LEARNING INTEGERS WITH REALISTIC MATHEMATICS EDUCATION APPROACH BASED ON ISLAMIC VALUES

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Abstract
Mathematical learning not only produces students who succeed in mathematical and procedural calculations but also develops religious thinking. Realistic mathematics education with the context of Islamic values makes students can imagine, which is one of the right ways to develop the skills of students' creativity, collaboration, and communication. This study aims to describe the learning trajectory that can help students understand integers with a realistic mathematics education approach based on Islamic values. It is hoped that student responses are positive, meaningful, and enjoyable. This research uses the design research method, which is a form of a qualitative approach. There are three stages in this research, namely: preliminary design, experimental design, and retrospective analysis. The results showed that the Hypothetical Learning Trajectory (HLT) trial with an Islamic value-based context showed significant progress based on student responses. Initially, students had difficulty understanding integers, but they felt delighted to follow the learning process along with the habituation. The HLT technique used in habituation was through pilot experiments, followed by teaching experiments. Students respond very positively and are happy to follow it by seeing the very significant development of their abilities during the learning process.

Keywords: Learning integers, Design research, Realistic Mathematics Education, Islamic values

Abstrak
Pembelajaran matematika tidak hanya menghasilkan siswa yang berhasil dalam perhitungan matematis dan prosedural, tetapi juga mengembangkan pemikiran religius. Pendidikan matematika realistik dengan konteks nilai-nilai keislaman membuat siswa berimajinasi yang merupakan salah satu cara yang tepat untuk mengembangkan keterampilan kreativitas, kolaborasi, dan komunikasi siswa. Penelitian ini bertujuan untuk mendeskripsikan lintasan pembelajaran yang dapat membantu siswa memahami bilangan bulat dengan pendekatan pendidikan matematika realistik berbasis nilai-nilai Islam. Harapannya, respon siswa bersifat positif, bermakna, dan menyenangkan. Penelitian ini menggunakan metode penelitian desain dalam bentuk pendekatan kualitatif. Terdapat tiga tahapan dalam penelitian ini yaitu: desain pendahuluan, desain eksperimental, dan analisis retrospektif. Hasil penelitian menunjukkan bahwa uji coba Hypothetical Learning Trajectory (HLT) menggunakan konteks berbasis nilai Islam memperlihatkan perkembangan yang signifikan berdasarkan hasil respon siswa. Awalnya siswa mengalami kesulitan memahami bilangan bulat, namun mereka merasa senang mengikuti proses pembelajaran beserta pembiasaannya. HLT yang digunakan dalam pembiasaan adalah melalui percobaan percontohan, dilanjutkan dengan mengajar percobaan. Siswa merespon dengan sangat positif dan senang mengikutinya dengan melihat perkembangan kemampuan mereka yang sangat signifikan selama proses pembelajaran.

Kata kunci: Pembelajaran bilangan bulat, *Design research*, Pendidikan Matematika Realistik, Nilai-nilai Islam

Mathematics has several characteristics, among others. It has abstract studies objects, logical and deductive mindset, and symbols that are empty of meaning. Furthermore, it is systematic and relies on the context. On the other hand, it is based on the agreement/definition and must be studied hierarchically. In connection with the plain meaning of mathematical symbols, the user use needs to concern about the context or scope of the conversation. In the study of mathematics, religion, and

culture, these characteristics provide opportunities that mathematical symbols can be given a specific meaning, especially when the scope of the conversation enters the realm of religion (Abdussakir, 2017).

Mathematics learning includes integers, whose starting point starts from context is Indonesia's realistic Mathematics Education. Integers are an essential requirement for human activities in daily life. Using these numbers, humans can mention many, few, less, more, the same, add, less, times, for, and give value to daily transaction activities. Abdussakir (2014) states that in the Qur'an, there are lessons about 38 different numbers, so there is no doubt that the Qur'an talks about mathematics, mostly about numbers.

That's why it is time for all teachers to start designing every learning, including integers, to enter Islamic values. By doing this, we hope to develop a good character for students. Education has an excellent opportunity to carry out the process of fostering and developing humanity. They do it by delivering teaching methods that are based on noble morals. We call it an education that combines aqidah, shari'ah, and morals in learning. The values are then integrated into the schools' learning process (Nihayati, Suningsih, & Abdullah, 2019). Education in schools must be able to develop students' values through religious education; we can do it by aligning the progress of science and technology (Maafi, 2011). This is then reinforced by the opinion of Abdussakir (2018), who states that Islam does not separate between science and religion. Declining morals and lack of religious values urged the need for reform in education (Rahman, Abdurrahman, & Kadaryanto, 2015; Irawan & Kencanawaty, 2017; Maryati & Priatna, 2017).

However, such ideals have yet to come into realization. Mathematics learning is usually done as a single subject, excluding Islamic values. As a result, mathematics learning becomes rigid, seems complicated, alienated with the reality of life, and ultimately tends to be a subject that students avoid. Mathematics is less able to provide the inculcation of both character and Islamic values. On the other hand, mathematics achievement tends to be low (Salafudin, 2015).

