

*Full Length Research Paper*

# Investigation of knowledge and usage levels of problem-solving strategies of prospective classroom teachers

Emine Ozdemir<sup>1</sup> and Burcu Sezginsoy Seker<sup>2\*</sup>

<sup>1</sup>Department of Mathematics Education, Necatibey Faculty of Education, Balikesir University, Turkey.

<sup>2</sup>Department of Classroom Teaching, Necatibey Faculty of Education, Balikesir University, Turkey.

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Today, it is important how much individuals acquire knowledge and use this theoretical knowledge in their daily life. A qualified training program is expected to train individuals who can solve problems. The prospective classroom teachers, who are expected to train individuals who can solve problems, should also have knowledge about the problem-solving process and applying it to the problem situations that they face in their own lives. This situation is thought to have a positive effect on the academic achievement of the students who will be trained. The study was conducted at the beginning of 2019-2020 academic year with 52 third grade prospective classroom teachers. Semi-structured interview form prepared by the researchers were used to examine the knowledge and usage levels of the prospective classroom teachers about problem-solving strategies. This study concluded that prospective classroom teachers could informally use some problem-solving strategies, even if they were not trained. However, the prospective classroom teachers failed to perform as expected.

**Key words:** Problem-solving in mathematics, problem-solving strategies, prospective classroom teachers, teacher training.

## INTRODUCTION

The importance of problem-solving in teaching and learning of mathematics has been emphasized since the 20th century. Based on a model by Pólya (1949), intuitive thinking is at the forefront in the first phase of research on problem-solving, especially in the 1960s and 1970s. In the 1980s, however, it was emphasized that problem-solving should be the focus of school mathematics

(NCTM, 1980). In order to teach and learn problem-solving in mathematics courses, the scope of problem-solving has been expanded by adding cognitive and intuitive thinking, as well as student-oriented opinions such as opinions, attitudes, emotions, and self-regulation behaviors (Schoenfeld, 1985, 1987, 1992). Mayer (1982), Schoenfeld (1982) and Silver (1982) state that preliminary

\*Corresponding author. E-mail: sezginsoy@balikesir.edu.tr.

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information is a key element in the problem-solving process. Preliminary information affects the problem solver's understanding of the problem and the choice of strategies to be called in trying to solve it. Prior knowledge and experience are decisive factors in a problem solver's first attempt to solve a problem. In his book "How to solve it?" Polya (1949), sets out the process of problem-solving based on prior knowledge and experience. He summarizes his four-stage process as follows: understanding the Problem, devising a Plan, carrying Out the Plan, looking Back. The idea of how much problem-solving individuals can use the theoretical knowledge acquired in school mathematics in their daily lives is directly related to the concept of mathematical literacy.

The concept of literacy is defined as the competence of students to use their knowledge and skills, analyze, make logical inferences and communicate effectively while defining, interpreting and solving problems encountered in various situations in the main subject areas. The increase in science and technology as 21st century skills in our daily lives also highlights digital competence. Digital competence, along with literacy, brings out the need to raise individuals with mathematical thinking and problem-solving skills. The general aims of the Mathematics Curriculum related to the subject are stated as;

*"3. Will be able to express their thoughts and reasoning easily in the problem-solving process and will be able to see the gaps or deficiencies in the mathematical reasoning of others. 9. Will be able to develop a positive attitude for mathematics through his / her experiences in learning mathematics and develop a self-confident approach to mathematical problems (MNE, 2019).*

However, international examinations such as PISA and TIMSS reveal that our students do not perform as expected. PISA research mainly evaluates students' skills in the fields of science, mathematics and reading skills. PISA aims to assess how students can make sense of what they know, and how they can apply mathematical knowledge, including new and unusual situations. To this end, PISA refers to real-life situations where many of its math units and questions require mathematical skills to solve a problem. In addition, students are expected to have the ability to formulate, use and interpret mathematical situations in the field of mathematics literacy. In TIMSS applications, concept and processing skills which are fundamental in learning mathematics are measured in routine and non-routine (in a context that students are unfamiliar with) problem-solving and reasoning skills. In order to measure these skills, questions are prepared by taking into account two dimensions: the learning area and the cognitive area. While PISA targets 15-year-old students, TIMSS evaluates the performance of 4th and 8th grade students. Turkey, with an average of 420 in mathematics literacy

field, is below the average in of all countries. In contrast to the results of PISA (2015), the average achievement of 31% of schools in Turkey, are above the international average according to TIMSS 2015 mathematics results. It is seen that approximately 30% of the students in the 4th grade in lower level schools do not reach the basic level of knowledge and skills in mathematics. In the 8th grade, this rate is approximately 40%. (TEDMEM, 2020).

The relationship between teachers' participation in professional development activities and student achievement is examined. For this purpose, in the TIMSS 2019 cycle, teachers were asked about professional development activities they have participated in the last two years. In the questionnaire, mathematics teachers' participation in professional development activities on mathematics course content, mathematics teaching, mathematics curriculum, integration of technology into mathematics teaching, developing students' critical thinking and problem-solving skills, measuring and evaluating mathematics, and meeting the individual needs of students were investigated.

Participation rates in development events, international is running lower than the average. Especially in the last two years, students' critical thinking and problem-solving in-service education to develop skills to be an average rate of 44% in international students with 4th-grade math teacher are noteworthy that despite 17% in Turkey. In line with the answers of the teachers, more than 80% of the students had both mathematics and science teachers' 'integration of technology into mathematics/science teaching' and the professional needs of 'developing students' critical thinking and problem-solving skills come to the fore (TEDMEM, 2020).

It is thought that the professional development of teachers can improve students' achievement by improving their teaching practices. In addition to the contribution of in-service trainings to professional development, it is important that programs that train classroom teachers should also train literate and problem-solving teachers. There is no direct course where problem-solving strategies are taught and non-routine problems are solved by using strategy in classroom education undergraduate program. Furthermore, problem-solving is introduced for the first time in the third grade of the undergraduate program in the scope of mathematics teaching 1-2 courses. This situation negatively affects the training of qualified teachers.

In order to increase the problem-solving success in mathematics course, first, it is necessary to examine the thinking and solution ways of the students (Krawec, 2014). It is thought that knowing the thinking styles of the students by the teachers will affect their applications in the classroom and their learning to a great extent (Bozkurt et al., 2011; Krawec and Montague, 2014). In the literature, it is recommended to examine the problem-solving skills of primary school students without being informed about the strategies. Thus, they are expected to

try alternative approaches and come up with creative solutions. Studies show that students use some problem-solving strategies informally, although they have not received any training on this subject (Yazgan and Bintaş, 2005). There are studies showing that problem-solving skills can be learned (Anzai and Yokoyama, 1984; Çelik and Güler, 2013; Artut and Tarım, 2009; Ersoy and Güner, 2014; Gökkurt et al., 2015; Verschaffel et al., 1999).

In the study conducted by Altun et al. (2007), the effects of the education on problem-solving strategies given to the students of classroom teacher training programs on problem-solving success were examined as well as their opinions on problem-solving strategies. The study was carried out on 120 prospective classroom teacher students. The students were given a 5 - week training and their learning strategies and problem-solving success levels were determined by applying pre-test and post-test. Teaching has been effective in teaching all strategies except equation writing and reasoning and has led to increase the problem-solving success. It was found that problem-solving success could be explained by three factors, and it was concluded that the strategies of looking for a pattern, working backwards, simplifying the problem, making systematic lists, reasoning and making a drawing or diagram strategies were strong in pointing the problem-solving success. All of the students stated that teaching strategies subject to the study should be included in teacher education.

