportunity to reflect on her own learning challenged Amy to reconsider what was most important in a math class: student reasoning and conceptual understanding.

This emphasis on students figuring out the math was also central to Amy‟s second mini-lesson which she enacted at the end of the Fall semester (see Table 6). She designed a collaborative, hands-on activity in which students investigated similar figures and "discovered a new application of the Pythagorean Theorem." The lesson plan emphasized student reasoning and ability to identify patterns (aligned with MP8), guided by the following questions: "what is true about corresponding angles in similar triangles?" and "what is true about correspond sides in similar triangles?" In the written reflection, she commented on her goal to reveal student thinking:

> *One of my peers commented that I asked good questions and fielded answers constructively. I was really proud of this comment because … I am trying to work on my active listening in order to make clear corrections and give better feedback.*

This quote illustrates that Amy saw her responsibility as a teacher as designing activities that scaffold and guide students to understanding and applying geometric criteria. Her attention to students "investigating" and "discovering," and reflection on her role as a teacher to "listen" illustrates her sensemaking about practice-based teaching.

Amy explicitly integrated her process-oriented goals with the math practices in her design of the ten-day mini-unit. The unit focused on the topic of ratios and proportions in a geometry class. In the introduction, she stated that content knowledge and process skills (understanding how and why) were equally taught and evaluated throughout the unit. Specifically, she focused upon the importance of reasoning, explanation, and communication: "Geometry is all about proofs and explaining why things are true," and "[t]he main thing I would want students to gain from this unit is the importance of communicating through mathematics." The unit addressed the following mathematical practices: constructing viable arguments and critiquing the reasoning of others (MP3), reasoning abstractly and quantitatively (MP2), and looking for and expressing regularity in repeated reasoning (MP8). These practices were explicitly scaffolded throughout the unit through discussion and writing activities. Amy observed, "[j]ournaling and reflecting are not a typical skill seen in a math classroom but communicating through mathematics is one way to make stronger connections. I decided to include these reflections…so I could monitor student thinking." Thus, Amy intentionally contrasted her goals of students figuring out the mathematics and expressing their conceptual understanding. Across the semester, Amy‟s experiences in Methods served to solidify her vision of math teaching.

STEM Contexts vs. Pure Mathematics

Amy also sought to incorporate real-world connections into her math instruction. Compared to Sabrina, her strategies were emerging during the Methods course, but provided a foundation to build upon during student teaching. Notably, Amy made these connections in her unit. She integrated a real-world task in the introductory lesson and in the end of unit assessment. These two lessons involved utilizing scales to develop a model dollhouse and city, respectively. For the introductory lesson, students were introduced to the importance of scale factors through a collaborative activity. Using a dollhouse as a model, students would determine the scale factor necessary to determine the dimensions for a real house. Then, each group would focus on calculating the dimensions of a real room. This introductory lesson provided students with the real-world application of these math skills in architectural design, and, as Amy stated, students will "unknowingly applying ratios and proportions by using a scale factor they create." Students then returned to a similar scenario at the end of the unit. Students were given the performance task to design their own city by applying their content knowledge, providing explanations, and communicating their understandings through a class presentation and individual journal. She also provided students with a more traditional test; however, it was a take-home test involving more applications than a traditional classroom test. Thus, with this two-pronged approach to assessment, she was able to balance more traditional products of mathematical knowledge with STEM applications:

> *I wanted my assessments to be more than just a test. Having two assessments allowed me to assess content goals through the take home packet, and the performance assessment allowed me to assess the communication and presentation skills that were incorporated in many lessons. Creating these two assessments was a personal success because I had to think outside of the tradition classroom structure and I am proud of my final products.*

This quote reflects Amy equally valued the process and the product as essential outcomes of learning math. Overall, designing the unit with these outcomes in mind helped her make sense of the intended goals of practice-based teaching.

Student Teaching

Due to scheduling challenges, Amy did not complete her student teaching in the same school she had observed in the Fall. She completed her student teaching at a Catholic high school, whereas she had completed her Fall observations in a public, high performing school. Consequently, she had little time to become acclimated to the school, her students, and her cooperating teacher before taking over teaching responsibilities. Her cooperating teacher gave her tremendous freedom, but also gave her little instructional support, guidance, or feedback. While Amy was overwhelmed with ensuring she covered the essential content, she also had the pedagogical freedom to implement the STEM approaches from Methods into her teaching. Thus, her pattern of only incorporating STEM into her instruction at the end of the semester may be explained by this adjustment to the cooperating teacher, students, and curriculum (see Table 6).

She directly contrasted her instructional approach with her cooperating teacher, who tended to implement more didactic methods. According to Amy, “[h]e was very into writing on the white board and everyone is going to copy.” In contrast, Amy emphasized student active engagement in mathematics. She discussed designing interactive lessons, aided by technology tools such as Pear Deck. She noted changes in student mathematical thinking because of this instructional shift; “[t]hey ask more questions than before because the material is already on their screen. The quality of their questions has also improved because I have to clarify less.”

Amy also implemented several math projects that provided students with real-world applications. One such project was the focus of the fourth observed lesson, which took place in a pre-calculus class. This project focused on cryptography, in which students applied their knowledge of matrix algebra to encode and decode messages. The lesson began with a movie clip of real-world code breaking during WWII and introduced examples of how students could use simple matrices (learned in a previous class) to accomplish the same goals. Then students began working on developing their own codes utilizing their algebraic knowledge. In the reflection for this lesson, Amy felt that this was successful, once again because the approach different from traditional math teaching and allowed students to communicate their knowledge in creative ways. “The typical style of this class is lecture, which is what my teacher prefers. I am glad that my students were able to do an application-based project because it extended their learning beyond the textbook.”