We can see the fact of the low mathematical achievement of Indonesian students in the example as follows. In the International Mathematics Olympiad, which Indonesian students attended every year, only a few students get medals. Likewise, it is found in TIMSS (Third International Mathematics and Science Study) and PISA (International Program for Students Assessment) (OECD, 2013). In TIMSS, our second-grade junior high school students ranked 34 out of 38 countries, while in PISA 2015, we ranked 63 out of 70 countries (Hadi, 2017). Students' ability to solve and interpret problems in various situations is still considered at a low level (Kamaliyah, Zulkardi, & Darmawijoyo, 2013). Such a fact is a significant concern to all parties; therefore, we need to put effort into improving the situation. One action that can be done by the teacher is to enhance learning patterns and the quality of learning. To such an initiative, teachers need to develop effective teaching strategies. Lesson study is a learning system to improve students' pedagogical abilities.

On the other hand, the teachers' pedagogical skill is the most concerning (Susanto, Rozali, & Agustina, 2019; Brandt, Burgener, Barth, & Redman, 2019). This is because pedagogical competency is teachers' ability to manage learning (Rahman et al., 2015; Kunter et al., 2013). This includes teachers'

ability to understand and develop, plan and implement learning, and evaluate learning (Dirgantoro, 2018). But teachers must try to manage the learning system to find the meaning of their education.

Johnson (2002) and Hadi (2017) said that when students find meaning from learning mathematics in school, they will understand and remember what they have learned. Contextual learning enables students to connect learning in school with the real context in everyday life. Contextual knowledge broadens their context. This is because providing new experiences for students can encourage their minds to make new relationships. Consequently, students can discover and construct the meanings on their own.

Constructivism in Mathematics Education approach is one view delivered by the Indonesian Realistic Mathematics Education (PMRI). The right learning strategies and methods are needed to develop students' thinking abilities. It is based on the orientation toward technical skills and mathematics education reform (Irawan & Kencanawaty, 2017; Sembiring, Hadi, & Dolk, 2008). Such reform is based on developing problem-solving skills in daily life. The Indonesian Realistic Mathematics Education recommends such the learning method (PMRI), an adaptation of Realistic Mathematics Education (RME).

One of the contexts used in PMRI was culture (Wahyudi, Zulkardi, & Darmawijoyo, 2016). Kuntowijoyo (2001) stated that religion and culture are two things that interact and influence each other to take shape, symbols, and content/values. Previously, it has been explained that religion and culture provide meaning to certain mathematical symbols. Especially when the context of speech enters the religious realm (Abdussakir, 2017), Islamic value-based context that students often do in their daily lives, this means it can also be used in PMRI with the principle that the real context does not mean concrete physical and tangible, but also includes what can be imagined by students' minds. Real-world problems are essential in RME, and it means students are offered problems they can visualize (Van den Heutvel - Panhuizen & Drijvers, 2014). Problems that students can solve by visualizing it will need students' pedagogical skills.

To improve students' pedagogical abilities, it is necessary to integrate the PMRI approach with lesson study and collaborative learning process (Asari, Fauziyah, & Uchtiawati, 2018; Putri, Dolk, & Zulkardi, 2015). Lesson studies are learning systems that use collaborative techniques (Sato, 2014a; 2014b). It is a learning system that was first carried out in Japan's education system (Darra & Kanellopoulou, 2019). It shows promising results in improving the teaching and learning process (Kusumah & Nurhasanah, 2017; Wessels, 2018). The use of lesson study in this learning environment dramatically helps develop a better learning environment. This is because the lesson study ensures that every student gets the same learning (Sato, 2014a). Students work in teams to create collaborative learning tools, and each step in the lessons helps them improve their pedagogical abilities (Coenders & Verhoef, 2019; Fauziah et al., 2019).

Research on the implementation of the use of this realistic mathematics education approach has produced encouraging reports (Ulandari, Amry, & Saragih, 2019). Students become more interested and happier to learn mathematics and show satisfying learning outcomes (Hadi, 2017). From the

description above, the question arises "how to design the learning path that can help students understand integers with a realistic approach based on Islamic values, meaningfully and pleasantly, characterized by positive student responses?"

METHOD

This research uses design research as our research method in this study. The design research consists of five characteristics, such as interventionist, process-oriented, reflective components, cyclic characters, and theory-oriented (Akker et al., 2006; Gravemeijer, 2004; Prahmana, 2017).

Research design is a form of qualitative approach because it emphasizes the socially constructed reality, has a close relationship between researchers and the subjects studied and has an emphasis on process and meaning (Denzin, 2017; Miles, Huberman, & Saldana, 2013). Besides, the design research method is a systematic and flexible method to improve the classroom's quality of learning. It does it by collaborating between researchers and teachers to develop learning designs (Gravemeijer, 1994). The design research method is also an appropriate way to answer research questions and achieve research objectives, namely developing Local Instruction Theory (LIT). It is based on existing theory (theory-driven) and empirically experimented (empirically based) through collaboration between researchers and teachers to increase the relevance of research to educational policies and practices (Gravemeijer & Cobb, 2006). There are four main reasons for using the design research method in this study.

