Effects of the enriched history of Mathematics course on prospective Mathematics teachers

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Abstract

This study aimed to determine how the primary school mathematics teacher candidates' subject matter knowledge about the history of mathematics and their attitudes and beliefs about the use of the history of mathematics in mathematics education changed after the history of mathematics lesson and to explain to what extent their subject matter knowledge about the history of mathematics predicted their attitudes and beliefs towards the use of the history of mathematics in mathematics education. Furthermore, it is aimed to better explain quantitative data by referring to the opinions of the teacher candidates. In this respect, the research was designed according to the exploratory sequential mixed methods in which quantitative and qualitative methods were used together. In the quantitative part, pretest/posttest experimental design without a control group was used, while in the qualitative part, teachers' opinions were considered. A total of 40 primary mathematics teacher candidates participated in the study. As a data collection tool, the knowledge test of history of mathematics (KTHM), Attitudes and Beliefs towards the Use of History of Mathematics in Mathematics Education (ABHME) Questionnaire, and interview form were used. Descriptive analysis, paired samples t test, correlation and regression analysis were used for the analysis of quantitative data. Also, In the analysis of qualitative data, content analysis was employed. After the experimental procedure, KTHM and ABHME scores of teacher candidates were significantly increased. However, it was concluded that the candidates' KTHM scores significantly predicted their ABHME scores.

1. Introduction

It is known that using the History of Mathematics (HoM) in the mathematics courses, or teaching it as a separate course will have positive effects on students, teachers and teacher candidates (Gulikers and Blom 2001; Tzanakis et al. 2002; Michalowicz et al. 2002; Panasuk and Horton 2012; Marshall 2000; Lit, Siu and Wong 2001; Jankvist 2009). It has been reasoned by various researchers why the HoM should be used based on these

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influences. These causes, which can be cognitive and affective (Arcavi and Isoda 2007; Calinger 1996), can be exemplified as follows; Increasing students' motivation to learn mathematics, helping teachers to identify where students will have difficulties by knowing the obstacles encountered in the developments in mathematics (Fauvel 1991), making math more interesting and understandable (Fried 2001), improving teachers ' didactic repertoire, increasing students' and teachers' interest and attitudes towards mathematics (Gulikers and Blom 2001; Tzanakis et al. 2002; Ho 2008), teaching students the worth of mathematics and understanding the reasons behind the development of algorithms and concepts (Barbin 2000). These reasons, which can be seen as common for students and teacher candidates, differ during the implementation phase. The teacher candidates take the HoM as a separate course, while students can learn the HoM through private lessons or activities integrated into the mathematics courses (Alpaslan, Işıksal and Haser 2014).

Students' transition from concrete arithmetic thinking to abstract algebraic thinking occurs at the end of middle school, and this can pose difficulties in learning for them (Filloy and Rajono 1989). It is important that the HoM provides multiple perspectives to middle school students to overcome these challenges (Hiebert and Grouws 2007). Furthermore, teaching mathematics from a historical perspective enables great strides to be made towards further problem solving and reasoning, two basic life skills of mathematics (Carter 2006). However, students can turn their fear and hatred towards mathematics into a positive direction by applying the HoM (Fried 2001). Nevertheless, for these changes to be observed on students, the HoM must be effectively included in the mathematics courses. To achieve this, teachers must have sufficient knowledge of the HoM. As a matter of fact, as shown in Figure 1, the HoM can change teacher's perceptions and understanding of mathematics, so it will shape the teaching method and thus affect students' perceptions and understanding of mathematics (Barbin, 2000).
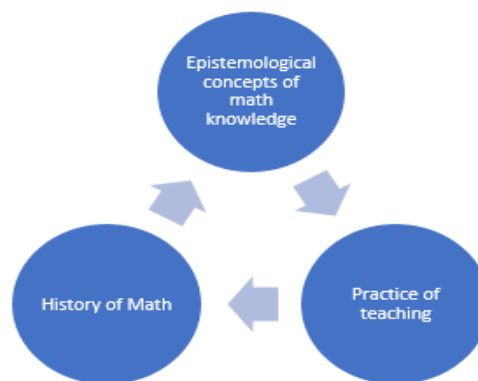

Figure 1. Linking history of mathematics to epistemology and practice (Barbin, 1996)

For teachers to effectively use the HoM in mathematics courses, they must first master the basic historical knowledge behind the concepts of mathematics taught in middle

school. However, having sufficient knowledge alone may not encourage teachers to use the HoM in mathematics courses (Ajzen 2001; Thompson 1992). Apart from the knowledge and teaching of the subject, scholars agree that teachers' beliefs in mathematics and attitudes towards teaching and learning mathematics will influence their approach to teaching (Phillip 2007). It is therefore important that teachers should have positive attitudes and beliefs towards the HoM (Alpaslan et al. 2014). This is because the emphasis and enthusiasm shown by a teacher in teaching the HoM can indirectly affect the attitudes and beliefs of the students towards mathematics (Horton 2011). Four years of university education is seen as a very good opportunity for teachers to have positive attitudes and beliefs about the HoM. Indeed, it is thought that the HoM courses that teachers take during their undergraduate studies can be very effective in gaining these qualifications. So, this study aims to investigate the direction and level of change in attitudes and beliefs of fourth-grade middle school mathematics teacher candidates towards the subject matter knowledge of the HoM and the use of the history of mathematics in mathematics education.

1.1. The theory of mathematical knowledge for teaching (MKT)

Discuss The question of what a teacher should know is an important issue that has been handled and discussed many times to educate good teachers on the field. Given that teachers need to know everything that students need to learn and more about it, what this "more" covers should be handled in a broad framework. By defining the concept of teacher knowledge, Shulman (1986) took the first step in explaining this issue. According to the framework he developed, he stated that teachers should have knowledge of pedagogy, subject matter knowledge, and pedagogical content knowledge (Shulman 1986). This framework, developed and elaborated later, has also been adapted to mathematics education by various researchers (Fennema and Franke 1992; An, Kulm and Wu 2004; Chick and Baker 2006; Ball, Thames and Phelps 2008). From there, Ball et al. (2008) developed a new framework called mathematical knowledge for teaching (MKT), which is unique to mathematics, by referencing Shulman's (1986) content knowledge and pedagogy. According to this framework given in Figure 2, MKT is divided into subject matter knowledge (SMK) and pedagogical content knowledge (PCK).
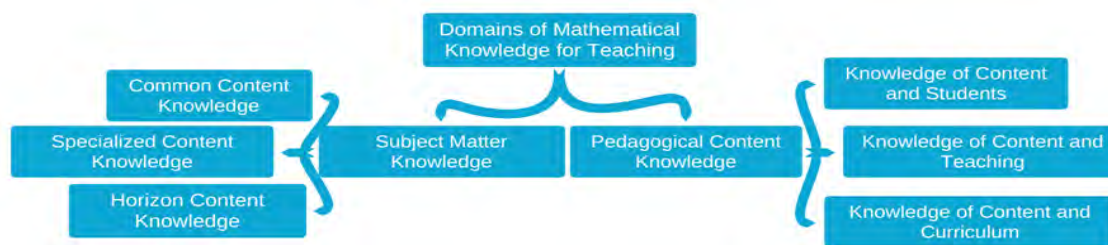

Figure 2. The Theoretical Framework of Mathematical Knowledge for Teaching of Ball et al. (2008)