Amy also designed a culminating real-world project for the Algebra 3 course that was observed in the fifth lesson. Algebra 3 was the lowest level math class, and consisted of primarily seniors who had failed Algebra 2. Amy took over responsibility for this course in March, the last of the five classes she taught. Amy explained that student motivation in this course was low due to student performance in the past, and there were numerous behavioral problems that were challenging even for her cooperating teacher to address. With this in mind, Amy developed a project focused on real world math skills utilized in eight distinct life phases, including planning for college and career. This project integrated content (e.g. creating a budget, applying for financial aid for college), process skills highlighting their ability to communicate mathematical explanations (e.g. a written report and a final presentation), and technological applications (e.g. using online interest rate calculators). Amy explicitly identified practice standards (e.g. make sense of problems and persevere in solving them, MP1; model with mathematics, MP4; use appropriate tools strategically, MP5; and attend to precision, MP6). Amy believed this project exemplifying her teaching, illustrating her commitment to the ideals of authentic STEM teaching. Amy reflected, “[t]his project was an exciting way for me to engage students who, in general, have a negative view of math.” Overall, as a student teacher, Amy maintained and built upon her pedagogical skills through the integration of technology to promote student reasoning and the development of several real-world projects.

Dawn

Figuring out vs. Learning about

Overall, Dawn‟s experiences in Methods challenged her prior conception that “math is math.” Similar to Sabrina and Amy, during Methods, she began to focus on developing instructional strategies to support student conceptual understanding. The "student hat" activities challenged her beliefs about how to best teach math (Table 5). In the end of year interview, she observed, "during the class I was exposed to new things like inquiry-based learning in math." She found the student hat activities emphasizing the STEM practices were useful alternatives to traditional lecturing in high school math. She commented, "[methods] showed me so many different teaching strategies and methods that I do not think I would have known about or would have been too scared to try if it had not been for your class." Through Methods, Dawn expanded her vision of math teaching to encompass additional inquiry-based activities. Specifically, the student hat activities provided her with a model for implementing alternative instructional strategies.

Dawn‟s beliefs about teaching math were further disrupted by the requirements of the instructional design activities and the act of planning to align with these goals. Specifically, Dawn identified designing the final unit as a critical moment challenging her beliefs about teaching math. She explained that the requirement to include practices,

> *challenged my way of thinking. I knew based on the assignment that I needed to include some sort of inquiry, and without that challenge I wouldn't have done it during student teaching. That's my critical moment was just making the unit plan and trying to find more exciting lessons than lecture and do this worksheet.*

Therefore, the act of engaging in instructional design with the requirement of addressing student engagement and the practices shifted her beliefs about what was possible in a math classroom. The requirements also forced her to creatively utilize these teaching strategies to teach specific math content in a novel instructional context.
Dawn‟s experiences planning with the practices placed students at the center of the instructional process, particularly on students figuring out and applying the content, rather than the teacher telling students what they should know. For the first mini-lesson, Dawn planned a traditional lesson involving lecture comparing parallel and perpendicular lines. This lesson centered around Dawn providing the steps, with practice problems, and a closing multiple-choice question. In her written reflection about this lesson, Dawn‟s comments reflected an emphasis on traditional, didactic teaching methods. For example, "I felt like I did everything I wanted to do/did not forget anything I planned to do," and "[s]omething I was really happy about my lesson was that my students actually seemed to understand the material and were able to follow what was going on." Furthermore, her objective also emphasized more didactic goals; "recognize the relationship between the slopes of parallel and perpendicular lines" as opposed to higher-level goals for analyzing or comparing. Thus, the lesson itself and her reflection emphasized the belief that teachers provide the procedures, which students follow.

In contrast, the second mini-lesson, which took place at the end of the semester, was required to be hands-on. Dawn designed and implemented an activity she named the "Triangle Investigation." The goal of this lesson was for students to discover the Triangle Angle Sum Theorem by observing patterns and engaging in reasoning to deeply understand the theorem. In the introduction to the lesson, she stated, "today we are going to see for ourselves that the total angle measurement of a triangle is 180°. This way they know that is true and do not just have to take me for my word." Thus, the emphasis was on students making the mathematical connections for themselves rather than being told by the teacher. Throughout the lesson, students engaged in several hands-on activities, using paper triangles, scissors, and rulers, to test their ideas in different ways. At the end of the activity, Dawn posed the question, "does this work with any type of triangle?" continuing to place the responsibility for figuring out the math on the students. This goal for promoting student mathematical thinking also evident in her written reflection on the lesson:

> *The ambiguity was kind of the point of my lesson, so while I recognize that the lesson may have been challenging and students may not have known exactly what to do right away, my goal was to challenge the class to work with their partner to figure it out.*

This intentional emphasis on open-ended problems is indicative of a shift in Dawn"s beliefs about the teaching and learning of mathematics. This comment indicates Dawn"s goals were for students to collaboratively figure out and make connections between their observations and mathematical theorems. Comparing these two mini-lessons offers insights into Dawn"s growth throughout the semester in seeing learning math as an opportunity for students to use math, as opposed to following procedures.

