Teaching Students With Learning Disabilities to Solve Secondary School Algebra Problems

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In this paper, we make a case for the importance of teaching secondary school level algebra to students with learning disabilities (LD). Furthermore, we illustrate how they struggle and present best-practices on how they are best supported. We demonstrate effective ways of how teachers can show students with LD how to solve challenging algebra problems. In particular, we depict how educators can help learners with LD show their work on paper in ways that support their thinking processes as they engage with challenging algebra problems.

Keywords: **mathematics, algebra, learning disabilities**

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In the United States, success in secondary school mathematics classes, like Algebra 1, is important for students' advancement to better opportunities in high school, in college, and for eventual employment; this course is critical for all students, including students with learning disabilities (LD) (Ysseldyke et al., 2004). Algebra 1 is taken in junior high school by some students, but often as a high school course by struggling learners. Students with LD, in the United States, are identified for special education services after a lack of improvement after receiving academic interventions in small group and individualized settings; these students have not responded to the extra interventions in a way that catches them up with students without disabilities (Gresham & Vellutino, 2010). This group of students often performs below the level of students without

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disabilities, but they have the ability to perform at similar levels in mathematics to other students if they are supported strategically by teachers (Andersson, 2008; Marita & Hord, 2017). With the right kind of support, students with LD can succeed with Algebra 1 content (Hord et al., 2016, Ives 2007).

In this paper, we will provide a brief description of the research that supports a set of interventions we recommend for specific Algebra 1 problem types that tend to cause difficulties for students with LD. This article is targeted toward pre-service and in-service teachers. The intention of the authors is to provide an accessible article for the teaching of some of the math problems students with LD will face in Algebra 1.

How Students with LD Struggle and How to Help

Many of the struggles of students with LD are related to working memory, which is the processing, storing, and combining of information (Baddeley, 2003; Swanson & Siegel, 2001). Students with LD often lose track of important information in short-term memory as they think about other parts of a problem; or, students may have trouble thinking about multiple pieces of information at one time (Swanson & Beebe-Frankenberger, 2004). Either way, students with LD often benefit from offloading information on paper (Hord et al., 2020). This is the process of storing information on paper rather than trying to remember this information mentally (Risko & Dunn, 2010).

Offloading can be especially helpful with difficult problems, which are more likely to overload working memory (Barrouillet et al., 2007; Hord et al., 2020). Students struggle more with processing, storing, and combining information when the pieces of information are more complex or unfamiliar, such as multistep problems or problems above the student's current skill level (Barrouillet et al., 2007; Swanson & Beebe-Frankenberger, 2004). In other words, it is harder to think more quickly and accurately about things we are not good at yet and/or when there are a lot of pieces of information to remember and think about all at once. When students with LD are faced with problems that are new, unfamiliar, and difficult and/or when these problems have more than one step, we recommend that teachers show students how to strategically offload information onto scratch paper (Hord et al., 2020). Working memory overload can be avoided in multistep situations if students put information from each step, and their thinking about each step, on paper as they work their way through a problem rather than trying to remember and think about lots of things in their minds all at once (Risko & Dunn, 2010; Xin, Wiles, & Lin, 2008). And, of course, in the beginning stages of learning something new and difficult, students will benefit from showing their work rather than trying to think through much of the problem mentally.

Ways to Offload

When working with elementary and middle school students, special education researchers recommend teaching students with LD to put information in diagrams. For example, Xin, Wiles, and Lin (2008) recommended a model for addition, □+□=□, for part-part-whole word problems with the smaller rectangles being used for each part and the larger rectangle being used for the whole. Once students place the two known parts of the problem in the blanks, they can then see which part is unknown and solve for it. For a problem such as, "Victor has 51 rocks in his rock collection. His friend, Maria, has 63 rocks in her collection. How many rocks do they have altogether?" (Xin et al., 2008, p. 167), the student could put 51 and 63 in the smaller rectangles and add those numbers to find the whole. For a problem such as, "Jamie and Daniella have found out that altogether they have 92 books. Jamie says that he has 57 books. How many books does Daniella have?" (Xin et al., 2008, p. 167), the student can place 92 in the larger rectangle and 57 in one of the smaller rectangles. Then, it is easier for the students to see there is a missing part and eventually subtract 57 from 92 to solve the problem. Xin and colleagues also recommended a model of multiplication, □×O=△, for equal groups word problems. They recommended teaching students to write the amount for each group in the rectangle, the number of groups in the circle, and the product in the triangle. In a study about the middle school level, Xin, Jitendra, and Deatline-Buchman (2005) used diagrams with similar principles for proportions word problems.