Common Content Knowledge (CCK), one of the SMK (subject matter knowledge) domains of the MKT is defined as mathematical knowledge that is not only unique to mathematics teachers but can also be used in non-teaching environments and is generally known to everyone. The knowledge required for the summation or division of two integers can be given as an example for CCK. (Smestad 2015). Specialized Content Knowledge (SCK), another of the CCK elements, is the mathematical knowledge specific to teaching mathematics that only mathematics teachers should have. The SCK, which is difficult to think about and not needed to be learned by others in everyday life, enables teachers to have the ability to accurately represent mathematical ideas, provide mathematical explanations for rules and procedures, and to study and understand unusual methods for problems (Ball et al. 2008; Hill et al. 2008). It can be considered SCK for the math teacher to find out where the error is, rather than finding only that the answer is wrong in a calculation (Smestad 2015). Indeed, having this knowledge is a very rare need outside the teaching profession. Horizon Content Knowledge, the ultimate domain of CCK, contributes to teaching school math subjects by giving teachers an idea of how the taught mathematical content is settled and connected to the wider discipline area (Jakobsen, Thames, Ribeiro and Delaney 2012). Ball and Bass (2009) described Horizon Content Knowledge (HCK) as the broader mathematical environmental vision needed in teaching. However, Horizon Content Knowledge clearly includes ways and tools to know in mathematics, types of knowledge, and where mathematical ideas come from. It provides all the resources to connect students to a wide and highly developed area by enabling teachers to hear students, judge the importance of specific ideas and questions, and handle mathematics with honesty (Jakopsen et al. 2012). For HCK, some examples can be cited such as teachers should know that the figures we use today are of Indo-Arab origin, and should know the origins of words like algebra and algorithms because these form the background of the taught mathematics.

The second component of MKT, pedagogical content knowledge (PCK), consists of three parts. The first of these, Knowledge of Content and Students (KCS), includes the knowledge that teachers have about what students know, how they think and learn, misconceptions, and their interest in any mathematics subject (Mosvold, Jakobsen and Jankvist 2014). Finding a useful counterexample to the sentence "dividing a number by another number reduces the divided number" is KCS. Because the teacher cannot use any counterexample, he\she needs exactly the required sample for his\her students. To find out that, he\she needs to know his students well (Smestad 2015). Knowledge of Content and Teaching (KCT), the second element of PCK, includes the ability of teachers to design a teaching environment using math knowledge for math teaching, select appropriate examples to be used in the course, to evaluate the advantages and disadvantages of the representations to be used for teaching, and to select the appropriate method for teaching (Ball et al. 2008). The last component of PCK is Knowledge of Content and Curriculum (KCC). The KCC is defined as a special understanding of programs and materials (Mosvold et al. 2014), a complete set of

programs designed to teach subjects at a certain level (Shulman 1986), and a knowledge of which mathematical subjects students will encounter first in future years and which resources are available. Examples can be given to the KCC such as math teachers' knowledge about which grade level and at what difficulty level the triangular area calculation must be and which materials are recommended in the curriculum.

1.2. Mathematics History in the Context of the Theory of Mathematical Knowledge for Teaching (MKT)

Given the importance of teacher's knowledge about the HoM, various opinions have been put forward and studies have been made on which component of this information should be in the component of MKT (Smestad 2015; Huntley and Flores 2010; Jankvist, Clark and Mosvold 2019; Molvold et al. 2014; Smestad, Jankvist and Clark 2014). Some of these views are that teachers can better understand subject rankings in teaching programs by learning how mathematics has evolved in the past (Schubring et al. 2002), that teachers can enrich teaching strategies with their knowledge of the HoM, and that it can be used as a tool to reveal the nature of mathematical activities (Liu 2003; Tzanakis et al. 2002).

The history of mathematics can be used in accordance with each component within the framework of MKT. Given examples of these components from the HoM; the mathematics teacher's knowing "that Egyptians use more unit fractions (Smestad 2015) or that Pythagorean theorem actually existed before Pythagoras" can be considered HCK or SCK. That is why the teacher can use this information when explaining fractions to his students or explaining the Pythagorean theorem.

Figure 3. Ancient Egypt's multiplication method

   Having knowledge of the multiplication method of ancient Egypt shown in Figure 3 and understanding whether this method works is not normally part of the mathematics that everyone needs. But it is important information for teachers who need it to be able to give feedback to students using different methods. Therefore, it can be considered in the SCK category. However, if this method is part of the curriculum then it can be treated as CCK (Smestad 2015).

"Pythagoras, a Greek mathematician, lived between 570 and 495 BC. He made studies of the relationship between the lengths of the sides of the right triangle. As a result of his work, his outcomes have been included in mathematics as the Pythagorean relation" (from MoNE 6th grade math textbook), such biographical information is a pedagogical tool belonging to the KCC of HoM. However, when the teachers aim to develop the feeling in students that mathematics is a human activity and to develop students' epistemological perspectives by giving this information, this biographical knowledge is accepted HCK, can be considered CCK only if everyone needs to know this information (Smestad 2015).

Another example is the interesting questions presented by the correspondence between Pascal and Fermat. Figure 4 is a visualized version of one of these questions. In solving this problem, it is known how mathematicians interact using trial-and-error methods and provide counterexamples to develop their theory. This knowledge content can be considered as HCK.

In the 17th century, two French mathematicians Blaise Pascal and Pierre de Fermat were impressed by the probability issue. They wrote letters to each other about a question that they called "the scores problem".

Pascal and Fermat imagine playing the game throwing money into the air in a café...

Pascal will get a score if heads come. If the tails come, Fermat will receive a score. Players add four metal coins to one box each round. The first player gaining 4 points earns the money in the box.

... Fermat had to go before the game was finished. How should they share the coins in the box?

Figure 4. Example of HCK (Mersin, 2020)

However, the knowledge of solving quadratic equations using Khwarizmi's method of completing the square, mentioned in Figure 5, can be considered in the KCT category, as it would be SCK since it offers an alternative solution method.

What is the number x that sum of 10 times of itself (10x) and it's squared (x^2) is 39?

$$x^2+5x/2+5x/2+5x/2+5x/2+25/4+25/4+25/4+25/4= (x+5)^2$$

$$39+25= (x+5)^2$$

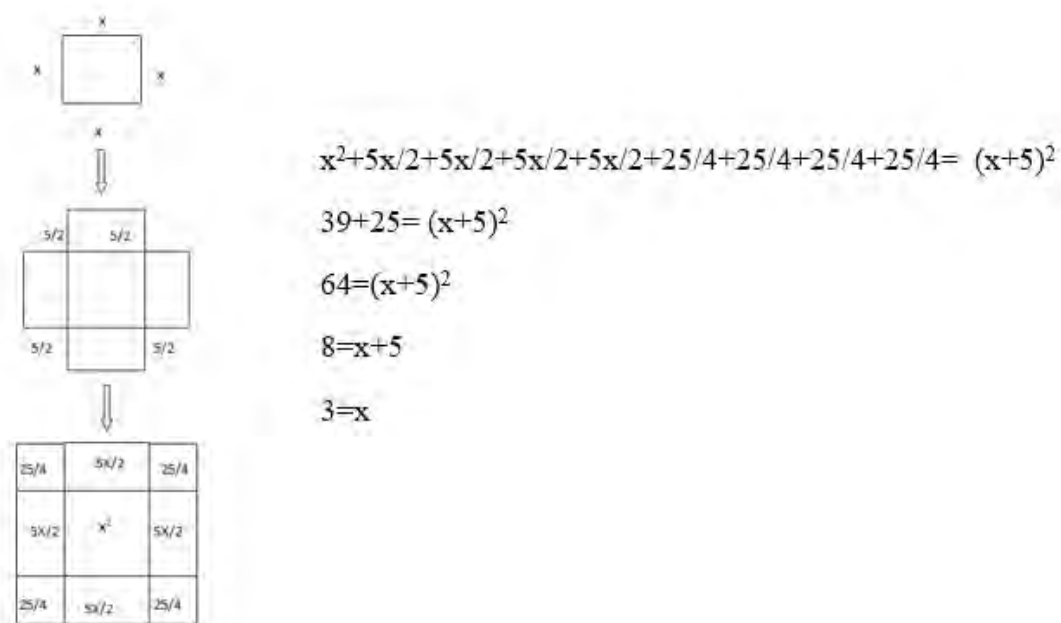$$64=(x+5)^2$$

$$8=x+5$$

$$3=x$$

Figure 5. (Mersin, 2019)

An example of HCK may be how the HoM is linked to different cultures in subjects such as units of measure, number systems, the history of algorithms, and geometric models. When the examples related to the HoM given within the scope of MKT are examined, it is seen that the same information can be under more than one category. This situation is thought to enable teachers and prospective teachers to use them under different objectives depending on their personal epistemology and mathematics teaching objectives.

