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Using Cartoons to Make Connections and Enrich Mathematics

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Reference

Cho, H., Osborne, C., & Sanders, T. (2015). Classroom experience about cartooning as assessment in pre-service mathematics content course. *Journal of Mathematics Education at Teachers College*, 6(1), 45-53. Retrieved January 10, 2018, from file:///C:/Users/st%20clairev/Downloads/1091-3251-2-PB%20(5).pdf.

Keywords

writing, finite mathematics, cartoons, teaching

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Using Cartoons to Make Connections and Enrich Mathematics

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Abstract

The article discusses the integration of cartoons into a finite mathematics college course. However, cartoon integration is appropriate for any educational level STEM course. Students and the author used an online comic strip creator, MakeBeliefsComix.com, to create cartoons that connected concepts to the real world and history. Following Cho, Osborne, and Sanders (2015), students wrote a paragraph about their cartoon and its mathematics. In addition to connecting mathematics to art and writing and unearthing students' creative side, cartoons helped show the humanistic side of mathematics and promote communication and excitement about mathematics. The author developed a rubric to evaluate students' cartoons. There was evidence that students who did cartoons were better able to explain a concept and give examples of its real-world connection than those who did not. The article has potential to encourage others to brainstorm about cartoon integration in their mathematics courses.

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The proverb, “A picture says a thousand words,” means that a picture gives as much information as the spoken or written word and maybe, more profoundly. What about a proverb relating cartoons and mathematics? Maybe, “A cartoon *sums* it up.” or “Mathematics; it’s all in the cartoon”. The possibilities about what cartoons can convey about mathematics are limitless. In addition to showing how to problem solve, cartoons in mathematics can convey such things as its beauty, its creativity, its history, its connection with the real world, its continual growth and change, and that its people – mathematicians – are real people who debate and revise. With respect to real people, I think about my favorite cartoon by Sidney Harris on a tee-shirt that I bought for my mother at an American Mathematical Society conference (Figure 1). My interpretation is that this cartoon shows a mathematician as a real person struggling to get the answer.

Figure 1: Sidney Harris mathematics cartoon tee-shirt.



Once untouchable in mathematics or any other school discipline for that matter (Cho, 2012; Toh, 2009), cartoons have made a fairly recent appearance in the mathematics world of teaching and learning. In addition to the benefits already listed, cartoons encourage students to express their thinking, motivate students to learn mathematics, reduce mathematical anxiety, help instructors detect students’ misconceptions and adjust instruction accordingly, promote understanding for algebraic symbolism (Cho, 2012; Cho, Osborne, & Sanders, 2014; Toh, 2009), and can be used to convey appropriate mathematical technical language and model how mathematicians really talk about mathematics. Cartoons are appropriate for any level of mathematics. Examples of ways that cartoons can be used in mathematics teaching (Cho, Osborne, & Sanders, 2015) are: (1) use an existing cartoon with mathematical content in a newspaper, for example, and develop activities related to it, (2) use cartoons specifically developed for mathematics (see for e.g., Ashbacher, 2015; Dabell, Keogh, & Naylor, 2008; Gonick, 2011; Gonick & Smith, 1993) and (3) have students create cartoons to convey their mathematical thinking.

The 2015 publication of *The Cartoon Guide to Statistics* by Larry Gonick and Woollscott Smith, sparked my interest in integrating cartoons in my mathematics classes. I decided to have students use an online cartoon maker and first e-mailed several sequential art professors at various universities in the country for their suggestions on a good one. They all said that because they had their students only draw cartoons by hand they were unable to recommend an online cartoon maker. Much googling, led me to decide on using the online comic strip maker at www.MakeBeliefsComix.com created by Bill Zimmerman, a journalist, book writer, and Pulitzer-prize nominee. The artist, for MakeBeliefsComix.com, Tom Bloom, draws for publications such as *The New*

York Times, *Wall Street Journal*, and *Business Week*. I plan to learn more about cartoon making through other sources that I found during my googling search (e.g., the annual Michigan State University Comics Forum (<http://comicsforum.msu.edu/>) and the Coursera course (<https://www.coursera.org/>), “How to Make Comic Book (Project-Centered) Course”).

In the sections that follow I start by discussing cartoons that I created and presented to my students to encourage them to realize the relevance of mathematics and that it is more than a collection of facts and skills. Next, I discuss cartoons that students created, showing how they gave me insight about their thinking about mathematical concepts and the nature of mathematics teaching and learning. Finally, I discuss the results of questionnaires that I gave students to determine whether and how mathematics cartooning benefited them and their opinions about it.

Integrating Cartoons in Teaching and Learning

Teacher Created Cartoons

In this section I discuss cartoons that I created to help students think about mathematics in real-world and historical contexts and how I integrated these cartoons into teaching and learning. When using cartoons to introduce objective concepts I encouraged students to interact with the cartoon characters by, for example, verifying cartoon characters’ ideas using a graphing calculator and having them complete Blackboard assignments involving real-world or historical ideas connected with concepts. Cartoon characters actually mention these Blackboard assignments in their dialogue. Other ways that I integrated cartoons that I created into teaching and learning include using them to help students review concepts and to encourage students to make hypotheses about problem solutions.

One way that I used cartoons was to introduce course objectives. Students were surprised when I told them that a Ph.D. dissertation has been written about cartoons in mathematics teaching and learning (Cho 2012). In the cartoons that I created, I made connections to the real world, to historical ideas, and sometimes to stories involving mathematics. Also, I included mathematical ideas and conventions that many students seem to overlook and showed that there are multiple ways to solve a problem. I created follow-up activities connected with the cartoons for further exploration of concepts. The cartoon characters mention these activities. I projected the cartoons on the projector screen and read them aloud, stopping at points to enter the cartoon’s world by expanding on or looking more deeply into a character’s thoughts or adding to or following up on their thoughts. I wrote notes on the board related to this. The students and I often went back and forth between the cartoon and the written remarks on the board related to ideas in the cartoon. Sometimes, I asked students to verify cartoon characters’ ideas by using the graphing calculator or to come to the board to verify characters’ ideas. And, sometimes I used ideas in the cartoon as a springboard for discussing other concepts not specifically addressed in the cartoon. My cartoon characters were Satchel and Tina, two students devoted to thinking and talking about mathematics. One of my students chose the name, Tina. To demonstrate the above ideas, I include cartoons that I created related to the objectives on graphing linear equations and solving systems of linear equations.

The “Graphing Lines” cartoon (Figure 2) starts with a reference to history: the
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ancient Egyptians and Descartes work related to our x-y coordinate system. I created the Blackboard assignment that Satchel mentions to involve students in reading and writing about the ancient Egyptian coordinate system (Lumpkin, 1997) and René Descartes. Students were surprised to learn that the ancient Egyptians had a sense of the rectangular coordinate system. Some questioned why many history of mathematics books do not include this. The cartoon characters discuss various ways to graph the equation, $2x - 3y = 12$. While reading the cartoon, students graphed the equation, $2x - 3y = 12$, on the graphing calculator to verify cartoon characters' ideas (e.g., the y-intercept, the slope). Satchel points out the connection between Robert Wadlow, the tallest man who ever lived (Jacobs 1994), and graphing lines. The cartoon ends with Satchel and Tina determining the equation of the line that gives the relationship between Wadlow's age and height. Satchel notes ideas that some of my students did not seem to realize: there are an infinite number of points on a line ("Graphing Lines" – Part 2) and the usual standard textbook notation and formula for slope (i.e., $m = \frac{y_2 - y_1}{x_2 - x_1}$ for (x_1, y_1) and (x_2, y_2) any two points on a line) does not mean that one has to "start with" the second ordered pair in the subtraction ("Graphing Lines" – Part 3). Also, Satchel notes that sometimes other variables instead of x and y (e.g., a for age and h for height) are used ("Graphing Lines" – Part 2). We talked about labelling the axes appropriately using a and b . With the cartoon characters "setting the stage," I asked the class to follow up on finding the equation for the Wadlow data in the cartoon ("Graphing Lines" – Part 3). We also, discussed the meaning of function using the Wadlow data and how to tell whether the graph of an equation will be a line. Figure 3 shows an excerpt of what I recorded on the board as we read the cartoon.

