Pre-service Teachers as Agents of Change in the Mathematics Classroom: A Case Study

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The pre-service teacher followed in this paper aimed to act as an agent of change during a time of reform in Irish secondary mathematics education. A teacher-researcher led action research project was conducted to depict the reality of the situation regarding the implementation of mathematical problem solving from a variety of perspectives-namely those of the pre-service teacher/teacher-researcher, the secondary school students, and the other mathematics teachers in the school. A problem-based approach to teaching and learning was implemented by the pre-service teacher using Van de Walle's teaching through problem solving framework in an attempt by the pre-service teacher to enact change at the classroom level. Following this intervention, participating students showed an improvement in engagement with mathematical problem solving and expressed positive opinions regarding the problem-based approach to teaching and learning. Meanwhile, other mathematics teachers in the school expressed concerns relating to the reality of implementing problem solving in their classrooms; however, these concerns were not issues faced by the author when implementing problem solving in the same setting. When these concerns were indirectly challenged by the pre-service teacher a change in conversation occurred.

Keywords · pre-service teacher · agent of change · problem-based learning · teaching through problem solving · teacher assumptions

Introduction

Over the past ten years the Irish secondary[1] mathematics curriculum has undergone a major reform. It included the introduction of a new curriculum, 'Project Maths' (National Council for Curriculum and Assessment [NCCA], 2011), which was designed with an aim of transforming mathematics education in Ireland. The pre Project Maths curriculum was criticised for being procedure focused, over reliant on textbooks (Lyons, Lynch, Close, Sheeran, & Boland, 2003), very abstract (Oldham, 2001), and for not producing quality mathematics students (PISA 2003, 2005,

[1] Secondary education in Ireland caters for students aged 11/12 – 14/15 in lower secondary and up to 18/19 in upper secondary.

2008). At the core of Project Maths is the philosophy that the teaching and learning of mathematics should provide students with the opportunity to:

> enhance their understanding of the subject, provide contexts and applications of mathematics that are meaningful and relevant, and enable them to develop problem solving skills and strategies that will serve them not only in their future study of mathematics but also their daily lives. (NCCA, 2008, p. 5)

Early evaluation reports following the initial implementation of Project Maths recorded improvements in relation to students' experiences of and attitude towards mathematics, and the ability of students to draw on their skills and knowledge from a variety of mathematical strands to solve a problem (Jeffes et al., 2013). The latest evaluations suggest, however, that the reform in curriculum has had no statistically significant effect on student performance in the PISA and TIMSS achievement tests and has created challenges in relation to teaching, learning, and assessment (Shiel & Kelleher, 2017). Teachers approve of the changes made in the reform and appear willing to embody the philosophies of Project Maths, but they are struggling to enact these changes in their classroom contexts due to perceived time constraints, lack of adequate resources, and the lack of choice in the final examination (Johnson et al., 2019).

This paper focuses on the possibility of pre-service teachers acting as agents of change in schools to assist teachers with these struggles in their particular contexts. Connor and Lake (1988) define an effective change agent as someone who is interested, visionary, persistent, organised, and a problem solver. This paper asks whether Irish pre-service mathematics teachers can act as change agents in the current era of reform in relation to Project Maths' focus on mathematical problem solving. This study follows the experience of one pre-service mathematics teacher aiming to create a change to a more problem-based mathematics learning environment in a placement school where the students claimed they had no experience of problem-based learning and who were disengaged in their current mathematics environment. This study asks what change, if any, did the pre-service teacher make in the given environment, and what impact did their actions have on the other mathematics teachers in the school? First, however, some context will be provided to highlight the importance of making this change.

Pre-Service Teachers as Agents of Change

In many schools pre-service teachers are partnered with cooperating/guiding teachers so that they can learn from the more experienced teachers and perhaps even replicate their practices. According to Lane, Lacefield-Parachini, and Isken (2003, p. 56) "… most models of the guiding teacher-student teacher relationship are unidirectional, based on the transmission concept of a mentor-mentee relationship where there is just one learner and one teacher." The validity of these kinds of unidirectional learning relationships have been questioned since the 1990s with the result that many researchers have advocated for the re-conceptualisation of the relationship into a more dynamic bi-directional one, where pre-service teachers can potentially act in the role of agents of change (Wenger, 1998). In Wenger's (1998) community of practice model, learning can occur both when "old-timers" narrate their past and share their experiences with "newcomers", and also when the newcomers bring an additional perspective to the conversation. This bi-directional relationship allows for the pre-service teacher to create and develop an identity that is connected to the past, and for the in-service teacher to imagine a future that is not simply a continuation of the past. While some argue that pre-service teachers are in relatively powerless positions while on school placement and often struggle to view themselves as teachers, let alone agents of change (Price & Valli, 2005), during eras of change within the education sector pre-service teachers have been seen as vitally important for successful reform. From the introduction of technology into the educational context in the 1990s (Weinburgh, Smith & Smith, 1997), to the move to active learning strategies in the primary curriculum (Murray & Stotko, 2004), to the changing of science instruction to have a more exploratory focus (Braund & Campbell, 2009), pre-service teachers have assisted cooperating teachers in enacting reforms in a variety of meaningful ways and in turn acted as agents of change. Regardless of the potential cognitive dissonance that may initially exist between pre-service and cooperating teachers, Field and Philpott (2000) reported that cooperating teachers noted benefits in terms of having to re-evaluate their current practices as well as feeling a kind of professional rejuvenation from working alongside pre-service teachers. Therefore, there is ample evidence to suggest that pre-service teachers can indeed act as change agents within a school setting by challenging in-service teachers to become more reflective practitioners and by altering the existing conversations around best practice.