In this study, it was attempted to improve the teacher candidates' knowledge of the HoM in all categories under MKT. However, among the types of knowledge listed under the subject matter knowledge (SMK) of MKT in the evaluation section, test items were prepared by concentrating mostly on HCK, and it was tried to determine to what extent the SMK of teacher candidates about mathematics history improved.

1.3. Attitudes and Beliefs towards the Use of History of Mathematics in Mathematics Education (MKT)

Attitude towards mathematics, which consists of positive or negative feelings towards mathematics and affects students' participation in mathematical activities (Ma and Kishor 1997), plays an active role in the learning and teaching of mathematics. Students' attitudes towards mathematics are influenced by many factors such as activities, used

materials, interesting subjects, the attitude of friends and teachers, knowledge of content (Duatepe-Paksu and Ubuz 2009; Yılmaz, Altun and Olkun 2010). With changes in some or all of these factors, students' attitudes towards mathematics can be expected to turn in a positive direction. It has been stated by many scholars that the use of the HoM in mathematics courses would be an application that could lead to this orientation. Nonetheless, to use the HoM in mathematics courses, teachers of mathematics should be a volunteer for it, therefore, it is necessary to have attitudes towards the use of the HoM in mathematics courses and to have high self-efficacy beliefs (Tschannen-Moran, Hoy and Hoy 1998) that express the degree to which teachers can influence students' learning. For this reason, it is thought that teacher training programs are a good option to increase teachers' beliefs and attitudes towards using the HoM in mathematics education and that the inclusion of the history of mathematics course among undergraduate program courses will contribute positively to this process. From here, it was aimed to investigate whether the prospective mathematics teachers' attitudes and beliefs in this direction changed or to what extent they changed.

1.4. Related Studies

1.4.1. Mathematics historical knowledge level

There are a limited number of studies in the literature which examine the historical knowledge levels of mathematics teacher candidates or teachers. Goodwin (2007) examined the relationship between high school math teachers' levels of knowledge in the HoM and their images in mathematics. A total of 193 teachers participated in the study. The teachers' knowledge of the HoM was determined by a test where mathematicians and their period were matched by giving explanations about the mathematicians, including who they were, and about the chronological ranking of the historical events and the date the events took place. About 67% of teachers answered half the test correctly. That's a pretty low rate.

In another study, Alpaslan (2011) developed an 11-point HoM test consisting of multiple-choice, short-answers, gap-filling, and right-wrong-answered questions that measured teacher candidates' knowledge levels and applied it to a total of 1593 teacher candidates. The average of the candidates in this test was 0.44, with correct and incorrect answers were scored as "1" and "0", respectively. This value is very low because it is below the middle.

In a similar study, Gazit (2013) applied a multiple-choice test consisting of 10 items related to the HoM to a group of 100 people including teachers and teacher candidates. Eight of the questions in the test are related to a concept used by Pythagoras, Descartes,

Euclid, Fermat, Fibonacci, Archimedes, and Al- Al-Khwarizmi, while 2 of them are related to the culture that contributes to mathematics (Arab culture, Egyptian culture). The test success of the participants was around 40%. Gazit evaluated his findings by dividing the results of the participants into three categories. The first group consists only of those who answer the fifth question. Also, 83% of the candidates correctly answered the question about who wrote the book based on plane geometry. The second group consists of questions (1-4 and 8) about the origin of the system of numbers, the period in which Pythagoras lived, the origin of the name of the system of Cartesian axes, also, who first used the simple fractions, and the source of the Fibonacci sequence. The success of participants in these questions ranged from 40% to 44%. The questions in the last group are (6, 7, 9, 10) questions that ask the first mathematician to calculate the number of pi, the origin of the "algorithm" word, and the first mathematician to investigate the properties of natural numbers. Respondents' correct answers to these questions ranged from 22% to 28%. This value shows that there is a general lack of knowledge.

In a different study in which the knowledge level of HoM was determined, Bütüner (2017) studied with 90 mathematics teachers and used a mathematical history knowledge test consisting of 11 items as a measurement tool. Six of the questions in the test are related to mathematicians and civilizations, while the others are related to the starting point of the Fibonacci sequence of numbers, sub-branches of mathematics, women mathematicians, and various mathematicians who prove the Pythagorean theorem. In the scoring of the test, the correct answers were rated "1" and the wrong answers were rated "0". He assessed teachers' test results by classifying them according to whether they used the HoM in their teaching. According to this, the test average of the teachers who used the HoM in mathematics courses was 5.33, while the average of those who did not use was 1.88, and the overall average was 3.5. As a result, teachers' level of knowledge of the HoM was found to be low.

In general, it is seen that teachers and prospective teachers have a low level of HoM.

1.4.2. Attitudes and beliefs towards the use of HoM in mathematics education

When we look at the literature, a limited number of studies examining the attitudes and beliefs of prospective teachers towards using the HoM in mathematics education have been found. Sullivan (2000), as a result of his experimental work with the prospective teachers on the use of the HoM in mathematics teaching, observed a significant increase in the attitudes of the prospective teachers in the experimental group towards using the HoM. Gönülateş (2004) found an increase in the attitudes of mathematics teachers towards using the HoM in mathematics courses after a semi-experimental process carried out with the historical activities of mathematics, but this increase was not significant. Gürsoy (2010), after a study conducted by using simple experimental method through mathematical history activities with mathematics teacher

candidates, concluded that the attitudes of the teachers towards the use of HoM in mathematics courses increased significantly. Alpaslan (2011) reported that teacher candidates' attitudes towards using the HoM in mathematics education increased as the grade level increased. But this increase is not meaningful.

As can be seen, there are a limited number of studies in the literature that examine the attitudes of mathematics teachers to use the HoM in mathematics education. The results of these studies are not consistent with each other. It is therefore important to conduct further research in this field and to share the results with teachers and mathematics educators and to make necessary arrangements for increasing attitudes and beliefs about the HoM among prospective teachers and to encourage its use in mathematics education.

1.5. Problems of the Research

The overall problem of the study is "what are the effects of the enriched mathematics history course on math teacher candidates' subject matter knowledge in the HoM and on their attitudes and beliefs towards the use of mathematics history in mathematics education?"

Sub-problems are as follows;

1. What are the scores of primary mathematics teacher candidates for the pretest and posttest of KTHM and ABHME?
2. Is there a significant difference between KTHM pretest and posttest scores of primary mathematics teacher candidates?
3. Is there a significant difference between the ABHME pre-test and post-test scores of primary mathematics teacher candidates?
4. Is there a significant relationship between the posttest scores of KTHM and ABHME of primary mathematics teacher candidates?
5. Are the KTHM posttest scores of primary mathematics teacher candidates a significant predictor of ABHME posttest scores?
6. What are the views of primary mathematics teacher candidates for enriched history of mathematics?

1.6. Importance of the Study

There is a limited number of studies in the literature examining the knowledge levels of teacher candidates on the HoM and their attitudes and beliefs for the use of mathematics history in mathematics education. As a study in which both are examined together, only Alpaslan's (2011) thesis study is available. In this research, survey studies were carried out to determine the current math history knowledge levels of teacher

candidates and their attitudes and beliefs for the use of mathematics history in mathematics education. We encountered no experimental study in the literature that examined the extent of the change in KTHM and ABHME scores of teacher candidates after an experimental procedure. In this respect, we think that the results of the research will contribute to the field. However, based on the results of the research, information about the effectiveness of the HoM in undergraduate programs will be gained.