In the study conducted by Dede and Yaman (2005), primary school mathematics and science textbooks were examined in terms of problem-solving and problem posing activities. In the examination of the textbooks, "problem posing and solving scale" consisting of 17 items was used. As a result of the research, it has been determined that problem-solving and problem posing activities are not adequately included in science and mathematics textbooks. In addition, it was found that science textbooks include more problem-building and problem-solving activities than mathematics textbooks. Tertemiz et al. (2014) revealed that the most preferred strategy of prospective classroom teachers was "writing an equation or inequality" strategy, and "looking for a pattern" and "making a drawing or diagram" strategies were among the mostly preferred strategies.

When the domestic literature on problem-solving is examined, there are studies on the problem-solving strategies, approaches and process choices of elementary school students and prospective teachers (İskenderoğlu et al., 2004; Yazgan and Bintaş, 2005; Artut and Tarım, 2006; Altun et al., 2007; Altun and Memnun, 2008; Avcu and Avcu, 2010; Tertemiz et al., 2014) the relation of problem-solving and mathematics success (Özsoy, 2005; Karataş and Baki, 2017), the effect of problem-solving education on the choice of problem-solving strategies (Arslan and Altun, 2007; Ramirez et al., 2016).

When the foreign literature is examined, there are studies on the problem-solving strategies of students and prospective teachers (Cai, 2003; Van Dooren et al., 2003; Elwan, 2016), the effect of the field knowledge of prospective classroom teachers on the evaluation of students' problem-solving strategies (Van Dooren et al., 2003), the relationship between problem-solving strategies and problem-solving success, the effect of problem-solving education on the choice of problem-solving strategies, (Verschaffel et al., 1999).

Problem-solving can be considered as one of the metacognitive strategies used to understand how an individual performs thinking and learning processes when faced with a mathematical problem and how he/she recognizes his / her knowledge. Metacognition is a form of cognition, it is a high level thinking process that involves active control over cognitive processes. In today's world, instead of individuals that memorize and store the information and are expected to reflect exactly what they have been taught; individuals that think, learn information by making sense of it, optimally make use of their thinking abilities and are able to reflect their own comments and understanding in addition to their acquisitions need to be educated. (Gama, 2005; Kazu and Ersözlü, 2008 cited by Koçak and Boyacı, 2010).

This study investigates the pedagogical knowledge and metacognition about problem-solving of prospective teachers and their abilities in the use of strategies. Change in education also includes the change in teacher education (National Council for Accreditation of Teacher Education (NCATE) 2008). The qualifications, competencies, and knowledge that teachers should have in the field of teacher training are constantly updated. With various approaches, theories, and models, the issue of what knowledge teachers have and how to train them with this information is constantly being examined. In this context, the type and quality of knowledge that the teacher to be trained gains importance.

As a result, it is expected to train individuals who can "solve problems" from a qualified training program. In this context, the aim of this study is to examine whether the prospective classroom teachers that are expected to train "problem-solving" individuals have knowledge about problem and problem-solving, problem-solving processes and problem-solving strategies and how they can use problem-solving strategies in the situations they encounter.

In this study, answers to the following questions were sought:

- 1) What is the level of knowledge of prospective classroom teachers about problem and problem-solving concepts?
- 2) What is the level of knowledge of prospective classroom teachers about problem-solving strategies?
- 3) What is the level of using problem-solving strategies of the prospective classroom teachers?

## MATERIALS AND METHODS

In the study, case study, which is one of the qualitative research designs, was adopted. Case study is defined as a document or a special case for a single issue (Bogdan and Biklen, 2003). With the case study, a rich description of the situations in the problem-solving process and the level of knowledge and use of the problem-solving strategies of the prospective classroom teachers (Hitchcock and Hughes, 1995) was provided.

The criteria sampling method was used when creating the study group. Problem-solving Basic mathematics subjects such as numbers and operations, ratio-proportion, relations, sets, data collection, and conversion to graphs are taught in the first year of the primary school undergraduate program in the full term within the scope of the Basic Mathematics 1 course. In the first grades, in the spring term, algebraic expressions, equations, and inequalities, basic geometry knowledge, and drawings are taught within the scope of the Basic Mathematics 2 course. In the third year, information is given about problem types, problem-solving strategies, and stages within the scope of the Mathematics Teaching 1 course in the full semester, limited to two weeks. In this context, while determining the study group, it was determined that the prospective teachers had not received any problem-solving training, but had taken basic mathematics lessons. For this reason, the study was conducted at the beginning of 2019-2020 academic year with 52 third grade prospective classroom teachers studying at Balıkesir University Necatibey Faculty of Education.

In the study, semi-structured interview form prepared by the researchers was used to examine the knowledge and usage levels of the prospective classroom teachers about problem-solving strategies. In the first part of the form, prospective classroom teachers were asked to express mathematics problem and problem-solving in mathematics with at least three concepts in order to determine whether they have knowledge about problem and problem-solving. The prospective classroom teachers were not asked to make a mathematical definition, but they were asked to list the first concepts that came to their minds.

In the second part of the form, they were asked to explain their problem-solving strategies in a few sentences. Then, two problems were addressed in order to see whether they use these strategies effectively in problem-solving. While selecting the problems, non-routine problems from the domestic and foreign literature and the strategies used to solve them were searched. As a result of this survey, it was found that the six most common problem-solving strategies were studied. These strategies are prediction and control, making a drawing or diagram, looking for a pattern, simplifying the problem, making systematic lists and working backwards. Two problems in which these strategies can be used were used as data collection tools in the study. The original versions of the problems are in the book titled "Problem-Solving in Mathematics (for 3-6th Grades) Powerful Strategies to Deepen Understanding" written by Posamentier and Krulik (2009) and translated into Turkish by Akgün et al., 2016.

There are two items in the third part of the form. The first item states, for the problem "Ali has a frog. This frog can jump 1 or 2 meters at a time. How many different ways can the frog jump 7 meters?", specify the strategy or strategies you will use to solve the problem. Strategies to be used in problem-solving are making systematic lists, acting-out, drawing figures or diagrams or finding correlations. The second item is "there is a snail at the bottom of a 16 cm tall glass. How many days can the snail climb up 4 cm during the day and slide 1cm back at night?", Specify the strategy or strategies to solve the problem and solve the problem. Strategies to be used in the solution; making a drawing or diagram, looking for a pattern, working backwards or acting-it-out. The solution to this problem also needs to be associated with daily life. There are many examples of the second problem, also known as the snail problem, in mathematics textbooks at different grade levels where only

modifying numerical data are used.

Qualitative data were analyzed using descriptive analysis technique. Qualitative data were analyzed separately by two researchers, and thematic coding was performed. The reliability of the study was tested based on the reliability formula proposed by Miles and Huberman (1994). Accordingly, the reliability was ensured as reliability coefficients for themes, and codes were calculated with the formula "reliability = number of agreements / (number of agreements + number of disputes)" were higher than 70%. In order to increase the reliability, prospective classroom teachers' solutions and opinions were directly quoted. Instead of the names of the prospective teachers participating in the study, the acronym PT was used and these abbreviations were numbered from 1 to 52.

## RESULTS

### First research problem

#### *Concept of mathematical problem*

In this study, where the level of knowledge and use of prospective classroom teachers in relation to their problem-solving strategies was investigated, prospective classroom teachers were asked to list the concepts that came to mind when math problem is first mentioned. The aim here is to examine whether the concept of problem and the concepts of problem-solving are confused or not. After the descriptive analysis of the concepts stated by the prospective teachers about mathematics problem, these concepts were collected under themes.

The concepts of problem and problem-solving were taken into consideration while themeizing the concepts. In the most general sense, problem is the case that the person wants to do something but cannot predict what to do immediately. Problem-solving is knowing what should be done in such situations where it is not known what to do. Polya's four-stage problem-solving process is widely used. This process includes understanding the problem, selecting the strategy related to the solution, implementing the strategy and discussing the solution.

