Research Report

A Learning Progression for Constructing and Interpreting Data Display

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Eun Mi Kim
Leslie Nabors Oláh
Stephanie Peters

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RESEARCH REPORT

A Learning Progression for Constructing and Interpreting Data Display

Eun Mi Kim, Leslie Nabors Oláh, & Stephanie Peters

Educational Testing Service, Princeton, NJ

K – 12 students are expected to acquire competence in data display as part of developing statistical literacy. To support research, assessment design, and instruction, we developed a hypothesized learning progression (LP) using existing empirical literature in the fields of mathematics and statistics education. The data display LP posits a progression of student understanding and learning of data display in terms of two progress variables: constructing data displays and interpreting data displays. An initial data display LP was revised through expert review and then supported the design of a set of data display tasks to elicit evidence of student knowledge and skills in constructing and interpreting data display. The data display LP and tasks presented in this research report can inform assessment development as well as classroom instruction, and they can be used for further studies on potential interactions between the two progress variables in students' development of data display knowledge.

Keywords Data display; learning progression; task design; statistics education; mathematics

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The visualization of information and data in graphical representations is prevalent in both scientific practices and everyday life situations (Åberg-Bengtsson, 2006; see also NGSS Lead States, 2013). Regarding statistical literacy, students are expected to develop competence in data display in mathematics and statistics education (e.g., see Common Core State Standards Initiative [CCSSI], 2010; Franklin et al., 2007). We conceptualize *statistical literacy* as the competency in reading and interpreting statistical data (presented in graphs, charts, and tables) to make decisions in solving different problems in everyday life situations; this conception is rooted in Wallman's (1993) definition of statistical literacy as "the ability to understand and critically evaluate statistical results that permeate our daily lives — coupled with the ability to appreciate the contributions that statistical thinking can make in public and private, professional and personal decisions" (p. 1).

Student competence in data display is one part of statistical literacy, and researchers in the fields of mathematics and statistics education have examined how students' learning and understanding of data display progress (e.g., Lehrer et al., 2014; Watson & Callingham, 2003; Wilson, n.d.). Following Smith et al.'s (2006) learning progression (LP) approach, we developed a hypothesized LP for data display (hereinafter referred to as the data display LP) using existing literature on student competence in data display to inform assessment development in this targeted content domain and to potentially support classroom instruction of data display at the K – 12 level.

According to Wilson (2009), "Devising means of measuring a student's location within or along a learning progression is a crucial step in advancing the scientific study of learning progressions, and for finding educationally useful applications of the idea" (p. 716). Rather than viewing student understanding as either "right" or "wrong," LPs portray student learning along a trajectory of increasingly sophisticated and often integrated ideas. In this view, early conceptions that have been seen as wrong, such as the tendency for students to add fractions by separately adding the numerators and the denominators together, are now seen as typical partial understandings on the way to developing a complete understanding of rational numbers. Continuing with the adding fractions example, it is important for teachers to learn that (a) this way of adding fractions makes sense given that students have been taught to treat numerals as integers (and not as part of a fraction) and (b) instruction to support students in learning how to add fractions will likely have to address the differences between integers and fractions (Petit et al., 2010). Similarly, LPs can be used to develop more nuanced assessment targets at different levels of an LP.

Corresponding author: E. M. Kim, E-mail: kimeun23@msu.edu

From this measurement perspective, we conceptualized two progress variables for the data display LP, constructing data displays and interpreting data displays, that describe "underlying assessment structures that one could build to undergird a learning progression" (Wilson, 2009, p. 729). Progress variables can also benefit the field of research in mathematics and statistics education by facilitating examination of levels of students' learning and understanding in constructing and interpreting data display as separate, but related, entities (cf. Wilson, n.d.).