2. Method

This section covers the model of the research, participants, data collection tools, application process, and analysis of the data.

2.1. Model of Research

Exploratory sequential mixed methods were used in this study. In this design, the collection and analysis of quantitative data were done before the qualitative data (Creswell and Plano Clark, 2017). In this design, qualitative data is used to explain the relationships within quantitative data. In the quantitative part of the research, pre-test and post-test experimental design without a control group was used to examine the change of teacher candidates' subject matter knowledge levels and attitudes and beliefs for the use of mathematics history in mathematics education. In the qualitative part of the research, teacher candidates were asked a qualitative question about the HoM course process.
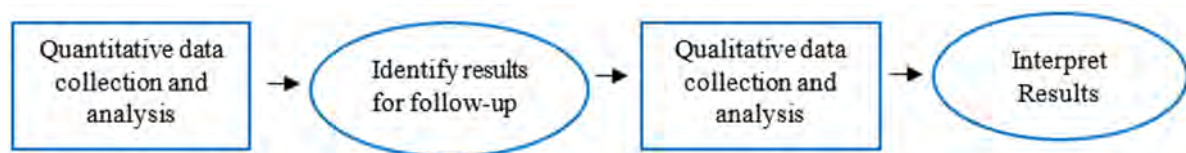

Figure 6. Research implementation stages

2.2. Participants

The participants of the study consisted of 40 primary mathematics teacher candidates, 30 of whom were female, and 10 of whom were male, who attended the fourth grade of a State University in the Black Sea region in the 2018-2019 academic year. Criterion sampling method was used in the selection of participants. The criterion was that students would take the HoM course. In addition, the participant group of the study took Private Teaching Methods 1 and Private Teaching Methods 2 and History of Science in the third grade. While pre-service teachers learn subjects such as learning and teaching

strategies, teaching activities through special teaching methods lesson, they become able to apply the methods, techniques, tools and materials used in mathematics education. In the history of science course, they learn subjects such as the nature of science, its characteristics, the main scientists and their features, and the contribution of different cultures to science.

2.3. Data Collection Tools

In the quantitative part of this research, the Knowledge Test of History of Mathematics (KTHM) and Attitudes and Beliefs toward the Use of History of Mathematics in Mathematics Education (ABHME) Questionnaire were used as tools of measurement. The knowledge test of the history of mathematics was developed by the researcher. ABHME is a previously developed scale that exists in the literature (Alpaslan 2011).

2.3.1. Knowledge test of history of mathematics (KTHM)

The research used a knowledge test of history of mathematics consisting of 30 questions. The items in the test are from various studies in the literature and consist of questions prepared by the researcher himself. Studies of Alpaslan (2011), Gazit (2013), and Bell (1992) were used in preparing the test. During the development process of the test, an item pool of 40 questions was created. In line with the views of the two experts, 5 questions were removed. The draft test was applied to a total of 120 students in the first, second, and third grades of the elementary mathematics teaching department. After the item analysis in the obtained data, another 5 questions with low distinctiveness power were removed. The remaining questions were prepared for the actual implementation process. The items included in the test are in the manner of multiple-choice, gap-filling, and short-answered. The correct answers were given a score of 1, the wrong and blank answers a score of 0. The maximum score that can be taken from the test is 30. The index of item difficulty and distinctiveness of the questions in the test are given below (See Table 1).

Table 1. Item Analysis Results For KTHM

Question Number	p	R	Question Number	P	R
1	.61	.54	16	.36	.38
2	.70	.69	17	.29	.73
3	.68	.60	18	.32	.47
4	.79	.48	19	.56	.60
5	.57	.73	20	.61	.57
6	.72	.63	21	.24	.44
7	.24	.44	22	.25	.44
8	.63	.44	23	.53	.82
9	.16	.38	24	.25	.38
10	.50	.85	25	.16	.34
11	.85	.38	26	.64	.73
12	.33	.73	27	.35	.51
13	.57	.50	28	.80	.57
14	.40	.44	29	.46	.44
15	.21	.34	30	.14	.31

p: Item difficulty index

r: Index of discrimination

Three of the items in the test are very difficult, 8 are quite difficult, 13 are moderate, 4 are easy, 2 are very easy. The average of item difficulties was calculated as 0.46. This shows that the test has an average difficulty level. When we look at the discrimination index of the items, it was determined that 7 items were very good and 23 items were highly discriminant. The average discrimination level of the test was calculated as 0.53. This value indicates that the test is distinctive at a good level. The internal reliability value of the KTHM was calculated with KR20. According to this, the KR20 value of the test was found to be 0.87. This value indicates that the test is fairly reliable. The following table shows what content the items in KTHM are in and which resources are used.

Table 2. Information on Questions in Math History Knowledge Test

No	Subject	Source	No	Subject	Source
1	Number system in Ancient Egypt Civilization	Alpaslan (2011)	16	Famous mathematicians (Archimedes)	Bell (1992)
2	Number system in Mayan Civilization	Alpaslan (2011)	17	Origin of numbers	Gazit (2013)
3	Number system in Babylonian Civilization	Alpaslan (2011)	18	Coordinate system	Gazit (2013)
4	Number system in Roman Civilization	Alpaslan (2011)	19	Fractions	Gazit (2013)
5	Famous mathematicians (Pascal)	Alpaslan (2011)	20	Plane geometry	Gazit (2013)
6	Famous mathematicians (Fibonacci)	Alpaslan (2011)	21	Natural numbers	Gazit (2013)
7	Ancient civilizations (Babylonian)	Bell (1992)	22	Possibility	Gazit (2013)
8	Decimal system	Bell (1992)	23	Fibonacci number sequence	Gazit (2013)
9	Pythagorean theorem	Bell (1992)	24	π number	Gazit (2013)
10	Number theory	Bell (1992)	25	Word origin	Gazit (2013)
11	Figurate Numbers	Bell (1992)	26	Thales theorem	Researcher
12	Discovery of zero	Bell (1992)	27	Analytical geometry	Researcher

As stated in Table 2, the knowledge test of mathematics history includes many topics such as ancient number systems, natural numbers, fractions, the first use of zero number, decimal system, numbers theory, figural numbers, analysis, probability, number sequences, number of pi, polygons, analytic geometry, and famous mathematicians. Some of the questions in the tests were used without any changes, and some with minor changes.

2.3.2. Attitudes and beliefs towards the use of history of mathematics in mathematics education (ABHME) questionnaire

The scale, consisting of 35 items, was developed by Alpaslan (2011). The attitudes and beliefs scale consists of three factors. These are positive attitudes and beliefs about the use of mathematics history in mathematics education, negative attitudes and beliefs about the use of mathematics history in mathematics education, and self-efficacy beliefs about the use of mathematics history in mathematics education. The reliability

coefficient of the scale was found as 0.90. The scale was applied primarily to 120 teacher candidates along with the knowledge test of mathematics history.

2.3.3. Semi-structured interview form

After the post-test applications, teacher candidates were given an interview form. In this form, they were asked what their views were on the mathematics course process from the pre-test to the post-test application.