The results of the descriptive analysis of the concept of mathematics problem are given in Table 1. It was determined that 10% of the 190 concepts stated by the prospective teachers were related to the definition of mathematics problem. 65% of 190 concepts are expressions of mathematical coverage. It has been revealed that the 37 concepts used for the math problem are related to problem-solving.

A problem is defined as being complex or the result is uncertain, needs to be solved and arouses curiosity. A problem is also a challenge, the solution requires research, discussion or mind gym. There is no preliminary preparation for the solution and it has not been encountered before.

Those who describe the problem informally, PT12 has defined the problem as "difficult but solvable, there are those who have nothing to do with daily life and some of them are wastetime for students" and provided an opinion

**Table 1.** Descriptive analysis results related to the concept of Mathematics problem.

Theme	Code	f	%
For the definition of the problem	Difficulty	9	4.74
	Thinking	2	1.05
	Struggle/speculate	3	1.57
	Logic	2	1.05
	Life	1	0.53
	It may not be related to daily life	1	0.53
	Taking time	1	0.53
<b>Subtotal</b>		<b>19</b>	<b>10</b>
For problem-solving	Four operations (addition/subtraction/ multiplication /division)	23	11.82
	Solution	5	2.57
	Result	2	1.03
	Setting up an equation	2	1.03
	Reading comprehension	1	0.51
	Interpretation of understanding	1	0.51
	Given /data	1	0.51
	Requested	1	0.51
	Processing from the mind	1	0.51
<b>Subtotal</b>		<b>37</b>	<b>19</b>
For mathematical content knowledge	Equation	27	14.27
	Numbers (Including four operations)	26	13.73
	Age, pool, worker, speed problems	26	13.73
	Unknowns	13	6.86
	Formula	4	2.11
	Ratio-proportion	4	2.11
	Geometry	3	1.59
	Probability	3	1.59
	Fractions	3	1.59
	Equality	3	1.59
	Inequality	2	1.06
	Identities	1	0.53
	Term	1	0.53
	Trigonometry	1	0.53
	Algebra	1	0.53
	Integral	1	0.53
	Digit	1	0.53
	Symbol	1	0.53
	Equivalent	1	0.53
	Relation	1	0.53
<b>Subtotal</b>		<b>123</b>	<b>65</b>
For positive attitude	Delight	1	0.666
	Happiness	1	0.666
	John Nash	1	0.666
<b>Subtotal</b>		<b>3</b>	<b>2</b>
For negative attitude	Paragraph (length of problem sentence)	3	1.5
	Higher education exam	3	1.5
	Abstract concept	1	0.5
	Mathematics teacher	1	0.5
<b>Sub total</b>		<b>8</b>	<b>4</b>
<b>Grand total</b>		<b>190</b>	<b>100</b>

about problem-solving rather than problems. Especially due to the nature of non-routine problems, their solutions are time consuming. Therefore, the expression "waste time for students" was not evaluated as a negative attitude, but was examined in relation to the definition of the problem.

PT24, provided an opinion of solving the problem more than problem by stating expressions "converting verbal language into mathematical language (equation), understanding what you read and the interpretation of what you understand". On the other hand, PT25 presents both problem and problem-solving data with the concepts of "numerical data, four operations and difficulty". Similarly, PT33 and PT35 use the concepts "equation, operation and difficulty" for the problem.

PT48 provides both mathematical coverage and problem-related data with the expressions "numbers, four operations, thinking, fractions", while PT49 provides data on both mathematical scope and problem with the expressions "four operations, equations, logic, numbers". PT52 develops codes related to problem, problem-solving and mathematical scope themes with the expressions "operation, algebra, number, occupation, difficult".

It has been determined that other prospective classroom teachers mostly use expressions that form the theme of mathematical scope. For example, the concepts used by PT39 are "age, pool, worker problems, unknowns, equations, inequality" whereas the concepts used by PM9 are "x, ratio, triangle, equality, equivalence". PT51, emphasized negative attitude and mathematical scope with the expressions "formula, difficulty, exam, equation, unknown", and PT1 emphasized problem and problem solution with the expressions "struggle, result, delight and happiness".

### **Concept of problem-solving in mathematics**

Prospective classroom teachers were asked to rank the concepts that came to their minds when problem-solving in mathematics mentioned. The 152 concepts obtained accordingly are collected under 6 themes and given in Table 2. The first four themes are about the problem-solving process, while the other two themes are about computational tools and attitudes. It is a remarkable finding that 94.1% of the concepts presented by prospective classroom teachers are directly related to problem-solving.

When determining the codes forming the theme of understanding the problem, it was examined whether the prospective classroom teachers wrote concepts that would answer two basic questions: "(1) What are the data, what are the conditions? (2) What is unknown / requested? Since if the student / teacher can write what is given and what is requested, he has fully understood the problem.

Other criteria of the theme of understanding the problem were also used as in the literature. These criteria

are;

- 1) can the student read the problem with appropriate emphasis considering meaning?
- 2) can the student know if the problem is missing or there is more information?
- 3) can the student see what information is obtained from the problem?
- 4) can the student make a drawing or diagram suitable for the cases and relations in the problem?
- 5) can the student divide the problem into sub-problems?

While the concepts forming the theme of selecting the strategy related to the solution were investigated, after the problem was understood, the appropriateness of the relationship between those given and unknown in the problem was investigated. There are criteria that can be used in this theme. If the prospective teacher / student cannot find an immediate relationship, prospective teacher / student should ask him / her some questions. These questions are:

- 1) Have I solved another problem like this before? What did I do there?
- 2) Do I know a correlation that will work in solution?
- 3) Do I use all the information in my designed solution?
- 4) Can I guess the answer to this problem? Which values can the answer be?
- 5) Can I solve the problem part by part? How close do I approach to solution each time?

While determining the codes that will form the theme of the implementation of the strategy, the actions taken by using the chosen strategy during the step-by-step solution of the problem were taken into consideration. Performing arithmetic operations is seen during the implementation of the strategy and related codes are gathered under this theme.

When assigning codes to the theme of discussing the solution, the actions to be taken at this stage of problem-solving are taken into account.

These actions are:

- 1) check the accuracy and appropriateness of the results,
- 2) solve the problem in other ways, if any and
- 3) express the different forms of the problem and think how the solution will be in this case.

The accuracy and relevance are also checked thanks to these questions. The expressions of the prospective classroom teachers about the concept of problem-solving in mathematics are settled as examples. PT5 has provided views on the themes of discussing the solution, attitudes, understanding the problem by using the words "failure to solve, low grade, complexity". The statements of the PT7 as "thinking, following steps, creating equations, concluding" are related to the themes of choosing the strategy for the solution and implementing the strategy. Similarly, PT50 has provided data on the themes of selecting the strategy and implementing the

**Table 2.** Results on descriptive analysis on the concept of problem-solving in Mathematics.

Themes	Codes	f	%	
Understanding the problem	Analysing	4	2.66	
	Complexity	2	1.32	
	Unknown	2	1.32	
	Struggle	2	1.32	
	Understanding logic	1	0.66	
	Given/Data	1	0.66	
	Requested	1	0.66	
<b>Subtotal</b>		<b>13</b>	<b>8.6</b>	
Choosing the strategy related to the solution	Reasoning	9	5.91	
	Formulas/rules	8	5.26	
	Tactical / method development	6	3.94	
	Thinking	5	3.29	
	Operation priority	4	2.63	
	Equivalent	2	1.32	
	Equality	2	1.32	
	Ratio/proportion	2	1.32	
	Step-by-step progress	2	1.32	
	Systematic thinking	1	0.65	
	Inequalities	1	0.65	
	<b>Subtotal</b>		<b>42</b>	<b>27.6</b>
Implementation of the strategy	Finding a result/solution	20	13.16	
	Operations	19	12.50	
	Creating equations	19	12.50	
	Looking for a pattern	2	1.32	
	Writing equation	2	1.32	
	Simplifying the problem	2	1.32	
	Making table	2	1.32	
	Replacement	1	0.66	
	Finding the unknown	1	0.66	
	Collecting the same graded terms	1	0.66	
	Simplifying the problem	1	0.66	
	Trial-error	1	0.66	
	Solving with drama	1	0.66	
	Using ratio-proportion	1	0.66	
	Pooling/grouping	1	0.66	
	X to the unknown	1	0.66	
	Elimination	1	0.66	
	Sorting data	1	0.66	
	<b>Subtotal</b>		<b>77</b>	<b>50.7</b>
	Discussion of the problem	Practice	3	1.97
Interpretation		2	1.31	
Different solutions		2	1.31	
Proof		2	1.31	
Success in solution		1	0.65	
Inability to solve		1	0.65	
<b>Subtotal</b>		<b>11</b>	<b>7.2</b>	
Calculation tools	Abacus	2	1.1	
	Calculator	1	0.55	
	Paper-pen	1	0.55	

Table 2. Contd.