Our conception of LPs is rooted in the following definition of *learning progression* used in the ETS *CBAL*® learning and assessment research initiative (Educational Testing Service, n.d.):

> a description of qualitative change in a student's level of sophistication for a key concept, process, strategy, practice or habit of mind. Change may occur due to a variety of factors, including maturation and instruction, and each progression is presumed to hold for most, but not all, students. As with all scientific research, the progressions are open to empirical verification and theoretical challenge. (Item 2)

This research report presents the data display LP and data display tasks designed to illustrate the potential use of the LP in assessment development. With this goal in mind, in this report, we first present our definition of data display to clarify our use of the term and the process of reviewing research literature to inform the data display LP. Next, we review core concepts in the domain of data display that undergird our conceptual framework for the data display LP. Then, we introduce the data display LP with its distinctive features. We also describe the revision process of the data display LP through expert review. Last, we present a set of data display tasks with their design goals. This sequence of reviewing existing literature, drafting and soliciting feedback on a hypothesized LP, and developing tasks to assess student performance along the LP are often the first steps in collecting validity evidence to support the use of an LP for assessment and instruction (see Graf & van Rijn, 2016).

Definition of Data Display

Lehrer et al. (2014) stated that "statistics measure characteristics of distributed data, and models of chance support inference about these statistics in light of variability inherent in chance events" (p. 35). In proposing a K–12 curriculum framework to support the development of statistical literacy, the American Statistical Association's the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report* (Franklin et al., 2007) promoted four activities that compose the process of statistical problem solving: (a) question formulation, (b) data collection, (c) data analysis, and (d) interpretation of results. For instance, in the process of investigating the nature of plant growth by comparing two plants of the same type, students need to formulate a statistical question such as "How does sunlight affect the growth of a plant?" (Franklin et al., 2007, p. 11). To answer this question, students need to collect data that acknowledge differences between two plants of the same type. During the process of data analysis, students construct statistical graphs in identifying the shape that distribution of data values takes (e.g., creating a dot plot to compare two sets of numerical data) and/or quantify statistical measures that describe the distribution of data values. Last, students make inferences or predictions about how sunlight affects the growth of a plant based on the interpretation of/about the shape of the data distribution displayed in the statistical graphs (e.g., symmetric or lopsided). During this process of statistical problem solving addressed in the *GAISE Report* (Franklin et al., 2007), data display is used for purposes of data analysis and interpretation of results. The data analysis component involves the selection and use of appropriate graphical representations of data for which the data are collected and the nature of given or collected data is known. The interpretation component includes the description, quantification, and comparison of data distributions in graphical representations of data.

Based on Lehrer and colleagues' (2014) view and the purposes of using data display in the process of statistical problem solving as addressed in the *GAISE Report* (Franklin et al., 2007), we conceptualize *data display* as the activities of constructing and interpreting graphical representations of data (i.e., statistical graphs) for the following purposes: visualizing and representing the shapes that distributions of data values take, interpreting statistical measures of data distributions, and making inferences or predictions about data in the process of statistical inquiry. In developing an LP for data display, we attend to students' growth in thinking and "*reasoning about shape of data*" (Bakker, 2004, p. 64) in statistical problem solving, related to central tendency or variability in data (both of which can be graphically represented in statistical graphs). With this focus on data display, we include a range of data display types in the data display LP (e.g., pictographs, dot plots, bar graphs, pie graphs, scatterplots, histograms, boxplots) and exclude tabular and other symbolic formats of displaying data in which the shapes of the distributions of data values are not graphically represented. This LP, therefore, incorporates findings from research on students' development of statistical reasoning and skills/strategies that are associated with construction and/or interpretation of statistical graphs.

Development of Learning Progressions through Literature Review

Starting from our definition of data display, we reviewed the literature in the fields of mathematics and statistics education by looking for "big ideas" (in the sense of Smith et al., 2006) in the domain of interest of data display. Our search of the literature was conducted as an iterative process. The research team identified an initial set of articles on student learning of statistics and selected those most closely tied to data display, including Lehrer et al. (2014), which was the most recently published work in the area. We also identified Wilson (n.d.) as a key paper because it was the most recent effort to create a trajectory of developing student understanding in data display. We used the reference lists from these two papers, as well as work produced by the same authors, to expand our collection of key articles with empirical research studies that address the development of K – 12 student learning and understandings of data display.

