Fear Not Early Childhood Teachers! You are Already Using Algebraic Strategies in Your Classroom

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Algebra! Just seeing or hearing the word can cause anxiety in early childhood teachers. However, this anxiety is socially acceptable, because in the United States some consider that it is okay not to like math (Isiksal, Curran, Koc, & Askun, 2009). For decades research has demonstrated that mathematical thinking is evident in children as young as preschool age (Bredekamp & Copple, 1987, 1998; Coople & Bredekamp, 2009; Curico & Schwartz, 1997; Hartley, 1952) and that algebraic reasoning is important for learning advanced mathematical concepts (Cross, Olufonke, Lee, & Perez, 2012). Moreover, Cross et al. state that young children are adept at developing spatial and algebraic reasoning, provided they engage in appropriate activities. Nevertheless, these algebraic activities may not take place in early childhood classrooms if teachers bring with them years of math anxiety and negative math experiences.

Many early childhood teachers state that they chose to teach young children (birth – age 8) because they do not like mathematics, are not good at mathematics, or will not have to know or teach a lot of math (Lake & Kelly, 2014). When asked to explain what algebra would look like in their classrooms, Elliott (2005) described preservice early childhood and elementary teachers' answers as vague, uncertain, tentative, and full of negative memories of how they struggled with the content and were forced to memorize facts and formulas. The fact is that most teachers of young children have had little experience dealing with algebra since they were in high school (Blanton & Kaput, 2003).

Teachers' negative experiences can be detrimental to how they teach math because their beliefs about mathematics affect how they see themselves, their instructional practices, and the level of appropriate mathematics activities offered in their classrooms (Hadley & Dorward, 2011; Mewborn & Cross, 2007). We know that positive beliefs about mathematics lead to increased student involvement and opportunities to learn, while teachers' "negative attitudes toward mathematics can produce negative results in mathematics" (Vinson, 2001, 90).

Along with our own experiences of working with, teaching, and observing teachers and preservice teachers in the area of math for over 25 years, research has shown that there are many classrooms where appropriate activities are happening but teachers do not recognize the math involved, and they may not understand how the activities promote algebraic understanding. With some coaching and math training, teachers begin to see that math is everywhere and can start to expand on the activities already occurring in their classrooms to promote algebraic thinking.

What is Algebra

Heddens and Speer (2001, 182) state that algebra is a "process of generalizing, abstracting, and representing functions and relationships." The Principles and Standards for School Mathematics (NCTM, 2000) state that "even young children can be encouraged to use algebraic reasoning as they study numbers and operations and as they investigate patterns and relations among sets of numbers" (p. 3). Additionally, the NCTM document provides a vision of pre-K-12 algebra around the following four themes:

- Understanding patterns, relations, and functions
- Representing and analyzing mathematical situations and structures using algebraic symbols
- Using mathematical models to represent and understand quantitative relationships
- Analyze change in various concepts

Van de Walle, Lovin, Karp, and Bay-Williams (2014) have further refined this vision of algebraic thinking for early childhood classrooms stating that it involves the following:

- Recognizing patterns and relationships and analyzing these relationships,
- Thinking about the use of symbols to generalize certain kinds of math operations, and
- Thinking about the props that children use to represent things and support their increasing ability to understand and use abstract symbols.

While the refinement of algebra is helpful, it still does not explain what early childhood teachers should do to foster algebraic thinking in their classrooms. Fortunately, Elliott (2005) provides

Table 1. Opportunities to Pose Questions

You are probably …	You can expand learning by…
Modeling your thinking when demonstrating or explaining a mathematical problem	Asking questions that prompt children to explain or justify their thinking, examples include: • How did you arrive at that answer? • How would you convince your classmates of your answer? • Is this always true? Explain. • Have you found all of the possibilities? How can you be sure?

five different ways teachers can help children engage in algebra: "pose questions about their world, precisely; look for patterns, continuously; represent structures, symbolically; model relationships, quantitatively; and analyze change, periodically" (p. 104). These five strategies will serve as the framework for this article. We describe each strategy and how it demonstrates algebraic thinking, tell what early childhood teachers are already doing in their classrooms, and share ideas they might try in order to expand algebraic thinking for their students. We acknowledge that the examples listed are not exclusive to one category; they have been classified in ways that make sense based on our experiences in the classroom; readers may see them as fitting in another category. The important thing is not what category they fit in, but rather that teachers include a variety of opportunities for algebraic thinking in their classrooms.