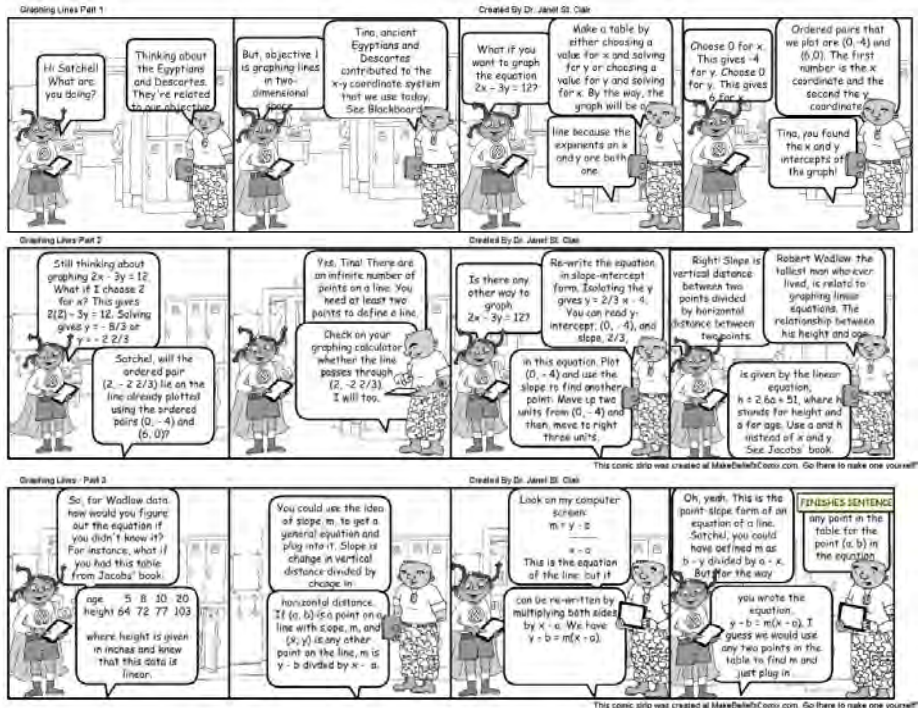


Figure 2: "Graphing Lines" cartoon that I created. https://digitalcommons.georgiasouthern.edu/stem_proceedings/vol2/iss1/12
DOI: 10.20429/stem.2018.020112

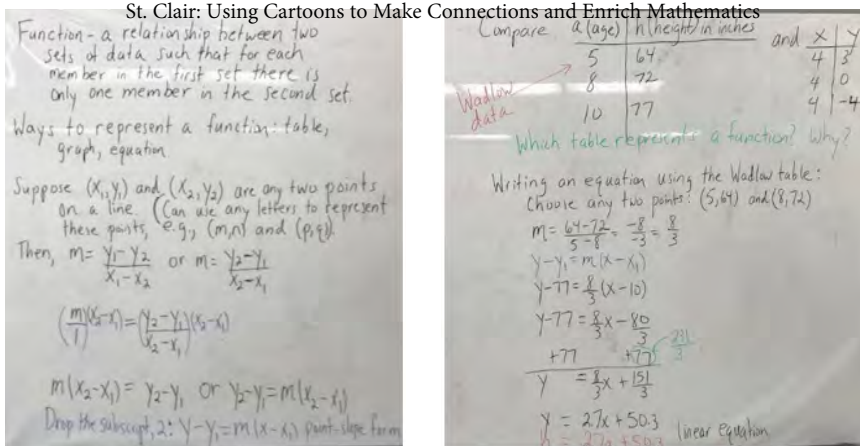


Figure 3: Board excerpt related to “Graphing Lines” cartoon (Figure 2) relates to cartoon characters’ discussion of finding the equation of the line for the Robert Wadlow age and height data. The table for age and height is expressed horizontally in the cartoon, but I wrote the table vertically on the board. Satchel and Tina give an idea of the path to take in determining the equation using information in the table, and we follow up on this by picking any two points from the table and using the point-slope form of an equation of a line to determine the equation. The cartoon is used as a springboard for discussing the concept of function. And, the board excerpt also includes a definition of function and a comparison between the Wadlow table, which represents a function, and a table that does not:

x	y
4	3
4	0
4	-4

In the “Systems of Linear Equations and the Tee-Shirt Sale” cartoon that I created (Figure 4) Satchel and Tina are trying to figure out how many student and how many community tee shirts they sold. They agree to create a system of linear equations and solve using the elimination method. Tina mentions a couple of other methods to solve their system. Satchel uses his graphing calculator to check their solution found using elimination, and the class and I viewed the graphical solution on the graphical calculator also. He indicates the appropriate forms for the equations in order to enter them into the graphing calculator. Satchel and Tina go on to talk about strategies for eliminating the x or y variables in another system of linear equations. Satchel doubts whether Tina will use the graphing calculator to check her result. This might show students that the graphing calculator is a useful tool for checking their own solutions rather than wondering or asking whether they are okay. I asked students to also verify the solution graphically as Tina, to Satchel’s surprise, did. Satchel and Tina eventually discuss the merchant problem in the story, The Tutor written in 1884 by Anton Chekhov (<https://www.ibiblio.org/eldritch/ac/tutor.htm>), that can be solved using systems of linear equations (Ochkov & Look, 2015):

The cartoon ends with Sachel and Tina planning to complete a Blackboard assignment that I prepared for the class. The assignment asked students to solve the merchant problem using a system of linear equations or any other strategy (including a non-algebraic one), write about what they thought about the story and its characters, and to extend the story or write a second part to it. It's interesting that the father in *The Tutor* solves the merchant problem without using algebra, but no details are given about his non-algebraic solution. All of my students solved the problem using a system of linear equations. I contacted Valery Ochkov, a professor at a university in Russia who wrote the article, "A System of Equations: Mathematics Lessons in Classical Literature" (Ochkov & Look, 2015), via e-mail to get his ideas on how the father solved the problem non-algebraically. He was kind enough to send his ideas. Students seemed to be very interested in this communication, and we discussed the non-algebraic solution in class. So, the cartoon was a springboard for communicating with another mathematician about another way to solve the merchant problem in *The Tutor*.

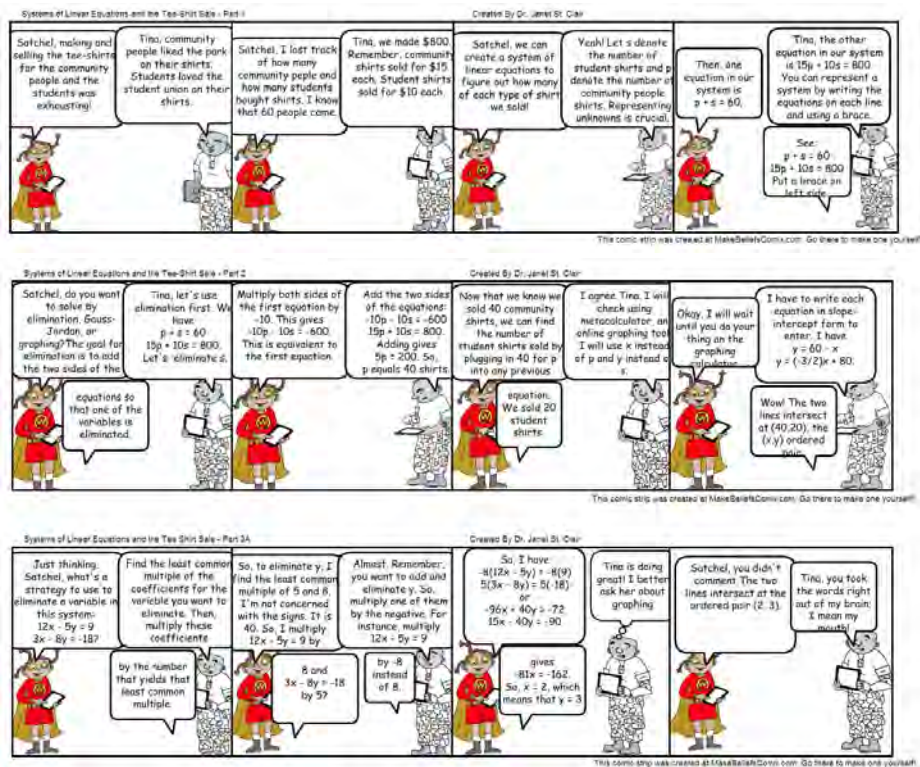


Figure 4: "Systems of Linear Equations and the Tee-Shirt Sale" cartoon that I created.