Through this process, we identified 20 key articles. Of these 20 articles, we categorized (a) 13 of them as high-priority articles covering empirical research on the development of K – 12 student learning and understanding of data display, (b) five of them as offering theoretical frameworks in developing an LP for data display, and (c) two of them as featuring overarching theories in developing a research-based LP in the domain of statistics education.

We then conducted qualitative analysis of these articles. Thirteen articles were uploaded into NVivo 11 as the basis for developing an initial set of codes. The first and third authors coded these research articles using two sets of codes: (a) one code set to analyze the developmental levels of students' learning and understanding of constructing and interpreting statistical graphs (e.g., prerequisite knowledge, key concepts, level distinction, empirical evidence, transition between levels, misconceptions) and (b) another code set to categorize information about research methods and task design (e.g., graph types featured, definitions of concepts, interview or assessment tasks). After calibration, these codes were applied to all 20 articles. After coding was completed, we queried the coded articles for level descriptions, corresponding representations, and other major codes. These excerpts were organized and summarized in an Excel spreadsheet to produce statements of student performance sorted into hypothesized levels. Once these levels were established, we added detail on the progress variables and statements describing transitions between levels as well as connections across the progress variables at each level. In writing descriptive statements at each level of the data display LP, we use the language from the original literature whenever appropriate.

Conceptual Framework: Core Concepts in Data Display

In building a conceptual structure for the data display LP, through the literature review, we identified five core ideas in data display: (a) context, (b) graphical components, (c) data reduction through data unit scaling, (d) proportional reasoning in comparing data sets, and (e) progression to an integrated view of data. Because these five ideas would need to be accounted for in the LP, we discuss each of them in the following pages.

Context

In distinguishing statistics from mathematics, Cobb and Moore (1997) stated that "statistics requires a different *kind* of thinking, because *data are not just numbers, they are numbers with a context*" (p. 801). Data analysts search for meaningful patterns, and the meaning and value of observed patterns are heavily dependent on their interaction with the applied context of data (Cobb & Moore, 1997, p. 803). In statistical inquiry, thus, students need to engage with the context of data (Pfannkuch, 2011; see also Cobb & Moore, 1997; Franklin et al., 2007).

From the standpoint of statistical problem solving, Pfannkuch (2011) defined the term *context* in terms of the context of the problem (i.e., data-context) and learners' prior statistical knowledge and the learning contexts in which they operate, such as physical and social contexts (i.e., learning-experience-contexts). Considering context as an important factor in the growth of informal statistical inferential reasoning (IIR), Pfannkuch conducted an exploratory study with two Web applications in a Grade 10 mathematics classroom in New Zealand. Using these applications, students plotted graphs of random samples obtained from a large, authentic database, and the teacher and students then discussed what they noticed about and found from the graphs (e.g., variation in the centers of box plots with differently sized random samples). Through the analysis of classroom dialog, Pfannkuch observed the role that context played in the development of learners' IIR. Thus the author suggested that "both data-context and learning-experience-contexts may need to be taken into account when developing learners' reasoning from data" (Pfannkuch, 2011, p. 43). Similar observations are reported in the literature on the relationships between data and context (e.g., Ben-Zvi & Aridor-Berger, 2016; Wilkerson & Laina, 2018).

The role of context in statistical inquiry and development of statistical reasoning appears to provide a grounding to consider why statistical graphs need to be situated in data contexts (see Friel et al., 2001). In designing graphs, Kosslyn (1989) emphasized that "the context and the semantic interpretation of the display must be compatible or comprehension of the display will be impaired" (p. 212). Through a review of the literature in graph comprehension, Friel et al. (2001) identified "contextual setting," or "semantic content of a graph (i.e., context)" (p. 134), such as titles of graphs or labels of axes, as one of three task characteristics that can elicit different levels of student competence in reading and interpreting statistical graphs.