For Dawn, it was important that abstract concepts from geometry, notably theorems about triangles, could be brought to life through hands-on activities, "because I think geometry has the most potential to use manipulatives and hands-on activities that show students the math rather than just tell them." This shift in her sensemaking about math teaching was also evident in the design of her final unit, which addressed the properties of triangles. For example, in the introduction to the unit, she stated, "[t]hroughout the unit students will become more motivated to learn math by seeing how interactive and fun it can be, as well as its importance in everyday life." She also emphasized students figuring out the math. For example, in the rationale for the unit, Dawn explained her overarching approach for scaffolding student exploration in math; "I will ask students to brainstorm ideas on how they might solve the problem, rather than me just telling them right away. This way, students are challenged to use previous knowledge to solve a problem." Thus, Dawn linked her sensemaking about the practices, emphasizing the student-centered nature of practice-based instruction which differs greatly from her initial emphasis on lecture. Additionally, she emphasized the importance of student engagement in her planning as a teacher, noting that these practices engaged students in deeper learning of mathematics:

> *I know that I could walk into class and just teach from the PowerPoint and note packet that my teacher has. However, I also know that this would not be engaging for my students and they would not get as much out of it as they would with more interactive activities that require active learning.*

These comments indicate that Dawn"s efforts as a teacher were to design authentic, engaging activities that promoted students" use of mathematics. As a teacher, her responsibility was to facilitate students" mathematical thinking.

STEM Contexts vs. Pure Mathematics

The shift in Dawn"s beliefs about how to teach math not only emphasized student conceptual understanding. She also sought to actively engage students with mathematics through real-world, integrated STEM applications. For Dawn, the fact that Methods emphasized interdisciplinary STEM connections challenged her beliefs about how to teach math:

In class you were coming from a science background. Science has labs. [You asked] how did you learn in math class, usually notes and a worksheet. I was predisposed to doing that style because that's how my math teachers did it.

Whereas Dawn previously believed these types of investigations were only appropriate in science class, the STEM emphasis helped her see how mathematics could be taught through a more real-world math applications. She featured elements of scientific inquiry into her unit as opportunities to engage students in exploring mathematical ideas.

Dawn incorporated STEM into her unit design in two key strategies: mathematical inquiry and engineering design. The unit she designed centered on exploring the properties of triangles with the overarching challenge of exploring why triangles are the strongest shape. The first lesson in the unit was an investigation. Students constructed different shapes from toothpicks and marshmallows and investigated strength by testing how much weight each shape could hold before it collapsed. Working in groups, students collaboratively designed, tested, and drew conclusions based on their data. Questions she developed to guide student conceptual understanding and engagement in the lesson included "Which shape is the strongest? Why?" and "What types of structures are shaped similar to the ones that you built? Why?" The stated intent of the activity was to "pique student interest" and to frame their learning throughout the unit about triangles to address the question of "what makes triangles so important and special and the strongest shape." In the reflection for the unit she commented that she wanted to include mathematical inquiry into her instructional design because "during the class I was exposed to new things like inquiry-based learning in math."

This investigation not only foreshadowed the math content to be covered, but also served as a preview for students to contemplate the unit assessment: an engineering design challenge in which they applied their knowledge of the properties of triangles to design a bridge. For the opening of this lesson, students were shown pictures of truss bridges featuring visible triangles. Then students are tasked with being the engineer on the design of a truss bridge for a railroad project, and independently design a blueprint for the bridge. The stated goal of this performance assessment on the student activity sheet was "to design scale models of truss bridges to witness a real-life example of the use of triangles and understand the importance of triangles as a part of the structure." The project rubric evaluated students on creativity, accuracy, practicality, stability, and mathematical reasoning. This integration of math with engineering represented a significant shift in Dawn"s approach to teaching mathematics towards a more authentic, integrated, problem-based approach.

Dawn was highly reflective of this shift in her beliefs about how to teach mathematics. In the unit reflection, she observed that challenging herself to apply what she learned in Methods about STEM and practice-based instruction to design the unit, shifted her beliefs and goals:

… I challenged myself to incorporate many of the pedagogical ideas that we spent time on in class into this unit, for example inquiry, problem-based learning and engineering design. This was difficult for me because it is not how I am used to math being taught. However, I feel like I got so much out of doing this. Now I feel like I will actually use these concepts when I am teaching because I know that I am capable of building lessons using them.

With this statement, she directly contrasted her new pedagogical vision with more traditional, teacher-centered math instruction. Thus, the process of planning was in and of itself a sensemaking activity for Dawn to reimagine the mathematics classroom as a student-centered learning environment in which students explore and use math to address real-world problems. The requirements for the assignments that students engage actively with mathematics pushed Dawn to focus more on how students were engaged with mathematics in her lessons.

Student Teaching

Dawn‟s student teaching placement was at an urban high school that served a racially, culturally, and linguistically diverse student population. Unlike Sabrina and Amy, Dawn encountered challenges in implementing STEM instruction in her student teaching experience. Specifically, the math department created standard unit assessments and curriculum packets consisting of notes, practice problems, and homework, to guide instruction. Thus, Dawn‟s student teaching experience can be characterized as challenging the traditional teaching is telling paradigm through more active learning opportunities in which students generated and applied their own knowledge. At the end of the year, Dawn reflected, "[w]hile these packets are useful and help to make sure all students learn the same concepts for the common assessments, the heavy lecturing can be very boring, mundane, repetitive, disengaging, passive learning for students." Contrasting the approach to instruction within her school compared to the instructional goals promoted in the Methods course deepened her learning about practice-based instruction. While her implementation in student teaching was uneven, the contrasts she was able to draw from the traditional math pedagogies and her preferred STEM approach deepened her learning and commitment to disciplinary integration.

At the end of the year, Dawn identified her greatest success as "planning lessons that my students enjoyed and learned from," because she, "developed many hands-on, engaging lessons and review activities to compliment the packets that students get." She justified these modifications with a similar rationale to what she expressed in her final unit; designing instruction that supports students figuring out, rather than being told what they should know:

> *I have found that these activities help to channel student's energy into learning …and they are more invested in doing the work because they are having fun playing the game or figuring things out on their own rather than me just telling them, so this leads to students retaining the information better than if it was just given to them in lecture or through independent practice at their desk.*

Thus, Dawn saw her goal as a student teacher to challenge the traditional pedagogies to improve student opportunities to learn math. While she did not enjoy as much freedom as she would have wanted, her cooperating teacher gave her some flexibility with lesson warm-ups and review activities (and for the scheduled observations by the university supervisor). Examples of activities she developed to actively engage students in mathematical thinking included: theorem discovery and review games.