2.4. Application of Data Collection Tools and Data Processing

Before beginning the HoM course, the candidates were given KTHM and ABHME and asked to fill them out. After the pre-test application, 28 hours of mathematics history were continued for 14 weeks. David M Burton's "introduction to the history of Mathematics" was used as a material throughout the course. However, Ancient number systems, Ancient Egyptian mathematics, Babylonian mathematics, Ancient Greek mathematics, Chinese remainder theorem, Islamic mathematics, and mathematicians, the emergence of modern mathematics were taught as part of the lesson. In addition to the content of the course, activities to use the HoM in secondary school mathematics courses were studied. These are;

1. The activities of converting numbers written with Egyptian, Maya, Babylonian, Roman, Greek number systems used in antiquity into today's number system;
2. Solving problems with ancient Egyptian multiplication and division methods
3. Problem-solving with lattice, line, and Russian multiplication methods
4. Estimating the area of the circle with the ancient Egyptian method and calculation applications
5. Activities of Pythagorean number triads creation by the Babylonian method
6. Activities related to finding general terms of figural numbers (triangular, quadrilateral, etc.)
7. Activities on the proofs of Pythagorean theorem made by different civilizations
8. Reviews on Euclid's book of elements
9. Activities on finding gcd by Euclid algorithm
10. The activity of finding prime numbers with the Sieve of Eratosthenes
11. Activities of finding the number of pi using Archimedes' method of calculating the number of pi
12. Activities for writing equations using algebraic representations in the book Arithmetica

13. Activities on solving quadratic equations using Al-Khwarizmi's Completing the Square Method

14. Activities using the Chinese Remainder Theorem

15. Activities on the study of the Fibonacci sequence

16. Activities related to special numbers such as amicable numbers, perfect numbers.

Various computer applications were also utilized during these activities. Discussions on how the HoM can be used in mathematics education were done. In addition, visual materials were used in the course processing, and documentaries were shown describing the lives of mathematicians and the development process of mathematics.

2.5. Analysis of Data

In the analysis of quantitative data, the first of all, the descriptive statistics were made. A formula developed by Alamolhodaei (1996) was used to determine what levels teacher candidates were at as a result of pre- and post-test. Accordingly, the scores of teacher candidates were evaluated at a high level if the score was greater than the summation of the average of the test and one-quarter of the standard deviation, and were evaluated as low if the score was less than the difference between the average of the test and one-quarter of the standard deviation, and moderate if the score was between the two (Karaçam and Ateş, 2010; Sarı, Altıparmak, and Ateş, 2013). According to this, the score of teacher candidates was evaluated as low if it was less than 12.36, moderate if it was between 12.36 and 15.52, and high if it was greater than 15.52. Nevertheless, the average score of teacher candidates on the ABHME was low if it was below 3.84, moderate if it was between 3.84 and 4.08, and high if it was greater than 4.08. After these, normality tests were done, and because the data was shown to provide normal distribution, related sample t-test, simple linear correlation analysis, and regression analysis were performed for the associated samples. Content analysis was used in the analysis of qualitative data.

2.6. Validity and Reliability

There are many factors in quantitative research that affects validity and reliability. One is the pre-test effect that affects internal validity, which is thought to also affect the post-test because participants will gain familiarity with the preliminary test. However, it is thought that having a period of 14 weeks between the pre- and post-tests keeps this effect to a minimum.

There was no loss of participants during the experiment. Nevertheless, the participants' interaction with each other during the pre- and post-test answering process was kept to a minimum with the participation of both researchers in the data collection process. Also,

participants were not felt in the experimental process. In this sense, it can be said that the study is valid. For its reliability, it was given importance to ensure that data collection tools were reliable. The analysis of data is described in detail.

For the qualitative dimension, credibility for the validity of the research is seen as important. In this respect, the question of the interview was clearly stated and the findings contained direct quotes of opinions. To ensure internal reliability, the findings were written directly without any comment, and quotes from the answers of the teacher candidates were given. However, the data was encoded by two experts and according to Miles and Huberman's (1994) formula, the consistency between encoders was found 0.91.

3. Results

In this section, primary mathematics teacher candidates' knowledge test of math history and findings regarding their level at the pre- and post-implementation of the Attitudes and Beliefs towards the Use of History of Mathematics in Mathematics Education (ABHME) Questionnaire are included, also covers the change in the level of subject matter knowledge and scale scores of the prospective teachers on the subject of the HoM after the pre- and post-test and findings on whether there is a significant relationship between the post-tests and their views on the HoM course.

3.1. Findings of the First Sub-Problem

In the findings of the first research problem, descriptive statistics for the scores from KTHM and ABHME and the level of teacher candidates at pre- and post-test are given.

3.1.1. Descriptive statistics for KTHM and ABHME

Table 3. Correct Answer Rates for Questions In KTHM

Item No	Prestest %	Posttest %	Item No	Prestest %	Posttest %
1.Number system in Ancient Egypt Civilization	47.5	92.5	16.Famous mathematicians (Archimedes)	40	47.5
2.Number system in Mayan Civilization	50	100	17.Origin of numbers	12.5	65
3.Number system in Babylonian Civilization	52.5	95	18.Coordinate system	27.5	45
4.Number system in Roman Civilization	60	95	19.Fractions	57.5	80
5.Famous mathematicians (Pascal)	45	90	20.Plane geometry	47.5	70
6.Famous mathematicians (Fibonacci)	65	100	21.Natural numbers	20	35
7.Ancient civilizations (Babylonian)	22.5	40	22.Possibility	10	47.5
8.Decimal system	62.5	75	23.Fibonacci number sequence	37.5	92.5
9.Pythagorean theorem	12.5	30	24.π number	22.5	35
10.Number theory	30	97.5	25.Word origin	7.5	30
11.Figurate Numbers	80	95	26.Thales theorem	52.5	92.5
12.Discovery of zero	10	77.5	27.Analytical geometry	17.5	55
13.Euclid's Book of Elements	42.5	82.5	28.Polygons	70	95
14.Euclid's Book of Elements	32.5	47.5	29.Calculus	47.5	60
15.Archimedes' discoveries	15	27.5	30.Female mathematician	15	17.5

The correct answer rates of the items in the knowledge test of the history of mathematics for the pre-test and post-test are given in Table 3. Accordingly, it is observed that the ratio of teacher candidates to correctly answer the items in the test increases in the post-test compared to the pre-test. In the preliminary test, it was

determined that teacher candidates had higher rates of answering the items related to figural numbers, concave polygons, decimal system, Fibonacci sequence, and the number system of Roman civilization. In the post-test, the items related to the number system of Maya civilization and Fibonacci were answered correctly by all teacher candidates. Nevertheless, it is seen that the number system in ancient Egyptian civilization, the number system in Babylonian Civilization, the number system in ancient Roman civilization, Pascal, the theory of numbers, the figural numbers, the Fibonacci sequence, Thales theorem, and concave polygons were mostly responded correctly.

Table 4. Descriptive Statistics For KTHM

KTHM	$\bar{X}$	N	SD	Skewness	Kurtosis
Pretest	11.12	40	4.02	-.094	.730
Posttest	20.12	40	2.89	-.960	2.620

According to Table 4, the average of the scores of teacher candidates from the pre-test of KTHM was found to be 11,12. Considering that a maximum of 30 points can be obtained from the total of the test, it can be said that less than half of the test was answered correctly. In the post-test, the average was increased to 20.12, so the knowledge level of the teacher candidates increased substantially after the HoM lessons. Also, the skewness and kurtosis coefficients of the pre- and post-tests are in the range of -3 and +3, which indicates that the data is normally distributed (Groeneveld and Meeden 1984; De Carlo 1997).

Table 5. Descriptive statistics for ABHME

ABHME	$\bar{X}$	N	SD	Skewness	Kurtosis
Pretest	3.88	40	.24	-.639	.227
Posttest	4.34	40	.52	.145	-.014

According to Table 5, the average score of teacher candidates from the ABHME pre-test was 3.88. In the post-test, the average increased to 4.34. From here, it is seen that the HoM course increases the ABHME scores of teacher candidates. Also, the skewness and kurtosis coefficients of the pre- and post-tests are in the range of -3 and +3, which shows that the data is normally distributed.