First, there is no theory about the learning trajectories of Islamic value-based realistic approaches to strengthen religious characters. By continuously presenting the context of the Quran-based and Hadith or Islamic values, it supports students to get accustomed to Qur'an and Hadith (Islamic values) in mathematical literacy. In return, this will strengthen the religious character of students with the Islamic values-based Realistic Mathematics Education approach. Second, this study allows researchers to study student learning processes to discover how activities -- with integer learning trajectories with Islamic value-based realistic approaches -- can have an impact on the growth of mathematics literacy based on Islamic values (the Qur'an and Hadith) in understanding integers. In doing so, it strengthens students' religious character, which is consistent with the objectives of this study. Third, to determine how an Islamic value-based context can help students understand integer operations in class VII of Muhammadiyah Palembang Middle School. Fourth, to ensure that students can learn happily and meaningfully.

The development of learning design is carried out in three stages, namely preliminary design, design experiments, and retrospective analysis (Bakker, 2004; Gravemeijer & Cobb, 2006; Simonson, 2006; Prahmana, 2017). The preliminary design aims to design the Hypothetical Learning Trajectory (HLT), which is then refined in the design experiment stage (Prahmana, 2017). Activities carried out in this stage are collaborating with model teachers to conduct a literature review of integer concepts, realistic mathematics education, and Islamic value-based contexts that can be used in integer learning. Also, researchers analyzed the idea of integers in the 2013 mathematics education curriculum.

Furthermore, the results of literature studies and curriculum analysis were used as a basis for designing learning trajectories and developing allegations into HLT.

In this case, the theory serves as a guide to increase students' activities and to think ability in every learning activity, so that it is flexible and can be revised during the experimental design stage. At the design experiment stage, the learning trajectory designed at the initial design stage is then implemented in the learning process (Prahmana, 2017). The purpose of this implementation is to explore and observe students' strategies and thoughts. There are two cycles in this stage; the first cycle is a pilot experiment that aims to evaluate and improve the learning trajectory that has been designed. The second cycle is a teaching experiment that attempts to apply the learning trajectory considered and revised in the first cycle. Implementation of integer learning activities with an Islamic values-based realistic mathematics education approach consists of five activities, shown in the Result and Discussion section.

The last step is a retrospective analysis. All data collected in the design experiment stage were analyzed by comparing the assumptions and HLT with the results of applying the learning trajectory carried out at the design experiment stage (Gravemeijer & Cobb, 2006). From the analysis, we will obtain the work in the form of a description of integers' learning trajectory with a realistic mathematical approach based on Islamic values.

RESULTS AND DISCUSSION

The results of this study describe the learning trajectory of integers with a realistic approach based on Islamic values. Islamic value-based context is a starting point in learning integers and their operations. Activities in implementing the learning path that has undergone improvement are written into five activities. We give a Student Activity Sheet (SAS). SAS is a teaching material format that contains instructions, a list of tasks, and guidance to do activities. There are five SAS according to the number of activities, namely: SAS 1 to SAS 5. We will show the results of the activities of the Islamic value-based context for writing, sorting, and comparing integers, with examples of activities 1 using five SAS, as shown in Figure 1.

Figure 1. Example of activities using SAS

Activities in SAS 1

We distributed SAS 1 to students. Its learning goals are that students can write integers, sort integers, and compare integers. In the beginning, the model teacher opens the lesson in a standard way, which is to draw attention by saying, "This time, you will learn integers by utilizing what is already in your mind and based on Islamic values." The teacher then motivates students by conveying learning objectives that are already listed on the first page of SAS 1 and the benefits after learning this material. Then the teacher gives a reference by reminding the main issues to be discussed. After that, the teacher making meaningful lessons by linking the initial knowledge that is already in the minds of students. The following are examples of the teacher's question to students.

"What can you say about the Holy Qur'an?"
"Can you all read the Qur'an?"
"Do you still lie often?"
"Who can write integers in front of the class?

In the end, the students start doing activities according to the SAS 1 instructions. The following are the student activities in SAS 1:

1. "This is the Book about which there is no doubt, a guidance for those conscious of Allah "(Surah. Al-Baqarah, 2). Indeed, the Qur'an is guidance and guidance for good. The Qur'an guides those who fear their Lord by obeying His commands, moving away from His prohibitions and leaving disobedience / sinning, then they benefit from it. Those are the three descriptions of the Qur'an. Why do you agree and be sure of the opinions above?

   Student answers:

   > Because everything written in the Qur'an must be true, considering the Qur'an itself was revealed to the prophet Muhammad SWT

   **Figure 2**. Student answers to question number 1 SAS 1

   Students' answers to the questions seen in Figure 2, as a sign that students can imagine the context based on the Quran. Students' answers turned out to be different, but all showed their confidence in the truth of the contents of the Qur'an.