Continued on next page.



A second way that I used cartoons was to review concepts. Sometimes I asked students to read my cartoon creations to review for tests. Once, I revised a student's cartoon from a previous semester to use for review of systems of linear equations. The student had provided an interesting context for systems, a girl buying plants from a man named "Mr. Panda." I made major revisions to the cartoon, naming the girl Inga Schmidt, making historical connections to the ancient Chinese method of solving systems as compared to Gaussian elimination many centuries later, and including Inga's description to Mr. Panda of her sister's trip to China. The trip idea came from my own younger sister's actual trip to China as part of the 2015 Bridge Delegation to China to help educators start or strengthen their institution's Chinese programs and partnerships (<https://www.collegeboard.org/all-access-tags/chinese-bridge-delegation>). I titled the cartoon "Systems, China, and Germany."

A third way that I used cartoons was to have students solve a problem posed in the cartoon before reading the characters' solution either during class or for homework. I noticed that some students became more interested and involved in making hypotheses about solution strategies than they normally were.

Student Created Cartoons

In this section I discuss students' cartoons, including paragraphs that they wrote to explain their cartoons. This section shows how students' cartoons enabled me to better see their creativity/imagination, their misunderstandings and the need to specifically address problem solving and metacognition in mathematics, and views that they might have about the nature of mathematics and mathematics teaching and learning. One student wrote that creating her cartoon helped her visualize her dream of owning her own business. Another student indicated that he learned from creating his cartoon that integrating life scenarios in cartoons can help learn mathematics.

I gave students several options to choose from to revolve their cartoons around:

- Connection of the concept to the real world
- Connection of the concept to the history of mathematics
- A problem of your choice related to the concept and solved using a method that was a part of the course objectives

I did not include my cartoons in Blackboard so that students will not be tempted to model their cartoons after mine. As I read more research about cartoons in mathematics (e.g., Cho, Osborne, & Sanders, 2015), the second semester that I integrated cartoons in my classes, I asked students to write a paragraph describing their cartoon and its mathematical content.

I developed a rubric using the Rcampus website, an education management system and a Collaborative learning environment (<http://www.rcampus.com/rubricshowc.cfm?code=G48C63&sp=true>), as a basis.

The rubric that I developed is as follows:

Your cartoon will be evaluated on number of panels, mathematical relevance, elements, presentation, and creativity. See the maximum 4 points for the maximum requirements. The points will be used for extra credit.

4 points

- Number of Panels** - Cartoon has the required 3 or 4 panels. Mathematical
- Relevance** - The cartoon provides a clear picture of the mathematical concept. One would be able to develop understanding of the concept by reading the cartoon.
- Elements** - The cartoon includes the required name/title and the author's name.
- Presentation** - The cartoon is presented in an attracting way, and the overall appearance is excellent. It includes backgrounds and objects in addition to such items as talk and thought balloons. Characters are scaled to realistic proportions in relation to backgrounds and objects.
- Creativity** - The cartoon sparks interest in the mathematical concept. Characters are well-chosen, and wording provides some humor or drama.

3 points

- Number of Panels** - Cartoon has 2 panels.
- Mathematical Relevance** - The cartoon gives a vague notion of the mathematical concept. One would have a difficult time developing understanding due to missing ideas.
- Elements** - The cartoon does not include either the required name/title or the author's name.
- Presentation** - The overall appearance of the cartoon is average. There might be some scaling problems with respect to character and background/object sizes. The cartoon seems to be done haphazardly.
- Creativity** - The cartoon generates little interest in the mathematical concepts. Choice of characters is good, but wording provides little humor or drama.

2 points

- Number of Panels** - Cartoon has 1 panel.
- Mathematical Relevance** - The cartoon does not provide information that would help one develop understanding about the mathematical concept.
- Elements** - The cartoon does not include neither the required name/title nor the author's name.
- Presentation** - The overall appearance of the cartoon is poor. Cartoon reflects that little to no thought was put into its plan.
- Creativity** - The cartoon does not generate interest in the mathematical concept. Characters are not well-chosen or seem to be unrealistic. Wording provides little to no humor or drama.

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As indicated in the rubric, cartoons were extra credit (4 points maximum).

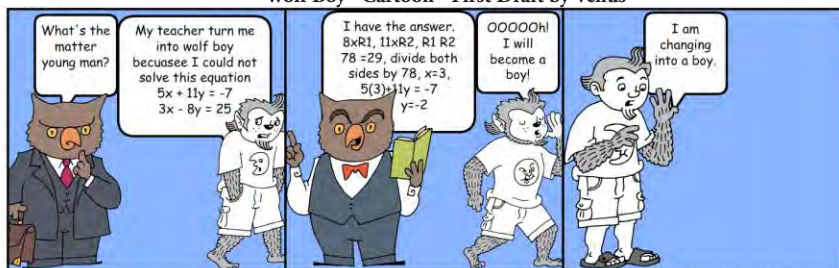
However, some of the follow-up assignments connected with the cartoons were counted toward quiz points. Normally, I don't give extra credit assignments and debated on whether the cartoons should count as extra credit or regular credit. Eventually, as students create a larger number of cartoons during a semester and become more familiar with using MakeBeliefsComix.com, I will probably count cartoons as regular credit.

MakeBeliefsComix.com allows one to both save their cartoon (e.g., on a flash drive) and e-mail it to themselves and someone else. Students e-mailed me their cartoons, and I sent them to campus duplication to be printed in color. It was nice to "get mail." Students did a draft cartoon and then revised using comments that I gave them. Revising conveys the idea of "writing (cartooning) as a process." I typed my reviews of each student's cartoon and gave students a hard copy along with a color copy of their original cartoon sometimes during class and sometimes via e-mail.

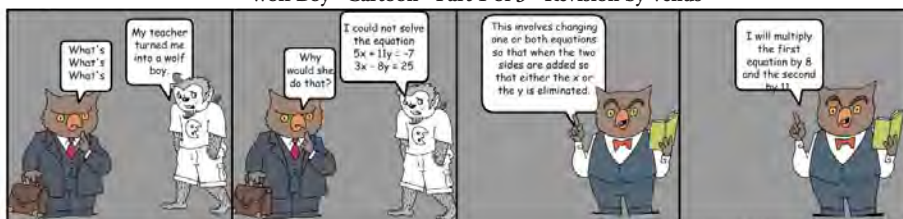
Examples for linear equations and systems of linear equations include small businesses that make head wraps and computers, a pedestrian and a police officer at parking meters, a chemist and his partner creating a new punch, and a grandmother asking her grandson to grocery shop for her.

Figure 5 is an example of a cartoon in which a student is able to integrate suspense, which is often a part of the cartoon genre, and manages the mathematics at the same time. Figure 6 is an example of a cartoon that more or less reflects equal authority between two peers (Cho, Osborne, & Sanders, 2015). Unlike the cartoon in Figure 5, the cartoon in Figure 6 embeds the mathematics in a real-world situation.

"Wolf Boy" Cartoon - First Draft by Venus



"Wolf Boy" Cartoon - Part 1 of 3 - Revision by Venus



Wolf Boy Cartoon - Part 2 of 3 - Revision by Venus



"Wolf Boy" Cartoon - Part 3 of 3 - Revision by Venus



Figure 5: Venus' "Wolf Boy" cartoon. The first cartoon is the draft. The last three cartoons are the three parts of the revised version. In part 2, panel 1 of the revised cartoon, "equivalent" should be "linear."

"A Great Summer" Cartoon - Part 1 of 2 by Hana - No Revisions Done



"A Great Summer" Cartoon - Part 2 of 2 by Hana - No Revisions Done



Figure 6: Hana's "A Great Summer" cartoon. She did not do a revision.