Pose Questions about Their World, Precisely

Asking questions allows children the opportunity to organize their thinking and create ideas about mathematics, developing children's *eyes* and *ears* for algebra (Blanton 2008). Open-ended questions can also prompt children to explain or justify their thinking (see Table 1) Hearing a counter example, one that does not match their idea, can place children in cognitive conflict. In order to resolve the conflict, they must either distort the new idea to fit their existing body of knowledge or change their body of knowledge to fit the new idea. It is through this continuous cycle of conflict and resolution that cognitive growth occurs (Amirshokoohi & Wisniewski, 2018).

The following scenario provides an example of how to pose questions to children.

Mrs. McCombs asks the children what two numbers added together are the same as 5? She writes the combinations children provide on the board: 2+3, 4+1, 3+2, 1+4. "Is that all the combinations?" The children say yes. "There are two more combinations. Use your counters, work with your partner, and let's find the final two combinations." Children work until they find 5+0 and 0+5.

Mrs. McCombs also keeps a list of mathematical terms posted on her classroom wall so that she is always reminded of these vocabulary words and can use them in real-world conversations naturally. Examples of first grade words are: compare, order, congruent, and symmetry. Her chart also includes math terms from kindergarten to 5th grade, which helps her differentiate and tailor her conversations with particular children or small groups. *When walking in the hall, the class notices another class' displayed work. "Are the papers vertically or horizontally placed? Compare why the author might have oriented the paper that way for their particular work*."

Look for Patterns, Continuously

Although patterns have other meanings in different contexts, mathematical patterns can be defined as "any replicable regularity" (Papic, Mulligan, & Mitchelmore 2011, p. 238) and begin to appear in the pre-k curriculum. Many teachers believe the "AB" pattern is the most basic type of pattern and start by teaching those. However, introducing children to "A" patterns will help them understand that a pattern is simply a repeating unit (McGarvey, 2013). Early childhood teachers can encourage children to find "A" patterns in their environment. One way to do this is to take children on a pattern walk around the school and neighborhood. They can identify "A" in many places including the square on the sidewalk, tiled floors and walls, brick or vinyl siding on buildings, or slats on a wooden fence; focus on more than one "A" pattern, so children see how it continually repeats. After children have experience identifying patterns, they can begin to generalize what "A" or "AB" stands for in each pattern (Kinach, 2014). Using concrete models and describing them with letters or symbols ultimately helps children understand abstract forms of numbers and symbols that will form the basis of later algebra (Lee, Collins, & Melton, 2016).

The NCTM (2000) pre-K-2nd grade section of the standards for algebra includes the expectation that children will generate and

Figure 1: "A" Patterns – Brick, Fence, and Sidewalk

Figure 2: Non-linear Growing Patterns

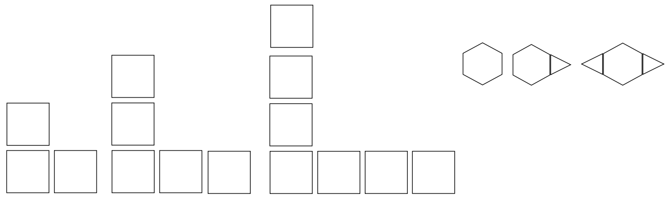
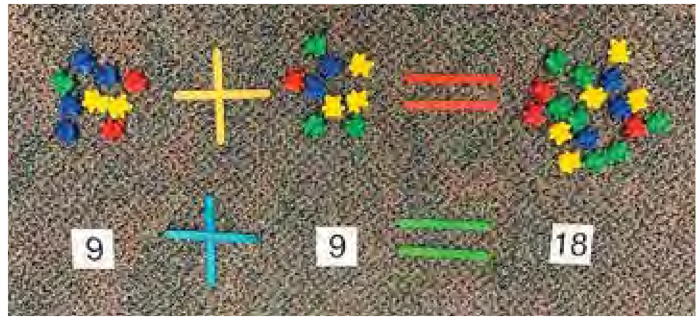


