# Lesson Study to Support Preservice Elementary Teachers Learning to Teach Mathematics

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This study sought to investigate what dimensions of didactic-mathematical knowledge preservice teachers discussed when engaged in lesson study and how lesson study supported the preservice teachers in implementing reform-minded teaching of mathematics. We employed an interpretive case study methodology with the intent of conceptualising the lesson study experience of the preservice teachers. The present lesson study took place in methods of teaching mathematics course at a state university in Turkey. We studied the experiences of two cohorts of preservice teachers during two consecutive years. The current study found that the lesson study experience engaged preservice teachers in discussions about the didactical dimension of teacher knowledge and supported their learning to teach from a reform-minded teaching approach.

**Keywords** · preservice elementary teachers · lesson study · didactic-mathematical knowledge · reformed teaching observation protocol · reform-minded teaching

## Introduction

For many years, teacher educators have discussed how to help preservice teachers become proficient teachers (Helgevold, Næsheim-Bjørkvik, & Østrem, 2015; Hiebert, Morris, Berk, & Jansen, 2007). Hiebert et al. (2007) suggested that the aim of teacher education programs is to help preservice teachers learn teaching from studying teaching. Preservice teachers could treat the lessons they teach as experiments and engage in a process of designing lessons based on explicit goals, monitoring the teaching process, collecting feedback, and interpreting the feedback for improving future teaching practices (Hiebert, Morris, & Glass, 2003). A complementary feature of such an approach is collaboration so that preservice teachers learn that a professional teacher draws from and contributes to a shared knowledge base upon which teaching is built. One such pedagogy for preparing teachers is lesson study, an approach that situates preservice teachers' learning in classroom practice. Through lesson study, preservice teachers plan, teach, observe, and revise lessons collaboratively.

The current study sought to examine the lesson study experiences of preservice elementary teachers as they focused on implementing reform-minded teaching practices of mathematics (Australian Association of Mathematics Teachers [AAMT], 2006; Greeno, 2003; National Council of Teachers of Mathematics [NCTM], 1991, 2000). To examine the preservice teachers' development of knowledge of reform-minded teaching, the model of Didactic-Mathematical Knowledge (Pino-Fan, Assis, & Castro, 2015) was used in this study. Although most current research on lesson study has examined inservice teachers' implementation of this professional development model (e.g., Fernandez, 2005; Hunter & Back, 2011; Meyer & Wilkerson, 2011),

recently efforts have focused on applying the lesson study approach in teacher education programs (e.g., Fernandez & Zilliox, 2011; Helgevold et al., 2015; Rasmussen, 2016). Further investigation of lesson study in preservice teacher education and how preservice teachers' knowledge and practices develop in these experiences are warranted (Fernández, 2010; Helgevold et al., 2015; Murata & Pothen, 2011). The current study will contribute to designing lesson study experiences for preservice teachers by examining their knowledge and practices as they engage in a mathematics education methods course in Turkey where the idea of lesson study was quite new. Understanding how lesson study worked in a new culture will add to the international knowledge base about lesson study. The following research questions guided this study: How does lesson study support preservice elementary teachers in implementing reform-minded teaching of mathematics? What dimensions of didactic-mathematical knowledge do preservice teachers discuss as they engage in lesson study?

## Reform-minded Teaching

Mathematics educators have long been advocating a vision of classrooms where students are at the centre of constructing meaning for mathematical concepts and procedures (Grant, Hiebert, & Wearne, 1998). Students are no longer perceived as passive recipients of knowledge during mathematics instruction. This vision of mathematics instruction, which we refer to as *reform-minded teaching*, uses teaching practices that build on students' prior knowledge, support students to develop conceptual understanding and procedural fluency, engage students in analysing multiple representation of a concept, encourage collaborative problem solving, and emphasise multiple solution methods for a given mathematical problem (AAMT, 2006; Cobb & Bowers, 1999; Greeno, 2003; NCTM, 1991, 2000). A teacher's role in reform-minded teaching involves posing carefully selected problems based on interpretations of student thinking to support students' conceptual understanding. Further, the teacher orchestrates whole class discussions and helps students negotiate mathematical meanings with classmates as well as internally making sense of the mathematical activity (Cobb, Stephan, McClain, & Gravemeijer, 2001).

Preservice teachers would need to develop skills and knowledge necessary to enact reform-minded mathematics instruction (Fernandez & Zilliox, 2011). Teacher educators might support preservice teachers' learning by providing them with necessary tools and by designing authentic learning tasks. For instance, classroom observation tools can scaffold preservice teachers' learning through highlighting certain instructional practices and fostering formative assessment (Boston, Bostic, Lesseig, & Sherman, 2015). Using classroom observation tools in authentic learning experiences might engage preservice teachers in collaborative problem solving and help them build knowledge of reform-minded mathematics instruction.

The Reformed Teaching Observation Protocol (RTOP), developed by Piburn and Sawada (2000), defines the characteristics of reformed teaching in mathematics and science lessons and guides the classroom observations. Boston et al. (2015) suggested that RTOP includes general indicators of reform-minded instruction and can be used both quantitatively to identify change in instructional practices over time and qualitatively to promote reflective discussions about the lesson. We utilised this observation tool in both ways in the current study. Some previous research in lesson study contexts revealed changes in preservice teachers' practices regarding specific aspects of reform-oriented teaching (e.g., focusing on reasoning mathematically (Fernández, 2010) and writing meaningful problems (Yu, 2011)). By using the RTOP as an observation tool, the current study aims to employ a holistic approach to examine the preservice teachers' knowledge and practices related to reform-minded teaching and contribute to literature from this perspective.

## Theoretical Notions

The current investigation is based on a theoretical perspective that views learning as a social, cultural, and historical activity (Cobb & Bowers, 1999; Polly, Allman, Casto, & Norwood, 2018; Vygotsky, 1978). From a sociocultural theoretical perspective, learning requires enculturation into a group's practice through participating in meaningful and collaborative activities. Development occurs as the individuals change their ways of participation in these activities. In this process, interaction with a more experienced person (knowledgeable person) or peers and using learning tools support development. Related to teacher learning, Putnam and Borko (2000) proposed that teacher development takes place within authentic activities that foster problem-solving skills of teachers. In the case of preservice teacher education, authentic learning tasks may include planning, teaching, and reflecting on lessons in collaboration with peers.

The idea of using authentic activities in teacher education leads to the issue of what professional knowledge should be developed in these activities. The knowledge base necessary for teaching has been studied extensively. Shulman (1987) proposed seven categories for the teacher knowledge base, three of which are specific to the subject matter being taught: content knowledge, curriculum knowledge, and pedagogical content knowledge. He emphasised pedagogical content knowledge (PCK) since it allows teachers to blend knowledge of content and pedagogy, and organise the content into an understandable form for learners. Building on Shulman's notion of PCK, Ball, Thames, and Phelps (2008) developed the construct of mathematical knowledge for teaching (MKT) that further divided Shulman's categories of content knowledge and PCK into subdomains.