The choice of characters in the cartoons reveal students' thoughts about how mathematics interactions occur between people (Cho, Osborne, & Sanders, 2015). The cartoon in Figure 5 shows that the student thinks interactions involve an expert or authoritarian of mathematics knowledge helping a young person figure out the mathematics. In this case, the young person suffered serious consequences for not being able to solve a system of linear equations in the classroom. His teacher turned him into a wolf boy! The student who did this cartoon might view mathematics as an invented,

non-changing collection of facts and skills transmitted by the teacher to students (Philipp, 2007; Thompson, 1992). This implies that more work needs to be done in the classroom to challenge some students' views of mathematics. For example, many years ago, Thompson (1992) gave this description of what might be done to help change this type of view:

... more purposeful activities that grow out of problem situations, requiring reasoning and creative thinking, gathering and applying information, discovering, inventing, and communicating ideas, and testing those ideas through critical reflection and argumentation (p. 128).

We want students to view mathematics as being socially constructed by real people and something that is revised and changes over time in the spirit of the characters in Lakatos' (1976) *Proofs and Refutations*.

To illustrate paragraphs that students wrote about their cartoons and to show how cartoons helped me realize students' misconceptions, I will present cartoons done by Evan and Nia (pseudonyms). Evan's cartoon, which he titled "Overthinking at its Finest," relates to our objective on graphing linear equations and finding equations of lines. See Figure 7. It involves a scientist, maybe a "mad" scientist, trying to figure out how the graph of $y = 5x$ would look. A baby helps the scientist visualize the graph. Evan's

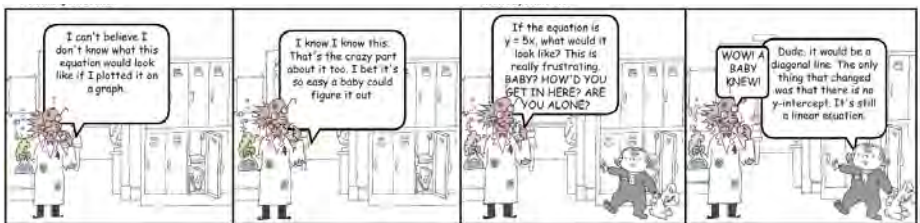


Figure 7: Evan's cartoon, "Overthinking at its Finest," relates to graphing linear equations.

paragraph is in Figure 8. He explains his cartoon, indicating that sometimes people make things more difficult than they really are by overthinking and implies that he uses humor to convey that overthinking causes problems. As Cho, Osborne, and Sanders (2015) note, incorporating humor requires an extra layer of thought in addition to the artistic demands and mathematics. Evan's idea of overthinking might also involve the scientist not being able to draw on his metacognitive knowledge or skills. It would have been interesting to weave this into the cartoon. Maybe, the baby could have given the scientist a lesson in metacognition (Schoenfeld, 1987)! Also, George Pólya's ideas about problem solving

I thought of doing this because its funny and people tend to make things more difficult than it really is. Overthinking will do that to you. I picked the scientist because sometimes the smartest people are the ones to over-think something so simple. They might think the answer has to be some extravagant answer. The baby was used to show that he needs to stop overthinking.

Figure 8: Evan's paragraph about his cartoon, "Overthinking at its Finest."
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one that one is having trouble with, is a strategy that might have helped the scientist. Unfortunately, Evan did not explain the mathematics in his cartoon as the instructions asked. It is not clear whether the scientist is comparing the graph of $y = 5x$ with the graph of $y = x$. The baby tells the scientist: “. . . The only thing that changed was not there is no y -intercept. It’s still a linear equation.” Based on this, Evan is comparing the graph of $y = 5x$ with some other graph. Evan might have been thinking about the line, $y = x$ (which bisects both Quadrants I and III) because the baby mentions a diagonal line. It would have been nice if Evan had included the idea of the scientist using the graph of $y = x$ to help him visualize the graph of $y = 5x$. Obviously, Evan was not aware of metacognition and Pólya’s problem-solving principles. This might suggest that time designated for instruction in these would help students. There is a possibility that Evan and other students incorporate in their cartoons the way they would handle solving a problem that they have difficulty with. In this case, Evan might have sought help from another person rather than figuring it out on his own. Another point is that I am not sure whether Evan realized that the y -intercept of the graph of $y = 5x$ (as well as the x -intercept) is $(0, 0)$. (The baby’s says, “. . . The only thing that changed was not there is *no* y -intercept . . .”).

I gave Evan typed suggestions for revising his cartoon, including a graph of $y = 5x$ that I did on the graphing calculator. But, cartoons were optional (worth a maximum of four points extra credit), and Evan did not revise. Here are my suggestions to Evan for revision:

In the first panel, tell what the equation is and maybe, add a little more: For example, let the scientist say, “I can’t believe what this equation, $y = 5x$, should look like if I plotted it on a graph. Would it be a line? Would it be a parabola? Would it be a hyperbola? Would it touch the x or y axes?”

In the third panel, add ideas about the scientist comparing $y = 5x$ with some other that he knows about, for example $y = x$. Evan, graph $y = x$ and $y = 5x$ yourself by hand on graph paper or using a graphing calculator. What do you observe about the comparison of these graphs?

In the last panel, re-word the baby’s talk balloon to convey this idea: “Dude, it would be a line through the origin. The x - and y -intercepts are both $(0, 0)$. Another point on the line besides $(0, 0)$ is $(1, 5)$. See, $(1, 5)$ makes the equation true. You can tell the graph will be a line by looking at the exponents on the variables. When these exponents are 1, the graph is a line. Check out the graph on the online graphing calculator, meta-calculator at www.metacalculator.com.”

Evan did a decent job of scaling his characters to sizes so that they were in proportion with the background and objects. However, the scientist appears slightly smaller in the first panel than in the last three panels.

Evan’s responses to a questionnaire that I gave at the end of the semester to get students’ thoughts about creating their cartoons is in Figure 9. Notice that he learned that life scenarios can be integrated with mathematical concepts (question 3) and that the most favorite aspect of creating his cartoon was incorporating humor (question 5).

1) Please explain whether the cartoon helped you in terms of your understanding of the concept you focused on. If it helped you, clearly explain how.

2) Did the cartoon motivate you in any way? If so, how?

3) What did you learn from creating your cartoon?

4) Did you enjoy the cartoon assignment? Tell why or why not.

5) What was your favorite part or thing about the cartoon assignment?

6) What was your least favorite part or thing about the cartoon assignment?

7) Did you enjoy the process of making the cartoon? Clearly explain why.

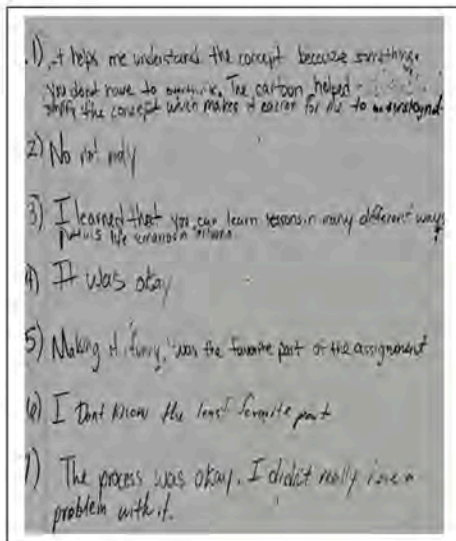


Figure 9: Questionnaire on students' experiences in creating cartoons given at the end of the semester (left) and Evan's responses to these questions (right).

Notice also that Evan's responses to questions 2 (motivation as a result of creating cartoon), 4 (enjoyment of cartoon assignment), and 7 (enjoyment of process of creating cartoon) would not inspire a person to make cartoons. This might suggest that I try different ways for students to do the cartoons. For example, they could work in pairs and create cartoons and publish their final products on a Website. Also, more time learning how to draw their own cartoons might help. I invited a sequential artist to visit the classroom to give a crash course in drawing cartoons, but due to time constraints, the artist came one time at the end of the semester.