Context is thus considered an essential underlying idea in the data display LP. In particular, the data-context is relevant to the constructing data display progress variable, including the selection and usage of appropriate statistical graphs and consideration of graphing goals (e.g., bar graphs for categorical data or line plots of measurement data, as stated in CCSSI, 2010; bar graphs for comparing specific, discrete data values or line graphs for looking for data trends and interaction, as stated in Kosslyn, 1985, 1989). The contextual setting is also relevant to interpreting the data display progress variable, regarding students' competence "to be aware of one's relationship to the context of the graph, with the goal of interpretation to make sense of what is presented by the data in the graph and avoid personalization of the data" (Friel et al., 2001, p. 146).

Graphical Components

Kosslyn (1989) stated that "displays are described as sets of components with specific relations among them" (p. 187). In considering the communication function of graph usage (Kosslyn, 1985) to deliver the information effectively, students need to see what graphical components are commonly used to form a statistical graph and how they are organized in the graph to represent the data accurately and efficiently. Thus students' knowledge and understanding of and about graphical components used for organizing a graph and relationships between and among the components (i.e., the structure of graphs) are essential for their construction and interpretation of statistical graphs (see Friel et al., 2001; Kosslyn, 1989).

Kosslyn (1989) categorized four general graphical components used for organizing a graph: (a) the graph *background*, (b) the graph *framework* (e.g., grid lines or axes), (c) a *specifier* that "conveys the particular information about the entities represented by the framework" (p. 188) of the graph (e.g., the bars of a bar graph), and (d) *labels*. These four basic graphical components can be considered when referring to graph design and purpose. Regarding Kosslyn's four graphical components, Friel et al. (2001) noted that each type of graph has "its own 'language' associated with" (p. 126) the graphical components, giving an example of the context of a line plot. When a line plot of the number of raisins in equally sized boxes is presented to consider the question, "Do all the boxes of raisins have the same number of raisins?" (Friel et al., 2001, p. 127), a graph reader might answer, "No, because if they did, all the Xs would be on the same number" (p. 127). This response, or the language used to describe the data distribution presented on the graph, reveals the graph reader's knowledge and understanding of the graph structure of a line graph in terms of specifiers and the framework.

According to Friel et al. (2001), students need to use the appropriate graphical components in constructing statistical graphs (e.g., choosing appropriate scales for given data sets) and demonstrate awareness of relationships between or across graphical components in interpreting statistical graphs (e.g., relating data values to the bar on a bar graph). Thus, in the data display LP, we attend to students' use of graphical components in both constructing and interpreting statistical graphs in terms of knowledge of graph structure.

Data Reduction Through Data Unit Scaling

According to Lehrer et al. (2014), the purpose of graphing data values is to display and communicate the trends of data values to readers at a glance. To efficiently display the shapes that distributions of data values take, graph makers need to conduct data reduction, "moving from tables and graphs that display raw data to those that present data that are grouped" (Friel et al., 2001, p. 146).

In Lehrer's (2011) study (as cited in National Research Council, 2014, pp. 69–73) on students' competence in structuring silkworm larvae growth data in a statistical graph, one group of students drew a case value graph, with 261 measurements of silkworms ordered by magnitude/size; however, specifying marks for each interval representing the larvae were not constant in length, and the length of the constructed graph was 5 feet long. This construction indicated that this group of students recognized the range of case values, but they were not aware of a need to represent values of the same size/magnitude in length using a constant-sized larvae icon (i.e., a *specifier*, in Kosslyn's, 1989, terms). A second group of students in the study used equal-width intervals (as a graded scale on the graph) to represent equivalence among groups of larvae lengths and the number of case values (as frequencies) within each interval. The second group's graph displayed the shape that distribution of case values takes more efficiently than the first group's did because it used scaled intervals on the graph to aggregate individual case values. Aligning with this observation, researchers in this field see data unit scaling (i.e., the use of scaled intervals or a conventionally graded scale) as "a tool for data reduction" (Friel et al., 2001, p. 141; see also Åberg-Bengtsson, 2006; Wilson, n.d.). Thus data reduction through data unit scaling becomes one of the framing concepts for our hypothesized data display LP.