2. "And establish prayer at the two ends of the day and at the approach of the night. Indeed, good deeds do away with misdeeds. That is a reminder for those who remember. (Surat Hud, verse 114). If there is no sin, we say it is denoted by 0 (zero), then write an integer by first observing the picture showing night and day seen in Figure 3!
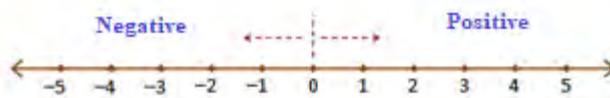
Figure 3. Earth night and day

Student answers:
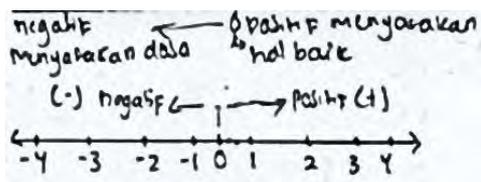
negative refers to sin ⬌ positive refers to good deeds

**Figure 4**. Students' answers to problem number 2 SAS 1

Generally, students can answer like in Figure 4, even though there are still other possibilities of how to write integers without using the help of a number line, as one of the students writing:

-5	-4	-3	-2	-1	0	1	2	3	4	5

Some others wrote, ..., -2, -1, 0, 1, 2, etc.

3. "No baby is born but upon Fitrah (as a Muslim). It is his parents who make him a Jew or a Christian or a Polytheist." (Sahih Muslim, Book 033, Number 6426). How do you sort integers according to the statement above?

Student answers:
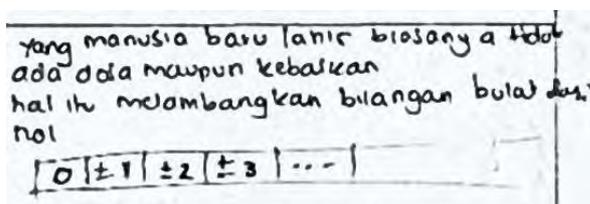
The newly born human usually has no sin or goodness, it symbolizes integers from zero

0	± 1	± 2	± 3	...

Figure 5. Student answers to question number 3 SAS 1

Figure 5 shows that student's answer is understandable because he has understanding that "For every child born, they are clean from sin, even though after adolescence, they may be doing good or bad". We found out that there were only who wrote: 0, ± 1, ± 2, ... while others wrote, 0, -1, + 1, -2, +2, etc., or, 0, -1, 1, -2, 2, ...

4. A 15-storey Medina hotel, has 3 underground floors. An employee was on the 3rd floor of the office. He went down 6 floors, then up 4 floors. Which floor number is bigger after going down 6 floors and then going up 4 floors? (use the "<" or ">" sign).

Student answers:
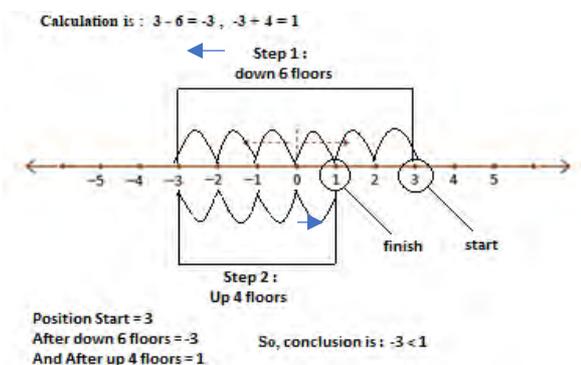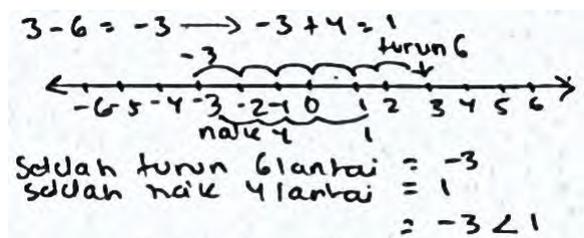
Figure 6. Student answers to question number 4 SAS 1

His answer, shown in Figure 6, was encouraging. It can be seen that he could explore the context of the problem of moving up and down the elevator, illustrated in the number line.

Activities in SAS 2

We distributed Student Activity Sheet 2 (SAS 2) to students. Its learning objective is that students can add integers. In the beginning, the model teacher opens the lesson in a standard way, which is to draw attention by saying, "This time, you will learn integers by utilizing what is already in your mind and based on Islamic values." The teacher then motivates students by conveying learning objectives that are already listed on the first page of SAS 2 and the benefits after learning this material. Then the teacher gives a reference by reminding the main issues to be discussed. After that, the teacher making meaningful lessons by linking the initial knowledge that is already in the minds of students. The following are examples of the teacher's question to students.

"Can you mention Fardlu (mandatory) prayers in the daytime and Fardlu (mandatory) prayers in night?"
"Can you name some good deeds?"
"Who can help answer 5 + (-2)?"