Nia did a cartoon, entitled "Isis's Dream," about a cartoon character named Isis who is planning a head-wrap business and thinks about ideas related to a linear cost function. See Figure 10. Nia's cartoon also related to the objective on graphing lines and writing equations of lines, and it was nice that she thought of a real-world context in which to embed the mathematics. Briefly, the linear cost function, $C(x)$, is defined in slope-intercept form as $C(x) = mx + b$ where $C(x)$ represents the cost to produce x items, m is the marginal cost – the cost to make one item, b is the fixed cost – the cost that doesn't change (e.g., cost to rent a place to make the product, cost to train workers), x is the number of items made. The cost equation can be expressed, of course in point-slope form as $C(x) - C(x_1) = m(x - x_1)$. Here, the point, $(x_1, C(x_1))$, represents the cost, $C(x_1)$, make a specific number of items, x_1 . The revenue, $R(x)$, made from selling the items made is given by the equation, $R(x) = px$, where p represents the price that an item sells for and x represents the number of items sold. Break even occurs when the cost to make the items equals the revenue: $C(x) = R(x)$. And, profit, $P(x)$, is revenue minus cost: $P(x) = R(x) - C(x)$.

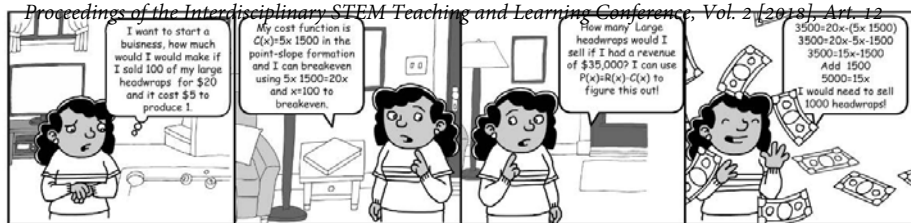


Figure 10: Nia’s cartoon, “Isis’s Dream,” related to the objective on graphing linear equations and finding equations of lines. She leaves out the addition symbol, +, in the equation, $C(x) = 5x + 1500$.

Nia typed the following paragraph in Figure 11:

I made "Iris' Dream" because I myself want to have my own business. I used cost function is the only math objective I have completely understood since the start of this class. If I did choose to have my own business I would sell head wraps and I think that having Iris think it through first was a good vision board. I choose the character I chose because she was a black girl trying to start a business which I support personally. This assignment neither helped or stagnated my learning process with math although it was cute in nature and fun as an extra credit it didn't further my success in my continuous struggle with Mathematics.

Figure 11: Nia’s paragraph that was supposed to explain her cartoon and the mathematics in it.

Nia notes that creating the cartoon was a “vision board” for her because she wants to start a head-wrap business one day. So, the cartoon may have helped Nia “live” her dream of owning her own business. Nia is frank when she points out that creating the cartoon did not help her in her “continuous struggle with mathematics.” At the beginning of the course, Nia told me that she would be asking a lot of questions because she felt that she usually has difficulty with mathematics courses. The main misunderstanding shown in Nia’s cartoon occurs in the third and fourth (last) panels. Some revision suggestions that I gave to Nia, in typed form, were:

The revenue representation, $20x$ (where x is the number of head-wraps sold), in panel 2 implies that the price of each head-wrap is \$20. So, panel 1 could be revised so that Iris says “. . . I sold 100 of my large head-wraps, making \$2000 . . .” instead of “. . . I sold 100 of my large head-wraps for \$20 . . .” (If 100 head-wraps sell for \$20, the price for one head-wrap would only be 20¢. You probably want to sell one head-wrap for more than 20¢ especially because you say that it costs \$5 to make one head-wrap).

In panel 3, instead of Iris asking, “How many large head-wraps would I sell if I had a revenue of \$35,000?” have Iris ask, “How many large head-wraps would I have to sell in order to make a profit of \$35,000?” Then, panel 4 would involve substituting into the profit equation, $P(x) = R(x) - C(x)$, as follows: $35,000 = 20x - (5x + 1500)$. (Note: Nia, I think, inadvertently substitutes 3500 instead of 35,000 for profit). Solving this for x gives approximately 2433.3, which should be rounded up to 2434 head-wraps that should be sold to make a profit of \$35,000. Have Iris give some explanation of the various equations that she uses.

In panel 2, with respect to the equation, $C(x) = 5x + 1500$, Iris could address why she thinks her fixed cost is \$1500. Note that you could also revise some of the mathematical ideas by having Iris say that it costs \$5 to make one head-wrap and then adding a cost to make a specific number of head-wraps. For example, Iris might find that it costs \$300 to make 50 head-wraps. Then, you could develop the cartoon by having Iris figure out a cost function using this information:

$$C(x) - C(x_1) = m(x - x_1)$$

$$C(x) - 300 = 5(x - 50)$$

$$C(x) - 300 = 5x - 250$$

$$C(x) = 5x + 50$$

Like Evan, Nia did not revise her cartoon. She actually ended up dropping the course at the time that I gave her my suggested revisions. Both Evan and Nia provided rich contexts that I and other students could revise to create interesting cartoons related to graphing linear equations and finding equations on lines.

Other misunderstandings that I noticed in students' cartoons included using inappropriate terminology, problems using algebraic notation, and difficulty modeling real-world situations mathematically or omitting mathematics in the cartoon. With respect to inappropriate terminology, I often noticed that some students referred to a system of linear equations as "equations." One student called it a "linear system elimination equation." With respect to algebraic notation, an example is a student who was not consistent is using the same case letters when defining unknowns. The student used X and Y when defining the unknowns and x and y when writing the equations in her system. Finally, with respect to difficulty in modelling, Nia had difficulty with ideas related to modelling linear equations in the context of the linear cost function. Other examples are two other students who had difficulty modelling ideas their cartoons related to systems of linear equations, one who embedded a problem in the context of buying flowers and the other in the context of buying ingredients for a pie. These students thought of rich contexts to embed their mathematics in but were unable to successfully connect their contexts with the mathematics. See Figure 12 for examples of the above misunderstandings. This shows me that I need to think of activities that will target these kinds of misunderstandings. For example, more readings and discussion related to equations, systems of linear equations, and use of variables to represent unknown values might have potential. This could include excerpts of historical readings (e.g., Grcar, 2011; Hart, 2011; Pycior, 1981) that show how these concepts developed over time. With respect to modeling, maybe engaging students in solving more real-world problems in pairs and/or as group projects will be helpful. For example, with respect to algebraic ideas, a function approach that embeds concepts in solving real-world problems using technology has great potential (Laughbaum, 2003; Laughbaum & Crocker, 2004). Another example is the Algebra Project, which involves relating everyday life of students to algebra (Moses, Kamii, Swap, & Howard, 1989; Wilgoren, 2001). Yet another is Realistic Mathematics Education based on Freudenthal's view of mathematics, which embeds mathematics in experiences that students relate to

(Proceedings of the Interdisciplinary STEM Teaching and Learning Conference, Vol. 2 [2018], Art. 12 (Precedentia), 1991). Felton (2014) makes an important point (<http://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Mathematics-and-the-Real-World/>). Felton (2014) discusses the value of two approaches to integrating real-world problems: using the real world as a stepping-stone to encourage students to think about mathematical concepts and using authentic real-world problems. The former approach includes problems that are “near” and ones that students will probably not exactly encounter outside of school. The latter includes problems that are open-ended and messy and have multiple ways of solving. This approach would include Realistic Mathematics Education.