3.1.2. KTHM and ABHME levels according to the pre-test scores of teacher candidates

As a result of the pre-test, the average score of the candidates who were high in KTHM was 17.75, the average of those who were moderate in KTHM was 13.83 and the average of those who were low in KTHM was 8.66. The average score of the candidates with the highest, moderate, and low levels in ABHME was 4.41, 3.91, and 3.18, respectively.
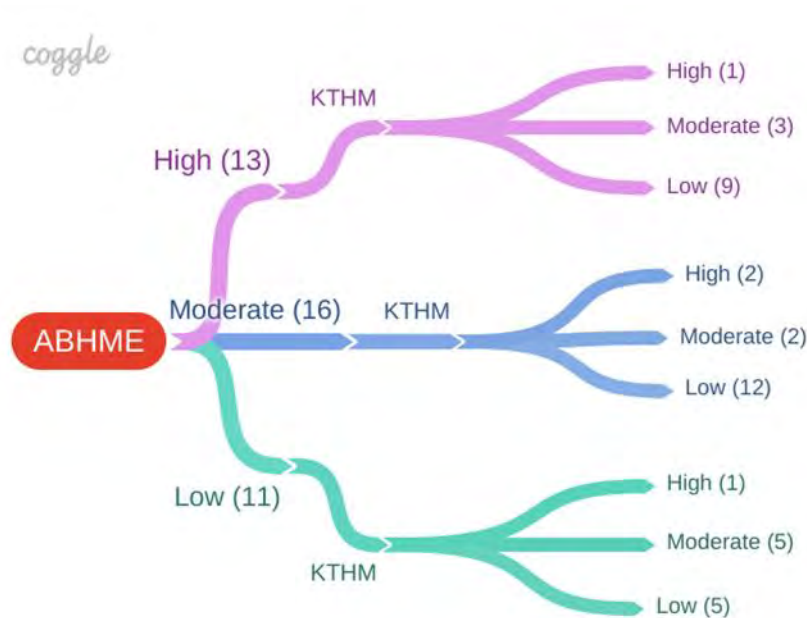

Figure 7. The Diagram on The Relationship Between KTHM And ABHME Levels Of Teacher Candidates According to the Pre-Test Results

As stated in Figure 7, as a result of the pre-test, it was determined that 13 of the prospective teachers had high, 16 had moderate and 11 had a low level of ABHME. Out of 13 candidates with a high level of attitude and belief, it was reached that 2 of them had a high, 3 of them had a moderate and 8 of them had a low knowledge level of mathematics history. However, of the 16 candidates with moderate ABHME, 1 has a high level, 4 have a moderate level, and 11 have a low-level knowledge of the history of mathematics. Of the 11 candidates with low ABHME, 1 was found to have high, 5 to moderate, and 5 to have low-level knowledge of the history of mathematics. As a result, in the knowledge test of history of mathematics, 4 candidates had high, 12 had moderate, and 24 had low knowledge levels of the history of mathematics.

3.1.3. KTHM and ABHME levels according to the scores of the post-test belonging to the teacher candidates

As a result of the post-tests, the average scores of teacher candidates with high-level, moderate level, and low level on the knowledge test of history of math were 20.52, 15, and 10, respectively. The average score of the candidates who were with a high-level and moderate level in ABHME was 4.39 and 3.93, respectively. There are no candidates with a low level of attitude.
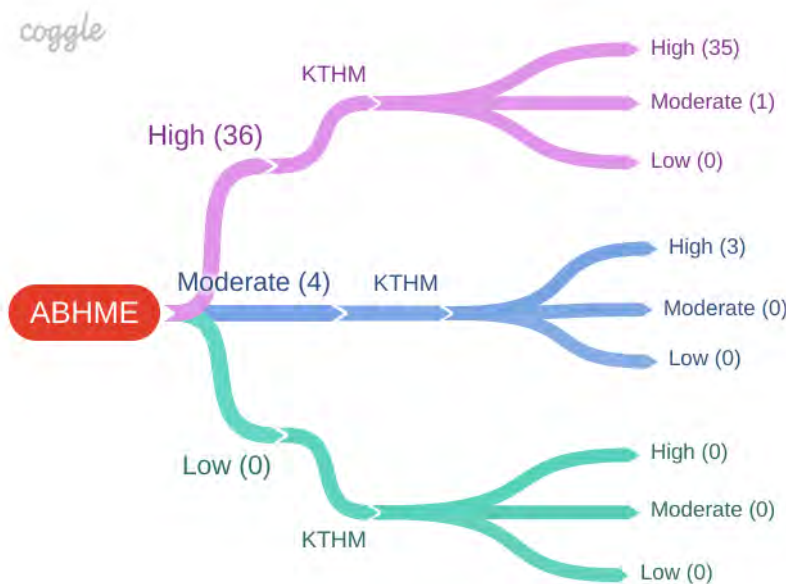

Figure 8. The diagram on the relationship between KTHM and ABHME levels of teacher candidates based on the post-test results

As shown in Figure 8, in the post-test, it was determined that 36 of the prospective teachers had high and 4 had moderate attitudes and beliefs about using the history of mathematics in mathematics education. There were no prospective teachers with low-order attitudes and beliefs. Of the 36 teacher candidates who have a high level of attitude and belief, 35 have a high level of knowledge of the history of mathematics, and 1 has a moderate level of knowledge. However, the result was that 3 of the 4 candidates

with moderate attitudes and beliefs had a high level and 1 had a low level of knowledge of the history of mathematics.

3.2. Findings of the Second Sub-Problem

Table 6. Paired Samples T-Test for the Difference Between KTHM's Pre- and Post-Test

KTHM	N	$\bar{X}$	SD	df	t	p
Pretest	40	11.12	4.02			
				39	-13.754	0.00
Posttest	40	20.12	2.89			

Paired samples t-test was performed to determine whether the knowledge of mathematics history of primary school mathematics teacher candidates underwent a significant change after the HoM lessons. According to Table 6, there was a significant difference between the pre- and post-tests of the teacher candidates (t(40)=-13.754, p<0.00). This difference is in favor of the post-test. Therefore, it can be said that the level of knowledge about the history of mathematics of the prospective teachers increased significantly.

3.3. Findings of the Second Sub-Problem

Table 7. Paired Samples T-Test for the Difference Between the Pre- and Post-Test of ABHME

ABHME	N	$\bar{X}$	SD	df	t	p
Pretest	40	3,87	.52			
				39	-7,509	0,00
Posttest	40	4,34	.23			

To determine whether there was a significant change in the scores of ABHME of teacher candidates, paired samples t-test was applied to the pre- and post-test. As shown in Table 7, there was a significant difference between the pre- and post-test scores of the teacher candidates. It can be said that the candidates' ABHME scores increased significantly after the history of mathematics course.

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3.4. Findings of the Fourth Sub-Problem

Table 8. Correlation Analysis Results for the Relationship Between KTHM and ABHME

		KTHM
ABHME	Correlation	.531
	Sig (2 tailed)	0.00
	N	40

Pearson correlation analysis was conducted to examine the relationship between KTHM and ABHME scores for the post-test. As a result, there was a significant relationship between knowledge and attitude scores as shown in Table 8 (r=0.531, p<0.05). This relationship is positive and at a moderate level.