These diagrams work as bridge between the word problems and the equation or math problem needed to solve the problem. The diagrams give the student a way to represent information from the word problem, which is not user-friendly for the students, in a way that makes it easier for them to think about as they work toward setting up an equation to solve the problem. In all of these cases with offloading into diagrams, information was strategically stored on paper in ways that supported students with LD when they needed to process, store, and combine information to solve a problem. Getting this information on paper in user-friendly ways seems to help students with LD not forget parts of the problem, see the relationships they need to see, and make the connections they need to make to solve the problems. These techniques are effective for students with LD with mathematics word problems ranging from addition and subtraction word problems up to proportions word problems (Hord & Xin, 2013; Marita & Hord, 2017).

Organizing information on paper in strategic ways also can be beneficial for algebra in secondary school settings (Hord et al., 2018; Ives, 2007). Algebra requires students to keep track of lots of numbers, variables, and symbols all at once, and students with LD sometimes seem to get lost in the struggle of trying to decipher what to do with all of these pieces of information (Hord et al., 2020). With algebra, students with LD sometimes struggle to not overlook parts of problems and see how all of these parts of the problems are related (Hord et al., 2016). For multi-step situations, Ives (2007) recommended teaching students to use simple graphic organizers (e.g., putting each step of work in a separate box) to stay organized while solving multi-step equations. For situations where students may have trouble processing the multiple things they need to see at once, Hord and colleagues (2016) recommended that teachers draw lines over parts of the students' work on paper to help students make the connections they need to make between parts of problems. For example, students with LD will sometimes make a mistake with distribution by forgetting to distribute to the second term in parenthesis (Hord et al., 2016). With a problem such as 3(x + 2), students will sometimes distribute the 3 to the x, but not to the 2 and write 3x + 2 on paper rather than 3x + 6. Teachers can draw arching lines from the 3 to the x and to the 2 to show the distribution that needs to happen (see Figure 1). After this adjustment, students will often make the necessary connection and write 3x + 6. Of course, it is better yet if the students learn to draw their own lines so they are more likely to independently make the connection from then on.

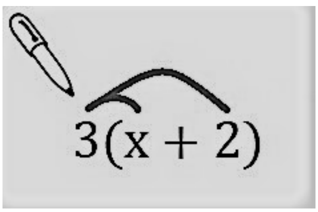

Figure 1. **Arches for Distribution**

This offloading and organization of information on paper that shows the relationships between parts of the problems works in similar ways in algebra as it does with word problems with younger students; the students can clearly see the parts of the problem and how the parts are related to one another. Most importantly, the students do not have to remember a lot of things in their mind and combine these things all mentally; the information is offloaded and organized onto paper and then students can focus their attention solely on the critical thinking that needs to be done to solve the problem. Once students get all of the parts of the problem displayed on paper in user-friendly ways, they can then take a step back and think about how all of these parts are related to solve the problem.

Patterns in Our Examples

In the following sections, we will focus on the content of Algebra 1, which is an important course for secondary level students with LD. From this level of algebra, we will identify some key problem types that are likely to cause students with LD difficulties and how teachers can teach students to offload and organize information strategically to support their thinking process. In these problems with our solutions, readers are likely to notice a pattern where we continue to suggest that students will be more likely to succeed if they can put pieces of information on paper, one piece or one step at a time (e.g., putting information into boxes), rather than trying to combine steps in their mind. Readers will also likely notice a trend where we use visuals to demonstrate relationships such as drawing arching lines or arrows for distribution and color coding or marking like terms with shapes. In these situations, the goal is to get information on paper rather than trying to hold it in memory storage and to get information displayed on paper in ways that students can easily see the relationships between parts of problems. When working with challenging algebra, students with LD often do better when they organize the information from a problem on paper so they can see everything clearly, take a step back mentally, look at each part of the problem, and then think critically about how each part is related to work toward finding an answer.