	The multiplication table	1	0.55
	Finger account	1	0.55
<b>Subtotal</b>		<b>6</b>	<b>3.3</b>
	High school	1	0.65
Attitudes	Low grade	1	0.65
	Abstract	1	0.65
	Intelligence	1	0.65
<b>Subtotal</b>		<b>4</b>	<b>2.6</b>
<b>Grand total</b>		<b>155</b>	<b>100</b>



**Figure 1.** The results of the descriptive analysis that reveal prospective classroom teachers' level of knowledge of problem-solving strategies.

she/he used are “equality list, finding unknowns and collecting terms of the same rank”. The PT3 presented data on problem-solving in mathematics using the concepts of “formula, abacus and finger calculation”. These concepts have been examined under the themes of analyzing and selecting the method related to the solution. Another prospective teacher who refers to “abacus and multiplication table” among the calculation tools is PT4. PT44 is the prospective teacher who states “paper-pencil”. The concepts that PT42 states as “systematic thinking, abstract, step-by-step progress, finding the result” are examined under the themes of attitudes, determination of the solution-related strategy and implementation of the strategy.

### Second research problem

The prospective classroom teachers were asked to explain the strategies in a few sentences in order to determine whether they have information about the problem-solving strategies. The qualitative data obtained were considered to be acceptable as definition and

not acceptable as definition. When the leaving blank rates for each strategy are analyzed, it is seen that strategies for working backwards and finding correlations are prominent. While the strategy that teacher trainees could define at least was to make a systematic list, the most successful strategy in defining was making a drawing or diagrams. The act-out strategy is the second acceptable definition. When the prospective classroom teachers' explanations about the strategies to simplify the problem and make a systematic list are examined, it is noteworthy that these strategies are by far the most unacceptable definitions. Similarly, most of the prospective classroom teachers' explanations to estimation and control and backward working strategies were considered as unacceptable definitions. Figure 1 shows the results of the descriptive analysis that reveal the level of knowledge of problem-solving strategies.

### *Making a systematic list strategy*

A systematic list-making strategy is very useful for problems that require finding all the possibilities for a



**Table 3.** Results on systematic list making strategy.

Acceptable	Unacceptable
<p>PT35</p> <p>Sistematik liste yapma</p> <p>Bir durumu bütün haliyle görmek için yapılır, çözümü böyle yapılır.</p>	<p>PT23</p> <p>Sistematik liste yapma</p> <p>Çözümüne ulaşmak için verilerin listesini yapma</p>
<p>PT21</p> <p>Sistematik liste yapma</p> <p>Problemdaki verileri liste halinde ilgili olarak yazma.</p>	<p>PT22</p> <p>Sistematik liste yapma</p> <p>Verilen bilgileri ve istenen bilgileret sıralamak</p>
	<p>PT26. "Systematic, regular list making"</p> <p>PT29. "Solving questions step by step by making a list"</p> <p>PT36. "The known to one side, the unknown to the other side."</p>

**Table 4.** Results on making a drawing or a diagram.

Acceptable	Unacceptable
<p>PT7</p> <p>Şekil veya diyagram çizme</p> <p>Problemden verilen verileri şekil ve diyagramla beraber daha açık şekilde gösterilebilir.</p>	<p>PT42</p> <p>Şekil veya diyagram çizme</p> <p>Öğrencilerin dikkatini çekmek için yapılır.</p>
<p>PT21</p> <p>Şekil veya diyagram çizme</p> <p>Verileri grafikçe ve görsel bir şekilde gösterme</p>	<p>PT22</p> <p>Şekil veya diyagram çizme</p> <p>Havuz probleminde soruyu anlamayı kolaylaştıran yöntemdir.</p>
<p>PT35</p> <p>Şekil veya diyagram çizme</p> <p>Verilerin aralarında ilişkileri göstermek için yapılır.</p>	<p>PT6</p> <p>Şekil veya diyagram çizme</p> <p>Problem akıldan canlandırılarak somut bir hale getirilir.</p>

case. A system that ensures that no possibility is repeated must be established. In this context, the following description of PT23 is not considered as an acceptable definition. Example cases are given in Table 3.

### Drawing a figure or diagram strategy

The strategy for drawing figures or diagrams involves the use of visually supportive drawings to solve the problem. It is visualizing what is given in the problem by making a

drawing or diagram. The problem is given by drawing a figure or diagram. It can make it easier to understand both the problem and to see the relationship in the problem and reach the solution. Example cases are given in Table 4.

### Looking for a pattern strategy

The strategy of finding correlations is the only strategy taught in both primary and secondary school mathematics

**Table 5.** Results on looking for a pattern strategy.

Acceptable		Unacceptable	
PT41		PT6	
Bağıntı bulma	Verilen tek veri arasındaki bağlantıyı bulmak mümkün kolaylaştırıcıdır	Bağıntı bulma	Problemin çoğunda yönelle bağlantılar kurulur.
PT21		PT42	
Bağıntı bulma	Verilerin arasındaki ilişkiyi ortaya çıkarma	Bağıntı bulma	Konular arasındaki ilişkiyi kurabilmek.

**Table 6.** Results on working backwards strategy.

Acceptable	Unacceptable
PT11.	PT3.
Geriye doğru çalışma	Geriye doğru çalışma
Bizden istenilen soruda verilenlere doğru gitme.	Büyükten parçaya gitme.
PT40.	PT40.
Geriye doğru çalışma	Geriye doğru çalışma
Mezela üniversite öğrencisinin tercihiyle bir problemi daha iyi anlamak için 11 yaş altındaki çalışmalarına bakması, aynı yeniden tekrar alması olabilir.	

programs as “Patterns and desing”. This strategy involves finding repetitive shape / number sequences or repeating events sequences. It also allows the person to reduce a complex problem to a relation and then use the relation to produce a solution. This strategy is often used in combination with the Table 5 or diagram strategy and the simplification strategy.

### Working backwards strategy

In some problems, the person is given the situation reached as a result of all the actions that take place in the problem and is asked to find out what the original situation is. The working backwards strategy (Table 6) is particularly useful for such problems. If the result of arithmetic operations is given in the problem, what is to be done is to reverse the operations. If the result of a series of events is given, but not the arithmetic operation, it is necessary to start from the last stage and then examine the previous stage, then the previous stage, and

continue until it reaches the initial state.

### Simplifying the problem strategy

Simplifying the problem strategy (Table 7) is a strategy that examines the version of the same problem with simpler or smaller numbers when faced with a problem that seems difficult due to the complexity or size of the numbers. There are two cases:

- 1) exploring the solution of the original problem by solving the simpler form of the problem
- 2) to examine the problem with the smallest number that makes it possible, then increase the numbers gradually and get the generalization from which to solve the real problem.