In the third observed lesson for student teaching, Dawn implemented a discovery activity as a warm-up to introduce the Triangle Midsegment Theorem. Similar to the mini-lesson she designed and implemented in

Methods, Dawn provided students with construction paper triangles to test out ideas expressed in the theorems before students were introduced to the theorem. Dawn expressed that she had not been able to do this type of activity due to restrictions on her teaching by her cooperating teacher, but she saw more value in actively engaging the students in an open exploration of the content. "I get a lot of questions, why does this matter… It is conceptually different from anything they are usually asked to do..." She explained that in planning this lesson, she explained that she believed this exploration would essentially prime students for the more traditional instruction, consisting of lecture and note-taking, that would take up the majority of the lesson:

> *I found an activity that I could use as a warm-up activity that would allow my students to explore the new material and hopefully engage them with something that is hands-on and they could physically see. Then, since this was just a warm-up, there would still be plenty of time left to formalize the material with notes as well as practice problems.*

In this quote, Dawn integrated her practice-based approaches to teaching as an engaging hook for student interest, which she intended to sustain student understanding of the notes.

Dawn also came up with creative games for students to review and apply content knowledge. For example, Dawn designed and implemented a warm-up activity about solving radicals using playing cards. Students were given the task to work in partners to solve for at least two different radicals. Dawn believed this activity helped students make connections between their math knowledge; in her lesson plan, she made a note to, "remind students that knowing how to simplify radicals is very important and something we will need to do frequently… with the Pythagorean Theorem." In her reflection on the lesson, Dawn noted that she was glad she was able to incorporate a more fun and engaging review activity as a warm-up for what ended up being a dense introduction to new material:

> *Also, I feel like on these days class is very note heavy, which I feel bad about because I know it can be kind of boring and not the most interesting or engaging. This is why I try to plan a different activity besides the packet/notes at least once or twice a week to switch it up (and ALWAYS do something fun [for] a review activity…)*

Thus, Dawn continued to prioritize active engagement despite the pressure she felt to cover the content in the packets.

In the final observed lesson, Dawn implemented a full-class review game before the unit assessment. While her cooperating teacher emphasized that the students must be able to successfully complete the practice problems in the packet, Dawn wanted to shift this activity to a more fun, active, and collaborative review and extension of the material. Therefore, she used the practice problems, which were presented in a typical worksheet format, and altered the activity to create a "Scavenger Hunt Activity" about the final challenge problems that featured real-world trigonometry applications. Students worked with partners to solve problems posted around the room. Students needed the solution from the first problem to identify the next problem to be solved. In the reflection, Dawn observed, "[t]he success of the lesson showed me how switching up one little thing (hanging the problems up around the room rather than just having them in the packet) can make a great, engaging activity." Therefore, while this lesson was not coded as aligning with the STEM criteria (Table 6), Dawn was successful in adapting traditional curricular resources (worksheets) to a more collaborative, engaging, and active learning experience.

She was able to continue to grow as a teacher by creatively adapting existing traditional materials to promote her instructional goals.

In sum, while Dawn"s implementation of STEM in her math instruction as a student teacher was limited by contextual pressures, she sustained her instructional priority on students figuring out and applying their content knowledge. Dawn creatively emphasized these goals in particular activity structures, notably warm-ups and review activities, and actively modified traditional curricular resources (e.g. the packet) for her instructional design. Moreover, it is important to note that Dawn was given more freedom on scheduled supervisory observations; therefore, the five observations measure her ideal practice as opposed to her routine pedagogy as a student teacher.

Grace

Figuring out vs. Learning about

Grace described that the Methods course had a large impact on her understanding about how to teach math. She reflected, "[i]n Methods last semester, you got us thinking a lot about straying away from direct instruction. That jump started me thinking about other ways to teach because direct instruction was the only way I had ever been taught math." Thus, similar to Sabrina and Dawn, Grace was exposed to alternative pedagogies for math instruction, forcing her to confront her preferences for lecture. Grace identified the instructional design activities as having the greatest impact on her sensemaking about the STEM practices. Specifically, through designing lessons and units, Grace integrated pedagogical concepts from other education courses, specifically disability studies, and the STEM practices to promote access to math content.

Grace"s proclivity for more traditional, didactic teaching methods was evident in her first mini-lesson (Table 6). The lesson focused on domain and range through direct instruction. In contrast, for the second mini-lesson, she designed a hands-on investigation about the perpendicular bisector theorem in which students created triangles and bisectors with construction paper and construct a viable argument, and critique the reasoning of others (MP 3) to arrive at a class definition of the perpendicular bisector theorem based upon their own observations. She believed this lesson was successful, "because students were able to successfully construct and identify perpendicular bisectors." Thus, this activity reflects a shift in her instructional goals; students were required to "construct" their knowledge and "explain" their understandings. Therefore, Grace placed the student at the center of instruction, and she utilized the STEM practices as a lever to enhance access to math rather than utilizing general instructional modifications.