A more recent model that has been used to analyse and characterise mathematics teachers' knowledge is Didactic-Mathematical Knowledge (DMK) (Pino-Fan et al., 2015). DMK builds on an onto-semiotic framework that integrates aspects of ontology of mathematical objects and epistemology with sociocultural theories in order to build a unified approach to mathematical knowledge and instruction (Godino, Batanero, & Font, 2007). The onto-semiotic framework views mathematics as having three aspects: a collaborative problem-solving activity, a symbolic language, and a conceptual system. Engagement in each aspect is considered as mathematical activity. Mathematical objects (e.g., concepts, procedures, arguments) are entities that emerge from and intervene during mathematical activity. Personal knowledge of mathematical objects is the result of individual thinking while the institutional knowledge is the result of communication and agreement within a community of practice. An important concept of onto-semiotic framework is semiotic function that refers to the dependence relation between an expression (signifier) and its components (represented). The richness of semiotic functions established between mathematical objects indicates variety of knowledge. Regarding mathematics instruction, the onto-semiotic framework emphasises students' shared knowledge construction through interacting with their peers and the teacher using available resources. This notion of teaching is aligned with a reform-minded mathematics instruction.

The DMK model reflects the sociocultural theory and highlights enculturation into a group's practice. Preservice teachers can appropriate the institutional knowledge of mathematics instruction by participating in authentic activities designed as part of the teacher education courses. In this study, *the reform-minded mathematics teaching* represents the institutional knowledge of mathematics instruction.

The current study's perspective of teacher knowledge is based on this DMK model since it allows a detailed analysis of each type of teacher knowledge and broadens previous models of teacher knowledge through articulating mathematical knowledge, objects, practices, and instruction (Pino-Fan et al., 2015). According to the DMK model, there are three dimensions of teacher knowledge: mathematical, didactical, and meta didactic-mathematical.

- The mathematical dimension is comprised of common content knowledge (knowledge of a mathematical object) and extended content knowledge (linking a mathematical object with another mathematical object that will be studied in future).
- The didactical dimension includes six subcategories: *interactional facet, mediational facet, ecological facet, cognitive facet, affective facet, epistemic facet.* Sample indicators for each facet is given in Table 4 in the methodology section.
- Meta didactic-mathematical dimension refers to teachers' reflection on their own teaching performance and suggestions for improvements.

This study examined the preservice teachers' knowledge of reform-minded instruction using both the DMK model and the RTOP. The RTOP was used as a classroom observation tool to help preservice teachers reflect on their teaching and build institutional knowledge of mathematics instruction as a community of practice. It also helped us understand whether there was a change in preservice teachers' instruction towards reform-minded teaching. The DMK model helped to conduct a fine-grained analysis of which dimensions and subcategories of teacher knowledge was focused on during an authentic learning experience, lesson study, and gave insight into emergence of institutional knowledge during this experience.

## Lesson Study

Originating in Japan, lesson study is a teacher-led professional development method that engages a group of teachers in working collaboratively in a cyclical process of lesson planning, teaching, reflecting, and revising (Baba, 2007; Lewis, 2002). Throughout the lesson study process, the planning stage includes transforming the curriculum into a lesson plan. The lesson study group develops a lesson focusing on an overarching goal that they chose based on student needs. Then, one teacher from the group teaches the lesson while the other members observe to collect evidence about student learning. Next, the group discusses student learning based on the data collected during the lesson observation and analyses any disparities between the overarching goal, the lesson plan, and the implemented lesson. Lesson study includes ongoing revisions. After usually several cycles, a reflective report is written by the lesson study group to inform teachers' practices. An outside knowledgeable advisor might also join the lesson study group to offer critical feedback.

In recent years, there has been an increase in the use of lesson study for learning to teach mathematics in teacher education programmes. Teacher educators have applied different versions of lesson study in different contexts such as having preservice teachers teach the lesson to their peers (Fernández, 2010) or to a small group of elementary school students (Yu, 2011); teaching the lesson once (Corcoran & Pepperell, 2011; Murata &Pothen, 2011); and involving cooperating teachers as knowledgeable persons (Rasmussen, 2016). Some of these studies investigated what preservice teachers learned through participating in these experiences. Analysis of preservice teachers' learning revealed that they expanded their mathematics knowledge (Fernández, 2010; Murata &Pothen, 2011); explored the meaning of mathematical practices (Corcoran & Pepperell, 2011; Rasmussen, 2016); revised their teaching approaches to be more student-oriented (Fernández, 2010); and improved their mathematical knowledge for teaching (Corcoran & Pepperell, 2011; Fernández, 2010; Leavy & Hourigan, 2016).

Among the latter group of studies, only Leavy and Hourigan (2016) focused on subcategories of mathematical knowledge for teaching and reported improvements in the preservice teachers' knowledge of content and students, and knowledge of content and teaching. Conducting fine-grained analyses of how preservice teachers develop institutional knowledge (reform-minded mathematics instruction) in each of the DMK dimensions has the potential to increase the research

community's knowledge of preservice teachers' learning to teach mathematics as they engage in lesson study. Understanding how the RTOP supports the knowledge development in a lesson study setting will provide insight into using observation tools in authentic experiences. Lesson study involves authentic collaborative experiences for the participants to reflexively examine their personal knowledge of teaching mathematics and build institutional knowledge in a community of practice. Thus, it could offer an appropriate context for analysing preservice teachers' knowledge development with the DMK model and the RTOP.

# Methodology

We employed an interpretive case study methodology with the intent of conceptualising the lesson study experience of the preservice teachers. Interpretive case study is used to develop conceptual categories to explain a phenomenon (Merriam, 1998). In this mode of investigation, a researcher collects rich data and develops a typology to understand the case. In the current study, we were interested in understanding how engaging in lesson study supported the preservice elementary teachers' mathematical and didactical knowledge and their reform-oriented mathematics teaching practices. We used the DMK model to uncover the content of preservice teachers' lesson study communications in the mathematical and didactical dimensions. Both the DMK model and the RTOP framed the analysis to understand how the preservice teachers co-constructed institutional knowledge about reform-minded instruction during the lesson study experience.