### Guess and check strategy

The student using the guess and check strategy (Table

**Table 7.** Results on simplifying the problem strategy.

Acceptable	Unacceptable				
<p>PT3.</p> <table border="1"> <tr> <td>Problemi basitleştirme</td> <td>Problemi aşamalaştırıp küçük problemler halinde basitçe çözmeye</td> </tr> </table>	Problemi basitleştirme	Problemi aşamalaştırıp küçük problemler halinde basitçe çözmeye	<p>PT41.</p> <table border="1"> <tr> <td>Problemi basitleştirme</td> <td>Çözümleri sadeleştirip sadece bizden isteneni bulmak için kullanacağımız verileri ayıklamak</td> </tr> </table>	Problemi basitleştirme	Çözümleri sadeleştirip sadece bizden isteneni bulmak için kullanacağımız verileri ayıklamak
Problemi basitleştirme	Problemi aşamalaştırıp küçük problemler halinde basitçe çözmeye				
Problemi basitleştirme	Çözümleri sadeleştirip sadece bizden isteneni bulmak için kullanacağımız verileri ayıklamak				
	<p>PT11</p> <table border="1"> <tr> <td>Problemi basitleştirme</td> <td>Problem içerisinde geçen kavramların kavram isimine yansıyan bilgileri ayıklama</td> </tr> </table>	Problemi basitleştirme	Problem içerisinde geçen kavramların kavram isimine yansıyan bilgileri ayıklama		
Problemi basitleştirme	Problem içerisinde geçen kavramların kavram isimine yansıyan bilgileri ayıklama				

**Table 8.** Results on guess and check strategy.

Acceptable	Unacceptable				
<p>PT7.</p> <table border="1"> <tr> <td>Tahmin ve kontrol</td> <td>Problemın sonucu tahmin ederek bir varsayımda bulunma ve doğruluğunu kontrol etme.</td> </tr> </table>	Tahmin ve kontrol	Problemın sonucu tahmin ederek bir varsayımda bulunma ve doğruluğunu kontrol etme.	<p>PT35.</p> <table border="1"> <tr> <td>Tahmin ve kontrol</td> <td>Problem hakkında işlenmemiş çözüme ulaşma.</td> </tr> </table>	Tahmin ve kontrol	Problem hakkında işlenmemiş çözüme ulaşma.
Tahmin ve kontrol	Problemın sonucu tahmin ederek bir varsayımda bulunma ve doğruluğunu kontrol etme.				
Tahmin ve kontrol	Problem hakkında işlenmemiş çözüme ulaşma.				
<p>PT8.</p> <table border="1"> <tr> <td>Tahmin ve kontrol</td> <td>Sarıda birineğin bulunması gereken sayıyı onun yerine bir sayı deneyerek bulmak. Kı, bir sayı deneyiz, yerine koyarız ve doğru mu diye kontrol ederiz.</td> </tr> </table>	Tahmin ve kontrol	Sarıda birineğin bulunması gereken sayıyı onun yerine bir sayı deneyerek bulmak. Kı, bir sayı deneyiz, yerine koyarız ve doğru mu diye kontrol ederiz.	<p>PT42.</p> <table border="1"> <tr> <td>Tahmin ve kontrol</td> <td>İleride istenecek kavramlar için öngöründe bulunmak</td> </tr> </table>	Tahmin ve kontrol	İleride istenecek kavramlar için öngöründe bulunmak
Tahmin ve kontrol	Sarıda birineğin bulunması gereken sayıyı onun yerine bir sayı deneyerek bulmak. Kı, bir sayı deneyiz, yerine koyarız ve doğru mu diye kontrol ederiz.				
Tahmin ve kontrol	İleride istenecek kavramlar için öngöründe bulunmak				

8), starts with a logical estimation and tests the estimation. If the estimation is inaccurate, he makes another prediction. It's more than a simple trial and error. However, the new estimation should take into account the results of the previous one. This process continues until the student reaches a possible result in the problem. It's more than a simple trial-and-error process.

### Act-it-out strategy

Acting-it-out strategy (Table 9) can be especially useful in small classes. Children can really own the roles in the problem and portray the action. They can use bottle caps, chips, written papers, models or drawings.

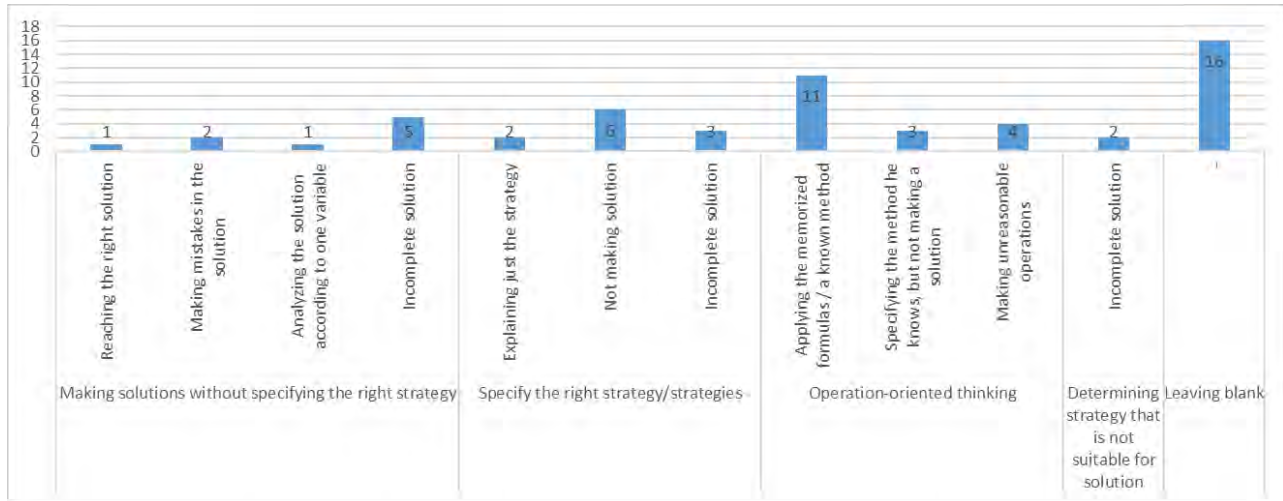
### Third research problem

#### Strategies used in the solution of the frog problem

The solution processes for each problem were examined and thematic coding was done in order to determine the level of use of problem-solving strategies of the prospective classroom teachers. According to this, five themes and sub-themes were obtained in relation to the solution of the frog problem. According to Figure 2, it is seen that even if the prospective classroom teachers determine the correct strategies for solving the problem, they are not able to make a complete solution by using these strategies. Only 1 out of 8 prospective classroom teachers who informally used the right strategy was able

**Table 9.** Results on act-it-out strategy.

Acceptable		Unacceptable	
PT39.		PT21.	
Canlandırma	Verilen problemlerdeki olay ve objelerin gözünde contlandırarak çözüm bulmaya çalışırım	Canlandırma	Problemin çözümünün doğruluğundan emin olma.
PT3.		PT6.	
Canlandırma	Problemi olasılıkla kılma adına sınıf ortamına entegre edip tiyatrolaştırma	Canlandırma	Problem dışınımda uygularım.



**Figure 2:** Strategies used in the solution of the Frog problem

to reach the right solution. Instead of understanding the problem, they focused only on the numerical values given in the problem and the structure of the problem, which showed that the prospective classroom teachers adopted more process-oriented thinking, Similar results are found in the literature.

The strategy of looking pattern is also used in the solution of the frog problem. The prospective classroom teachers would form the Fibonacci sequence informally as soon as they could find out the rule of the pattern in solving the problem. There have been prospective classroom teachers who have indicated that a correlation strategy could be used to solve the frog problem, but they have not presented data on the solution. Two prospective classroom teachers who have stated that they are using the correlation finding strategy are actually using the systematic list making strategy.