Similar to Amy, Grace began to reflect upon her mathematical knowledge, and was able to make conceptual connections between college math and the math she would be teaching in high school. She utilized these connections to better scaffold and extend student learning of the content. For example, in the end of year interview, Grace observed she engaged in metacognition about the content while she considered how to design an effective lesson to support student learning. She stated, "So much comes back to you because you don't understand how the math concepts you learned in college helped to understand why things work the way they do.” This quote illustrates that the act of engaging in instructional design prompted Grace to integrate STEM practices to facilitate student conceptual understanding of mathematics.

She emphasized students figuring out the mathematics in her final unit. The unit focused on the topic of relationships in triangles and explicitly addressed MPs 1, 3, and 5 (making sense of problems and persevere in solving them, construct viable arguments and critique the reasoning of others, and use appropriate tools strategically). Grace emphasized the fact that no entire lesson focused solely on transmitting knowledge; each lesson either incorporated a warm-up supporting student application of knowledge or a collaborative activity. For example, in a lesson about identifying the centroid of a triangle, students worked in groups on an engineering design challenge to discover the balance point of a triangle. As stated in the lesson plan, “[s]tudents have not yet learned what a centroid of a triangle is, but by building a mobile, they discover that the balancing point of a triangle is what will hold their structure up and balance their weights.” At the end, Grace planned for groups to share with the class, “why they believe they were successful or unsuccessful” in order to collaboratively come up with a classroom strategy to define the centroid. In another lesson, about triangle inequalities, students used pipe cleaners to create triangles using sides of different lengths. Students made predictions and conduct their own investigation to prove or disprove their initial ideas. For the lab, each student worked individually with pipe cleaners of different lengths to collect data and create a data table documenting when they made a triangle. Then, students work in groups comparing data tables to identify patterns and “to come up with a statement about the relationship between the three sides of a triangle.” This lesson illustrated Grace‟s utilization of the practices (constructing viable arguments, MP3) to promote student access and engagement with the mathematics.

In the unit reflection, Grace commented upon how designing the unit promoted changes in the type of math teacher she wants to be:

> *The creation of this unit taught me a lot about what it means to be a teacher and having a student-centered learning environment. I could have just made this unit entirely direct instruction and taught geometry the same way I was taught, through repetition, worksheets, and practice problems. However, I went a completely different route… I wanted students to construct their own learning and I wanted to put to use the theory that I have been learning about these last four years.*

With this statement, Grace reflected on her professional growth, indicating her uptake and interpretation of the practices from a student-centered pedagogical stance. Grace emphasized students “figuring out” the math to increase student self-efficacy. As she explained in the unit reflection, “[m]y hope is that in doing activities like these, students gain confidence in defending their arguments and develop into articulate speakers who are able to give coherent reasonings when asked.” Therefore, Grace was thoughtful about her design of instruction, with the goal of empowering students to think mathematically.

STEM Contexts vs. Pure Mathematics

Grace also emphasized math as a tool to explain natural phenomena or to design solutions to problems. These connections were most evident in instructional hooks she used at the beginning of lessons to frame student learning of the content. For example, the second mini-lesson about perpendicular bisectors began with the following question to connect the math content to architectural and interior design: “I wanted to put an island somewhere in this kitchen that would make food preparation most efficient, where should I tell the architect to put it?” In her reflection, she specifically noted the value of the real-world hook to engage students. “I think that my energy as well as the opening kitchen example made the lesson engaging and made students want to listen.”

Her efforts to connect mathematics to STEM more broadly to promote student access and engagement to mathematics is also evident in the two-week mini-unit. Both of the lessons described above (centroid and triangle inequality) began with a hook and included active and applied engineering projects not typically seen in a math classroom. The Mobile Design lesson focused on the centroid of a triangle through the real-world phenomena of a balancing point. For the hook, students watched a video of Chinese plate spinners, and were prompted to consider how the acrobats were able to balance the plate. The stated purpose of this video was to guide “students thinking about the learning activity for that class as well as the relationship between balancing point and triangles.”

The second lesson, Pipe Cleaner Lab, explored the topic of triangle inequalities. For the real-world hook, students were given the task to brainstorm a solution to the following problem:

> *… I want to add a tassel cording around the perimeter of the two triangular pillows that I am making… I use scraps from a previous project because I am on a budget. I select the scraps at random. The first scraps are 3 inches, 8 inches, and 6 inches. These three scraps form a triangle. However, the next three scraps I select are 3 inches, 3 inches, and 8 inches. They do not form a triangle. Why? How can this be?*

Beginning the lesson with this problem prompted students to consider the relationship between the sides of triangles before engaging in open-ended exploration previously described. Thus, these STEM connections used as instructional hooks for student inquiry illustrate how Grace ensured student access and engagement by showcasing a natural phenomenon and real-world applications, largely through architecture and design.

Student Teaching

Despite this shift to STEM teaching goals, Grace was unable to consistently incorporate STEM practices her instruction as a student-teacher (Table 6). Grace completed her student teaching in an upper-middle class community where the majority of students attended college. She worked with two cooperating teachers to teach two different courses: Geometry for the College Prep (CP, lower level) track and Algebra 2 for Honors students. wither cooperating teachers also had different instructional styles: Geometry was taught via more traditional methods, whereas Algebra 2 was much more student-driven learning. Moreover, while she was provided with a curriculum to follow in geometry, her cooperating teacher gave her more freedom to design her own instruction, and she immediately began to implement more STEM-focused math lessons.

> *I was told, you are teaching these lessons. And for me, I was coming out of [Methods], and I knew all these different ways we could incorporate technology and connect it to life, and geometry could be really*

hands-on. So I took the initiative to think of what I was going to do and we would do notes secondary.