Proportional Reasoning in Comparing Data Sets

Proportional reasoning is often used as a way of discerning data trends (e.g., Watson & Moritz, 1999). To interpret information and data from statistical graphs, students need to perceive an overall data trend rather than attending only to individual data values presented in the graph (Friel et al., 2001). The perception of an overall data trend presented in statistical graphs may involve decoding all graphical components used in the statistical graphs (Lowrie et al., 2011) and making sense of the shape of the distribution of data values in terms of the central tendency and variability of the data values that are represented on the graphs (Lehrer & Schauble, 2000; Watson & Callingham, 2003, 2005).

Friel et al. (2001) suggested a taxonomy of data display judgment tasks to examine students' performance in graph comprehension. First, when attending to one quantity, their judgment task asks students to locate a specific data value presented on a statistical graph. Second, when attending to more than one quantity or focusing on relationships between or across data values, their judgment task asks students to use the information drawn from (a) calculation of sum or mean of data values and a ratio between two data values; (b) part-to-part or part-to-whole comparison among data values to determine numerical or relative differences or proportions; and (c) identification of data trends, such as an increase, decrease, or fluctuation. Friel and colleagues' judgment task taxonomy becomes a framework to incorporate different research findings on development of student competence in reading and interpreting statistical graphs.

Related to this judgment task taxonomy (Friel et al., 2001), we account for proportional reasoning in comparing data sets presented in statistical graphs. Franklin et al. (2007) noted the role of proportional reasoning in converting "from counts or frequencies to proportions or percentages" (p. 34) in interpreting the data values in a statistical graph. Watson and Moritz (1999) saw the appearance of proportional reasoning as evidence of a transition into the upper levels of competency in reading and interpreting statistical graphs. In comparing two bar graphs of different group sizes, for instance, Watson and Moritz found that students who began to resolve the difference between the two graphs used proportional reasoning with a visual comparison strategy. Other researchers in this field also view the emergence of proportional reasoning in students' interpretations of statistical graphs as a key feature of upper levels of competence in interpreting and comparing statistical graphs (see, e.g., Franklin et al., 2007; Wilson, n.d.).

Progress to an Integrated View of Data

Bakker (2004) noted that in the context of data modeling and analysis, students tend to have a case-oriented view of data (i.e., perceiving data as a series of individual cases) rather than having an aggregate view (i.e., perceiving data as an aggregation of cases). To reason about the shape of data, students must develop a conceptualization of data sets as aggregates. Many researchers in the domain of data display and the fields of mathematics and statistics education have focused on students' progression from a case-oriented view to an aggregate view of data as a crucial part of the development of knowledge about data display (Bakker, 2004; Casey, 2015; Friel et al., 2001; Lehrer & Schauble, 2000).

For instance, Casey (2015) examined eighth graders' conceptions of the line of best fit when they placed the line of best fit on a scatterplot and found a range of understanding from a case-oriented view of data to an aggregate view of data. Casey identified the progression of conceptions of the line of best fit from thinking of the line of best fit as (a) the line halfway between the lowest and highest points or through the first and last points on a scatterplot—a case-oriented view since only attending to the specific points on a scatterplot; to (b) the line through the most points among all the points on a scatterplot—the transitional stage from a case-oriented view to an aggregate view of data since in looking for a set of points on a scatterplot that satisfy their criteria, the line goes through as many points as possible; to (c) the line placed to have an equal number of points on both sides—revealing an aggregated view since it examines the entire set of points on a scatterplot. The findings lead us to hypothesize a progression from a case-oriented view to an aggregate view of data with a transitional stage between them.