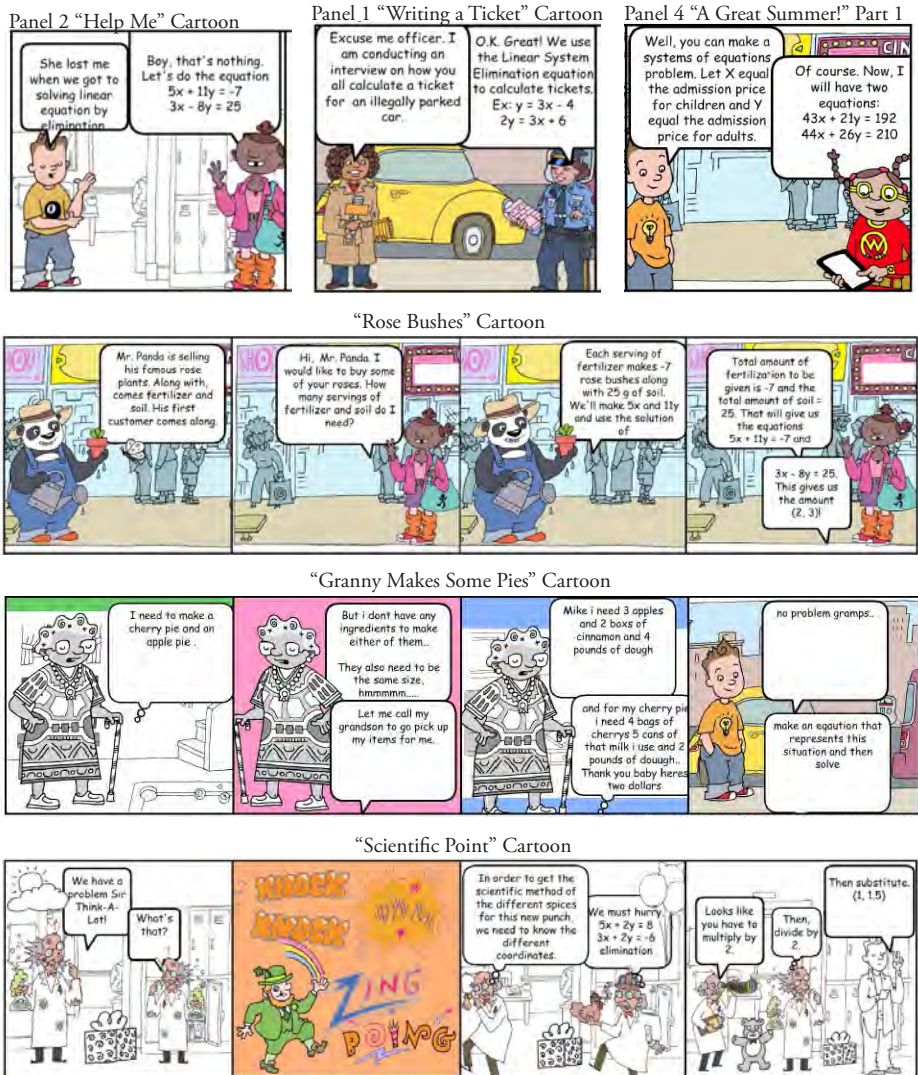


Figure 12 (continued on next page): Examples of students’ misunderstandings including using inappropriate terminology – “Help Me,” “Writing a Ticket,” and “A Great Summer;” problems using algebraic notation – “A Great Summer;” difficulty https://digitalcommons.georgiasouthern.edu/stem_proceedings/vol2/iss1/12

“Off-Call” Cartoon



Figure 12 (Continued)

Findings Related to Students’ Understanding and Thinking

In this section I discuss students’ thinking and opinions about cartoons and their understanding of concepts as a result of reading/discussing cartoons that I created and creating their own cartoons. Also, I discuss the results of a pre- and post-questionnaire that measures changes in student motivation, interest, and anxiety given to students in one class. Students tended to have positive opinions about the cartoon experience. There was evidence that students who created cartoons were able to answer questions related to systems of linear equations more successfully than those who did not create cartoons. There was also evidence that cartoons helped students with mathematics anxiety.

The previous section gives examples of misunderstandings that I found as I read students’ cartoons. Questionnaires that I created to find out how cartoons influenced students’ knowledge and motivation also helped to shed light on students’ understanding. I gave a few questions related to each objective before discussing it and again toward the end of the semester and compared students who did cartoons with students who did not. Figure 13 gives results for questions related to systems of linear equations. One question asked students to write everything they knew, including definition, examples, and real-world connections. The other question asked students to solve the following two systems of linear equations using any method:

$$\begin{cases} 2x + y = 9 \\ 3x - y = 16 \end{cases}$$

$$\begin{cases} 4x + 3y = -1 \\ 5x + 4y = 1 \end{cases}$$

Students who did cartoons tended to be able to answer these questions more successfully than those who did not.

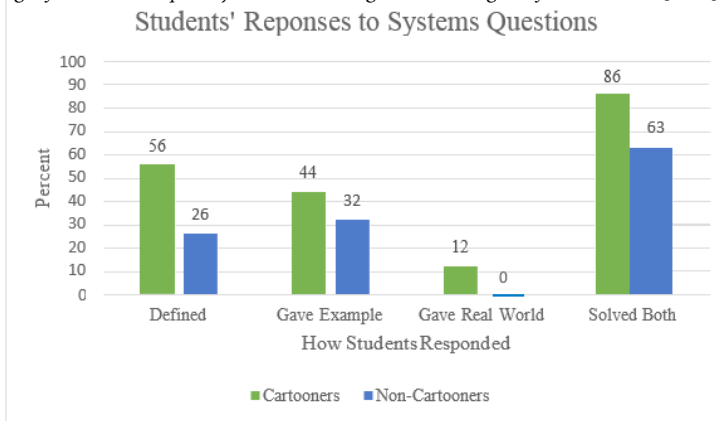


Figure 13: Results of students’ responses to two questions: 1) Write everything you know about systems of linear equations. Examples of things you might include are: definition, example of a system of linear equations, and how a system of linear equations is used to solve real-world problems. 2) Solve the following systems of linear equations:

$$\begin{array}{l}
 \text{a) } \begin{cases} 2x + y = 9 \\ 3x - y = 16 \end{cases} \qquad \text{b) } \begin{cases} 4x + 3y = -1 \\ 5x + 4y = 1 \end{cases}
 \end{array}$$

I also asked students to answer questions about what they thought about the various cartoons that I created and presented in class. All students’ responses were positive. Figure 14 gives these questions for the “Graphing Lines” and “Venn Diagram – What’s the Fuss” cartoons that I created along with two students’ responses. Students indicated such things as appreciating historical information in cartoons, appreciating learning about how concepts related to real life, and appreciating learning more about particular course-related concepts. An education major indicated that she appreciated learning that there are different ways to teach a lesson, i.e., use cartoons.

Questions for “Graphing Lines” Cartoon That I Created	One Student’s Responses to Questions for “Graphing Lines” Cartoon
<ol style="list-style-type: none"> 1) Please explain whether the cartoon helped you in terms of your understanding of graphing and finding equations of lines. If it helped you, clearly explain how. 2) Did the graphing lines cartoon motivate you in any way? If so, how? 3) What did you learn from the graphing lines cartoon? 4) Did you enjoy the graphing lines cartoon? Tell why or why not. 5) What was your favorite part or thing about the graphing lines cartoon? 6) What was your least favorite part or thing about the graphing lines cartoon? 	<ol style="list-style-type: none"> 1. The cartoon helped me by explaining in detail how graphing and finding equations of lines and the history of it. 2. It motivated me to do more graphing lines and I can understand them better. 3. I learn more about graphing lines and how it works. 4. Yes, I enjoyed the cartoon it was very informing. 5. My favorite thing about the cartoon was how informing it was. 6. I really did not have a least favorite part in the cartoon I enjoyed it very much.

Figure 14 (Continued on next page)

St. Clair: Using Cartoons to Make Connections and Enrich Mathematics
 Questions for "Venn Diagrams – What's the Fuss?" Cartoon That I Created

- 1) Please explain whether the cartoon helped you in terms of your understanding of Venn diagrams. If it helped you, clearly explain how.
- 2) Did the Venn diagram cartoon motivate you in any way? If so, how?
- 3) What did you learn from the Venn diagram cartoon?
- 4) Did you enjoy the Venn diagram cartoon? Tell why or why not.
- 5) What was your favorite part or thing about the Venn diagram cartoon?
- 6) What was your least favorite part or thing about the Venn diagram cartoon?

Two Students' Responses to Questions for "Venn Diagrams – What's the Fuss?" That I Created

- 1) Yes, it was helped very much. It was helpful to include the paper plate. Example in the cartoon, because it became even more alive to me.
- 2) I think the Venn diagram cartoon motivated me to appreciate the many ways to explain and teach a lesson.
- 3) I learned that Charles Dodgson was a Mathematician, also John Venn developed Venn diagrams in 1841.
- 4) Yes, I enjoyed the Venn diagram cartoon. I enjoyed the characters. I also enjoyed how everything is explained.
- 5) The names of the characters were very creative.
- 6) I enjoyed reading about the two stories.