Distribution and Combining Like Terms

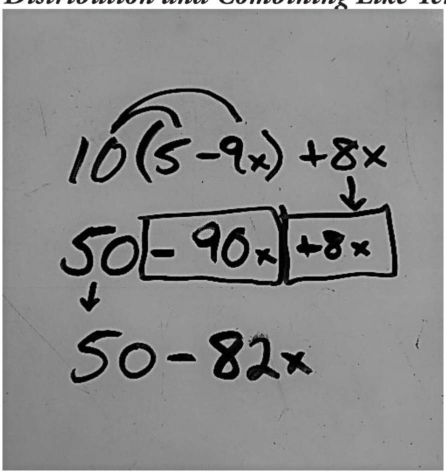

Figure 2. **Distribution and Combining Like Terms**

This type of problem shown can be difficult for students for a few reasons (see Figure 2). First, students will often forget to distribute the 10 on the outside of the parenthesis to both the 5 and the -9x on the inside of the parenthesis, sometimes only distributing to one term instead of both. Notice how we recommend drawing lines (or arrows if preferred; both tend to work well; Hord et al., 2021) from the 10 to both terms in the parenthesis so students remember to distribute to both terms. While doing this, students may also not recognize that the 9x is negative, and this will cause them to get a positive 90x on the second step instead of the correct negative 90x. Once this step is complete, we recommend working with students to combine like terms. The boxes around the -90x and +8x show a student that these two terms must be combined since they are like terms. After combining like terms, students may feel the need to still subtract 82x from 50, but this is not possible because they are not like terms.

Tips for Combining Like Terms

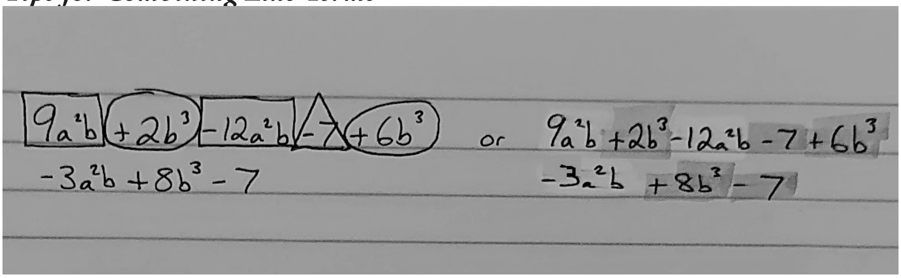

Figure 3. **Tips for Combining Like Terms**

When problems have a lot to keep track of all at once and students need to combine like terms, we recommend having students use visuals to stay organized (see Figure 3; note that it in color it would be yellow for a^2b, green for b^3, and blue for -7). Students who struggle with understanding combining like terms often find one of these two methods helpful when they are given problems with multiple variables. If a student has access to various colors, the method on the right allows them to highlight like terms in the same color and then simply combine the same colors into their answer. A student who does not have access to colors can put different shapes around each like term and then combine circles, squares, triangles, etc. This is shown in the left example in the picture.

Putting the Use of Arrows and Combining Like Terms Strategies Together

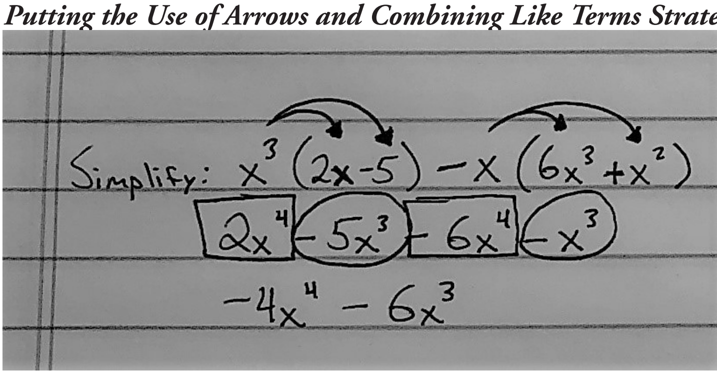

Figure 4. **Putting the Use of Arrows and Combining Like Terms Strategies Together**

If students have to work with a lot of these challenges at once, teachers can teach them to combine what they have learned from their teachers to navigate challenging problems that have a lot of distribution and combining of like terms (see Figure 4). When working with more challenging distribution, we teach students to draw arrows from what is on the outside of the parenthesis to what is being multiplied inside the parenthesis. This helps them make sure they are multiplying to both terms inside the parenthesis. The most common mistakes on this type of problem are for students to forget negatives when multiplying or overlook exponents (e.g., remembering that x has an exponent of 1) when multiplying variables with exponents. For example, students need to remember to take negative x (rather than positive x) times $6x^3$ and x^2 in the previous example. Also, for example, when multiplying the -x times x^2, they need to realize that this becomes $-x^3$ because the variable x has an exponent of 1.