In addition to the transition from a case-oriented view to an aggregate view of data, Lehrer and Schauble (2000) stated the need for students "to maintain a sense of the relation between individual cases and the aggregate" (p. 114). Wilson (n.d.) suggested that the case-oriented and aggregate views are integrated at the most sophisticated level of understanding data display in that students develop the ability to see cases as examples of data regions and use the aggregate in evaluation of individual cases regarding the general data trend. This may imply that a goal of data display in mathematics and statistics education should be to move students from a case-oriented view to an aggregate view of data and ultimately to an integrated perspective of both views. The idea of progressing to the integrated view of data presented by Wilson (n.d.) provides a conceptual foundation to frame our hypothesized data display LP.

In sum, according to the research literature in this domain, the following impact student competence in constructing and/or interpreting statistical graphs: reasoning of and about context; knowledge and use of graphical components forming a statistical graph; data reduction with data unit scaling; and proportional reasoning. Furthermore, the development from a case-oriented view to an aggregate view of data to an integrated perspective of both is suggested to support development of students' knowledge about data display in mathematics and statistics education. We include these big ideas in the LP to describe students' conceptual development of data display in terms of

- Construction of appropriate graph forms for a given data context (Pfannkuch, 2011) and interpretation of the information presented in statistical graphs regarding their contextual setting (Friel et al., 2001).
- Appropriate construction of components (Kosslyn, 1989) in graphing statistical graphs and interpretation of graphical components or the shape of the data distribution (Friel et al., 2001).
- Sufficient data reduction through data unit scaling (Friel et al., 2001) in graphing data values.
- Use of proportional reasoning skills in comparing data sets presented in statistical graphs (e.g., Watson & Callingham, 2003, 2005; Watson & Moritz, 1999).
- Progress to an integrated view of case-oriented and aggregate views of data (Bakker, 2004; Wilson, n.d.) in both constructing and interpreting statistical graphs.

While these overarching concepts describe students' conceptual development of data display in a general sense, to inform assessment development, we wanted to be more precise about the areas of instructional focus in which we would expect to see student growth. Kennedy and Wilson (2007) promoted the specification of "progress variables," or "representations of the knowledge, skills, and other competencies one wishes to increase through the learning activities associated with a curriculum" (p. 272), as one way to make LPs more tangible and useful for teachers. For example, they stated that teachers can attend to the specific knowledge and skills described in progress variables as a way to interpret student performance on assessment tasks. We propose that progress variables can also be used in assessment design as a way of specifying the types of evidence that assessment developers want to elicit. In the case of this proposed data display LP, we propose two progress variables, constructing data displays and interpreting data displays, as detailed in what follows.

Hypothesized Data Display Learning Progression

Organizational Structure of the Hypothesized Data Display Learning Progression

Drawing from the review of empirical research on K–12 students' learning and understanding of statistical graphs, we propose an LP for data display consisting of two distinct but connected progress variables: one for constructing statistical graphs and the other for interpreting statistical graphs (hereinafter referred to as constructing data displays and interpreting data displays, respectively; see Figure 1 for our conceptualization of the constructing and interpreting data display progress variables). The two progress variables contain vertical transitions through the levels as well as lateral connections between them.