In the end, the students start doing activities according to the SAS 2 instructions. The following are the student activities in SAS 2:

1. Look again at the meaning of the Q.S. Huud verse 114 this below:

"And establish prayer at the two ends of the day and at the approach of the night. Indeed, good deeds do away with misdeeds. That is a reminder for those who remember. (Q.S. Surat Hud, verse 114).

a. Can prayers erase bad deeds (sins)? Why?

b. Which sins can be removed by good deeds?

Student answers:
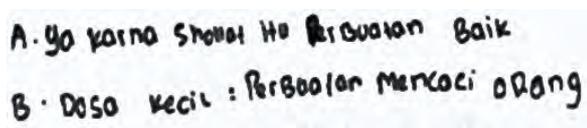
a. yes, because prayers do good

b. minor sin: insulting people.

Figure 7. Student answers to question number 1 SAS 2

One other student answered like one in Figure 7. Some answered: a. yes, because prayer is obligatory for every Muslim as a good deed that can wash away small sins, b. Minor sins, lying, insulting friends, stealing, and many others they can name of.

2. Read and comprehend the word of Allah in Surah Al-Kahf, verse 25.

"And they remained in their cave for three hundred years and exceeded by nine".

a. Which mathematical operation expressed above?

b. From question a), why Allah expressed the verse using such mathematical operation? Why didn't Allah show the exact number instead?
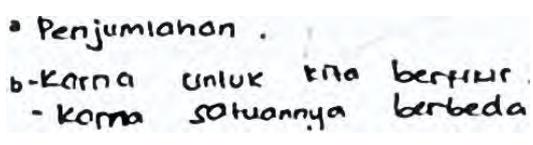
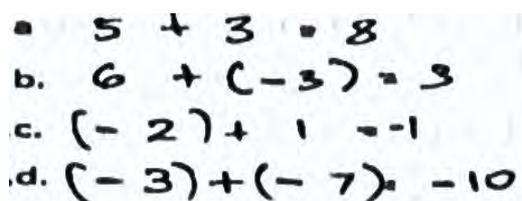Student answers:

a. Addition

b. – It is such that Allah invites humans to think.

- because the units used were different

Figure 8. Student answers to question number 2 SAS 2

Using the context shown in Figure 8, we can invite students to discuss about addition, as well as a to think about the meaning of the verse.

3. Express the integer addition operation below by making examples from these ideas.

a. Good deeds followed by another good deeds are better than the sum of both deeds

b. Good deeds followed by bad deed will decrease the good deed.

c. Bad deeds followed by good deeds will decrease the bad deed.

d. Bad deeds followed by another bad deeds are worse than the sum of both deeds
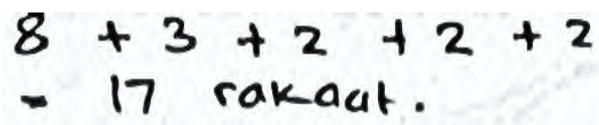
Student answers:

Figure 9. Student answers to question number 3 SAS 2

Question number 3, continuing from the context of Surah Hud:114, was used to guide students to find the concept of addition. It turned out students could answer it as shown in Figure 9.

4. Amir was doing the midnight prayer, ended by witr. He then went to masjid to pray in congregation. He entered the masjid, then doing Tahiyatul Masjid prayer. After that, he stood up to do Qobliyah Subuh prayer. Last, he prayed Subuh.

How many raka'ahs did Amir pray that morning?

Student answers:

Figure 10. Student answers to question number 4 SAS 2

The teacher should start by explain the Islamic value-based context with open-ended answers, such as in the in Figure 10 above. Even though there are many possible answers because the midnight prayer can be done by the combination of Tahajjud (2,4, a multiplication of 2 rakaah, up to 20 rakaah) and Witr (1, or 3 rakaah). As in Subuh, Tahiyatul Masjid, and Qobliyah Subuh, they have exact number of rakaah.

Activities in SAS 3

We distributed Student Activity Sheet 3 (SAS 3) to students. Its learning objective is that students can subtract integers. In the beginning, the model teacher opens the lesson in a standard way, which is to draw attention by saying, "This time, you will learn integers by utilizing what is already in your mind and based on Islamic values." The teacher then motivates students by conveying learning objectives that are already listed on the first page of SAS 3 and the benefits after learning this material. Then the teacher gives a reference by reminding the main issues to be discussed. After that, the teacher making meaningful lessons by linking the initial knowledge that is already in the minds of students. The following are examples of the teacher's question to students.

"Why did Allah order humans do good?"
"What sins can fardlu(mandatory) prayers erase?"
"Who can help answer 5- (-2)?"