Student Engagement and Its Importance

Mathematics permeates all aspects of society and has a fundamental role to play in both enabling and sustaining cultural, social, economic and technological advances. Additionally, it is critical for success in many career pathways (Martin et al., 2012). Despite this, larger and larger numbers of students are disengaging from mathematics during their time in school (Smyth & Hannan, 2002). Thomson et al. (2010) showed how in Australia a declining trend in middle years students' mathematical performance could be traced back to their disengagement in mathematics. Furthermore, disengagement with mathematics is a contributing factor towards lower participation rates in intermediate and high-level mathematics courses at higher secondary school levels and at university (Barrington, 2011). In Ireland, Smyth and Hannan (2002) reported how a significant proportion of students who achieved high grades in their lower secondary school examinations did not continue to study mathematics at the highest possible level in upper secondary school, unless they felt it was needed for university. These findings would therefore suggest that student engagement is an essential component of success in school and there exists a significant body of research that highlights how increases in student engagement levels can lead to increased academic achievement (e.g., Bobis et al., 2011) or reduce the risk of students dropping out of school (e.g., Rumberger & Lim, 2008).

In Ireland, the secondary school mathematics curriculum was designed with the aim of developing students' mathematical proficiency. It defines proficiency as an interconnected construct consisting of five integrated components: conceptual understanding, procedural fluency, strategic competency, adaptive reasoning, and productive disposition (Kilpatrick, Swafford, & Findell 2001). Kilpatrick et al. (2001) define productive disposition as "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p. 116), which according to several researchers are positive indicators of student engagement with learning (e.g., Bobis et al., 2011). Therefore, the employment of teaching strategies that enhance students' productive disposition are essential as these strategies can have a positive effect on student engagement. One such teaching approach, proposed by Bobis et al. (2011), is the use of open-ended questions and tasks, which involve students engaging in problem solving. This would suggest that the effective utilising of open-ended questions and problem-solving activities could therefore enhance student engagement in the mathematics classroom.

Mathematical Problem Solving

The ability to solve problems has become a vital competency for all citizens to survive and excel in our modern world (National Research Council, 2012). In an educational context, curricula and exams are becoming more and more problem-based requiring students to be efficient problem solvers to succeed, particularly in the subject area of mathematics. Problem solving is widely regarded as an integral part of mathematics and is at the core of mathematics curricula worldwide, including both Ireland (NCCA, 2011) and Australia (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2019). However, evidence suggests that this focus is not permeating into the mathematics classroom (Anderson, 2003; Schoenfeld, 1992), although teachers do recognise its importance (Anderson & White, 2004).

Many definitions of "a problem" have been proposed over the years but the following is the definition selected for the purpose of this paper - any set question for which a specific rule or method has not been taught, nor for which students believe there is a single method of solution (Hiebert et al., 1997; Szetela & Nicol, 1992). Problem solving is the act of creating a method and finding a solution for such a question. Meanwhile, efficient problem solvers will be deemed as those who think flexibly and fluently, are comfortable in using their knowledge and skills, and are prepared to patiently continue working towards a solution (Rigelman, 2007).

In recent times, problem solving has become a central concept in educational reform; however, the call for problem solving to be the basis for curriculum reform in the subject of mathematics has a long history (Hiebert et al., 1996). While traditional approaches to teaching and learning tend to avoid higher order questioning, student investigation, and a focus on understanding, problem solving can provide students with the opportunity to engage in a process of enquiry that encourages them to question, think, make sense of mathematics, and develop their mathematical understandings (Diggs, 2009). A problem-based approach to teaching and learning has been shown to produce confident, flexible, motivated mathematics students (Boaler, 1997). In addition, it is argued that interaction with a problem-based approach to mathematics leads to a deeper understanding of concepts (Cobb et al., 1991; Hiebert et al., 1996). Furthermore, it is noted that students who are regularly exposed to problem solving activities tend not to have a separation between conceptual and procedural knowledge (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fuson, Smith, & Lo Cicero, 1996), and over time develop a positive attitude toward the subject of mathematics (Carpenter et al., 1989; Cobb et al., 1991). It is no surprise, therefore, that problem solving is now an integral part of many mathematics curricula globally (e.g. NCCA, 2011; ACARA, 2019), and is central to international standardised testing of students' mathematical achievement (e.g., PISA, TIMSS).

"Teaching" problem solving is widely regarded as a difficult, complex task that often does not lead to the desired outcomes (Schoenfeld, 2013). How the teacher incorporates problem solving into their curriculum, how the classroom environment and norms are structured, what problem tasks are chosen, students' knowledge of both mathematical concepts and problem solving strategies, student affective attributes such as mindset, beliefs, interest and motivation, in addition to time available and teacher knowledge all impact on how successfully problem solving can be incorporated into mathematics classrooms and how successful the teaching and learning of problem solving is (Lappan & Briars, 1995; Liljedahl, 2019; Rowland, 2007; Schoenfeld, 1992; Schroeder & Lester, 1989).

Barriers Facing Mathematics Teachers in Implementing Problem Solving

Secondary school mathematics teachers face many daily challenges that could inhibit their ability to adequately include problem solving as part of their classroom routine. Firstly, current teachers of mathematics would have experienced a very different approach to the teaching and learning of mathematics when they themselves were students. The traditional approach to teaching and learning that they likely experienced is in stark contrast to the type of classroom environment and approach needed to teach through problem solving. In times of indecision or uncertainty teachers have been seen to revert to teaching the way they were taught (Tomlinson, 1999), a cycle that is likely to be detrimental to the successful implementation of a new teaching approach such as problem solving. Additionally in the Irish context, there has been a high incidence of out-of-field teaching of mathematics. A national survey of secondary mathematics teachers found that 48% of respondents were unqualified to teach the subject (Ní Ríordáin & Hannigan, 2011), which suggests that some of these teachers do not have sufficient mathematical content or pedagogical knowledge to teach mathematics for understanding effectively. Clarke (1997) highlighted how inadequate teacher knowledge can be a factor that influences the implementation of a newly introduced reform and thus can act as a barrier to the inclusion of problem solving in the classroom.