Questions for "Venn Diagrams – What's the Fuss?" Cartoon That I Created

- 1) Please explain whether the cartoon helped you in terms of your understanding of Venn diagrams. If it helped you, clearly explain how.
- 2) Did the Venn diagram cartoon motivate you in any way? If so, how?
- 3) What did you learn from the Venn diagram cartoon?
- 4) Did you enjoy the Venn diagram cartoon? Tell why or why not.
- 5) What was your favorite part or thing about the Venn diagram cartoon?
- 6) What was your least favorite part or thing about the Venn diagram cartoon?

Two Students' Responses to Questions for "Venn Diagrams – What's the Fuss?" That I Created

1. The cartoon did help me because through different problems in math I found that I can use Venn diagrams to make it easier to answer them.
2. No. It didn't motivate me.
3. I learned that not only are they used for math problems but real life problems as well.
4. The cartoon was okay. I like to see funny cartoons.
5. I don't have a favorite part.
6. I don't have a least favorite

Figure 14 (continued): Examples of students' responses to questions about the "Graphing Lines" and "Venn Diagram – What's the Fuss?" cartoons that I created and presented in class. Note: One student (Evan) e-mailed responses to me, and the last set of responses is a copy of what *he typed* in his e-mail.

In addition to answering questions about cartoons I created and presented, I asked students to respond to similar questions in Figure 14 for the cartoons they created. See Figure 9 in the previous section for an example.

A questionnaire that I gave students in one class at the end of semester showed that students thought the cartoons were helpful. Figure 15 gives questionnaire items along one student's responses. This student e-mailed me her *typed* responses.

Figure 15 (on next page): A student's answers to questions about their experiences in creating cartoons. The student's actual *typed, e-mailed* responses are included. Published by Digital Commons@Georgia Southern, 2018

1) Please clearly explain how the cartoon helped your understanding of systems of linear equations and how to solve systems of linear equations.

2) Did the cartoons motivate you in any way? If so, how?

3) What did you learn from creating your cartoons?

I like the fact that doing the comic was fun! I had fun taking the time to be creative and learn about the concept of solving linear equations. I never made my own comic before until I started this class and I see that it can be quite challenging. Sometimes it was hard making sure that I used the same characters, backgrounds, term of knowledge, and still be able to teach the concept. This project helped me incorporate my own way of doing a comic without having an actual restriction on what I could do. I really enjoyed doing this project.

Creating the cartoons did motivate me. I never truly was good at math because I felt I was too busy to sit down and actually learn it, but now that I am in college and I know I can't wait on anyone to do my work for me, I decided to learn on my own and teach myself. This cartoon motivated me to work hard at the things I am not so good at, math especially.

I learned from making my own cartoon that creating a comic is not easy. I had to triple check my work to make sure it was done to the best of my ability and I had to make sure that the problem was done correctly. I had to also make sure that people would be able to understand my work without getting confused.

Interestingly, for the first question and also the third question in Figure 15 the student wrote a comment that coincides with a point Cho, Osborne, and Sanders (2015) made: They found that their students' cartoons not only presented mathematical concepts, but also showed students' ability to handle the mathematics coupled with a "complex narrative genre" and their thinking about what constitutes mathematical interactions. In Figure 15, my student wrote:

. . . I never made my own comic before . . . it can be quite challenging. Sometimes, it was hard making sure that I use the same characters, backgrounds, term[s] of knowledge and still be able to teach the concept . .

Also interesting is that the student said she was teaching herself as she created the cartoon, noting also that the cartoons encouraged her to persevere with mathematics. Robert A. Heinlein, an American novelist and science fiction writer, expressed the saying between teaching and learning in a nice way: "When one teaches, two learn."

For one of my classes, I gave students a pre- and post-questionnaire entitled Student Motivation, Interest, and Anxiety Changes that Cho (2012) used in his dissertation study. The questionnaire measures changes in student motivation, interest, and anxiety. Table 1 gives the items. Responses to items were on a 5- point Likert scale: "strongly disagree" (1), "disagree" (2), "don't know" (3), "agree" (4), "strongly agree" (5). Five out of eight students (about sixty-two percent) in this particular class chose to do cartoons. Only students who did cartoons completed the pre- and post-questionnaire. Graphs showing changes in mean scores for motivation, interest, and anxiety are in Figure 16. Motivation pre- and post-score means remained the same for all items except item 5 (I don't give up easily when I don't understand a mathematics problem). For this item, the mean score decreased from 4 (agree) to 3 (don't know). Interest pre- and post-score means increased for items 2 (Mathematics is a very interesting subject than other subjects) and 4 (New ideas in mathematics are interesting to me). But, mean scores for interest item 6 (I see mathematics as a subject I will rarely use) went from 1.6 to 2.4. And, for interest item 1 (I am interested in learning mathematics), mean scores went from 4 to 3. Also, there was a small drop in mean scores (0.2 change) for interest items 3 (I will need mathematics for my future work) and 5 (I find that many mathematics

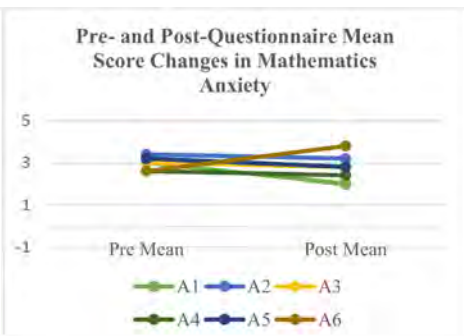
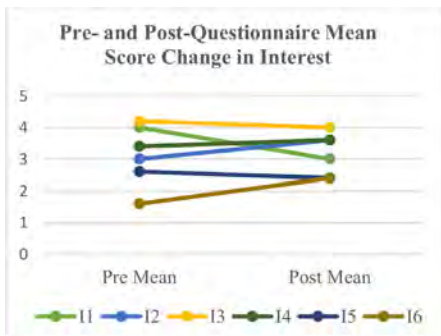
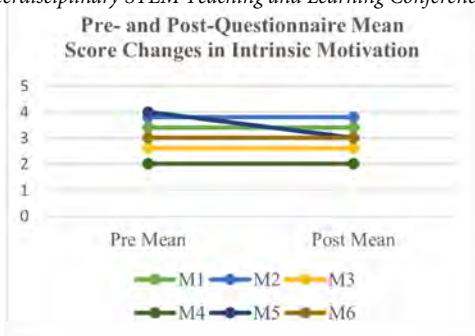
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Type of Item	Item
Motivation (M)	M1. I like mathematics even if I make a lot of mistakes. M2. I enjoy learning new things in mathematics. M3. I enjoy doing an assignment in mathematics. M4. I like difficult problems because I enjoy trying to figure them out. M5. I don't give up easily when I don't understand a mathematics problem. M6. I would like to learn more mathematics in school.
Interest (I)	I1. I am interested in learning mathematics. I2. Mathematics is a very interesting subject than other subjects. I3. I will need mathematics for my future work. I4. New ideas in mathematics are interesting to me. I5. I find that many mathematics problems are interesting. I6. I see mathematics as a subject I will rarely use.
Anxiety (A)	A1. When I hear the word mathematics, I have a feeling of dislike. A2. I have usually worried about being able to solve mathematics problems. A3. Mathematics usually makes me feel uncomfortable and nervous. A4. Mathematics is boring. A5. Mathematics makes me feel uneasy and confused. A6. I usually have been at ease in mathematics classes.

Table 1: Questionnaire items related to motivation, interest, and anxiety.

problems are interesting). The most positive evidence was for anxiety. Mean scores for anxiety items 1 (When I hear the word mathematics, I have a feeling of dislike), 2 (I have usually worried about being able to solve mathematics problems), 3 (Mathematics usually makes me feel uncomfortable and nervous), 4 (Mathematics is boring), and 5 (Mathematics makes me feel uneasy and confused) decreased, and those for item 6 (I usually have been at ease in mathematics classes) increased from 2.6 to 3.8. Cho's (2012) motivation and interest results gave a more positive influence of cartoons on students' motivation and interest. I should also point out that I noticed that all students in this class, except one, created cartoons that did not connect mathematics to the real world or to history as the instructions indicated. The one student who did withdrew from the course early and *did not* complete the post- questionnaire.