In the end, the students start doing activities according to the SAS 3 instructions. The following are the student activities in SAS 3:

1. Read and pay attention of following:

"And We certainly sent Noah to his people, and he remained among them a thousand years minus fifty years, and the flood seized them while they were wrongdoers" (Surah Al Ankabut:14)

a. Which mathematical operation expressed above!

b. From question a), why Allah expressed the verse using such mathematical operation? Why didn't Allah show the exact number instead?
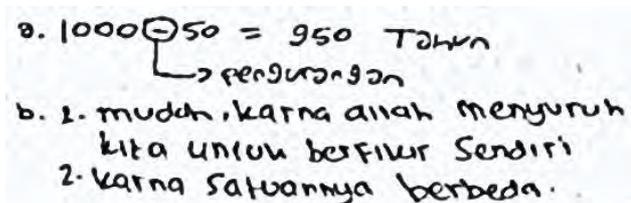
Student answers:

a. 1000 – 50 = 950

subtraction

b. 1) easy, because Allah tells us to only exercise our thinking ability;

2) because the units are different

Figure 11. Student answers to question number 1 SAS 3

Using the context shown in Figure 11, we can invite students to discuss about addition, as well as a to think about the meaning of the verse.

2. Read and pay attention to below hadith!

" Have taqwa (fear) of Allah wherever you may be and follow up a bad deed with a good deed which will wipe it out and behave well towards the people ". (HR. Tirmidhi).

Express the integer addition operation below by making examples from these ideas.

a. Someone whose good deeds is taken away from him, it is worse for him.

b. Someone who has good deeds and Allah take away his bad deeds, it is better for him.

c. Someone who has bad deeds and Allah take away his good deeds, it is worse for him.

d. Someone whose bad deeds is taken away from him, it is better for him.

Student answers:
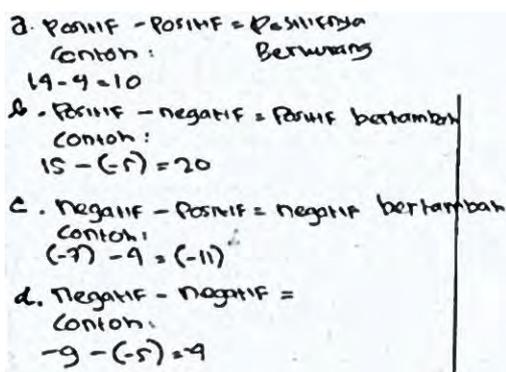
a. positive - positive = positive decreases

example: 14 - 4 = 10

b. positive - negative = positive increases

example: 15 - (-5) = 20

c. negative - positive = negative increases

example: (-7) - 4 = -11

d. negative - negative = negative decreases

example: -9 - (-5) = -4

Figure 12. Student answers to question number 2 SAS 3

Teachers could use the context shown in the hadith above to guide students to understand subtraction. It turned out students can answer it as shown in Figure 12.

3. The temperature in Mecca is 37^0C (above 0^0C). At the same time, the temperature in Japan is 40^0C below the temperature of Mecca. What is the temperature in Japan now?

Student answers:

The temperature in Makkah is 37 ^{0}C (above 0 ^{0}C) and

The temperature in Japan at that time is 40 ^{0}C below the temperature of Makkah.

= 37° C – 40° C = - 3° C

Figure 13. Student answers to question number 3 SAS 3

Problem number 3 was used to evaluate students' understanding of subtraction. In general, students answered as shown in Figure 13.

4. If the Fajr prayer time is at 04.43 and the Dzuhur prayer time is at 12.02, how long do we have to wait from Fajr to Dzuhur time?

Student answers:

Figure 14. Student answers to question number 4 SAS 3

The student's answer in Figure 14 showed an effort to express time subtraction, from 12.02 to 11.62 (since 1 hour = 60 minutes, then the student subtracts 11.62 with 04.43 = 07.19. It appears that the student only completed the calculation step buy yet to answer the question, which is by stating that the time needed to wait from Fajr to Dzuhur prayer is 7 hours 19 minutes.

Activities in SAS 4

We distributed Student Activity Sheet 4 (SAS 4) to students. Its learning objective is that students can subtract integers. In the beginning, the model teacher opens the lesson in a standard way, which is to draw attention by saying, "This time, you will learn integers by utilizing what is already in your mind and based on Islamic values." The teacher then motivates students by conveying learning objectives that are already listed on the first page of SAS 4 and the benefits after learning this material. Then the teacher gives a reference by reminding the main issues to be discussed. After that, the teacher making meaningful lessons by linking the initial knowledge that is already in the minds of students. The following are examples of the teacher's question to students.

"Can you accuse someone? Explain your opinion."
"Do you know, how much Allah reward humans for doing a good deed?"
"Why is multiplication called repeated addition? Give an example."

In the end, the students start doing activities according to the SAS 4 instructions. The following are the student activities in SAS 4:

1. Read and pay attention to the following verse:

    "Whoever comes [on the Day of Judgement] with a good deed will have ten times the like thereof [to his credit], and whoever comes with an evil deed will not be recompensed except the like thereof; and they will not be wronged" (Surah Al-Anam: 160). From the verse, how much will Allah reward humans for doing good deed? And how much for doing rewards taken away for doing bad deeds?