## *Research Context and Participants*

The preservice elementary teachers in Turkey take two required courses on methods of teaching mathematics, each being three hours per week for fifteen weeks, during the third year of undergraduate education. The first author taught both methods of teaching mathematics courses. In the current study, the first course focused on developing preservice teachers' DMK for teaching each strand of elementary school mathematics. For instance, the preservice teachers analysed video-recorded lessons featuring examples and non-examples of student-centred mathematics instruction and discussed issues related to interactional and mediational facets; participated in and reflected on mathematics activities to increase knowledge related to epistemic facet; and interviewed an elementary school student to probe the student's thinking about a specific mathematics topic and reflected on their knowledge in the cognitive and affective facets. The second course focused on developing preservice teachers' skills in planning and implementing reform-minded mathematics lessons. We integrated a 6-week lesson study project in this second course in conjunction with the practicum course taken by the preservice teachers in the same term to have an authentic teaching context. Two unique elements of the current lesson study design are allowing preservice teachers to teach both in the university classroom and in the elementary school classrooms and using RTOP as the lesson observation instrument. Table 1 presents a summary of the lesson study process used in this study.

This study focused on the experiences of two cohorts of preservice teachers during two consecutive years. The participants in the first year were 26 preservice teachers aged 20-23. The participants in the second year were 31 preservice teachers aged 20-24. In the preparation phase, the preservice teachers formed cooperative groups for their lesson study project. There were eight groups in the first year and nine groups in the second year. In the lesson planning phase, the course instructor provided examples of overarching goals from previous lesson studies detailed in Lewis (2002) and suggested that they focus on one of the mathematics process standards

(NCTM, 2000). Each group integrated mathematics process standards into their overarching goal. As an example, one group's overarching goal was as follows: *Enhance students' problem-solving skills in subtraction problems and help them value different solution methods*.

Table 1
*The Lesson Study Process Used in This Study*

| Lesson Study Phase | Procedures |
|---|---|
| Preparation | • The preservice teachers formed groups of 3 to 4 members.<br>• The course instructor introduced the lesson study process to the preservice teachers.<br>• The preservice teachers practiced using the RTOP. |
| Lesson Planning | • Each group determined an overarching goal that integrates mathematical processes.<br>• Each group wrote a lesson plan using the lesson plan format by Meyer and Wilkerson (2011). |
| Teaching | • The lesson study groups taught their lessons three times.<br>o The first teaching took place in the university classroom.<br>o The second and third teaching took place in two different elementary school classrooms.<br>• The preservice teachers took turns in teaching the lesson with the other group members observing and taking notes.<br>• Each lesson was video-recorded.<br>• The course instructor observed or watched the video of the lessons. |
| Reflecting and Revising | • The lesson study groups watched the video of their lesson.<br>• Each group member completed one RTOP form individually.<br>• Then, the group completed one RTOP form collectively, discussing how their lesson worked and how it could be improved for future teaching.<br>• The course instructor met with each group after the first and second teaching.<br>• Each group wrote a revised lesson plan noting the reasons for the revisions. |
| Reporting | • Each group submitted a group report. This report included an explanation of how the overarching goal was determined, all lesson plans, reasons for the revisions on each lesson plan, video recordings of the lessons, individual and group RTOP forms, elaboration of how each group member contributed to the project, and a group reflection paper on their lesson study experience. |

As part of the *reflecting and revising phase*, after each lesson, each group member completed the RTOP first individually and then one RTOP form was completed as a group to promote collegial

discussions and reflections on their teaching from a reform perspective. In the first and second lessons taught, the instructor observed or watched the video of the lessons and met with the group to provide feedback. In these meetings, the instructor, as a knowledgeable advisor, asked group members' reflections on the lessons and posed questions to help them consider how the teaching influenced students' mathematical learning.

## *Data Sources and Analytical Process*

In this study, the data were used to answer two research questions (see Table 2) by employing three analytical approaches (RTOP scoring, DMK coding, and lesson plan coding). Table 2 presents how each research question is related to the data sources and the analytical procedures employed. Data sources included 17 group reports and transcripts of audio-recorded lesson study discussions of four volunteer groups (group-A and group-B from the first-year cohort and group-C and group-D from the second-year cohort). Each group had four meetings. Since group-A did not record two of their meetings, there were 14 recorded group discussions in total. Four group discussions were planning meetings; the remaining group discussions were post-lesson reflection meetings. The length of the recorded discussions ranged from 10 to 40 minutes.

Table 2
*Data Sources and Analytical Procedures*

| Research Question | Primary Data Sources | Analytical Procedures |
|---|---|---|
| 1. How does lesson study support preservice elementary teachers in implementing reform-minded teaching of mathematics? | • 17 groups' first and third video lessons<br>• RTOP forms completed by the preservice teachers<br>• 17 groups' first and third written lesson plans | • RTOP scoring<br>• Lesson plan coding |
| 2. What dimensions of didactic-mathematical knowledge do preservice teachers discuss as they engage in lesson study? | • 14 recorded group discussions by four groups<br>• 17 groups' written reflections on the lesson study process | • DMK coding |

To answer the first research question, each group's first and third video lessons were scored using the RTOP to determine any changes in the preservice teachers' teaching approaches. Prior to using RTOP for the current study, the first author had completed a workshop for using RTOP in research studies; therefore, she conducted this analysis. This instrument has been adapted into Turkish context and was shown to be valid and reliable (Temiz & Topcu, 2013). RTOP has five parts. The first two parts are for collecting contextual information such as class size, sitting plan, and observation notes. The next three parts consist of 25 items adding up to 100 points. Table 3 provides brief descriptions for these three parts and sample items. To obtain an RTOP score, the

observer views the video-recorded lesson, taking notes on the first two parts of the form. Then, the observer assigns scores from 0 to 4 for 25 items.

- A score of 0 indicates that the behaviour was not observed.
- A score of 1 indicates that the behaviour was observed at least once.
- A score of 2 indicates that the behaviour was observed more than once but it loosely describes the lesson.
- A score of 3 indicates that the behaviour was frequently observed.
- A score of 4 indicates that the behaviour describes the lesson very well.

Adding the scores of 25 items results in an RTOP score for the lesson (MacIsaac & Falconer, 2002). Wilcoxon matched-pairs signed-ranks test was used to determine whether there was a significant change between the RTOP scores for the first and third lessons.

Table 3
*A Summary of RTOP*

| RTOP Categories and Subcategories | Description | Sample Items |
|---|---|---|
| Lesson Design and Implementation | Assesses whether the students participate in the learning process as members of a learning community. | The focus and direction of the lesson was often determined by ideas originating with students. |
| Content | | |
| Propositional Knowledge | Assesses the significance of the content and students' conceptual understanding of it. | The lesson promoted strongly coherent conceptual understanding. |
| Procedural Knowledge | Focuses on the process of inquiry. | Students made predictions, estimations and/or hypotheses and devised means for testing them. |
| Classroom Culture | | |
| Communicative Interactions | Aims to capture the nature of classroom discourse during instruction. | Student questions and comments often determined the focus and direction of classroom discourse. |
| Student/Teacher Relationships | Assesses classroom climate regarding how the teacher supports students' inquiry-based learning. | The teacher acted as a resource person, working to support and enhance student investigations. |

The authors analysed the first and third written lesson plans to determine any change towards involving more student-centred procedures. To analyse the lesson plans, a coding scheme was developed drawing on the existing literature in mathematics education (AAMT, 2006; Fernández, 2010; NCTM, 2000). The codes used in the analysis of lesson plans were collapsed under two categories as follows.