Using the Box Method for Distribution

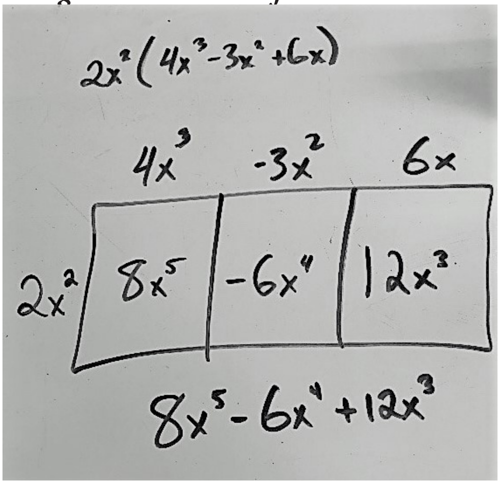

Figure 5. **Box Method for Distribution**

In a problem that involves exponents with variables and more than two terms to distribute a number to, we recommend working with students on setting up a box to keep their work organized and be sure each variable and number is used (see Figure 5). Without the boxes, students can lose track of the many pieces of information they have to think about and organize. The first term is set up to the left of the box with the numbers from inside the parenthesis going across the top. Students then use their knowledge of multiplication and multiplying variables to come up with their answers in each box. Once students have their answers organized in each box they must simply put them together in an equation as their final answer. A student's mistakes in this type of problem will come from not understanding that when variables are multiplied we must add their exponents. In addition, students will often forget to bring a negative into their answer box, like in the middle answer of the picture provided. Be sure to watch for these mistakes from students.

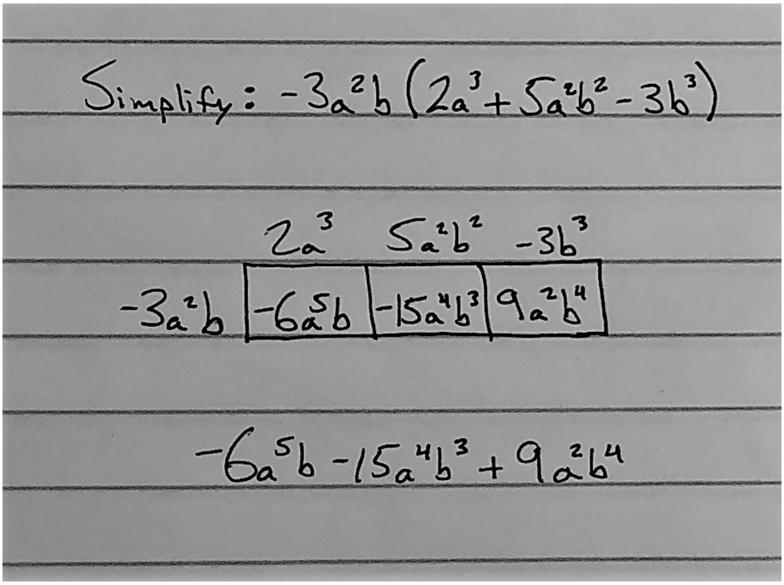

Figure 6. **Box Method for Complicated Distribution**

The hardest part of challenging distribution problems such as this one is that students must make sure that they have the variables correct (see Figure 6). Using the box method can help students be sure they have the correct number of terms in their answer. Some students will forget to include both variables in their answer boxes. They may only multiply the a's in the first box, but think that since there is no b on the top they do not need a b in their answer box. This problem makes students use knowledge of multiplication and multiplying variables. Teachers need to make sure that they have helped students double check their variables.