Furthermore, designing and implementing problem-based lessons can be, initially, a time consuming task. In relation to preparation time, Irish secondary teachers spend over 95% of their working time in school teaching lessons, leaving little time for planning. In contrast, Australian secondary teachers spend 66% of their working time in school teaching lessons, with the OECD average being 63% (OECD, 2017). Additionally, Irish teachers do not feel they have the class time to allow students to engage in inquiry, exploration, reflection, and discussion while also covering the prescribed content that can be examined in the standardised State Examinations (Shiel & Kelleher, 2017). Irish students spend between 120 – 300 minutes per week in lower secondary school, and 175-290 minutes per week in upper secondary school in mathematics class, with 62% and 82% of lower secondary and upper secondary teachers, respectively, deeming this time insufficient to cover their syllabus (O'Meara & Prendergast, 2017). Additionally, 92% of lower secondary teachers and 48% of upper secondary teachers report that they do not have a scheduled double period in the week for mathematics (O'Meara & Prendergast, 2017), with most periods being 35-40 minutes in length. This, coupled with most Irish students studying 10 subjects in lower secondary school and 7-8 subjects in upper secondary school, creates many potential barriers for teachers.

The study reported in this paper aimed to increase the level of engagement among students in a mathematics classroom by incorporating problem solving activities in the context of the constraints and barriers outlined above. The following research questions will be used to guide this study:

1. According to teachers in the study school, what factors influence the adoption of a problem-based approach to the teaching and learning of mathematics?
2. What change, if any, did the pre-service teacher make in the given environment regarding the implementation of mathematical problem solving, both at the classroom level and the wider school level?

Methodology

Selection of Problem-Solving Framework

Based on the evidence gleaned from the literature review, a framework was chosen that supported the teaching *through* problem solving approach – namely, Van de Walle's (2004) *teaching through problem solving* framework. The framework was used to design and implement four problem-based lessons (two 40 minute class periods focusing on a single problem solving task) in a second-year secondary school mathematics classroom (students aged 13/14). This framework was selected as it offers structure, support, and guidance to the teacher and students who are engaging in problem solving activities for the first time.

The *teaching through problem solving* framework consists of three phases: before, during and after. In the 'before' phase the teacher introduces the students to the problem and helps to position this new problem within students' prior knowledge and problem solving experiences. As well as establishing connections between the new problem and their prior knowledge, the teacher also assists students to understand the problem setting, the mathematical context and the challenge, as well as making sure that they establish clear expectations for themselves. At this stage it is critical that the teacher does not provide too much problem specific information to the students as this could lessen the overall challenge of the problem, yet they must ensure that they have adequately motivated their students to attempt to solve the problem. The 'during' phase happens next. In this phase the students work to solve the problem using appropriate problem solving heuristics. During this phase the teacher moves about the classroom, observing student performance and encouraging on-task behaviour. The teacher challenges students by asking appropriate questions and monitors their overall progress within the problem. At this phase also, for students who are capable of deeper investigation, the teacher may provide extra questions related to the problem. The third and final phase is the 'after' phase. This phase can commence when the students have made sufficient progress toward solving the problem. The teacher encourages the students to present and discuss their solution strategies whilst listening actively throughout this process. The teacher may also summarise the main ideas and mathematical knowledge utilised throughout the problem solving activity as well as identify extension or future problems.

The structure of the *teaching through problem solving* framework supported student-centred exploration of mathematically rich problems while allowing teachers to use continuous assessment to inform instructional practices.

Action Research

The overall design of the research study used an action research approach. Action research follows a cycle – identify a problem, gain a theoretical understanding of the problem, design a solution, trial the solution, and evaluate, adjust, and retrial the solution until a conclusion is reached (Lewin, 1948). Unlike many other forms of research, action research is designed as "a small-scale intervention in the functioning of the real world and a close examination of the effects of such an intervention" (Cohen, Manion, & Morrison, 2007, p. 297). This "small-scale" characteristic makes it very tangible and realistic for both practising and pre-service teachers to implement as part of their daily routine, which in turn leads to many benefits for the development of the teacher as a professional (Noffke & Zeichner, 1987).

> To proceed unthinkingly is to be caught up in the flux of things, to be 'caught up' in dailyness, in the sequence of tasks and routines. Of course, we have to proceed that way a good deal of the time, but there should be moments when we deliberately try to draw meaning out of particular incidents and experiences. This requires a pause ….(Greene, 1984, p. 55)

Action research is a practical example of such a "pause". It allows for theory-based action to be taken to a problem witnessed first-hand by the researcher.

The Action Research Cycle of this Project

In the case of this research study, the problems of student academic disengagement and the absence of problem solving in Irish mathematics classes were witnessed personally by the pre-service teacher (first author), both in her own class and the classes of others. Finding a solution

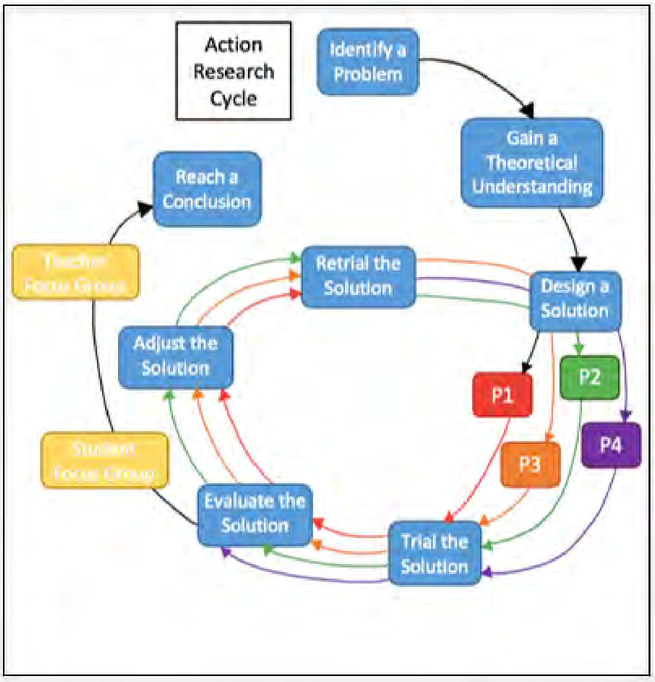

Figure 1. The action research cycle of the project.