- Codes reflecting reformed teaching procedures: Students discover relationships, students engage in problem solving, students use multiple representations, students communicate with each other, students explain relationships, students make connections, and teacher guides student exploration.

- Codes reflecting teacher-centred procedures: teacher presents concepts and students rote practice.

For example, the code *students explain relationships* was observed in group 1's third lesson plan as the students were asked to justify why the commutative property of multiplication holds true using unit cubes. The code *students rote practice* was observed in group 1's first lesson plan as the students solely practiced the commutative property of multiplication using pencils and balloons without providing a justification. Three groups were randomly selected and their first and third lesson plans were coded individually by the authors by examining whether each code was present in each lesson plan or not. The inter-coder reliability on the lesson plans was 89%. Since the inter-coder reliability was high (Miles & Huberman, 1994), the first author coded the remaining lesson plans.

To answer the second research question, we applied the DMK model (Pino-Fan et al., 2015) to analyse the groups' written reflections and transcripts of audio-recorded lesson study discussions. This analysis helped us understand what the preservice teachers' attention was being drawn towards during the lesson study process and, in turn, what they thought about, reflected on, and learned. During this phase of the analysis, written reflections and transcripts of the lesson study discussions were unitised into distinct ideas expressed in a phrase, sentence, or paragraph. Then, all units of data were classified into mathematical dimension or didactical dimension of DMK. Data coded under the didactical dimension was further classified into the facets of this category. Table 4 lists sample indicators. Communications falling under each subcategory were colour coded. Using a word counting function of a word processor, we calculated percentages for each subcategory.

Table 4
*Indicators of Subcategories of Didactical Dimension*

| Knowledge Subcategories | Sample Indicators |
|---|---|
| Interactional Facet | Organising interactions among the teacher and the students and between the students. |
| | Selecting pertinent representations to illustrate the concept. |
| Mediational Facet | Assessing the pertinence of the use of materials to foster students' learning. |
| | Managing time in teaching activities. |
| Ecological Facet | Linking mathematics curricula at different grade levels. |
| | Identifying contextual, social, political, and economic factors that influence teaching and learning processes. |
| Cognitive Facet | Anticipating/analysing student misconceptions when solving a mathematical task. |
| | Interpreting students' mathematical ideas. |
| Affective Facet | Selecting tasks that motivate students to do mathematics. |
| | Dealing with students' behavioural aspects and mood changes. |
| Epistemic Facet | Comprehending multiple meanings of a mathematical object. |
| | Providing mathematical explanations. |

We strove to increase validity by using rigorous data analysis methods and presenting rich descriptions (Patton, 2002). The findings were triangulated by using multiple data sources such as lesson study discussions, group reflections, or video records (Patton, 2002). Collecting data from multiple sources corroborated conclusions drawn from a single source, increasing reliability of findings.

## Results and Analysis

This section is organised into two subsections. To answer the first research question, we present the findings about how the teaching of the preservice teachers changed throughout the lesson study experience based on reform-minded teaching and the RTOP coding. Next, to answer the second research question about what dimensions of didactic-mathematical knowledge the preservice teachers discussed as they engaged in lesson study, we provide the findings related to DMK analysis. All names used are pseudonyms.

### *Changes in the Teaching Approaches toward Reform-Minded Teaching*

The RTOP scores assigned to the first lesson videos ranged from 37 to 64 with a mean of 50.1 while the scores for the third lesson videos ranged from 51 to 70 with a mean of 59.7. An RTOP score greater than 50 indicates a teaching approach containing features of reformed teaching (MacIsaac & Falconer, 2002). As the RTOP score increases, the lesson is more aligned with reform-oriented teaching. The RTOP scores of the preservice teachers' lessons indicate that even the first lessons included elements of reformed teaching, possibly due to knowledge and experience gained in the first methods course. Throughout the lesson study project, the lessons were revised to include more features of a reform-oriented teaching approach. According to Wilcoxon matched-pairs signed-ranks test results, RTOP scores increased significantly (p=.000) from the first lesson to the third lesson.

Figure 1 contains procedures included in the first and third lesson plans. Preservice teachers' lesson plans also indicated a shift towards reform-minded teaching. The first lesson plans included procedures such as using multiple representations and students' communications with each other, procedures that are promoted by RTOP. Preservice teachers' discussions on student-centred procedures through RTOP seem to affect their lesson planning as we found more evidence of reform-minded teaching in the third lesson plans. For example, the third lesson plans included more instances of *students engage in problem solving* and *students discover relationships*. In summary, both RTOP scores and Figure 1 show that the preservice teachers changed their teaching approaches to be more aligned with a reform-oriented mathematics instruction. This finding was confirmed by the preservice teachers in their written reflections. For instance, in their group report, group 2 wrote "We completed a project that connected the theories of university courses with practice. We understood the importance of being a teacher as listener and experienced the joy when students made discoveries. It helped us teach different from traditional teaching approach."

The RTOP scoring and lesson plan coding results present an overall picture about the changes in the preservice teachers' mathematics teaching practices. The next section provides a more detailed analysis into how the preservice teachers co-constructed didactical-mathematical knowledge throughout the lesson study process.

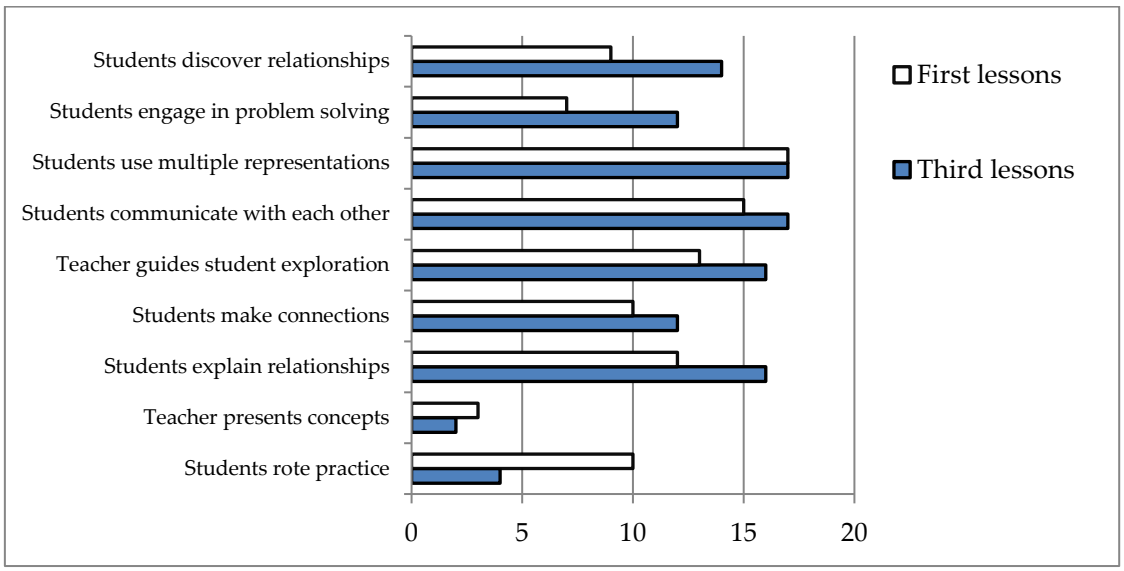


*Figure 1.* Number of procedures included in the first and third lesson plans.