In her second observed lesson during student teaching, Grace designed a hands-on investigation for her geometry course about the Pythagorean Theorem. The goal was for students to see "why the Pythagorean Theorem worked." Utilizing Unifix cubes as a physical manipulative for "creating the equal squares, students will see how the two side lengths of a triangle squared are equal to the hypotenuse squared." In the written reflection following this observation, Grace explained that her underlying rationale for this activity was to make the content more accessible for students with diverse needs:

> *I thought it would be a great way to engage all different types of learners and hopefully make students excited about the lesson. I believe that I was successful in the engagement piece because students participated in the building of the right triangle.*

This example illustrates that Grace emphasized instruction designed to engage students in figuring out the math ideas.

While she had less freedom in Algebra 2, and "Algebra 2 is cut and dry," and therefore more challenging to engage students in real-world, practice-based activities, Grace developed a hands-on activity for Algebra 2 course that was inspired by the aforementioned lesson. about the lesson addressed quadratic equations utilizing Unifix cubes to show why the squares were equal to each other. In the end of the year interview, she identified this lesson as a critical moment that helped her come to the realization of the power of enacting these types of investigations in math classes to support student learning:

> *I saw first-hand was when I did a hands-on activity with my Honors Algebra 2 kids…. They saw it and it was like, legendary. They were all like "oh my gosh that's so cool, that's why this works. They had never seen completing the square before. The grades on that quiz were so high. I was like wow, hands on works. It was so cool. They saw it in action, it made sense to them.*

She explained that while she learned and practiced these ideas in theory in the Methods course, having the experience of enacting this activity with her students was powerful visual to promote conceptual understanding.

However, in the end of year interview, Grace revealed two pedagogical challenges that shifted her teaching to more direct instruction. First, there were several lessons in which she engaged students in an investigation, but found students lacked the content and skills to complete the activity. She explained,

> *They did not understand anything from the investigation. I had to spend another extra day going over the trig[onometry]. I didn't know that was going to be hard. I should have done direct instruction first and then done the lab. I'm still working that out and seeing how things connect.*

The second challenge arose with respect to student engagement. She was frustrated when she felt students still did not apply themselves and lacked the motivation to engage in open-ended tasks as opposed to more didactic methods:

> *These kids couldn't genuinely care less about math. So, how do I make it exciting? In my mind it's exciting, but for them, they don't care. And we talked about it in Methods. But then I saw it. I saw my students be like this is dumb, why am I doing this. I had a student crumple up his guided note sheet that I*

had spent 3 hours making and literally throw it in the garbage, and I was like I can't believe he did that.

These quote reveals two potential challenges. First, Grace‟s instruction may have been driven by the activity as opposed to promoting student conceptual understanding of mathematics. Second, Grace‟s approach was unfamiliar to students, and so different from the math instruction that they experienced that students needed additional scaffolding to engage in these learning experiences.

By the end of the semester, Grace became discouraged and implemented more traditional, didactic math lessons. Grace began to feel pressure to cover content. For the fifth observed lesson, she explained, "[b]ecause we are beginning something completely new topic, there is no warm up, but the front board will have the graphing calculator projected on it" and the "[m]ajority of this class period will be through direct instruction because it is a completely new concept." Therefore, Grace‟s utilization of instructional hooks and investigations connecting math to broader STEM phenomena was shaken based upon challenges she encountered challenges in supporting student understanding and engagement. Similar to Dawn, Grace‟s experience illustrates that the student-teaching context, including the students, the cooperating teachers, and the school curriculum, have a strong impact on student-teachers‟ implementation of practice-based strategies for teaching mathematics following a Methods course. Interestingly, the pressures discouraging STEM integration were observed in a high performing school (Grace) and low performing schools (Dawn) (see Table 4).

Discussion

This study used sensemaking as a framework for exploring how pre-service secondary mathematics teachers interpreted and incorporated STEM into their teaching. Examining the experiences of pre-service teachers through the sensemaking framework revealed they acquired new pedagogical perspectives as well as mathematical knowledge by teaching math through STEM. This occurred in two ways that have implications for pre-service teacher preparation. First, with respect to pedagogy, the STEM framework in the Methods course *enhanced pre-service teacher pedagogical content knowledge (PCK)*. Second, the STEM framework facilitated pre-service teachers *developing knowledge at the mathematical horizon,* a domain of mathematical content knowledge not covered in their content courses. This discussion considers the study findings in light of teacher experiences in coursework and student teaching with implications for secondary mathematics teacher education.

Enhancing Pedagogical Content Knowledge

One key contribution of this study is how STEM learning experiences in the Methods course intentionally disrupted (Weick, 1995) teachers‟ preconceptions of math teaching and spurred pedagogical growth. In particular, learning experiences promoting interdisciplinary STEM through the practices and real-world connections enhanced prospective secondary math teachers‟ pedagogical content knowledge (PCK). PCK may be considered a fusion of subject matter knowledge and teaching practices in order to help learners of various backgrounds, prior knowledge, culture, and experiences learn concepts and skills during instruction. This specialized knowledge base involves the knowledge of students, orientations toward teaching, knowledge of instructional strategies, knowledge of the curriculum, and knowledge of the teaching context necessary to effectively craft instruction to promote student learning of specific topics (e.g. Carlson, Daehler, & Alonzo, 2019; Gess-Newsome, 2015; Shulman, 1986). The findings of this study have explicit implications for three of the aforementioned domains of PCK: knowledge of students, orientations toward teaching, and knowledge of context.

Across all four cases, pre-service teachers identified experiences in which they were positioned as learners as essential to their sensemaking about how to teach mathematics from the lens of STEM. Notably, these included experiences in which they engaged with STEM learning experiences while wearing the "student hat" (Lowell & McNeill, 2020; Marco-Bujosa, González-Howard, et al., 2017). Sabrina and Dawn both identified these learning experiences as instrumental to their own instructional practice. Amy and Grace intentionally reflected on their experiences as learners in college math classes, also indicating this metacognition is essential. This finding shows providing pre-service teachers with opportunities to reflect upon student thinking and engagement with STEM and mathematics (Lowell & McNeill, 2020) over the course of the semester is essential to promoting PCK of students.