to this problem became important, and action research allowed for this to happen while balancing the workload of teaching, planning, and data collection. Figure 1 summarises the cycle through which this action research project progressed. After identifying the problem, a comprehensive review of the related literature was conducted in order to gain a theoretical understanding of the problem. The "solution" to the problem came in the form of problem-based lessons – two class periods dedicated to allowing students to work collaboratively on a single mathematical problem. After a problem-based lesson was designed and implemented, the pre-service teacher evaluated the success of the problem-based lesson and used this new knowledge to adjust and retrial the solution with a new problem-based lesson. This cycle occurred four times with four different problem-based lessons designed, trialled and evaluated. Classroom level, student data was collected throughout the four lessons to monitor changes in regard to student engagement on an ongoing basis. Focus groups with participating students were held after the evaluation of the fourth lesson to further analyse any changes that occurred as a result of the pre-service teacher's actions. An additional focus group with other mathematics teachers in the school took place after this, in an effort to share the knowledge gained by the pre-service teacher with these colleagues, an action supported by the action research methodology.

Selection of problems

Four problems, one for each problem-based lesson, were chosen, or designed, for this study based on section one of Van de Walle's framework and the supporting research on problem selection. All problems met, or were adapted to meet, the necessary requirements of beginning where the students are, connecting with students' curiosities and skills, and providing multiple entry points for all levels of abilities. The first problem, titled In-N-Out Burger (Figure 2), was sourced online from the work of Robert Kaplinsky (2013). It focused on forming and solving algebraic equations. Participating students were familiar with In-N-Out Burger from American television shows and through social media. The second problem, titled Ski Slopes (Figure 3), was adapted from an idea presented by Dan Meyer in his TED talk entitled "Math class needs a makeover" (Meyer, 2010). It was used to introduce the concept of slope in coordinate geometry. Skiing is a popular winter foreign holiday activity in Ireland and many Irish secondary schools organise ski trips to mainland Europe for students each year. Problems three (Figure 4) and four (Figure 5) namely, A Syrian Journey (discovering the midpoint formula) and Colliding Planes (calculating the intersection point of two lines), were designed by the pre-service teacher guided by Van de Walle's framework. All problems aligned with the content being taught at the time and the curriculum objectives for each topic outlined by the national mathematics curriculum (NCCA, 2011).

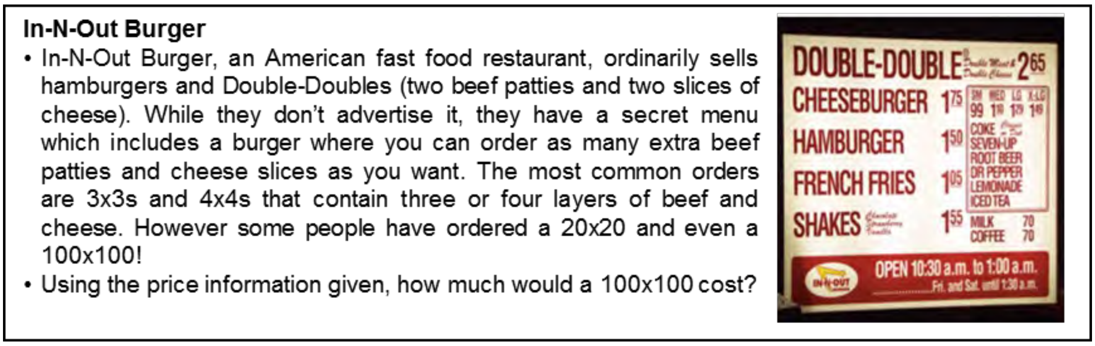

Figure 2. In-N-Out Burger Problem.

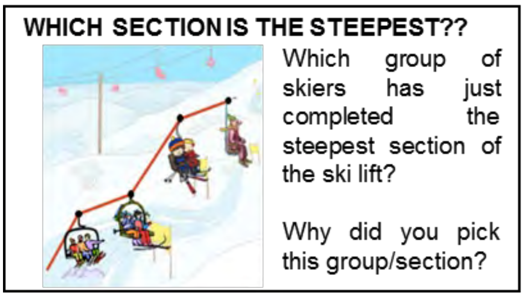

Figure 3. Ski-Slope Problem.

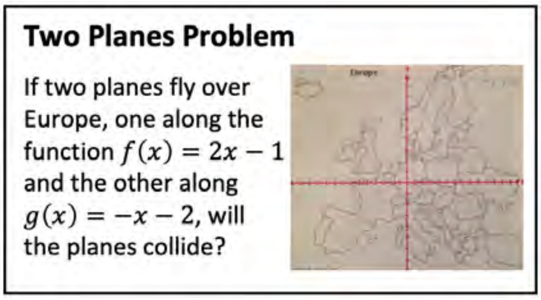

Figure 4. Two Planes Problem.

A Syrian Journey

Depicted on the next slide are the journeys of four families from the danger of Syria to the safety of where they have landed. They are wanting to find the midpoint of their journey from where they started to where they are now. The point will for them symbolise the point where they were equidistant from safety and horror. The challenge you face is to find this point of the map for each group, first using a method of your choice, before devising a method using only coordinate geometry and algebra.

Journey 1

Hosein and his family originated in a city called Damascus located at (13, -6) before fleeing to Europe. They lived in a refugee camp for 2 months before being moved to Ireland. They are now happily settled in Waterford at point (-7, 2).

Journey 2

This is 18 year old Hamid who fled his city Safta (11, -7) after ISIS tried to recruit his as a soldier. He travelled alone across Europe before finding refuge in Sweden at point (3, 5).

Figure 5. A Syrian Journey Problem.