Figure 3. Picture of number sentence



analyze both repeating and growing patterns. A repeating pattern is one that repeats and stays the same; there is never anything added to the sequence. Most children become interested in repeated patterns about age 4. A growing pattern is one that demonstrates a relationship in every sequence. An example of a growing shape pattern is ●, ●■, , ●■▲, ●■▲◆. In each sequence the same shapes are repeated, while one more is added. A growing number pattern is 1,12,123,1234. In this case the next numerals added to the series would be 12345. The previous numerals are repeated plus the next numeral in the sequence. Experiences should not be limited to linear patterns. Non-linear growing patterns can be created by the children using a variety of materials including square tiles, pattern blocks, or even toothpicks (Figure 3). Growing patterns can be introduced in first grade if children have a firm foundation of repeated patterns. When introducing growing patterns, teachers should provide the first three examples of the pattern before letting children explore the pattern. Examples of how teachers can expand children's knowledge of patterns are summarized in Table 2.

Represent Structures, Symbolically

Early childhood teachers can use a homemade or commercially purchased bucket balance, balance scales, or pan scale as a concrete way to demonstrate equivalence. Children can manipulate with Unifix® cubes, bear counters, or other manipulatives to try to make the two sides balance. Children are using algebraic thinking when they understand the relationship that is occurring between the two sides of the balance (Warren, Mollinson, & Oestrich, 2016). The children can represent what happens with the balance using terms such as *greater than*, *less than*, *equal*, and *not equal*. Working with the balance can also help children develop a concrete understanding of the equal sign (Knuth et al., 2016). While many children believe that the equal sign means, 'the answer is coming up' or 'do something,' activities with the balance help children understand that the equal sign means both sides of the equation are the same (Lee, Collins, & Melton, 2016).

Let's return to Mrs. McComb's classroom to see an example of ways to represent structure. Mrs. McCombs places two hula hoops on the ground. She invites four children into the hoop (three girls and one boy) and asks the class, "How do I make the second hoop the same?" One child says, "we need to add four friends." Another child adds, "we need to add three girls and one boy to make it match." This activity engages the class in a conversation of equal, same, and match.

Model Relationships, Quantitatively

Early childhood children are expected to use concrete, pictorial, and verbal representations of mathematical situations (NCTM, 2000). In many early childhood classrooms, children count how many children are at school as part of their daily routine. Teachers can extend this by encouraging children to categorize (e.g., boy/girl, 6-year olds/7-year olds, wearing tie shoes/Velcro shoes, or wearing glasses/no glasses) to determine who is present. As an extension, the children can determine other categories. At the beginning of the year, the teacher should model how to create a concrete representation by counting actual children. For example, all of the boys can stand in one group and the girls in another. One child can volunteer to count each group (not forgetting to count themselves, of course). Next, the teacher models how to create a pictorial representation using one of these categories. For example, 12 + 7 = 19 would represent that there are 12 boys and 7 girls in class for a total of 19 children. After the children master creating pictorial representations, they should be encouraged to find other ways to represent how many children are at school (see Table 4).

Mrs. McComb's class provides an addition example of how to model relationships. Two children in Mrs. McCombs class used counting bears, popsicle sticks, and number tiles to represent their understanding of number sentences (see Figure 4). They first used the bears to show quantity and the popsicle sticks for the operations (+ and =). Underneath the bears, they put the number tiles to show the quantity and duplicated the operation signs with popsicle sticks.

Table 2. Ways to Expand Patterning

You are probably …	You can expand learning by…
Helping children recognize and create simple repeating patterns such as AB, ABC, etc.	Encouraging children to identify "A" patterns
Focusing on linear, repeating patterns	Giving children opportunities to explore (identify and create) growing patterns, both linear and non-linear

Table 3. Representing Structures

You are probably …	You can expand learning by…
Comparing groups of objects using vocabulary such as same, different, more, and less	Modeling the relationship of algebraic equality using greater than, less than, equal, and not equal

Table 4. Modeling Relationships

You are probably …	You can expand learning by…
Counting the number of students who are at school	Creating concrete, pictorial, and verbal representations of how many children based on a variety of categories

They said, "Mrs. McCombs, we just made a number sentence."