In addition, Methods disrupted pre-service teacher orientations toward math teaching. While most research on math teacher education has found pre-service teachers tend to reproduce the type of instruction they themselves experienced as students (Baxter & Williams, 2010; Clarke, & Ellis, 2005; Singleton, 2015), an education phenomenon referred to as the "apprenticeship of observation" (Lortie, 1975, p. 61), the findings of this study indicated their experiences in Methods shifted their pedagogical stance by disrupting their preconceptions about the learning and teaching of math. The disruption was first created with readings, video, sample lesson plans, and classroom learning experiences that emphasized STEM education (Tables 1 & 2). These teachers exhibited growth when they were provided with multiple opportunities to confront their preconceived notions of teaching with the pedagogical goals of practice-based teaching (e.g. Allen & Penuel, 2015; Marco-Bujosa, González-Howard et al., 2017, Berland et al., 2016). Specifically, learning experiences provided "images of practice" that allowed pre-service teachers to identify and contrast features of instruction that differed from their own experiences as learners (Marco-Bujosa, González-Howard et al., 2017). Moreover, the instructional design activity requirements, including a hands-on activity for the second mini-lesson and featuring disciplinary practices as learning goals in the final unit, *forced* pre-service teachers to challenge their pedagogical choices. The pre-service teachers were highly reflective of their instructional design process, and all four identified the instructional design activities as critical incidents in their understanding of how to teach math. In particular, their design choices, and interpretations about what they made these choices, illustrated attention to designing instruction aligning with the overarching goals of STEM education. They designed classroom structures and activities that tapped students into this type of thinking in a variety of ways, ranging from activities in which students explored geometric theorems (Dawn), engaged in design challenges (Dawn), engaged in activities to express their reasoning, such as Card Sorts (Sabrina) and journaling (Amy), or investigations (Sabrina). They were creative in their pedagogical design, applying and extending their knowledge in novel ways (Brown, 2009).

Despite evidence of pedagogical growth for all four teachers by the end of Methods (Table 6), student teaching contexts limited their ability to incorporate practice-based, real-world STEM into their math instruction. The most recent model of PCK explicitly elevates the importance of teachers developing knowledge of context (Carlson et al., 2019). Effective teachers must "have a deep knowledge of the learning context in which they teach, including knowledge of contexts that are both distal and more proximal to their students" (Carlson & Daehler, 2019, p. 87). Notably, Sabrina had a supportive learning environment in which to apply what she learned in Methods. Amy did not receive pedagogical support, but once she became familiar with the curriculum and her students, she was also able to implement STEM-aligned instruction (Table 6). In contrast, Dawn, working in a low-performing school, faced the restrictions of a structured, traditional curriculum. Grace, working in a high-performing and high-income school, faced similar pressure to cover content, but also faced resistance from students. These contradictions in pre-service teaching between the university and student teaching are well documented in the literature in math and science education (e.g. Marco-Bujosa, McNeill, & Friedman, 2020; Brown, Jones, & Bibby, 2004; Frykhom & Glasson, 2005; Nolan, 2012). For example, Brown and colleagues (2004) observed that these school norms are particularly powerful in reproducing traditional math instruction. They argued, "[s]chool mathematics, as it is presently conceived, seems to have a habit of deflecting people from creative engagement into more rule governed behaviour" (p. 177). These same barriers exist for STEM in student teaching. In a study for pre-service teacher integration of science and mathematics as student teachers, Frykholm and Glasson (2005) found "fostering understanding of the relationships and connections between mathematics and science is contextually based" (Frykholm & Glasson, 2005, p. 129). This had implications for pre-service teacher continued learning and implementation of the STEM framework; while Sabrina and Amy continued to grow and reflect, Dawn‟s energy was focused on adapting existing restrictive materials, and Grace found these contextual challenges insurmountable. Thus, the "apprenticeship of observation" (Lortie, 1975, p. 61) may be more challenging for teachers with a STEM approach unless cooperating teachers and the mathematics department are open and supportive of designing instruction to make these connections.

STEM at the Horizon of Mathematical Content Knowledge

Another contribution of this study is to offer insight into the construct of "knowledge of mathematics at the horizon" (Hill et al., 2008). While an extensive amount of research has examined the knowledge of effective math teaching, including the sources of knowledge, content expertise, and knowledge of students (Hill et al., 2008), that the findings indicate effective math teachers require not only knowledge of specific mathematical topics, but also but the special forms of mathematical knowledge, including applied mathematics and knowledge of mathematical connections (Deborah, Ball, Hill, & Bass, 2005; Hill et al., 2008; Hill, Rowan, & Ball, 2005). The findings of this study indicate that the STEM perspective of the Methods course facilitated pre-service teacher ability to make mathematical connections, both across mathematical topics and to other disciplines.

While originally conceptualized as illustrating the interconnectedness between mathematical ideas, Guberman and Gorey (2015) found teachers who were skilled at connecting also had different views of mathematics. The observed teachers, "did not teach mathematics as a collection of techniques, but rather as a connected discipline that has intrinsic logic and connections between its topics” (p. 178). Frykholm and Glasson (2005) also highlighted the ability to make connections as central to STEM teaching. They argued STEM teaching would be better promoted if, “the notion of connections between science and mathematics – connections that are situated authentically in the respective practices of each field and in the common experiences of learners,” (p. 130) that individual teachers could make, rather than a larger endeavor of interdisciplinary or integrated teaching. The findings of the present study indicated The STEM framework helped teachers critically examine their mathematical knowledge to consider both student engagement and motivation as well as the conceptual understandings necessary to develop mathematical expertise. Mathematical horizon knowledge developed and employed by the pre-service teachers in this study included the use of triangles in bridge design (Dawn), mathematically modeling the motion of bouncing balls (Sabrina), calculating interest on student loans (Amy), designing mobiles (Grace), and scaffolding student understanding of triangle congruence through transformations (Amy).