Selection of participants

The school in which this study took place is a DEIS (designated economically disadvantaged) secondary community college located in an economically disadvantaged area of an Irish city. Student participants consisted of those students in the pre-service teacher's placement second year (13-14 years of age) mathematics class who were willing to partake. This cohort of students was chosen for convenience and due to their lack of prior experience in a problem-based mathematics classroom. The class was a mixed ability group of fifteen females and three males, with a class average achievement of 60% (ranging from 33% to 85%) at the end of the previous school year. This class average is based on the percentage grade each student is given at the end of a term, which is calculated using scores from in-term and end of term summative assessments based on the mathematical content covered during that time. These assessments contain both procedure/skill-based questions and problem solving questions. Teachers for the focus group consisted of those mathematics teachers assigned to the school's numeracy team.

Problem-based lessons

The four problem-based lessons were designed to follow the three stages of Van de Walle's framework - before, during and after - and were implemented in the mixed ability second year class during the first term of the school year. Each problem-based lesson spanned two non-consecutive 40-minute class periods. For each specific problem students worked in mixed-ability groups of four students, with the groups changing slightly each time due to student absences and personal conflicts among some students. A problem-based/inquiry approach to learning was also implemented in all other lessons with these students by the pre-service teacher through the use of continuous higher order questioning, class discussions, and mathematics puzzles/games. This approach differed greatly from the traditional "demonstration and practice" approach to which the students had become accustomed.

Data Collection

Observations and reflections

According to Van de Walle's framework, students are to be left to approach, and potentially solve, the problems with scaffolding from the teacher. To scaffold, the teacher provides clues, reminders, encouragement, and prompts without revealing the solution to the problem, reducing this support over time so that the students can grow in independence as problem solvers (Van de

Walle, 2004). The teacher scaffolds the students' learning when needed and spends the remainder of the time moving around the room observing the groups. This approach provided an opportunity for the pre-service teacher to monitor students' engagement with the problem by noting the number of times groups were witnessed off topic, the number of questions a group asked/the number of times a group needed to be scaffolded, and whether a group reached a final answer complete with explanation (either written or verbal, correct or incorrect). Discussing topics unrelated to the problem, stating that the group had "given up", or having two or more group members not knowing what the group was doing resulted in an "off topic" observation. Groups who were found to be off topic after they believed they had reached a solution to the given problem, and before receiving feedback or an extension activity from the teacher, did not receive an "off topic" noting. An observation sheet was completed by the pre-service teacher during each of the four lessons. She also maintained a reflective diary throughout the process, keeping notes on both the planning and implementation processes to supplement the observations. In doing so, a more comprehensive view of each of the lessons was depicted for analysis.

Focus groups

Three focus groups occurred following the final problem-based lesson. Focus groups one and two consisted of participating students from the study class, while focus group three consisted of mathematics teachers from the study school. Student participants were chosen based on their perceived level of engagement in mathematics lessons over the course of the ten weeks they spent in the pre-service teacher's mathematics class. Both highly engaged students (those who asked/answered questions and rarely needed to be refocused by the teacher) and poorly engaged students (those unwilling to seek help and often found to be off topic when working individually or in groups) were chosen to allow for the opinions and feedback from a variety of student types to be heard. Both student focus groups consisted of four students and took place in a separate classroom. The students were asked a series of open-ended questions about their experience of mathematics both before and during the research study.

The third focus group consisted of four mathematics teachers from the school with a variety of teaching backgrounds. Teacher One was an engineering teacher who recently completed the Professional Diploma in Mathematics for Teaching (a two-year part-time mathematics teaching qualification for teachers of other subjects) and had 5 years experience teaching secondary mathematics; Teacher Two was a qualified mathematics teacher who had been teaching secondary mathematics and applied mathematics for almost thirty years; Teacher Three was a science teacher with experience teaching secondary mathematics and recently completed the Professional Diploma in Mathematics for Teaching but currently teaches lower-secondary science and coding; and Teacher Four was a science teacher with no mathematics teaching qualification but had five years' experience teaching lower-secondary lower-level mathematics courses. It is important to note that all four of these teachers had recently attended a problem solving workshop presented by the Project Maths Development Team (a subsidiary of the national Professional Development Service for Teachers team). All three focus group interviews were audio recorded, with the permission of participants, and later transcribed with no significant details omitted.

Data Analysis

Quantitative and qualitative data from a variety of perspectives were gathered during this research study. Quantitative frequency data obtained from observations made during the problem-based lessons were analysed and compared through the use of histograms in Excel.

Focus group interview transcripts were read and key terms/themes were identified through the process of content analysis. Bailey (1978) summarises the purpose of content analysis as "to take a verbal, non-quantitative document and transform it into quantitative data" (Cohen, Manion, & Morrison, 2007, p. 197), which was the aim of the analysis for this study. Brenner, Brown, and Canter (1985) outline a series of steps to be followed when completing content analysis, which were used to structure the data analysis for this project. These steps can be summarised as *immersion* (reading and rereading the transcripts), *categorising* (identifying and labelling mutually exclusive themes), *incubation* (reflecting upon the data and themes to develop interpretations), *synthesis* (reviewing the labels and categories), *culling* (combining similar themes), *interpretation* (making sense of the themes), and *writing* (preparing this paper to make connections with quantitative results and also current literature in the field).

Multiple perspectives were examined, with both quantitative and qualitative data gathered, to increase the reliability and validity of this study. Jick (1979) refers to this as triangulation and maintains that multiple viewpoints lead to greater accuracy. Three viewpoints were examined in this study—the pre-service teacher as researcher, the participating students, and the other mathematics teachers in the school—with analysis of both quantitative and qualitative data providing evidence to support claims about the framework and its application in an Irish secondary mathematics classroom setting, and the potential barriers that may prevent mathematics teachers from successfully implementing problem solving in their classrooms.

Results

Student Perspective

Teacher observations

Throughout each of the four problem-based lessons, the pre-service teacher conducted observations on each of the five groups under four headings – number of times groups are found to be off topic, number of times groups ask for help/received teacher scaffolding, percentage of groups with a final answer complete with explanation, and percentage of groups with a correct final answer complete with explanation. These observations took place throughout the *during* section of each lesson and were taken in the form of a simple tally chart. The class averages for each problem-based lesson are shown in Table 1.