## *DMK Analysis in Lesson Study Discussions and Written Reflections*

This section includes the findings from the DMK analysis of lesson study communications of four groups (group-A, group-B, group-C, and group-D) and reflections written by all 17 groups. The data analysis revealed that the lesson study groups did not discuss or write about mathematics itself. On the other hand, we found that the preservice teachers spoke and wrote about the didactical dimension of DMK. Table 5 shows the percent of communications in each knowledge category across four meetings. We primarily focus on group-C since it is a typical case that illustrates the preservice teachers' thinking and learning during lesson study discussions. Group-C members will be named PT1, PT2, and PT3.

**Interactional facet [IF]**

Table 5 shows that the groups spoke a considerable amount about classroom interactions. The preservice teachers intended to create a class culture where students actively engaged in meaningful lesson activities and the teacher listened to the students' ideas and guided their exploration of mathematical concepts. An example for such intention occurred in group-C's second lesson in terms of not correcting a student's error right away. This group taught a measurement lesson with nonstandard units to the first-grade students to help them connect mathematics with real life. The lesson started with comparing the lengths of two paper arm models stuck to the board, not allowing direct comparison. A girl claimed that they had the same length while in fact they were not equal in length. She measured the arm models with her handspan. The group had the following discussion about this incident in their second post-lesson meeting. PT1 taught the lesson.

PT1: The class said it [one of the arms] was longer. [Cognitive facet [CF]] In fact, if Sema did not say that they were equal, my job would have been difficult. Because in that case I couldn't have a class discussion. [IF]

PT2: Yes, it was good for you. [IF]

PT1: It was good for me that Sema thought that they were equal. [CF] I quickly recognised her response. [IF]

PT3: You had several students measure the arms. That was also good. They discovered themselves. [IF]

PT1: In fact, yes, we did not plan to have that many students measure, but the class teacher liked it, too…In fact, when Sema made the error, the teacher wanted to interfere. I said "wait; let her leave space [between hand spans]." [IF]

PT1 used this opportunity to discuss measurement errors with students. Instead of correcting the student's error right away, as the class teacher who was observing the lesson attempted, the preservice teacher asked other students' opinions and invited them to share their solutions with the whole class.

After an overall reflection on the lesson, group-C assessed the lesson using the RTOP. RTOP provided a forum for the preservice teachers to analyse and reconsider their knowledge about classroom interactions. For example, one item of RTOP asks preservice teachers to determine whether "the focus and direction of the lesson was often determined by ideas originating with students." PT2 commented that the teacher (PT1) determined the focus and direction of the lesson. PT1 disagreed "the focus and direction of the lesson, in fact things came up that we did not plan, Ali made measurements, and others too [after Sema's comments]." PT3 agreed that the lesson procedure changed according to student ideas. Group-C wrote in their reflection paper "one of our main goals was to lead the direction of the lessons according to what students say." RTOP includes items focusing on forming a learning community in the classroom particularly in the *lesson design and implementation* and *classroom culture* sections. Designing and assessing their lessons using RTOP helped the preservice teachers focus on building productive classroom discourse in their instructions.

Other groups expressed similar goals in their reflection papers as well and planned lessons that were student-centred and encouraged student-student interactions. This is evident in groups' lesson plans. Fifteen of the 17 groups initially integrated group work into their first lesson plans and all the groups included group work in their third lesson plans. The preservice teachers not only included group work in their plans, but also tried to integrate elements of cooperative learning such as positive interdependence (e.g., assigning a role to each group member) and individual accountability (e.g., giving individual mini test at the end of the lesson). Such classroom interactions are aligned with a teaching approach that would get high scores on RTOP.

Another finding about the interactional facet was that initially, many groups planned too many activities for their lesson. The course instructor (the first author) helped groups focus on their learning goal by frequently asking them how each activity in the lesson contributed to their mathematical learning goal. Six of the 17 groups reduced the number of tasks used in their initial lesson plans to teach fewer but deeper content. For instance, after their first lesson, in their group report, group 9 wrote "We removed the egg activity that was designed for teaching the making union meaning of addition. We decided to focus on adding-on meaning of addition only. There were too many activities for one lesson." This group's initial lesson plan had several tasks one of which included adding numbers using base ten blocks. The revised lesson plan focused on teaching decomposition-to-10 strategy using base ten blocks and included a worksheet for students to represent their work using multiple representations such as pictures, numbers, and words. Instead of implementing several tasks, the group decided to implement fewer tasks with a specific content focus that allowed students to make richer connections. The course instructor's involvement in each cycle of lesson study helped preservice teachers focus on students' conceptual learning of mathematics aligned with reform ideas.

Table 5
*Distribution of Lesson Study Communications over the Didactical Dimension*

| | | Planning Meeting | First Post-lesson Meeting | Second Post-lesson Meeting | Third Post-lesson Meeting |
|---|---|---|---|---|---|
| Interactional Facet | Group-A | 51% | | 31% | |
| | Group-B | 41% | 66% | 42% | 60% |
| | Group-C | 21% | 54% | 50% | 40% |
| | Group-D | 57% | 49% | 58% | 57% |
| Mediational Facet | Group-A | 6% | | 20% | |
| | Group-B | 12% | 15% | 18% | 0% |
| | Group-C | 38% | 9% | 9% | 5% |
| | Group-D | 8% | 15% | 7% | 0% |
| Ecological Facet | Group-A | 5% | | 5% | |
| | Group-B | 11% | 1% | 0% | 0% |
| | Group-C | 6% | 11% | 14% | 10% |
| | Group-D | 21% | 5% | 10% | 6% |
| Cognitive Facet | Group-A | 5% | | 27% | |
| | Group-B | 2% | 5% | 13% | 4% |
| | Group-C | 2% | 11% | 17% | 26% |
| | Group-D | 6% | 7% | 7% | 0% |
| Affective Facet | Group-A | 18% | | 8% | |
| | Group-B | 20% | 11% | 24% | 31% |
| | Group-C | 25% | 9% | 8% | 12% |
| | Group-D | 1% | 13% | 17% | 23% |
| Epistemic Facet | Group-A | 7% | | 0% | |
| | Group-B | 7% | 0% | 0% | 0% |
| | Group-C | 7% | 2% | 0% | 0% |
| | Group-D | 2% | 7% | 0% | 0% |

**Mediational facet [MF]**
Table 5 shows that the percent of communications in this category ranges from 0% to 38%, with lower percentages in the third post-lesson discussions. Mediational facet involves knowledge about classroom resources. All the groups included a material (e.g., unit cubes, fraction circles, and rules) in their lessons. The following conversation from group-C's planning meeting illustrates how the preservice teachers discussed about using materials for teaching mathematics.