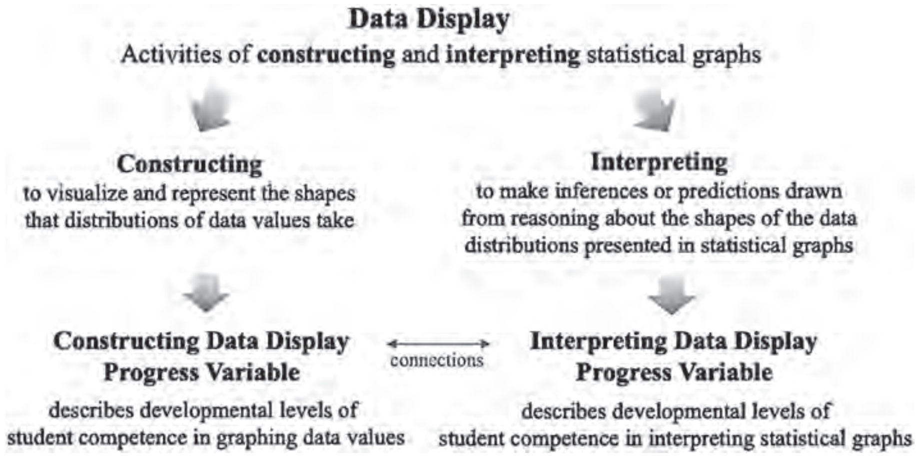


Figure 1 Conceptualization of the constructing and interpreting data display progress variables.

Within each of the constructing and interpreting data display progress variables, transition points between each level of the LP are hypothesized to reflect a transitional stage between two neighboring levels as well as a continuous progress along levels of student learning in data display. Other researchers have used these notions to further specify student learning. Casey (2015) addressed the progression from a case-oriented view to an aggregate view of data with the transitional stage between the two views in conceptualizing the line of best fit on a scatterplot. In a different content area, but supporting the argument of continuous progress, Gutiérrez, Jaime, and Fortuny (1991) reported that the students in their study showed two consecutive van Hiele levels of geometrical reasoning at the same time in response to an assessment item of three-dimensional geometry.

Constructing and Interpreting Data Display Progress Variables

To describe the development of student competence in graphing to visualize and represent the shape of data, we propose six hierarchical levels with transitions between adjacent levels. Figure 2 presents an overview of the organizational structure of the constructing data display progress variable with transition points between each level.

For the development of student competence in interpreting information and data from statistical graphs to make inferences or predictions about data in the process of statistical inquiry, we propose six hierarchical levels with transitions along the levels. In Figure 3, we present an overview of the interpreting data display progress variable with transition points between each level.

Connections Between the Two Progress Variables

The Common Core State Standards for Mathematics (CCSS-M; CCSSI, 2010) recommended that instruction for data display starts with generating data (i.e., counts) and organizing the data by category (Measurement and Data, K.MD), followed by reading the categorized data to answer questions of “the total number of data points, how many in each category, and how many more or less are in one category than in another” (p. 16, Measurement and Data, 1.MD). The instructional sequence of generating and representing data followed by reading and interpreting data consistently appears in standards through the upper grades (see, e.g., 2.MD, 3.MD, 4.MD, and 5.MD in the content domain of Measurement and Data for Grades 2–5 and 6.SP, 7.SP, and 8.SP in the content domain of Statistics and Probability for Grades 6–8). This approach to instruction in data display featured in the CCSS-M motivated us to present the constructing data display progress variable prior to the interpreting data display progress variable in this report. However, it is not our intention to suggest that students’ development in constructing statistical graphs precedes the interpretation of statistical graphs in the overall knowledge development of data display (or vice versa). In their study on middle school students’ interpretation of box plots, for example, Edwards, Özgün-Koca, and Barr (2017) stated that