Table 1
Class Averages from Teacher Observations

	Off Topic	Scaffold	Final Answer %	Correct Answer %
Lesson 1	1.8	2.4	60	20
Lesson 2	1.6	2	80	40
Lesson 3	1.0	1	100	40
Lesson 4	0.4	0.6	100	80

From the data there is evidence to suggest that over the course of the four problem-based lessons group engagement improved with a noticeable reduction in the average number of times groups were found to be off topic (recall that off topic behaviour is classified as students discussing topics unrelated to the problem, stating that the group had "given up", or having two or more group members not knowing what the group was doing). Figure 6 shows each group's progress through the four lessons regarding this engagement. Four of the groups are shown to be off topic less often during lessons 3 and 4 in comparison to during lessons 1 and 2. Groups 2, 3 and 4 all managed to avoid being off topic, as per the criteria mentioned previously, in one or more of the 4 lessons.

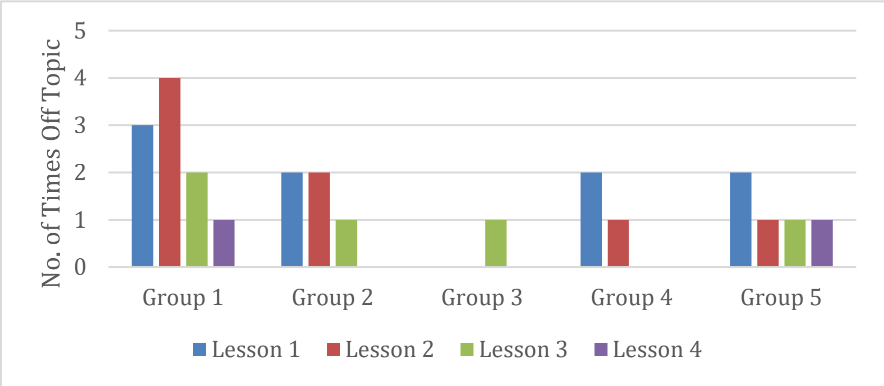

Figure 6. Group engagement during problem-based lessons.

Student focus group

Two student focus groups took place at the end of the school term. By this time, all students participating in the focus groups had experienced four problem-based lessons designed using Van de Walle's framework. The focus group interviews produced qualitative data in the form of students' answers to the following questions:

- What did you like about the problem lessons?
- What didn't you like about the problem lessons?
- Do you think you learned anything new in those lessons?
- Do you like learning in this way?
- Can you compare this style of learning to any other ways that you learn?
- What would your ideal mathematics class be like?

Participating students shared their thoughts and opinions of their experience during these problem-based lessons, and mathematics class in general. Students' answers were read and analysed with three themes being extracted from the data following the grouping of terms and phrases uttered by the participants into subject groups. These themes were understanding, engagement, and enjoyment.

The terms understand/learn/make sense/explain were mentioned 18 times by half the students interviewed (n = 4) over both focus groups. Students recognised that *understanding* mathematics is essential to learning and expressed that partaking in problem-based lessons, where new content was introduced, helped them to understand the new content.

> I thought it was easier to learn in the lessons. It helped explain it more and just made the maths easier. (Student 1)

This approach to learning was compared by the students to their negative experience of mathematics class in the past where they were expected to "just sit down and read from a book and do the sums" (Student 3) or "do our homework, correct our homework and then get our other homework and we basically never learned anything" (Student 7). Students also commented on the control they had over the pace of the lesson which allowed them to explain material to others in their group and ensure everyone understood the material before moving on to a new topic.

Similar to the observations made by the pre-service teacher discussed above, students also noted that they were more *engaged* in the mathematics lessons when they were working in groups to solve problems, with 5 students referencing teamwork and engaging with their peers 6 times. Students liked the concept of working in teams as it helped them to stay focused and learn from their team members.

> We were all putting in stuff instead of just reading from a book individually. (Student 3)

They also noted the importance of the activities in this regard when describing their ideal mathematics class.

> More activities so it would be more engaging so people wouldn't be just zoned out. (Student 6)

The students also found that the context of the problems helped them to engage more meaningfully with the mathematics as they could see a new found relevance between their own lives and the subject of mathematics, something which they are not accustomed to.

> It was made more relevant to real life because people are always saying that maths is all around us but we never use examples or stuff like that to actually show us how it is in real life. Like in some books it's like "if Kevin went to the shop and bought 89 potatoes" and you're like "no, nobody buys 89 potatoes". (Student 4)

Finally, the theme of fun and *enjoyment* was evident throughout the focus groups, both in what the students said and how they said it. It was obvious from their tone that "correcting homework and taking down homework" (Student 4) and "just like sitting there doing the sums, like, reading from a book" (Student 1) were not deemed enjoyable experiences. When discussing the problem-based lessons, however, the first response from group one was:

> They were fun and, like, it makes maths more funner. (Student 1)

It is necessary to acknowledge that the students may have been influenced by social pressure to please the pre-service teacher who taught the problem-based lessons and conducted the focus group interviews. However, students' responses were consistent and aligned with the observations of their behaviour made during lessons.

Teacher Perspective

Teacher focus group

At the end of the first school term a focus group with a selection of mathematics teachers in the school took place. Each teacher present currently teaches, or has recently taught, secondary mathematics. The focus group was divided into three parts, opening with a discussion on problem-solving in mathematics in general, followed by an explanation of Schroeder and Lester's (1989) teaching *through* problem-solving approach and Van de Walle's framework that was implemented by the pre-service teacher during that term, and closed with a discussion on these. The focus group produced qualitative data in the form of the teachers' answers to the following questions:

- What is your understanding of the concept of problem solving?
- What is your current approach to incorporating problem solving in the mathematics class?
- What barriers do you face when trying to incorporate problem solving?

- What are your initial reactions to the framework?
- Do you think this framework would help you incorporate more problem solving into your lessons?
- Is there any part of the framework that you think needs clarification or expansion?