This group designed a ruler made of ladybug stickers put end to end to make a measuring tool with a nonstandard unit.

PT2 (teacher): …I will not count as one ladybug ruler, two ladybug rulers, and so on. I will count [the stickers] one by one. [MF]
PT1: Are you going to count one by one or just call it a ruler? … [MF]
PT2: We will count the ladybugs [stickers]. This is a ladybug ruler but, for instance, the length of this arm is 27 ladybugs. [MF]

The preservice teachers discussed how to use classroom materials to foster student learning. They sometimes revised the materials after getting feedback from their peers, elementary school students, or the course instructor. For instance, in her meeting with group-C, the course instructor asked them whether they had a mathematical purpose for choosing the number 9 as the number of ladybugs on each ruler. They responded that it was a random selection. One group member suggested revising the ruler to include 10 ladybugs to support students' transition to standard units. In their group report, group-C wrote "In our first teaching there were 9 ladybugs on each ruler. We realised that students might better grasp the concept if there are 10 ladybugs, and so we increased the number of ladybugs to 10."

**Ecological facet [EF]**

Table 5 shows that the percent of communications in this category ranges from 0% to 21%. Communications about ecological facet were about curricular and contextual factors. In regard to curriculum, the groups discussed about which standard to focus on, what topics the students had learnt, students' grade level, and the alignment of the tasks to the standards. Regarding the contextual factors, the preservice teachers referred to the existing class culture in their elementary classrooms. For instance, during the practicum experience, they observed that the class teachers did not use manipulatives frequently in their mathematics lessons. As a solution, some groups introduced the materials used in their lessons such as base ten blocks prior to their lesson. Also, some groups had problems in implementing aspects of cooperative group work such as students taking turns in completing a worksheet or following their role as a group member. They discussed that some of the problems were due to the students' lack of experience with group work. For instance, group-C reported that the students in their class had not engaged in group work before. Similarly, group-D wrote "One challenge for us was that our students do not often use manipulatives and are used to direct teaching method." These challenges were revealed during the second and third lessons in elementary classrooms. The preservice teachers noticed contextual factors effective in mathematics instruction through teaching in elementary classrooms and sought solutions in cooperation with their peers when they had challenges.

**Cognitive facet [CF]**

According to Table 5, there were more conversations about students' mathematical thinking and understanding in the post-lesson meetings than in the planning meetings. The higher percentages in this category were observed in the second and third post-lesson meetings that took place after the lessons taught in elementary classrooms. For example, in group-D's second post-lesson meeting, they discussed about students' conceptual understanding as asked by an RTOP item. PT1 who observed the lesson spoke

> A weakness that I observed was that, for example for adding 14 and 26, our goal was for them to say 6 plus 4 is one ten, ten ones make up one ten, but some of them could say it and some could not since they had not learned it before. [CF] Shall we not interfere there in our third lesson? I mean, when they added 6 and 4 and put 10 ones together, we should not say this is one 10 and should accept their way of doing it? [IF]

The group agreed to provide several examples without interfering the student explorations and then to ask students which patterns they observe in the examples solved. The preservice teachers had observed the students' solution methods in the previous lesson and based on their observations they revised the teaching activities to allow more opportunities for student exploration and thinking for the future lesson. Through collaborative reflection on student thinking in the post-lesson meetings using the RTOP, the preservice teachers negotiated the meaning of student-centred instruction. Group-D reported "We planned and revised our lessons according to students' cognitive, affective, and psychomotor development levels...We experienced that we should pay attention to what students say and we can learn a lot from them."

At times, the groups explained students' conceptions or misconceptions based on curricular or contextual knowledge (ecological facet). For example, group-C had the following conversation about students' failure to provide mathematical explanations in their second post-lesson meeting. The conversation is about three types of ladybug rulers shown to students, with two of them having space between the ladybugs.

PT1: They all showed the correct one among the 3 rulers. [CF]
PT3: This shows us that they knew the principles of measurement. [CF]
PT1: It does, at least when they encounter them in real life, they are …
PT3: Familiar…
PT1: Familiar with them. After pushing a little bit, they were able to say "there are spaces between" I mean, they can speak about the principles of measurement, they expressed that it wouldn't work, but they couldn't explain the reason. [CF]
PT2: That's because they did not finish the measurement unit yet. [EF]
PT1: They did not, and their age level is low, we cannot expect them that much. [EF]

This excerpt illustrates that the group had low expectations from the students and excused students' failure to provide mathematical explanations with reasons in the ecological facet category such as students' age, instead of revising their teaching approach to have rich communications about the principles of measurement.

**Affective facet [AF]**

Table 5 shows that the percent of communications in this category ranges from 1% to 31%. Communications and reflections about students' emotional and behavioural aspects fell into this subcategory. The preservice teachers referred to emotional aspect of affective facet in terms of motivating students for learning mathematics in their group reports and discussions. Examples included introducing the lesson with a puppet, using drama, making connections to cartoons, and playing games. Ten of the 17 group reports included reflections on motivation. For instance, group 4 wrote "The motivation at the beginning of the lesson is crucial for attracting students' attention. For example, students liked the pictures used in Gargamel task [a character in the Smurf cartoon] and the real-life connections made in our teaching… They all wanted to express their ideas." Group 15 reported "We realised that in order to spark students' interest and increase students' relationships with each other, we need to make our teaching engaging." Most lesson study groups used active student learning as a strategy for increasing motivation, a recommendation by recent reforms in mathematics education (AAMT, 2006; NCTM, 2000).

Regarding behavioural aspect of the affective facet, we observed that, in some of the classrooms, the preservice teachers experienced classroom management problems. According to Table 5, the percent of communications in the affective facet category for the groups B and D are high in the second and third post-lesson meetings. Both group-B and group-D planned games as

part of their lessons and had classroom management problems as students were playing the games. The lesson study meetings helped them reflect on the reasons for the problems and develop strategies to resolve them in their next or future teaching. They used strategies such as using a puppet to lower the noise level in the classroom, discussing about communication skills with the students, using the white board effectively or playing the game in several stages to help students focus on the lesson, strategies that were discussed in the methods courses.