I also gave the same class that did the motivation/interest/anxiety questionnaire a questionnaire entitled Opinions about Cartoons also used by Cho (2012) in his Ph.D. dissertation study. This questionnaire contains three types of items: enjoyment/interest doing cartoons, value/usefulness of cartoons, and pressure/tension while doing cartoons. Table 2 gives the items. Responses to items were on a 5-point Likert scale: "strongly disagree" (1), "disagree" (2), "don't know" (3), "agree" (4), "strongly agree" (5). As the bar graphs in Figure 17 indicate, students had positive opinions about their experiences doing cartoons.



	M1	M2	M3	M4	M5	M6	I1	I2	I3	I4	I5	I6
Pre Mean	3.4	3.8	2.6	2	4	3	4	3	4.2	3.4	2.6	1.6
Pre Standard Deviation	.8	.4	.8	.63	.63	.63	.7	.89	.4	.8	1	.5
Post Mean	3.4	3.8	2.6	2	3	3	3	3.6	4	3.6	2.4	2.4
Post Standard Deviation	1.2	.75	1.2	.63	.89	.89	.89	.8	.63	1	1	.49

	A1	A2	A3	A4	A5	A6
Pre Mean	3	3.4	3	2.6	3.2	2.6
Pre Standard Deviation	.89	1.4	.89	.49	.98	1.2
Post Mean	2	3.2	2.8	2.4	2.8	3.8
Post Standard Deviation	0	.98	.98	.49	.98	.98

Figure 16: Changes in mean scores for motivation, interest, and anxiety.

Type of Opinion Item	Item
Enjoyment/Interest (EI)	EI 1. While I was doing cartoon activities, I enjoyed it. EI2. I thought cartoon activities were a boring activity. EI3. I would describe cartoon activities as enjoyable. EI4. Cartoon activities were fun to do. EI5. I thought cartoon activities were an interesting activity. EI6. I would describe cartoon activities as fun.
Value/Usefulness (VU)	VU 1. I believe that doing cartoon activities could be of some value for me. VU 2. I would be willing to do cartoon activities again because it has some value for me. VU 3. I believe that doing cartoon activities is useful for improved concentration. VU 4. I think cartoon activities are an important activity. VU 5. I am willing to do cartoon activities again because I think it is somewhat useful. 6. VU I believe doing cartoon activities could be somewhat beneficial for me. VU 7. It is possible that cartoon activities could improve my studying habits.
Pressure/Tension (PT)	PT 1. I felt tense while doing cartoon activities. PT 2. I was anxious while doing cartoon activities. PT 3. I felt relaxed while doing cartoon activities. PT 4. I did not feel at all nervous about doing cartoon activities. PT 5. I felt pressured while doing cartoon activities.

Table 2: Questionnaire items related to opinions about cartoons.

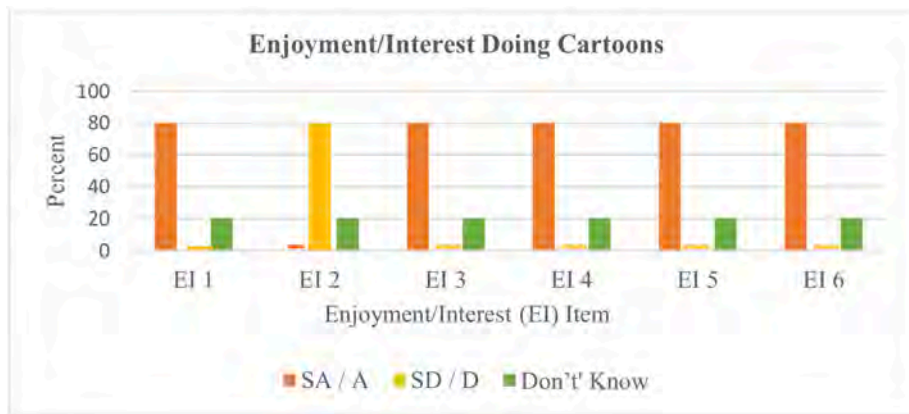


Figure 17 (continued on next page): Students’ responses to opinion questionnaire.

Conclusion

Cartoons lowered students’ mathematics anxiety; unleashed their imagination and creativity; enabled them to draw on their prior knowledge and experiences and in one case, supported their dreams; encouraged them to pose problems; showed them that their ideas are valued; and helped them see that mathematics teaching and learning is not about giving “correct” short answers but involves rich dialogue. With respect to

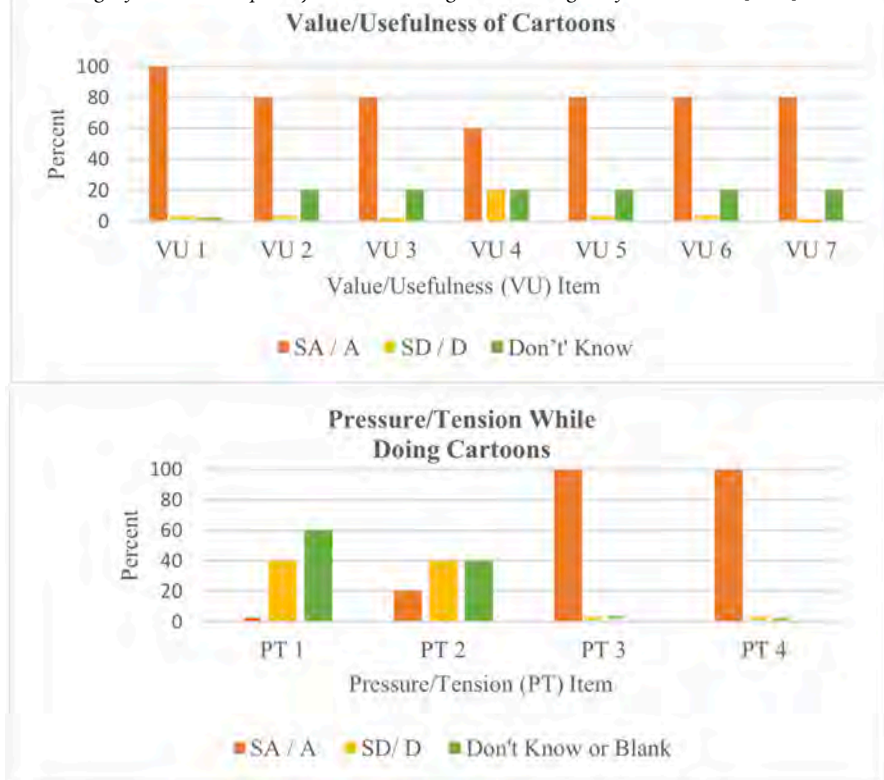


Figure 17 (continued): Students’ responses to opinion questionnaire.

the last point, I have noticed that students began to ask more deep-rooted questions involving such ideas as alternative strategies to solve problems and even questions such as “Why do many people not like mathematics?” Further, it encouraged another form of communication (i.e., e- mail). I overheard one student telling another in an excited tone, “I’m going to do my cartoon now!” Also, students seemed to be more apt to give hypotheses about solutions to problems. There was also some evidence that students who did cartoons learned certain concepts more deeply than those who did not. For example, students who did cartoons were able to answer several questions related to systems of linear equations more successfully than those who did not. It is important that there was evidence that most students had positive opinions about doing the cartoons.

Cartoons helped me better understand not only their misunderstandings and ability to use appropriate mathematical language and symbols but also their views about mathematics. This was useful in adjusting instruction. Cartoons also encouraged me to examine concepts more deeply, including their historical roots, their appearance in literature, and new happenings related to them. For example, with respect to Venn diagrams I developed Blackboard assignments that involved students in reading about other ways to show the relationship between sets such as Carroll diagrams and the 11-set Venn diagram done by Khalegh Mamakani and Frank Ruskey at the University of Victoria in British Columbia, Canada (Aron, 2012).

This was the first time that I used cartoons in teaching and learning. In the future, I would like to make them a more integral part of learning and include more authentic real-world problems in the cartoons. This should help give more positive results with respect to motivation and interest similar to Cho's (2012) findings. It would also be useful to get more students' thoughts on how they feel about the cognitive demand of handling both the mathematics and creating a cartoon, which Cho, Osborne, and Sanders (2015) label as a "complex narrative genre".

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