Analyze Change, Periodically

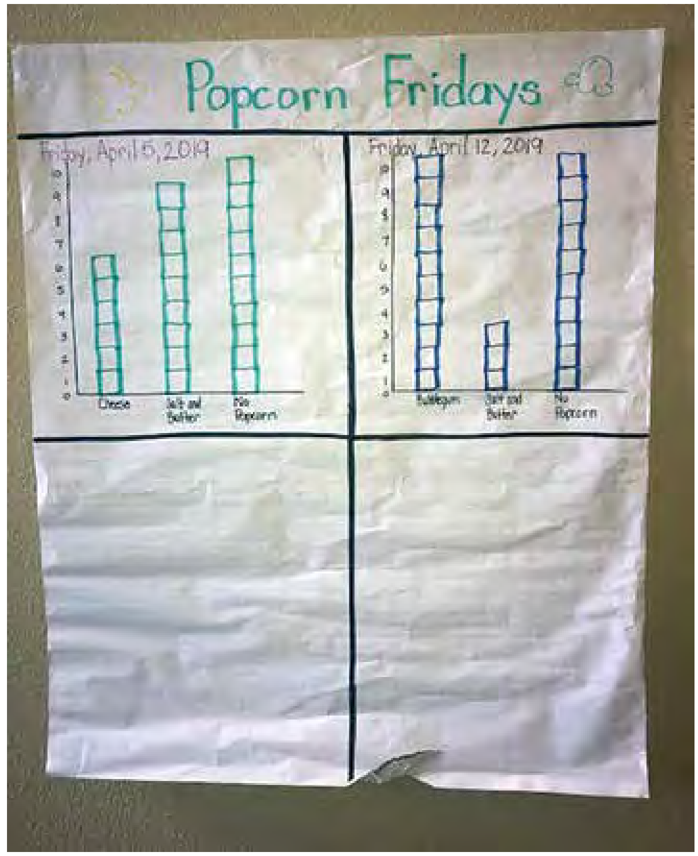
Understanding change can help children to better understand the world around them, a major goal of mathematics (Björklund, 2010). Early childhood teachers keep track of many events that occur in the classroom, such as lost teeth, but do not necessarily encourage children to make mathematical meaning. Young children need opportunities to describe change using both quantitative and qualitative language. Quantitative changes, as the name implies, include looking at quantities. In the example of lost teeth, the children can describe the decreases or increases in the number of teeth lost from month to month. In one classroom, 7 teeth were lost in August, 3 in September, and 5 in October. The children could describe this quantitatively as a change of 4 from the first to the second month and then a change of 2 from the second to the third month. A qualitative description of the change would be that more teeth were lost during than first month than the second and third months.

You are probably …	You can expand learning by…
Keeping track of how many children lost teeth	Using quantitative and qualitative mathematical language to describe the changes in the number of teeth lost

Mrs. McCombs class also works to analyze change. This scenario shows what she does.

In Mrs. McCombs's class, they graph everything. A piece of chart paper becomes an anchor chart for popcorn Fridays, which is a school fundraiser where children can purchase flavored popcorn for $1.00. Using ¼ of the chart paper, the children graph how many bought each of the two flavors of popcorn and who did not buy popcorn. This portion of the chart is dated then compared to the previous week's popcorn selection. This anchor chart allows for the class to compare four weeks of popcorn buying, flavor selection, and the amount of money spent each week and cumulatively (see Figure 4).

Figure 4. Picture of anchor chart



Conclusion

Young children are very capable of spatial and algebraic reasoning (Cross et al., 2012); it is a disservice to them not to be provided opportunities for engagement in algebra. Whether they know it or not, many early childhood teachers utilize basic strategies that promote algebraic thinking; it all starts with concrete pre-algebra activities (Lee et al., 2016). Research by Knuth et al. (2016) has shown that young "children can develop critical algebraic thinking skills that are foundational to the successful study of algebra in the secondary grades" (p. 68), and that this early algebra exposure may decrease some of the difficulty children experience in middle and high school.

Our goal was to highlight algebraic strategies from Mrs. McCombs' first grade classroom so other teachers might identify similar strategies they are using and deepen their understanding of how each one promotes algebraic thinking. In our work with preservice and practicing teachers, we have found that once they connect a strategy to algebra, they are more willing to provide similar activities. This understanding also lessens teachers' anxiety and fear of teaching algebra, thus increasing time spent on algebraic learning.

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[1] All names of children as well as of teachers are pseudonyms.