**Epistemic facet [EpF]**

The groups mainly spoke about mathematical knowledge shaped for teaching in the planning meetings as the percentages in Table 5 indicate. The conversations that fell into this subcategory was about meaning of operations, correct use of mathematical terminology, elaborating mathematical processes such as problem solving, and representing mathematical concepts. For instance, in their planning meeting group-C spoke about integrating conceptual foundations of measurement such as unit iteration into their lesson plan by designing different types of rulers.

PT1: We could make rulers and put stickers on them by leaving space in between. [MF]

PT2: We could stick them but how are we going to integrate the principles [of measurement]? [EpF]

PT1: one will have space between the stickers, one will have space at the beginning, and one will have space at the end. [EpF]

The group continued to talk about the design of the activity without further elaborating conceptual foundations of measurement. Typically, the conversations in the epistemic facet category were not long and detailed.

## Discussion

In this study, we examined the development of institutional knowledge (reform-minded mathematics teaching) in the context of a lesson study design. The findings revealed that the participating preservice teachers revised their teaching approaches toward reform-minded teaching. The RTOP scores significantly increased from the first lesson to the third lesson and the lesson plans included more instances of student-centred procedures, potentially indicating development of reform-minded teaching as institutional knowledge. The DMK coding provided a fine-grained analysis of preservice teachers' knowledge in the didactical dimension. In the following section, we draw on all three analyses (the RTOP scoring, lesson plan coding, and the DMK analysis) and discuss the emergence of institutional knowledge in the lesson study experience of the preservice teachers. Then, in the next section, we discuss the elements of the current lesson study design that supported the preservice teachers' learning.

### *Preservice Teachers' Learning about Reform-Minded Teaching*

The current lesson study design engaged the participating preservice teachers in discussions and experiences that supported their learning about reform-minded teaching as the institutional knowledge since their attention was on instructional processes that are important from a reform perspective (Cobb & Bowers, 1999). For example, the DMK analysis revealed that the preservice teachers had detailed and extensive communications that fall into the interactional facet category as the percentages in Table 5 indicate. The interactional facet is about sequencing instructional tasks and organising classroom interactions. In terms of sequencing instructional tasks, we found that the groups reduced the number of tasks (e.g., focusing on teaching decomposition-to-10

strategy only) or reorganised them (e.g., asking several addition with regrouping questions to help students explore patterns and make mathematical connections) throughout the lesson study cycles and focused on students' conceptual learning through meaningful activities and interaction with others. About classroom interactions, we found that the procedures that promoted classroom interactions such as *students communicate with each other* increased from the first lesson plans to the third lesson plans. The preservice teachers realised the importance of improving the communication of mathematical ideas in the classroom and designing inquiry-based instructional tasks to better support their students' conceptual understanding, aligned with the recommendations of mathematics education communities (AAMT, 2006; NCTM, 2000). This awareness can benefit their personal knowledge development as they implement and reflect upon reform-minded teaching practices in their future career, and as suggested by Leavy and Hourigan (2016), they could transfer the knowledge gained through lesson study to other teaching contexts.

The participating preservice teachers aimed to design and use appropriate learning materials and give sufficient time to their students, aspects of the mediational facet category, for allowing students to contribute to shared knowledge construction in the classroom. Using learning materials and managing time effectively to foster student learning is one of the core ideas in reform-minded teaching (Grant et al., 1998) and therefore preservice teachers' attention to these aspects is important for their development as reform-minded teachers. The content category of the RTOP has items asking about students' use of materials to represent concepts. Using the RTOP as the classroom observation tool possibly influenced preservice teachers to use learning materials in their lessons. All 17 groups used a material in their lessons as evidenced in the lesson plans. The materials were used to scaffold students' conceptual learning. For instance, group-C designed three different ladybug rulers and asked students which could be used to measure length correctly with an aim to promote discussion of principles of measurement. Throughout the lesson study cycles they changed the number of ladybugs from 9 to 10 in the correct ruler to help students' transitioning to standard measurement units after having a meeting with the course instructor. These types of experiences during the lesson study indicate learning about how to design and use materials effectively in a reform-oriented classroom.

Teaching in elementary classrooms gave the participating preservice teachers opportunities to expand their knowledge in the ecological facet category. The preservice teachers' learning seemed to focus on the contextual aspect of the ecological facet. They noticed that the existing class culture influenced their mathematics teaching practices. The groups reported that the existing class culture reflects a teacher-centred learning environment. Their teaching practices aimed to challenge this culture towards reform-minded teaching where students had more voice in determining the direction of the lessons. For the challenges they faced in the elementary classrooms, they generated solutions in cooperation with their colleagues, a culture that we aim to foster both in preservice and inservice teachers (Fernandez, 2005; Hiebert et al., 2003).

Another aspect of the institutional knowledge that the participating preservice teachers learned about is cognitive facet that refers to the knowledge about student thinking. The reform-minded teaching emphasises using student thinking as a resource to adapt instruction (AAMT, 2006; NCTM, 1991). RTOP has items asking about students' conceptions of a topic, solution methods for problems, and reflections on their learning (Piburn & Sawada, 2000). In order to answer these questions, the preservice teachers carefully observed the students during the instruction and afterwards reflected on student thinking and learning in the post-lesson debriefings. The communications about students' lack of understandings or misconceptions often triggered communications about alternative teaching strategies for future lessons, indicating that the preservice teachers recognised the need to revise the instructional decisions in order to reach the desired learning outcomes (Hiebert et al., 2007). The groups reported that they revised their

lesson plans based on student thinking. Even though the preservice teachers' attention to student thinking is important for the development of reform-minded teaching, there were instances where the preservice teachers explained student thinking based on ecological factors and demonstrated low expectations from their students. This finding might inform future knowledgeable advisors in lesson study settings to guide preservice teachers' discussions involving student thinking. Research studies that elaborate and exemplify development of students' mathematical thinking might be utilised (e.g., Yu, 2011).

One finding about the cognitive facet category is that group-B and group-D did not speak about students' mathematical thinking to a greater extent, perhaps due to experiencing classroom management problems. Fernandez and Zilliox (2011) reported a similar finding that some preservice elementary teachers focused on classroom management issues without formative assessment from a knowledgeable person. Our study suggests that some lesson study groups might still need more support for paying closer attention to students' mathematical thinking even when a knowledgeable advisor was involved in the lesson study cycles. RTOP is effective in promoting most components of reform-oriented teaching; however, it is not specifically focused on mathematics (Boston et al., 2015) and could be enhanced with adding open-ended sub-items. For instance, sub-items asking about students' conceptions and misconceptions of the mathematical concepts might be added to an RTOP item that assesses conceptual understanding. Such a revision might further support the development of institutional knowledge in regards to the cognitive facet.