They could not complete the solution. In Table 10, there are examples of solutions to the frog problem of prospective classroom teachers'and evaluations made by

researchers on these solutions.

**Strategies used in the solution of the snail problem**

In order to determine the prospective classroom teachers' level of using problem-solving strategies, the solution processes related to the snail problem were examined and thematic coding was conducted. Accordingly, five themes and sub-themes were obtained for the solution of the snail problem. The results are presented in Figure 3.

According to Figure 3, it was revealed that 6 of the prospective classroom teachers who determine the right strategies make a complete solution using these strategies and 6 of them use the right proportion as a memorized method, but they do not pay attention to day-night distinction. 9 out of 10 prospective classroom teachers who informally used the right strategy achieved the right solution. While the prospective teacher should take into account the day-night variables given in the

**Table 10.** Examples of solutions related to Frog problem and evaluation of solutions.

Theme	Sub-theme	f	Example Solution	Evaluation of the Solution
			<p>ya da stratejileri belirleme ve problemi çözme.</p> <p>örneğin <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p><math>5 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p><math>3 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p><math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p><math>(6) = \frac{6 \cdot 5}{2} = 15</math></p> <p><math>(2) = \frac{2 \cdot 1}{2} = 1</math></p> <p><math>(4) = \frac{4 \cdot 3}{2} = 6</math></p> <p>21 farklı şekilde sırayabilir.</p>	PT12 was able to finish the solution with the correct reasoning and reach the correct conclusion.
Solving without specifying the right strategy	Making mistakes in the solution	1	<p>5. "Ali'nin bir kurbağası var. Bu kurbağa bir seferde 1 ya da 2 metre sırayabiliyor. 7 metreyi kaç farklı şekilde sırayabilir?" şeklinde verilen problemin çözümünde kullandığınız strateji ya da stratejileri belirtiniz ve problemi çözünüz.</p> <p>1 1 1 1 1 1 = 7</p> <p>1 1 1 1 2 = 6</p> <p>1 1 2 2</p> <p>1 1 2 1 2</p> <p>1 2 1 1 2</p> <p>2 1 1 1 2</p> <p>1 1 2 2 1</p> <p>1 2 1 2 1</p> <p>2 1 1 2 1</p> <p>2 2 1 1 1</p> <p>2 2 1 1 1</p> <p>2 2 2 1</p> <p>2 2 1 2</p> <p>2 1 2 2</p> <p>1 2 2 2</p> <p>9 + 7 + 6 + 4</p> <p>26 //</p> <p>6. "16 cm'lik bir kurbağaya bir seferde bir ya da iki metre sırayabilir. 7 metreyi kaç farklı şekilde sırayabilir?" şeklinde verilen problemin çözümünde kullandığınız strateji ya da stratejileri belirtiniz ve problemi çözünüz.</p>	PT1 did not specify what strategy s/he used in the solution. However, it can be said that s/he used a systematic list-making strategy from his solution. The first of the errors in the solution is that it always completes 7m in the event of a 1m jump. The other is that there are not 9 but 10 different situations in the cases of 1m in the event of 3 jumps and 2m in the event of 2 jumps.
	Analyzing the solution according to one variable	1	<p>1 1 1 1 1 1</p> <p>1 1 1 1 2</p> <p>1 1 2 2</p> <p>1 2 2 2</p>	PT14 has evaluated 2m jumps in its strategy to be none, once, twice and 3 times, but examines each situation once. S/he could not list all the situations.
			<p>ya da stratejileri belirleme ve problemi çözme.</p> <p>örneğin <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>1m 2m 2m 2m</p> <p>2m 1m 2m 2m</p> <p>2m 2m 1m 2m</p> <p>2m 2m 2m 1m</p> <p>1 1 1 1 1 1</p> <p>1 1 2 2</p> <p>Bütün olasılıkları hesaplayarak problemi çözebilirim</p>	PT10 uses a systematic list making strategy informally in its solution.
Making solutions without specifying the right strategy	Incomplete solution	5	<p>1. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>2. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>3. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>4. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>5. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>6. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>7. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>8. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>9. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>10. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>11. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>12. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>13. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>14. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>15. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>16. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>17. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>18. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>19. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>20. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>21. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>22. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>23. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>24. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>25. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>26. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>27. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>28. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>29. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>30. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>31. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>32. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>33. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>34. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>35. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>36. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>37. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>38. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>39. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>40. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>41. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>42. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>43. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>44. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>45. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>46. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>47. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>48. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>49. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p> <p>50. <math>1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1 \rightarrow 1</math></p>	PT28 is the only prospective teacher who, unlike others, tries to solve the problem using a tree diagram. However, when the solution of the problem is examined, it is seen that s/he does not specify every situation and that s/he makes mistakes.

Table 10. Contd.

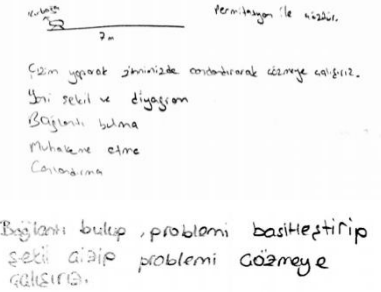
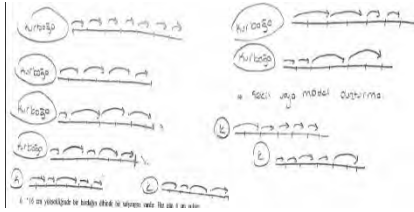
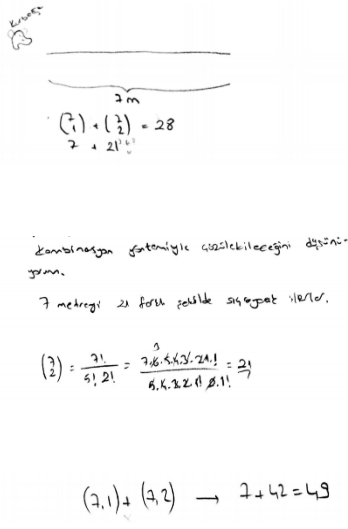
<p>Just explaining the strategy</p>	<p>2</p>	<p>5. "Ali'nin bir kurtbağsı var. Bu kurtbağa bir seferde 1 ya da 2 metre sırayabiliyor. 7 metreyi kaç farklı şekilde sırayabilir?" şeklinde verilen problemin çözümünde kullandığınız strateji ya da stratejileri belirtiniz ve problemi çözünüz.</p> <p>Çantlandırma, Tahtaya 1 öğrenci çizdirilir. 7 metre sırayılır. Öğrencinin bir sırayısı 1 metre olarak kabul edilir. Öğrenci önce birer metre sırayarak gider. Daha sonra farklı şekillerde deneneret öğrenci çözümce ulaştırılır.</p>	<p>PT11 has identified a strategy that can be used to solve the problem. It was determined that the the prospective teacher had information about the strategy to be appropriate to the solution of the problem from his / her statements, but s/he did not provide any data on the solution.</p>
<p>Specify the right strategy/strategies</p>	<p>6</p>	 <p>Çözüm yaparak zihninizde canlandırarak çözümü çalıştır. Yeni şekil ve diyagram Başlangıç bulma Muhakeme etme Canlandırma</p> <p>Başlangıç bulup, problemi basitleştirip seçtiği adımla problemi çözmeye geçtiğini.</p>	<p>PT23 lists the strategies that can be used. Unlike other prospective classroom teachers, it is seen that he has a strategy of reasoning. However, no finding on the solution has been reached.</p>
<p>Making incomplete solution</p>	<p>3</p>		<p>PT31 uses the making a drawing or a diagram strategy, which s/he states as a form or model creation statement, to solve the problem. This statement is acceptable and the solution is successful, but only 10 situations are illustrated. PT31 is the only prospective tearcer who uses the figure/diagram strategy correctly.</p>
<p>Operation-oriented thinking</p>	<p>11</p>	 <p><math>(7,1) + (2,2) = 28</math></p> <p><math>(2) = \frac{7!}{5! \cdot 2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 21</math></p> <p><math>(7,1) + (2,2) \rightarrow 7 + 42 = 49</math></p>	<p>PT4 thinks that the problem can be solved by combination. However, it was determined that the prospective classroom teachers has deficiencies in the process of meaning of the problem. Indeed, repeated permutations are used to solve this problem.</p> <p>Similar solution of PT8 is observed. The main point to focus on here is that prospective classroom teachers 1-4 as the grade level of the. to be able to teach mathematics at grade level. Here, the main point to be considered is that these prospective classroom teachers are to be teaching grade level of 1-4 mathematics at classroom level.</p> <p>In this context, teachers are expected to use methods appropriate to the level of their students.</p> <p>However, a third of the prospective classroom teachers tried to use a method they knew or memorized. PT19 thinks that the problem can be solved by permutation method, as well.</p>