Teachers' answers were read and analysed with four themes emerging following the synthesis of terms and phrases used by the teachers: designing/sourcing suitable problems; time to incorporate problem solving; students' ability; and teacher collaboration.

Finding/designing a suitable problem is the first key step to successfully incorporating problem solving into a mathematics classroom. However, this is also the first key barrier the teachers say they face. Each of the teachers expressed the difficulty they face regarding where and when they can source the "right problem". They stated that if a "bank of problems" (Teacher 2) existed that they could readily tap into, then incorporating problem solving into their classrooms would be possible, with Teacher 1 suggesting that the Project Maths Development Team could provide these resources for Irish teachers. In relation to designing their own problems, the teachers all agreed that they simply do not have the time to do this.

Time was a major discussion point throughout this focus group. In addition to the time needed to design problems, teachers also expressed concern over the time needed to implement problem solving lessons in addition to covering the necessary content, with one teacher stating:

> Time is huge … it's just such a huge task to undertake initially that it's just so much simpler - you know you're going to get through the stuff going through the book. (Teacher 1)

Other teachers agreed, adding:

> Especially for maths, by the time you've homework corrected and you move on to the next topic, within 40 minutes [it] would be hard. (Teacher 4)

However, after the concept of teaching *through* problem solving and Van de Walle's framework were discussed, some teachers began to see how this approach to teaching and learning may, in fact, save time.

> I think if the kids could understand it then they can move forward much quicker. And they're engaged. Something like that very well could speed it up almost. (Teacher 3)

Teachers also saw *students' ability* as a barrier to incorporating problem solving in their mathematics class. They noted that they "don't give them out these problems because you know you're going to have to do it for them anyway" (Teacher 1), but also recognise that this is a "defeatist attitude" (Teacher 1). They recognised that while, at first, students may find problem tasks difficult, the only way for them to improve is to regularly engage in problem solving exercises. The teachers also suggested that students would struggle with the language of problems and thus expected students to be unable to engage in problem solving activities as a result.

> A lot of the time it is language as well, they don't understand the question - the English that's in the question or the way the question is worded. (Teacher 4)

The final element in relation to student ability discussed was students' ability to assimilate their knowledge from a variety of areas of mathematics to solve a single problem. The example provided by the teacher was:

> These problems that are cross topic … they're looking at it going "which chapter do I use?" They can't do that. (Teacher 1)

One teacher expressed concern regarding consistency in a school's approach to teaching and learning. A student may have several different mathematics teachers while in secondary education, and a smooth transition between teachers is necessary. The teacher was concerned that if they personally began teaching content through problem solving, and incorporating a discovery style of learning, this transition would not be smooth for the student. The teachers agreed that for Van de Walle's framework to be effective long term, all mathematics teachers in the school would need to design their teaching in this way, and *collaborative planning* would be necessary. They stated that currently they do not do enough collaborative planning, and do not do any collaborative lessons. One teacher concluded that collaborative planning could, in fact, be the solution to the time problem regarding problem design.

> Every teacher taking a topic and developing problems or a series of problems for that topic, or problem solving activities, and then we could just bank them all. (Teacher 1)

During this focus group, these teachers' assumptions were challenged indirectly through the presentation of the pre-service teacher's experiences implementing problem solving with students from the teachers' school. The pre-service teacher did not experience many of the constraints outlined by the teachers and during this focus group discussion the teachers' attitudes towards their ability to implement problem solving took a noticeable shift. At the beginning of the conversation, the teachers focused on why they could not implement problem solving in their particular setting. By the end of the conversation, the teachers began discussing solutions to the constraints they had outlined initially and admitted that they should be doing more in this regard. The conversation among the teachers changed from one relating to barriers and challenges, to a conversation centred on solutions to these challenges and planning together how they could incorporate more problem solving into their teaching by working together to find problems and plan lessons. Anecdotally, outside of this focus group, two of the four teachers in this group approached the pre-service teacher asking for copies of the problem solving resources she had used and spoken about in the focus group. This change in conversation and engagement with the pre-service teacher in this regard is in contrast to both how the teachers spoke of mathematical problem solving in their context at the beginning of the focus group and what the pre-service teacher witnessed in the school in regards to mathematical problem solving at the beginning of the school placement.

Discussion & Conclusion

The first research question of this study sought to investigate the factors that the teachers believed influenced their adoption of a problem-based approach to mathematics instruction. The beliefs and constraints outlined by the teachers in this study directly align with the current research on implementing mathematical problem solving. Many research studies have questioned teachers on the elements that restrict them from implementing problem solving in their classrooms, with the main responses being: lack of preparation time (Anderson, Sullivan & White, 2004), lack of class time (Cavanagh & Prescott, 2007), an over filled curriculum (Anderson, 2008), and teachers' beliefs regarding student ability (Anderson, 2008; Cavanagh & Prescott, 2007), which were similarly expressed by the teachers in this study. In the context of this research study, teachers in the school expressed concerns over a number of elements that were either not met, or were overcome by the pre-service teacher. Before beginning this research study, the pre-service teacher had a very positive attitude towards problem solving and was proactive in finding a method for easily including problem solving into the mathematics class. Constraints were not defined prior to the study, and few constraints were encountered during the study. Any difficulties that were faced were viewed as challenges to be solved rather than constraints to be allowed inhibit the inclusion of a problem-based approach. Over the course of this relatively short intervention, these challenges were defined and overcome and problem solving was effectively included into the mathematics classroom. The ability to overcome these potential barriers is an important finding as it shows the need for teachers' assumptions to be identified and challenged if change is to occur (Brookfield, 1995). While some constraints may be linked to school contexts and systems, some may be mere assumptions which, if not challenged, will restrict teachers from linking the theory to their practice. It may also force teachers to revert to how they were taught as these assumed constraints create a sense of uncertainty and indecision (Tomlinson, 1999). These beliefs held by teachers are a major factor influencing the implementation of problem solving in the classroom (Anderson, 2003; Cavanagh, 2006), and thus need to be questioned and challenged.