3.5. Findings of the Fifth Sub-Problem

Table 9. Simple linear regression analysis regarding the post-test scores obtained from KTHM to predict ABHME scores

Dependent variable	Independent variable	B	Standard Error	Beta	t	p	R^2	
ABHME	KTHM	3.47	.229		15.180	.000		
		.043	.011	.531	3.860	.000	.282	$F_{(1-39)}=14.89$ $P=.000$

According to Table 9, the level of subject matter knowledge about the history of mathematics is a significant predictor of the attitudes and beliefs about the use of the history of mathematics in mathematics education ($F(1-39)=14.89$, $p<0.05$). The scores of ABHME account the scores of KTHM by 28% ($R=0.531$, $R2=0.28$). It can, therefore, be said that as the students' subject matter knowledge of mathematics increases, their attitudes and beliefs about using the history of mathematics in mathematics education increase. The regression equation is as follows:

ABHME score = (0.043 X KTHM score)+ 3.47

3.6. Findings of the Sixth Sub-Problem

In the qualitative section following the quantitative section of the study, the diagram showing the codes and categories of the answers to the question "what are your views on the process of the history of mathematics course from pre-test to post-test?", which was asked to the teacher candidates in the qualitative section following the quantitative section of the study, was given in Figure 9.
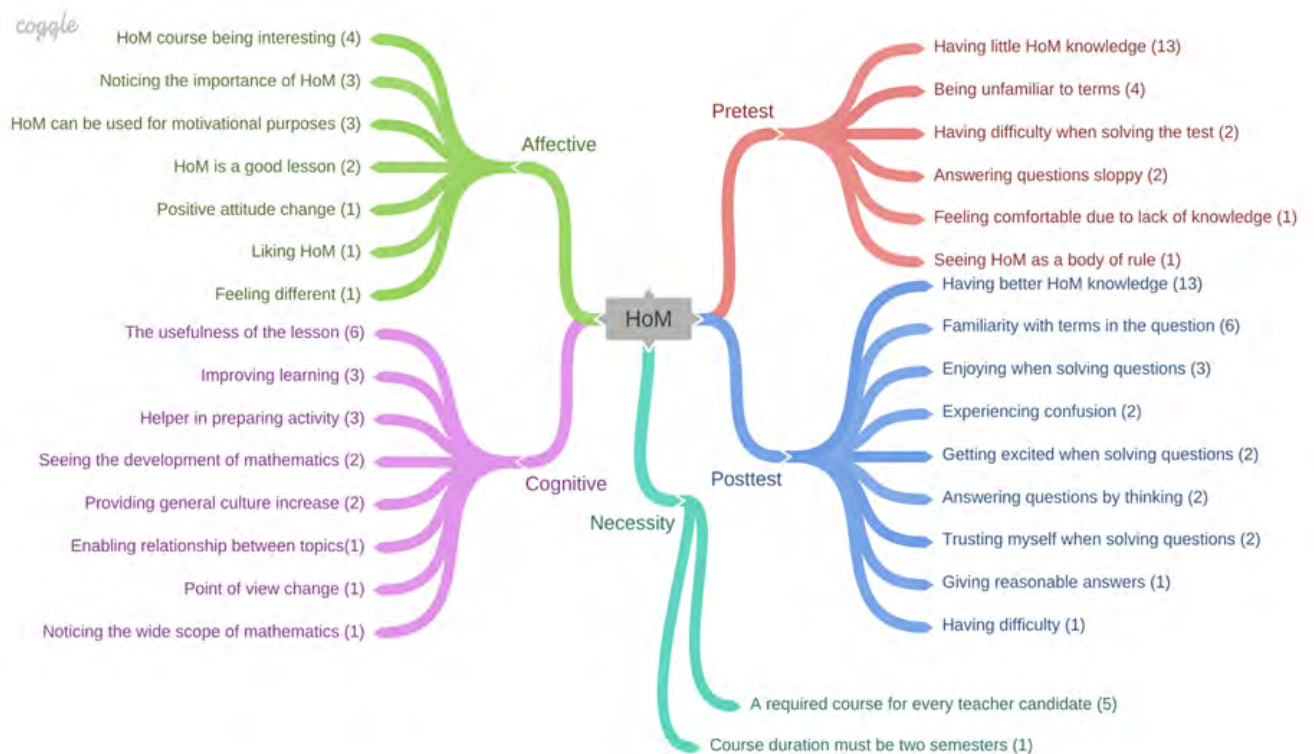

Figure 9. Diagram of The Views of The Teacher Candidates for The History of Mathematics Course

After the experimental procedure, the teacher candidates were given an open-ended opinion form consisting of one question to answer. With this form, primary mathematics teacher candidates were asked to evaluate the HoM course including the pre- and post-test. The answers of the teacher candidates were collected under 5 categories. These categories are pre-test, post-test, affective, cognitive, and necessity as shown in Figure 9. Under the pre-test category, teacher candidates' views on pre-test applications were collected. The stated opinions are "realizing that they have little knowledge when answering the preliminary test, being unfamiliar with the terms of the history of mathematics, having difficulty in answering the test, answering the questions sloppy, seeing HoM as a body of rules, and feeling comfortable when solving the test due to lack of knowledge". Under the post-test category, there are codes for what the teacher candidates think about the post-test. According to this, the teacher candidates said that they had more knowledge when solving the post-test, that the questions were more familiar to them, that they answered the questions by thinking, that they gave reasonable answers and that they trusted themselves more. However, two teacher candidates experienced confusion and one candidate stated that he had difficulty solving the test. It is generally understood that teacher candidates are more comfortable solving the post-test than the pre-test and have more knowledge. In this sense, it supports quantitative data. As a matter of fact, the pre-test scores of teacher candidates are lower than the post-test. The opinions of the candidates for the pre-test and post-test categories are given below.

PT2: I was very interested in the HoM as a subject. Although some subjects are very detailed, the knowledge I learned was beautiful in general. Comparing the beginning and the end of the lesson, I realized that my attitude towards the lesson had changed in a positive way.

PT6: When I first solved the test, I didn't have a lot of knowledge. So, I didn't even understand some of the questions. And I didn't feel anything, and I just predicted because I didn't know. But the last time I did, I had more information than the first, and I answered the questions by thinking, excited, I mean, happy to see what I knew.

PT38: When I solved this test again, I realized I had more information. I think that by taking advantage of what I learned in this course when I become a teacher, I can create different activities for my students to increase their motivation for mathematics.

Another category for the answers of the teacher candidates is named "affective". This category includes codes such as that the HoM is interesting, candidates recognize the importance of the HoM, HoM increases motivation and can be used to increase the motivation of their students, HoM is a good course, they love HoM, and it provides positive attitude change.

Under the cognitive category, there are codes for the HoM such as being useful, enabling better learning, helping in preparing activities, enabling to see the development

process of mathematics, increasing the overall culture, enabling to establish relations between subjects, and providing a change in perspectives. The most widely expressed view under this category is that the HoM course is beneficial. Examples of teacher candidates' views on the affective and cognitive category are given below.

PT12: I had very little knowledge of the history of mathematics. Thanks to the course we have taken, I intend to make historical activities of mathematics to provide the motivation of my students during my teaching years and to see the conveniences of today's mathematics.

PT20: I saw mathematics as a body of rules and realized that I had memorized it. After this course, it was very impressive to see how mathematics came about and how it was going through a process of development. I realized I didn't really know anything about math. I think it's a very useful lesson for me.

PT38: When I solved this test again I realized I had more knowledge. I think that by taking advantage of what I learned in this course when I become a teacher, I can create different activities for my students to increase their motivation for mathematics.

The final category regarding the opinions of the teacher candidates is the category of necessity. This category mentions that each mathematics teacher candidate should take the HoM course and the HoM course should be two semesters. Student views for this category are given below.

PT27: The history of Mathematics course is a very useful course to be taken. It's great to see how the area we've been dealing with has been explored. But there should be at least two terms in the 4-year education process.

PT33: The history of Mathematics course is a course that everyone who attends this school should take. It helped me a lot. Learning the point of origin of the subjects made it easier and better to understand the subjects.

4. Discussion

In this study, it is aimed to investigate the level of HoM subject matter knowledge (SMK) levels of primary mathematics teachers and their attitudes and beliefs about using HoM in mathematics education after the experimental process and aimed to investigate the opinions of prospective teachers about this process. First, KTHM was developed to determine the HoM subject matter knowledge (SMK) levels of the teacher candidates. In this context, the current mathematics history knowledge tests in the literature were investigated and a pool of items consisting of the questions in these tests and the questions prepared by the researchers were prepared. The 30-question KTHM was prepared for implementation after the regulations were carried out within the framework of expert opinions and reliability studies. In the test, correct answers were

scored as 1 and wrong answers as 0. Therefore, the maximum of 30 points can be obtained from KTHM.