    Student's answers:

one good deed = 10 reward

one bad deed = balanced with the bad he made, which is 1 evil

Figure 15. Student answers to question number 1 SAS 4

It turned out that in Figure 15, it is true that each good will be crime with 10 rewards, but one bad will get a penalty of sin in proportion to his crime. It may be that students were in a hurry, such that they wrote goodness instead of bad deed.

2. The example of those who spend their wealth in the way of Allah is like a seed [of grain] which grows seven spikes; in each spike is a hundred grains. And Allah multiplies [His reward] for whom He wills. And Allah is all-Encompassing and Knowing. (Surah Al Baqarah:261)

    How do you express the verse above into the mathematical symbol?

    Student's answers:

1 tree = 100 seeds.

So, 7 trees = 700 seeds

7 x 100 = 700 seeds

100 + 100 + 100 + 100 + 100 + 100 + 100 = 700 seeds

Figure 16. Students' answer to question number 2 SAS 4

Figure 16 shows that the student's answer showed that he had absorbed the meaning of the verse. Although he had yet to express that $100 + 100 + 100 + 100 + 100 + 100 + 100 = 7 \times 100$ (Muslimin et al., 2018).

3. But whoever earns an offense or a sin and then blames it on an innocent [person] has taken upon himself a slander and manifest sin. (Surah Annisa:112). We imply "And whoever earns an offense or a sin" as negative expression and assign negative symbol (-). We imply "blames it on an

innocent [person]" as positive expression and assign positive symbol (+). We imply " … has taken upon himself a slander and manifest sin " as negative expression and assign negative symbol (-). Note the order of the symbols, "negative, positive, negative". Observe the patterns formed. Express the four different possible patterns using multiplication!

Student answers:

Figure 17. Student answers to question number 3 SAS 4

Teacher used the context from the verse above to guide students explore the concept of multiplication. It turned out students can answer it as shown in Figure 17.

4. Express the statements below using multiplication. Give examples.

   a. The act of saying a fact truthfully, is a good deed

   b. The act of saying a fact falsely, is a bad deed

   c. The act of saying a lie as if it's a fact, is a bad deed

   d. The act of saying truthfully, that something is a lie is a good deed

   Student's answers:

Figure 18. Student's answer to question number 4 SAS 4

Students' answers to question number 4 shown in Figure 18 above, are analogous from Figure 17.

Activities in SAS 5

We distributed Student Activity Sheet (SAS 5) to students. Its learning objective is that students can multiply integers. In the beginning, the model teacher opens the lesson in a standard way, which is to draw attention by saying, "This time, you will learn integers by utilizing what is already in your mind and based on Islamic values." The teacher then motivates students by conveying learning objectives listed on the first page of SAS 5 and the benefits after learning this material. Then the teacher gives a reference by reminding the main issues to be discussed. After that, the teacher makes meaningful

lessons by linking the initial knowledge that is already in students' minds. The following are examples of the teacher's question to students.

"Do you still remember that multiplication is called repeated addition? Give an example!"
"Mom makes a small circle shaped cake. If she divides the cake equally to her three children, how many pieces does each child get?"
"How much is 1/3 of Rp 15,000.00?"

In the end, the students start doing activities according to the SAS 5 instructions. The following are the student activities in SAS 5:

1. Read the following verse

   "Whoever believes in Allah and the Last Day, he should glorify his neighbors" (HR. Bukhari 5589, Muslim 70)" or

   " O Abu Dharr, when you cook a stew, put more water in the broth and take care your neighbors." (HR. Muslim).

   Pay attention to the words of the Prophet above which are narrated by Bukhari and Muslim, then:

   a. Write your opinion from reading the hadith above. Then, think the following context.

   b. Mom buys 20 liters of cooking oil. She wants to share with her neighbors, so she pours the oil into small 5-liter bottles. How many small bottles she needs?

   c. So, what do you think division is?

   Student' answers:
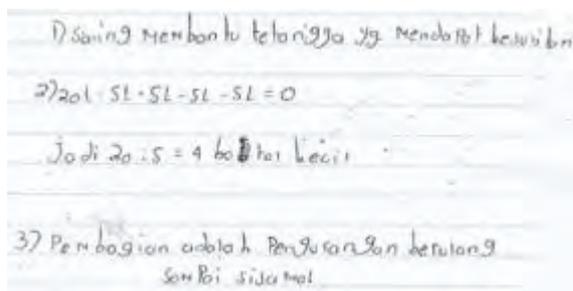
1) Often helps neighbors who get into trouble

2) 20 liters - 5 liters - 5 liters - 5 liters - 5 liters
= 0

3) The division is a recurring subtraction to remainders zero;

Figure 19. Student Answers Number 1a in SAS 5

Students' answers in Figure 19 showed that a hadith-based context let students to imagine in their minds. One of them answered, "helping neighbors who are in trouble", even though many other answers available. When students visualized the action of pouring cooking oil into small bottles, then he, indirectly, performed a repeated subtraction. this is used to strengthen the understanding that the positive integer division operation is a repeated subtraction operation. Finally, students can calculate as shown in Figure 19.