The second research question considered the potential change, if any, the pre-service teacher made in the given environment regarding the implementation of mathematical problem solving. The concept of teachers being viewed as 'change agents' is nothing new. Fullan (1993, 1999) places the responsibility of facilitating the growth and development of students at the core of the job description of a teacher. While many acknowledge that it may be difficult for a pre-service teacher to act in this role and portray the characteristics of an agent of change (Connor & Lake, 1988; Price & Valli, 2005), it is not an impossible task. While a pre-service teacher may not transform the school environment in which they work, they can still be given the opportunity to act independently and develop their agency as an emerging teacher. In doing so, they can be viewed by other members of the school community as agents of change as they increase their awareness of new teaching strategies or simply challenge their assumptions by performing tasks, or enacting reform, in a context in which in-service teachers did not deem it possible.

That is what occurred in this case study. A pre-service teacher incorporated mathematical problem solving and a teaching through problem solving approach while not being inhibited by the barriers assumed by the school staff. The pre-service teacher, while present in the school, created a change in the classroom that was evident in both the observations of the participating students made during the problem-based lessons and by the comments the students expressed in their focus groups regarding their enjoying of mathematics. When these observations were brought to the attention of the other mathematics teachers in the school, the conversation changed from one filled with problems and difficulties to a conversation on solutions and possibilities. The pre-service teacher potentially created a change in the conversation of the cooperating teachers. Although there are no data to track any changes after the pre-service teacher's placement ended, the possibility for pre-service teachers to act as agents of change in their placement schools has still been shown. With appropriate levels of support in place, pre-service teachers can be encouraged to be interested, visionary, persistent, organised problem solvers who challenge the environments in which they are placed and open the eyes of their cooperating teachers to the possibilities and potentials of the classrooms in which they work. Initial teacher education programmes can highlight the potential for pre-service teachers to enact change within their education system, and support them in designing lessons with approaches to teaching and learning that will help them to achieve this. Meanwhile, teacher in-service training can be provided to help teachers use the placement of pre-service teachers within their schools as an opportunity to accurately analyse their context and find resources to meet their own needs as the classroom teacher and also the needs of their students. Such approaches might support a reconceptualization of school placement as a bi-directional learning experience for both pre-service teachers and their cooperating/guiding teachers, in line with Wenger's (1998) notion of communities of practice comprised of newcomers and old-timers.

Many of the elements of action research that make it attractive and attainable for teachers to complete also led to limitations in this study. Action research is unique as it offers insights into the researcher's specific situation. However, this also implies that the researcher has no control over the participating sample or external factors which make the generalisation of results difficult, but as the number of action research projects on a specific topic increases, this difficulty decreases (Cohen et al., 2007). In relation to this research study, time and sample size were both limitations. This study was a short-term study based on one pre-service teacher and her eighteen students. These students were chosen based exclusively on their membership of the pre-service teacher's class and were not a representative sample of Irish mathematics students. The time scale of this project was also limited to the short time spent by the pre-service teacher in this particular school and the lack of access to both students and teachers following the conclusion of the school placement. However, interesting insights into the potential for pre-service teachers to create change and challenge the assumed constraints in the mathematics classroom have nevertheless been shown and should be further studied as a possible solution to successfully enacting educational reforms.

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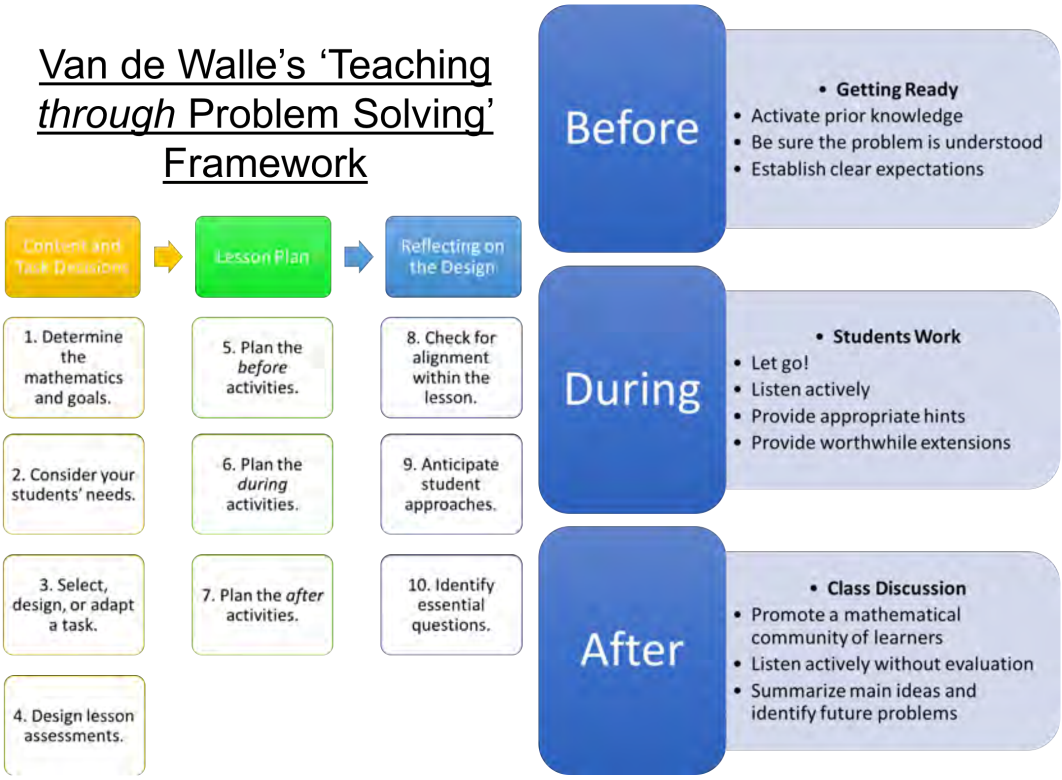
Appendix



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