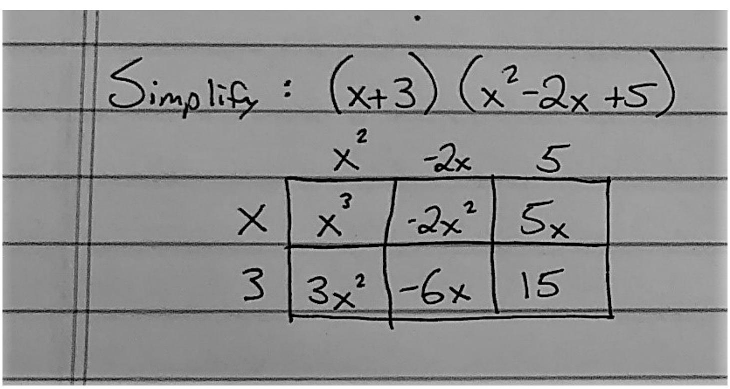

Figure 7. **Box Method for a Binomial and Trinomial**

When multiplying a binomial by a trinomial, we also recommend teaching students to use the box method (see Figure 7). This box method allows students to set up a box with two spaces on the left and three spaces on top. When students multiply, they use the 6 boxes to organize their answers. Students' mistakes happen when they do not multiply correctly to start. To get their final answer, students have to combine like terms and recognize that the boxes with x^2 can be combined and the boxes with x can be combined. In this situation, students can color code like terms using a highlighter to help with combining like terms (e.g., highlight the x^3 pink, the x^2 terms yellow, the x terms green, and the 15 blue).

FOIL or Box Method for Multiplying Binomials

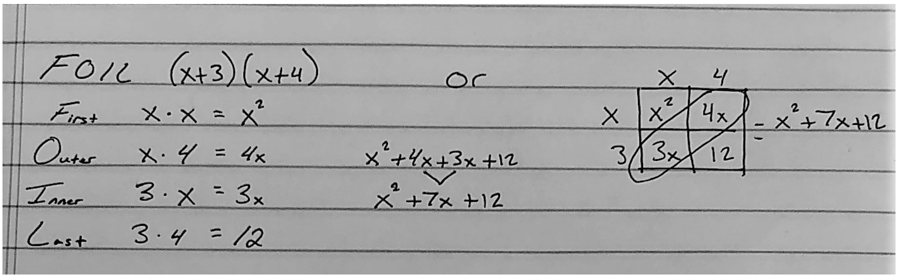

Figure 8. **FOIL and Box Method**

When teaching FOIL for multiplying two binomials, we recommend teaching students to list out each step separately for the acronym of FOIL, so they know exactly what is being multiplied (see Figure 8). Once students have completed each of the four multiplication problems, they list their answers in order and then combine like terms. Notice the V-shape below 4x and 3x to help the students see that those are like terms that can be combined. On the right side of the picture, we used the box method to solve the same problem. Multiplying each piece into its box is for organizational purposes. Notice how we recommend circling the like terms in the box that can be combined. This method can help students stay consistent since we teach them how to use the box method for multiplying monomials, binomials, and trinomials in the future.

Teaching Summary

Students who struggle with a learning disability in math often have trouble keeping their work organized and remembering to carry out each important step. To help these students, as teachers, we have come up with different strategies that students can use time and time again to solve for the correct answer. In the end, we want a student to have a way to keep their work organized and come to a correct answer. By having students show their work, we can see documentation of their thinking processes, and we can assist when there are errors and help them realize what mistakes they have made. It is important to be able to show a student their mistake and have them learn from it. This way, a student can try it again and be successful. The most important part is to remember that not every method or strategy works for every student. Some students appreciate and use the strategies on all problems and other students may ignore our strategy and use another. This is the struggle and the beauty with teaching students that have a learning disability in math. It is not a one size fits all method. And, fortunately, these strategies are useful for all students, not just students with LD.

Connections between Research and Practice

Much like special education researchers recommend putting information into diagrams for intermediate steps with word problems for elementary and middle school students, we demonstrated a pattern of making sure that students offload intermediate steps in algebra problems on paper into boxes. The box methods work similarly to the part-part-whole diagram by Xin and colleagues (2008). The information from the initial presentation of the problem (e.g., a word problem or an equation) is offloaded and organized strategically into the boxes so students do not have to remember a lot of pieces of information and think about those pieces all at once. The diagramming and the box method also help students organize information on paper in accessible ways that often represent the relationships between parts of the problems. In other situations, we recommend that the information is offloaded on paper not in boxes, but just below the equation such as with our demonstration of offloading while using the FOIL method. Students seem to be more successful when they organize their thinking with arrows for distribution and with shapes or color coding for combining like terms rather than when they try to do this all in their head. Overall, the key pattern that we are demonstrating is that students have a better chance of success with solving challenging algebra problems if we teach them to minimize how much work they do in their head and get their work organized in accessible ways on paper.

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