Table 10. Contd.

Specifying the method s/he knows but not solving	3	<p>Kombinasyon ve Permutasyon kullanmak için gerektirir. Ancak bu konuda da eksiklikler olduğunu bu problemi nasıl aşmam gerektiğini biliyorum.</p>	<p>PT22 and PT17 state that the problem can be solved by one of the methods they memorized, but it is clear that they do not have the knowledge of these methods. However, if s/he could not focus on the process but visualize the problem situation in the simplest sense, s/he would be able to develop his own method. What is stopping the prospective teacher from mathematical thinking in this problem?</p> <p>Why do we encounter similar situations in our students? The Results of another recent study revealed a similar situation in secondary school students</p>	
Making meaningless operation	4	<p>Ösükle kullanılarak ösükle diye tahmin ediyorum. -İma hatırlanıyor</p>	<p>The meaningless operations and meaningless explanations made by Ö39 and Ö50 for the solution of the problem are given as examples.</p>	
Determining strategy that is not suitable for solution	Making the incomplete solution	2	$\frac{1}{2} \cdot 7 = \frac{7}{2}$	<p>The meaningless operations and meaningless explanations made by Ö39 and Ö50 for the solution of the problem are given as examples.</p>
Determining strategy that is not suitable for solution	Making the incomplete solution	2	<p>1 gelme sonucu <math>\frac{1}{2}</math> 2 prime sonucu <math>\frac{1}{2}</math> ?</p>	<p>Although PT20 has stated a strategy that can be used to solve the problem, s/he has tried to make a solution suitable for a different strategy. In his/her solution, it appears that s/he does not examine all situations.</p>
Determining strategy that is not suitable for solution	Making the incomplete solution	2	<p>1 7.1 → Başarılı bulma 2 5.1 + 2 3 2.2 + 3.1 4 2.1 + 5.1 5 2.3 + 1</p> <p>5 farklı şekilde</p>	<p>Although PT20 has stated a strategy that can be used to solve the problem, s/he has tried to make a solution suitable for a different strategy. In his/her solution, it appears that s/he does not examine all situations.</p>
Leaving blank	-	16	-	<p>It was found that the prospective classroom teachers made no explanation for the solution or simply wrote the phrase "I could not solve it".</p>

problem, only making progress over one day and therefore using the correct proportion showed that the prospective teachers adopted more operation-oriented thinking. In the solution of this problem, prospective classroom teachers should associate their solutions or their results with daily life. Although more positive results were obtained for the solution of this problem than frog problem, operation-oriented thinking could not be prevented. Table 11 shows the examples of the solution of the prospective classroom teachers about the Snail problem and the evaluations made by the researchers about these solutions.

## DISCUSSION

Prospective classroom teachers have associated the math problem most with mathematical scope. In addition, 19% of problem-solving was associated with the problem, while only 10% identified the concept of problem recognition. From this point of view, it is concluded that the prospective classroom teachers did not have enough information about mathematics problem.

A large majority of teachers (92.5%) associated problem-solving in mathematics with the problem-solving process. From this point of view, it is concluded that the

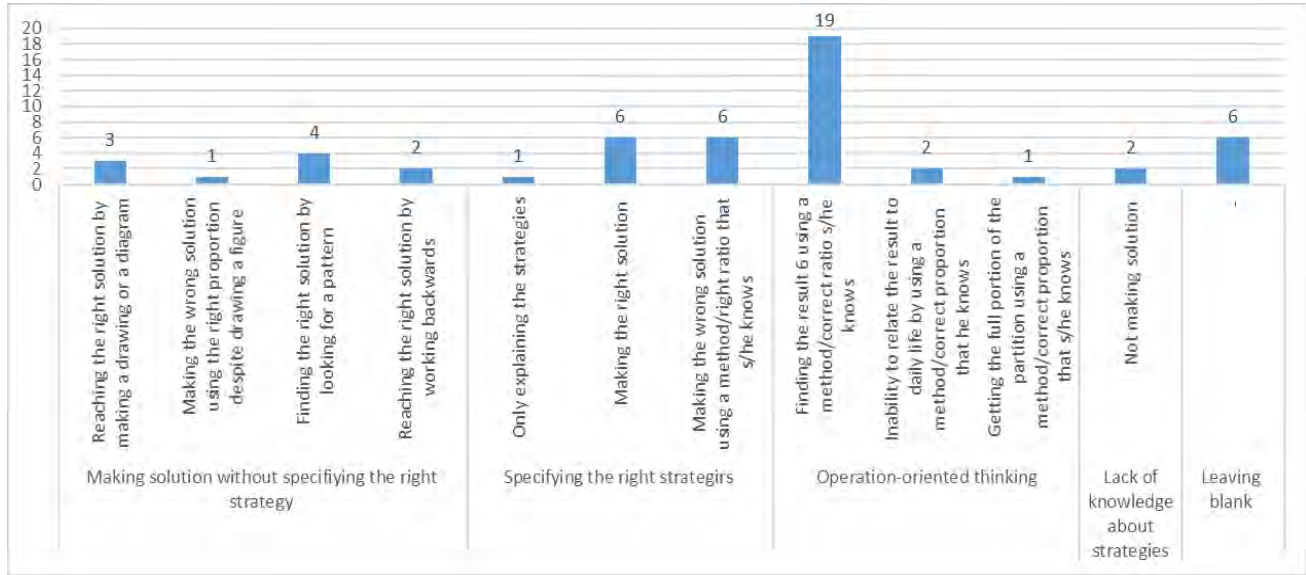


Figure 3. Strategies Used in the Solution of the Snail Problem.