KTHM and ABHME were applied to the candidates before they started the HoM courses which formed the experimental process. The pre-test KTHM average of the teacher candidates was calculated as 11.12. Similarly, the total correct answer rate of the test was 37%. Given that the maximum score taken from the test is 30, it can be said that the candidates' HoM content knowledge was initially below the moderate level. This result is similar to the results of Gazit's (2013) third-grade teacher candidates, Bütüner's (2017) math teachers who did not use HoM in their courses, Alpaslan's (2011) first and second-grade math teacher candidates, and Goodwin's (2007) teachers who did not take HoM course. The ratio of teachers' candidates to answer questions about which civilization first used the Zero, which civilization was aware of the Pythagorean theorem before Pythagoras, the origin of the word "algorithm" and the origin of the numbers we are currently using was very low. The post-test KTHM averages of the teacher candidates were calculated as 20.12 and the total correct answer rate was around 67%. This value was found to be quite high compared to the pre-test. This result was similar to the results of math teachers and candidates who took the HoM course of Goodwin (2007), Alpaslan (2011), Gazit (2013), and Bütüner (2017), but the KTHM average of teacher candidates was higher in this study. It was observed that all the teachers answered correctly the questions about Fibonacci and the number system used by the Maya civilization. It was determined that the rate of the correct answer to the question of the Pythagorean theorem, the question related to Archimedes' discoveries, and the question related to the female mathematician increased compared to the pre-test, however, it was still found to be at a lower level than other questions.

It was determined that the answer rate of the questions related to the origin of numbers, the theory of numbers, the first civilization to use zero, probability, Fibonacci sequence of numbers, the origin of the word algorithm, analytic geometry increased more than 3 times compared to the preliminary test, therefore, the candidates have more knowledge in these subjects. According to the scores obtained from the pre-test of KTHM, we concluded that 4 teacher candidates had a high level, 12 candidates had medium level and 24 candidates had low-level subject matter knowledge of the history of mathematics. After the post-test, we found that 38 teacher candidates had high, 1 teacher candidate had moderate, and 1 candidate had a low level of knowledge in the history of mathematics. After all, there was a significant difference between the pre-test and post-test KTHM scores of teacher candidates in favor of the post-test. Of course, after the history of mathematics course, the candidates are expected to increase their level of knowledge. However, while there were 4 high-level teacher candidates on the pre-test, this number rose to 38 in the post-test and nearly doubled the test average, which suggests that the implementation process is quite effective.

Another feature of mathematics teacher candidates which is intended to be developed, is their attitudes and beliefs about the use of the history of mathematics in mathematics education. Before the experimental procedure, ABHME was applied to the candidates, and the status of the candidates was described. According to this, the average score of the teacher candidates for ABHME was 3.88. According to the classification based on the formula of Alamolhodaei (1996), 13 of the candidates have a high, 16 have a moderate, and 11 have a low level of attitude and belief. As a result of the post-test, ABHME averages were calculated as 4.34, and 36 of them were at a high level, and 4 were at a moderate level. As a result of the post-test, we determined that there were no low-level candidates for ABHME scores and that many candidates with low and moderate levels in the pre-test had high attitudes and beliefs. There was a significant difference between the average score of the pre-test and the post-test of ABHME in favor of the post-test. Therefore, the 14-week HoM education process can be said to be effective in increasing teacher candidates' attitudes towards the use of HoM in mathematics education. This result is similar to the results of Sullivan (2000), Gürsoy (2010), and Alpaslan (2011).

The level of knowledge in HoM subject matter knowledge and ABHME score levels were compared. In this context, while there were 2 teacher candidates who had a high level of knowledge and attitudes and beliefs in the pre-test, we observed that this number increased to 35 in the post-test. After all, as a result of the pre-test, there were 5 teacher candidates with low HoM subject matter knowledge (SMK) and attitudes and beliefs, but in the post-test, these candidates shifted to a moderate or high level.

Between KTHM scores and ABHME scores of teacher candidates, a moderate positive correlation was found. This result is similar to the results of Alpaslan (2011). Furthermore, KTHM accounts for 28% of the candidates' ABHME scores. Therefore, it can be said that, as the HoM subject matter knowledge (SMK) of candidates increases, ABHME scores increase. However, since this ratio is not very high, we think that there are different variables other than subject matter knowledge (SMK) that accounts ABHME.

In accordance with the answers given in the qualitative study conducted with teacher candidates through the experimental procedure, the opinions for the pre- and post-test were collected under five categories: pre-test, post-test, affective, cognitive, and necessity. The teacher candidates emphasized that they had less HoM knowledge when solving the pre-test, and were unfamiliar with the terms in the test, and had difficulty solving the test. These views of the candidates are paralleled by quantitative research results. As a result of the pre-test results of the teacher candidates, HoM subject matter knowledge was found to be quite low. However, they stated that they liked and found HoM interesting under the affective category, that they realized the importance of this course and that they could use it to increase the motivation of their students in mathematics lessons. Therefore, it is in line with the post-test results for the use of HoM in lessons and explains these results. Furthermore, these results are similar to some research findings

in the literature (Yenilmez 2011; Tol, Çenberci and Yavuz 2016; Yıldız and Baki 2016; Genç and Karataş 2018; Başıbüyük and Şahin 2019; Bell 1992; Philippou and Christou 1998; Fraser and Koop 1978; Charalambous, Panaoura and Philippou 2009; Fadlelmula 2015).

Under cognition, which is another category, the candidates stated that they found HoM course useful, that it would help them to learn mathematics better, that it would help them to prepare activities, that it would increase the general knowledge and provide a change of perspective. These results can be used as a justification for candidates to benefit from HoM for increasing their knowledge of the HoM. Also, it shows similarities with the results of Gürsoy (2010), Yenilmez (2011), Tol et al., (2016), Yıldız and Baki (2016), Genç and Karataş (2018), Başıbüyük and Şahin (2019), Furinghetti (1997), Fadlelmula (2015), Guillemette (2017).

Under the post-test category, the candidates emphasized that they had more HoM subject matter knowledge when solving KTHM and were more familiar with the words in the questions and enjoyed doing the solution. These results support the KTHM test results of the post-test. Indeed, candidates' post-test KTHM scores rose significantly. Finally, their opinion that HoM is a course that every teacher candidate should take and that it would be better to have two semesters are included in the necessity category. These thoughts explain the belief that HoM should take part in mathematics courses. This result is similar to the findings of Yenilmez (2011).

5. Conclusions

In general, we observed that the HoM subject matter knowledge of the teacher candidates, which is low before the HoM lesson, can be raised to a higher level with HoM lesson. Also, it is revealed that the candidates of teachers can gain HoM subject matter knowledge during the teaching process up to a certain level so that they can use HoM in mathematics courses, but this is still not at the desired level. Although the attitudes and beliefs of the candidates towards the use of HoM were not very low before the HoM course, their knowledge of the subject area was determined to be low, however, significant increases occurred in both HoM subject matter knowledge and attitudes and beliefs after the HoM course. Also, the increase of the HoM subject matter knowledge had a certain effect on the increase of the attitudes and beliefs of the candidates regarding the use of HoM in mathematics education, therefore the importance of the HoM course requirement was understood. After all, for teacher candidates to use HoM more effectively in mathematics courses during the teaching process, in-service training should be conducted to increase their knowledge of HoM subject matter knowledge. Also, the attitudes and beliefs of mathematics teachers to use HoM in their lessons are predicted to influence not only their knowledge of the subject matter but also their pedagogical content knowledge. In this context, it is considered necessary to investigate the extent to

which HoM pedagogical content knowledge affects the attitudes and beliefs about using HoM in their courses and to add new content on how HoM can be used in mathematics courses in line with their results. This study was conducted with one group since there was no second branch. Further research to be carried out on the effectiveness of HoM courses can be done by creating a design with a control group. Also, by preparing questions for the different domains of teaching within the framework of mathematical knowledge theory, and by searching more thoroughly the HoM knowledge levels of candidates, it can be comparatively investigated what level of knowledge they are lacking or sufficient.

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