Implications

The findings of this study have implications for mathematics teacher education and future research. While emerging research indicates replacing traditional didactic pedagogies in mathematics is challenging (Abel, Search, & Salinas, 2020; Nolan, 2012; Selling, 2016), the findings of this study illustrate how this can be accomplished in teacher education. First, Methods courses should intentionally disrupt the knowledge pre-service teachers hold about math and how math is best taught. Students must have the opportunity to engage in learning math through practice-based instruction to shift their pedagogy (Selling, 2016). Teachers must also be supported and encouraged to apply this knowledge in instructional design and reflect intentionally on their pedagogical choices (Marco-Bujosa, González-Howard et al., 2017). Mathematics content courses must help prospective math teachers develop expertise beyond specialized knowledge. Coursework should feature common knowledge, utilized in the real-world, as well as knowledge at the horizon (Hill et al., 2008) as central components of the course, or as extensions for education majors.

Based on the findings of this study, a number of modifications could be made to the Methods and student teaching sequence to strengthen pre-service teacher learning and continued growth in STEM teaching. First, given the importance of pre-service teachers engaging in metacognitive reflection on their learning, either through student hat activities or in their own math classes, additional purposeful reflections on learning should be incorporated. While many reflections were conducted as whole class conversations following the student hat activities, findings indicate individual reflection is valuable. Individual journal prompts could be incorporated throughout the semester to encourage introspection on their learning from the student hat activities and to intentionally reflect on their own learning experiences in mathematics. As described by Anderson, Boaler, and Dieckmann (2018), "[t]he resolution of dissonance is not as easy as replacing ones‟ prior knowledge with new knowledge. As active participants in their sense making, teachers must participate in a deep analysis of current beliefs and proposed practices." (p. 4). Thus, analyzing their own experiences as learners is essential to developing knowledge of how students learn math. While pre-service teachers‟ instruction reflected the goals of STEM, these goals must be made more explicit in instructional design (Selling, 2016). Given that being *forced* to incorporate their new pedagogies promoted growth, stating the practices as instructional objectives and providing a STEM context should be required in Methods instructional artifacts and in student teaching to provide consistency across the contexts of Methods and student teaching.

Moreover, in student teaching, there is a role for sustained content-based support beyond the Methods course. Specifically, pre-service math teachers must be supported and encouraged to take pedagogical risks (Nolan, 2012), and to reflect on these experiences both individually and as a group (Van Driel & Berry, 2012). Findings indicate content-based support should be provided as pre-service teachers enact these new pedagogies, through peer collaboration among prospective math teachers and university supervision. Content-team meetings extending and building upon the strategies learned in Methods, facilitated by the instructor, could facilitate and enhance learning, and provide opportunities for teachers to discuss content-based challenges to enacting STEM. This sustained support could have assisted Grace in working around her contextual challenges, as she was the only pre-service teacher to not have the Methods instructor (and author of this manuscript) serve as her university supervisor for student teaching. There is also an essential role in student teaching for the placement coordinator to identify schools that would encourage creativity in math teaching.

While the findings of this study are supported by other research that teachers must experience new pedagogies as learners before they can change their instruction (Marco-Bujosa, González-Howard et al., 2017; Selling, 2016), more research is needed to identify additional strategies that create the type of dissonance necessary to shift their instructional goals. Moreover, the findings of this study indicated that pre-service teachers‟ shifting beliefs about mathematics were linked to learning how to teach. Documenting and understanding these beliefs and mindsets about mathematics, or dispositions, is complex and challenging (Nolan, 2012) yet is essential to transforming mathematics education and promoting authentic STEM connections in high school coursework. Finally, in order to assess the long-term impact of STEM in the Methods course, and to understand the role of teaching context, additional research is needed to explore the instructional practice of these teachers at different stages in their careers.

Conclusion

STEM education has the potential to prepare students with the skills and mindsets to address complex global challenges and economic needs (e.g. English 2016; Marginson et al. 2013; NAE & NRC, 2014; NSF, 2016; U.S. Congress Joint Economic Committee, 2012). However, the disciplinary silos of American high schools and the preparation of teachers have made authentic STEM education challenging to implement (Banilower et al., 2018). Moreover, despite education reforms promoting more practice-based and interdisciplinary thinking (AAAS, 1989; NGA & CSSO, 2010), most K-12 and higher education math classes continue to be taught through more traditional, didactic methods (Bolden, Harries, & Newton, 2010; Nolan, 2012; Selling, 2016). The findings of this study indicate that teacher education programs have a role to play in transforming math education and promoting STEM education. Learning to teach math from the framework of STEM shifted pre-service teacher beliefs about the nature of mathematics and instructional practice, and they designed and implemented instruction that promoted student conceptual understanding and ability to make mathematical connections, skills that promote student learning of mathematics (Boaler & Staples, 2008; Kazemi & Stipek, 2001) and the type of interdisciplinary thinking called for in STEM (NSF, 2016; U.S. Congress Joint Economic Committee, 2012). Pre-service teachers were able to accomplish this through the mathematical practices called for in the Common Core (NGA & CCSO, 2010), indicating that STEM education is aligned with disciplinary frameworks.

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