Data analyses revealed that the participating preservice teachers emphasised motivating students to learn mathematics and dealt with student behaviours, aspects of the affective facet. Both of these aspects were discussed in the lesson study meetings and group reports. The preservice teachers noticed that when students are active participants of the lesson, their motivation increases. This awareness is important from a reform perspective (AAMT, 2006; NCTM, 2000). One RTOP item is "There was a climate of respect for what others had to say." This and similar items in the Classroom Culture section of the RTOP seemed to help shape how the preservice teachers solved behaviour problems. For instance, they assigned roles to group members and emphasised taking turns in group work to have a collaboration where students respect each other and have responsibility to achieve the group goal. Aligning with previous research (Fernández, 2010), one of the benefits of the current lesson study was to provide opportunities to preservice teachers to discuss about and reflect on the classroom management problems.

One aspect of the institutional knowledge that took less part in lesson study communications is epistemic facet. The DMK analysis showed that the preservice teachers had limited conversations in this facet (see Table 5). Nevertheless, the lesson plans indicate that the preservice teachers possibly enhanced their knowledge in this facet as well. For instance, group-C built their tasks on principles of measurement. Group-D focused on adding with regrouping meaningfully using base ten blocks. In their final lesson, group-1 engaged the students in exploring why the commutative property of multiplication holds true using unit cubes. Similar revisions in lesson plans were done by the other groups as well. These revisions require knowledge of mathematics needed for conceptual teaching. Therefore, even though there could be improvements in the lesson study design to have more communications about the epistemic facet, we could claim that the preservice teachers learned about this facet as well. In prior research studies of lesson study experiences, teacher educators supported preservice teachers in making mathematical explorations through assigning challenging secondary school mathematics topics to the lesson study groups (Fernández, 2010) or guiding the selection of topics that are challenging for elementary school students (Murata & Pothen, 2011). In the current study, most groups selected the topics that will be taught during the implementation cycle according to the curriculum. The

mathematics topics were not challenging for the participants. This could be a reason for the limited amount of conversations about the epistemic facet category. In future lesson study designs for preservice elementary teachers, the mathematics topics could be limited to challenging topics for students (e.g., Murata & Pothen, 2011) and/or the preservice teachers could be required to build their lesson plans on research findings about their mathematics topic and learning trajectories in this topic (e.g., Leavy & Hourigan, 2016).

## *Elements of the Current Lesson Study Design*

In this section, we discuss what components of the current lesson study design possibly contributed to the participants' learning of reform-minded teaching. First, the repeated cycle of lesson study gave the preservice teachers opportunities to plan, teach, reflect, and revise lessons and change their teaching to be more student-oriented. This aspect of lesson study was found to support preservice teachers' learning to teach in previous studies as well (Fernández, 2010; Helgevold et al., 2015; Sims & Walsh, 2009). Unlike previous studies, we used RTOP to support preservice teachers' learning about reform-minded teaching throughout the lesson study cycles. We found that RTOP influenced the preservice teachers' thinking about lessons and teaching practices. Specifically, RTOP helped the preservice teachers have productive discussions about the didactical dimension of teacher knowledge. Discussing each RTOP item collaboratively helped the preservice teachers revise their lessons to allow students to determine the direction of the lesson and to include more student-student interactions, student explorations, and useful tools to promote conceptual understanding. Hence, RTOP is a promising tool for lesson study approaches used in teacher education programs with an aim to enhance the preservice teachers' reform-minded teaching practices.

Secondly, another important aspect of the present study is that the preservice teachers taught their research lesson in both university and elementary school classrooms as a result of collaboration between the methods and fieldwork courses. This design allowed the preservice teachers to receive feedback from their peers and the instructor before teaching to elementary school students. At the elementary school site, the preservice teachers experimented with their lessons and valued this experience since it offered them a more authentic learning context. They had opportunities to observe and reflect on student thinking as evidenced by higher percentages in the cognitive facet category of Table 5 for the second and third post-lesson meetings. Teaching at the university classroom helped the preservice teachers focus more on the interactional and mediational subcategories of teacher knowledge without having to deal with challenges of a typical classroom while teaching at the elementary school helped them attend to student thinking and supported their knowledge falling within the cognitive and affective subcategories. Teacher educators have been designing programs to link university courses with clinical experiences (Darling-Hammond & Bransford, 2005). The current design of lesson study seemed to help preservice teachers transfer what they learned in the methods courses to actual teaching practice by building coherence between methods and fieldwork courses.

Thirdly, the role of the course instructor as a knowledgeable advisor was another aspect of the current lesson study that supported preservice teachers' learning. By asking the lesson study groups to focus on mathematics process standards as they chose their overarching goal, the course instructor drew the groups' attention on planning, teaching, and revising a research lesson aligned with reform-oriented instruction. Thus, from the very beginning of the lesson study experience, the lesson study groups were engaged in appropriating the institutional knowledge. In addition, throughout the lesson study cycles, the instructor met with each group multiple times and asked probing questions to help the groups assess their progress in achieving their overarching goal and reflect on student learning. Meetings with the course instructor helped the

preservice teachers revise their teaching tasks or tools to align them with the group's overarching lesson study goal. This finding corroborates previous studies that pointed out the importance of knowledgeable advisors in drawing the preservice teachers' attention to overarching goal of the lesson study and student learning, guiding the conversations focus on important dimensions of teacher knowledge, and making connections between theory and practice (Fernández, 2010; Rasmussen, 2016; Sims & Walsh, 2009; Takahashi, 2014).

Although our findings are promising, we recognise that this study has several limitations. First, the data were collected in the context of a graded project. The preservice teachers might have spoken/written to please the course instructor. To overcome this limitation, the course instructor told the groups that she would listen to the audio recordings after the course grades were assigned. Second, the audiences of the first and third lessons were different and possibly affected the quality of teaching. To overcome this limitation, we used multiple data sources to triangulate the findings (Patton, 2002).

## Conclusion

Lesson study has been suggested as an approach for preservice teachers to enhance their knowledge needed for teaching mathematics (Fernández, 2010; Leavy & Hourigan, 2016). The current study confirms the prior research suggestions as we found that the lesson study experience provided the participants with opportunities to learn the institutional knowledge in the didactical dimension. The participants' lesson plans and teaching practices reflected reform-minded teaching approaches. The current study suggests that designing lesson study experiences that combine aspects of methods and fieldwork courses might engage preservice teachers in learning to teach mathematics from a reform-oriented perspective in an authentic context. The support mechanisms such as using RTOP in lesson planning and debriefing, and the course instructor's role as knowledgeable advisor might increase the learning opportunities for preservice teachers. Future studies might investigate learning opportunities for both preservice and inservice teachers by including the teachers in the practicum classroom in the lesson study process.

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