2. $4 : -\frac{25}{10}$ $\leftrightarrow$ 4 x $(-2,5) = ...$?

3. $6 : 2 = 3$ $\leftrightarrow$ $\frac{1}{2}$ x $6 = 3$. Calculate $7500 : 5 = ...$? $\leftrightarrow$ $\frac{1}{5}$ x $750 = ...$?

Student's answers:

Figure 20. Student answers to question number 1b SAS 5

As shown in Figure 20, students understood that division is an inverse multiplication operation. To support students' understanding, multiplication can also be interpreted as a repeated addition (Hendriana, Prahmana, & Hidayat, 2019).

4. Suppose a husband dies, leaving a wife, a mother and a son. His inheritance is 48 million rupiah. It is known that the share for a wife is 1/8, mother gets 1/6 share, son got the remaining share. How much do each of them get?

Student's answers:

Heir	Share	Siham (divider) = 24	Amount of assets obtained
Wife	1/8	3	3/24 x Rp 48 million = 6,000,000
Mother	1/6	4	4/24 x Rp 48 million = 8,000,000
Son	Remaining	17	17/24 x Rp 48 million = 34,000,000
		24	= Rp 48 million

Figure 21. Student answers to question number 2 SAS 5

Figure 21 shows that student understood that division is an inverse multiplication operation and vice versa. In this exercise, student uses two number operation namely division and multiplication to solve this problem.

5. Suppose a wife dies, leaving a husband, a mother, and a son. Her inheritance is 60 million rupiah. It is known that the share of each heir; husband ¼, mother 1/6 and remaining for her son. How much does each of them receive?

Student's answers:

Husband = 1/4 x 60 million = 15 million

Mother = 1/6 x 60 million = 10 million

son = 60 - (15 + 10)

     = 60 - 25 = 35 million

Figure 22. Student's answer to question number 3 SAS 5

Figure 22 explains that student understood that division is an inverse multiplication operation and vice versa. Furthermore, student also understands that addition is an inverse subtraction operation and vice versa. Lastly, based on this exercise, student can use four number operation namely addition, subtraction, division, and multiplication to solve this problem.

By closely evaluating the implementation of Learning Trajectory (LT) above, results shown from SAS 1 through SAS 5 activities are by the five characteristics of PMRI. This is the adoption of Realistic Mathematics Education (RME) (Treffers, 1987; Wijaya, 2012; Daryanto, 2012; Maslihah, 2012). From the results and the previous discussion, several findings were obtained as follows.

Firstly, every time a meeting begins with the activity of reading the Qur'an or Hadith, students who can and especially those who are fluent in reading the Qur'an have tried to get them appointed. While those who have not and cannot read the Qur'an fluently, they choose to be set to read the meaning. Secondly, at the initial meeting, students have not been active in doing activities. They may not be accustomed to active thinking themselves or are not accustomed to learning given a context problem, let alone the context based on the Qur'an and Hadith.

Third, from the activity results in the second step, in this third step, they continue the group discussion. It turned out that the discussion activity showed quite well, even though at the beginning of the conversation, the students were still confused and awkward. Yet, after a few minutes of discussion with the teacher's direction, they had begun to be active. The results of this study add to the empirical evidence which states that Islamic values can be integrated with the mathematics learning process which can emerge student understanding (Lubis, 2015; Masduki, Khotimah, & Sutarni, 2014; Muhsinin, 2013; Muslimin et al., 2019; Piliang, Daulay, & Siddik, 2017).

Lastly, at the end of each meeting, there is a class discussion. That is, they present the results of each group's discussion. At the first meeting, students are asking "Why at night there are many crimes (sinning)?", All members of the group from each group fell silent. Then the teacher told them to open the Q.S. Al-Falaq, please open the Qur'an translation, they open the Qur'an translation, and some are reading from their handphone. Suddenly someone half shouted "I know sir!", Without being appointed, he immediately called "Q.S. Al-Falaq verse 3 sir, meaning" and from the crime of the night when it was pitch black". It shows that starting to enjoy the activity of discussion towards the action in the activities,

as mentioned earlier, is inseparable from the words do good and do evil (sin). If that is done by the teacher, especially the mathematics teacher, the students will learn really and enthusiastically. Sometimes, they will indirectly realize that learning requires compulsory education (Asy-Syaimaa' Hussain & Ramli, 2017). There is interference from their God, so they do not dare to be arrogant, let alone not be polite to the teacher.

CONCLUSION

The learning trajectory of integers through context-based on Islamic values was effective in helping students understand integers. The proven learning trajectories consist of four phases, namely beginning the presentation of the context-based on Islamic values, *Iqra* (literate) to understand the context, resolving the context individually continued in groups to develop an informal model to formal, and communicating with the presentation.

In addition, the learning trajectory of integers with a starting point context based on Islamic values, in addition to sharpening students' minds, focus, meaningful, and fun as well as developing good character. If the teacher wishes to do mathematics learning in general with a learning trajectory using Islamic value-based contexts, then he must accept the challenge of learning to understand the Qur'an and Hadith.

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