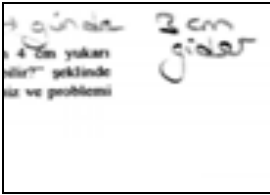
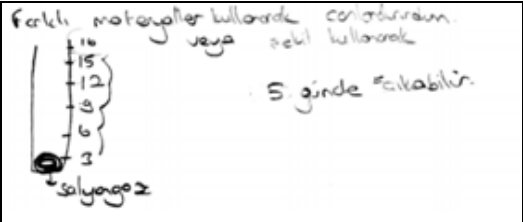
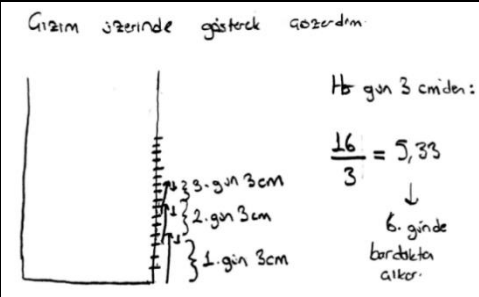
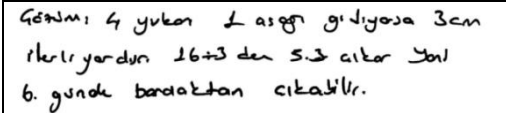
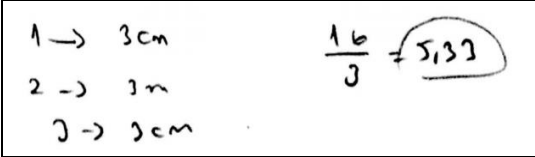
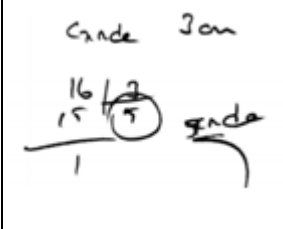
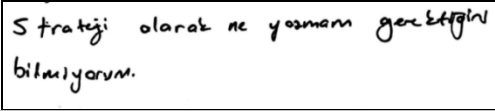
Table 11. Results on the strategies used in the solution of the Snail Problem.

Theme	Sub-theme	f	Example solution	Prospective teacher
	Reaching the right solution by drawing a figure	3		PT1
Making solutions without specifying the right strategy	Making the wrong solution using the right proportion despite drawing a figure	1		PT28
	Finding the right solution by finding the correlation	4	<p>4-1=3cm Her gün 3cm artırıyor</p> <p>16cm'ye <u>5</u> günde ulaşır</p> <p>5.gün 4cm ilerler</p>	PT7





Table 11. Contd.

				
				PT49
Specifying the right strategy	Making the wrong solution using the right proportion despite drawing a figure	6		PT3
	Finding the result 6 using a method/correct ratio he knows	19		PT22
Operation-oriented thinking	Inability to relate the result to daily life by using a method/correct proportion that he knows	2		PT4
	Getting the full portion of the partition using a method/correct proportion that it knows	1		PT17
Lack of knowledge about strategies	Not making solution	2		PT22
Leaving blank	-	6		

prospective classroom teachers have knowledge about problem-solving.

Although no training was given to prospective classroom teachers about problem-solving strategies, it was determined that the most acceptable strategy was the figure or diagram strategy. It is concluded that the

prospective classroom teachers have knowledge about the strategy of drawing figures or diagrams. The prospective classroom teachers explained this strategy by giving examples such as graphs and schemas in sets.

Rosengrant et al. (2007) emphasizes the importance of visualizing the problem while solving the problem. The

use of the most meaningful expressions in this strategy also predicts that prospective classroom teachers should use this strategy effectively. However, the rate of achieving the right solution by using the strategy of drawing figures and diagrams is quite low among the prospective classroom teachers. Similar results are also found in secondary school students.

Yazgan and Bintaş (2005) concluded in a study of the problem-solving strategies of elementary school fourth and fifth grade students that the students learned the use of some strategies informally, although they did not receive such an education. The results of the studies of Ersoy and Güner (2014) and Bal-İncebacak and Ersoy (2016) are also similar. Similar results were obtained in this study, as well.

- 1) None of the 11 prospective classroom teachers who stated the right strategy in the problem reached the right solution. Only 1 prospective teacher who informally made a solution with the right strategy without specifying the right strategy was able to reach the right result.
- 2) There are 6 prospective classroom teachers who specify the right strategy and reach the right solution in the problem. 9 prospective classroom teachers made solutions using strategy informally without specifying the right strategy and reached the right result.

This study did not include strategies of creating equations or inequalities, making tables, and reasoning that require high-level thinking. The strategy of establishing an equation or inequality can be used in high school, especially by 7th and 8th grade students. It is one of the strategies taught under the normal curriculum in connection with algebra and used best by students.

The main idea of a table-making strategy is to write the given data into the table in such a way as to reveal a relation and, thus to find the missing information. Tables usually consist of rows and columns that are list important variables. Usually the first cell of a row or column of the table becomes a starting point, from which the data progresses regularly.

At this point the table reveals the answer. This strategy is often used in conjunction with the strategies of drawing figures or diagrams, simplifying the problem, and finding correlations to produce the data in the table.

The solution of some problems is based on reasoning as the basic strategy. These problems may be problems that require simple logic, such as which product is better to take, or they may be more difficult problems that involve a chain of inferences. You make one inference, that inference gives rise to a second inference, and so on. This process will continue until the problem is solved.

Follmer (2000) investigated the effect of problem-solving skill on thinking ability. He studied with the 4th grade students. It has been found that thinking skills are very effective in problem-solving and students are more successful when they solve problems by recognizing their

thinking ability.

In literature, a study which reached a similar conclusion, it was revealed that the prospective classroom teachers were prone to operation-oriented thinking. Operation-oriented thinking was an identified problem in 18 prospective classroom teachers in the 1st problem while in the 2nd problem this number was found to be 22. As a result, operation-oriented prospective classroom teachers were not successful in reaching the correct result.

In their studies Bozkurt and Karşlıgil-Ergin (2018), they concluded that about one-third of the students in 4th and 5th grades, while a significant number of the class students in 6th grade was making meaningless operations with numbers.

It has emerged that the vast majority of the students in the classroom are not proficient in problem-solving and determining solution strategies correctly. It was found that the majority of 4th, 5th and 6th grade students were insufficient to determine problem-solving and solution strategies correctly. So much so that when they encounter a problem, they tend to take a look at the problem and quickly apply the operations that need to be applied to the numbers and find the result. In the Singapore study (Cai, 2003), when solution types are examined, it is observed that students use numerical solutions, and this usage increases in proportion to grade level.

Similar cases was found in this study. Among those who stated that permutation/combination could be used in the solution of the first problem, there were cases of making operation mistakes, making meaningless operations and leaving the problem unsolved. Prospective classroom teachers think that the frog problem can be solved by a method they have memorized or already known. However, it is seen that these thoughts do not motivate them to solve the problem, but rather cause them to make no attempt. As a matter of fact, leaving the frog problem blank without any solution was observed in 16 prospective classroom teachers, while in the snail problem this situation was observed in 6 prospective classroom teachers.

## Recommendations

Based on the results of this study, elementary school students, like classroom prospective classroom teachers, should encounter more frequent problems that are not routine. Non-routine problems should be included in textbooks that are an effective resource in teaching mathematics. Thus, students will be able to develop solutions to everyday situations from a mathematical perspective.

This study concluded that prospective classroom teachers could informally use some problem-solving strategies, even if they were not trained. However, the prospective classroom teachers failed to perform as

expected. At this point, it can be examined by another study whether classroom prospective classroom teachers' awareness of problem-solving strategies will change the results of the practice.

Teachers should ensure that students develop their operational skills, recognize different problem types and solutions with the various problems examined, and recognize and address various real-life problems appropriate to their level. In this context, it is important to include applied problems that reflect real-life that will enable students to think metacognitive, open-ended problems for conceptual understanding, and mathematics research/projects in addition to problems that require four operations (Foong, 2002; cited by Tarım and Hacıömeroğlu, 2019).

In order to improve math literacy and achievement in international exams mathematics applications were added to the 5th and 8th grades. It may be suggested to include them in the curriculum starting from the 1st grade and to include problem-solving practices within the scope. Seeing that different strategies are used and solutions are reached in problem-solving will also enable the student to determine the best strategy for the solution. Individuals who can solve problems are expected to understand the problem well, develop strategies for the solution, make solutions with this strategy and discuss the solution. Although understanding the problem is very effective in solving the problem, every stage of the problem-solving process is important. Especially in the final stage, problem writing skills are also gained. As a continuation of this study, problem writing skills of the prospective classroom teachers who have problem-solving training can also be investigated.

## CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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