Constructing Data Display Progress Variable
Level 6: Students construct statistical graphs with an integrated view of case-oriented and aggregate views of data.
↑ Transition from Level 5 to Level 6: Integration of case-oriented and aggregated views of data has developed, and data reduction, in connection with a graphing goal and context, has developed.
Level 5: Students construct statistical graphs with an aggregate view of individual cases.
↑ Transition from Level 4 to Level 5: Sufficient data aggregation (and data reduction) emerge and continue to mature into the upper levels of this progression.
Level 4: Students construct statistical graphs with an attempt to aggregate individual cases to address graphing goals.
↑ Transition from Level 3 to Level 4: Aggregation of individual data cases, with regard to a given graphing goal and context, begins to emerge and continues to develop in the upper levels of this progression.
Level 3: Students draw statistical graphs associated with individual cases (i.e., with a case-oriented view of data).
↑ Transition from Level 2 to Level 3: Sufficient data unit scale/scaling usage (i.e., the use of a conventionally graded scale) for a given data set, and appropriate construction of graphical components in graphing (by different types of statistical graphs) emerge and continue to mature into the upper levels of this progression.
Level 2: Students draw informal, unconventional statistical graphs with partial recognition of scale usage on a graph; namely, students are at least consistent in their representations even though they are not using standard scales/visuals.
↑ Transition from Level 1 to Level 2: Early data unit scale/scaling usage for graphing (i.e., the use of an unconventionally graded scale on a statistical graph), in connection with the noticing of any appearance or trend of data values (Lehrer et al., 2014), begins to emerge and continues to develop in the upper levels of this progression.
Level 1: Students draw idiosyncratic statistical graphs.

Figure 2 A simplified version of the constructing data display progress variable with transitions.

Interpreting Data Display Progress Variable
Level 6: Students interpret statistical graphs from the integrated view of data relations to make predictions about general trends of data values.
↑ Transition from Level 5 to Level 6: Integration of case-oriented and aggregate views of data to make predictions associated with the overall distribution of data values has developed.
Level 5: Students interpret data relationships displayed in statistical graphs to infer the relationships implicit in data values.
↑ Transition from Level 4 to Level 5: Sufficient integration of explicit and implicit information presented in statistical graphs, regarding the shapes that the distributions of data values take, emerge and continue to develop in the upper levels of this progression.
Level 4: Students analyze information presented in statistical graphs to identify the relationships among data values.
↑ Transition from Level 3 to Level 4: Early integration of information (i.e., graphical features, data trends, and graphing goal and context) presented on statistical graphs emerges. Recognition of the shapes that the distributions of data values take begins to emerge and continues to mature into the upper levels of this progression.
Level 3: Students translate graphical features shown on statistical graphs to find the relationships between/among data values.
↑ Transition from Level 2 to Level 3: Decoding and translation of graphical features shown on statistical graphs emerge and continue to mature into the upper levels of this progression. Early recognition of the relationships between/among data values through reading of the data values shown emerges.
Level 2: Students read statistical graphs to locate specific data values with insufficient attention to a given graphing goal or context.
↑ Transition from Level 1 to Level 2: Locating of the most perceptually salient data values shown on statistical graphs begins to emerge and continues to develop in upper levels of this progression.
Level 1: Students read statistical graphs idiosyncratically.

Figure 3 A simplified version of the interpreting data display progress variable with transitions.

Constructing Data Display Progress Variable		Connections Between Two Progress Variables		Interpreting Data Display Progress Variable
Level 6: Students construct statistical graphs with an integrated view of case-oriented and aggregate views of data.	←	**Level 6 Connection:** Integrated view of data, moving back and forth between the case-oriented and aggregated views of data by the purpose and context of statistical inquiry.	→	Level 6: Students interpret statistical graphs from the integrated view to make predictions about general trends of data values.
Level 5: Students construct statistical graphs with an aggregate view of individual cases.	←	**Level 5 Connection:** Aggregated view of data with attention to the shape of the data distribution and perception of an appropriate data display fit for data displaying goals or contexts.	→	Level 5: Students interpret data relationships displayed in statistical graphs to infer the relationships implicit in data values.
Level 4: Students construct statistical graphs with an attempt to aggregate individual cases to address graphing goals.	←	**Level 4 Connection:** Transition from case-oriented to aggregate view of data by conceiving a set of individual data cases/values (in equal sized groups or scaled intervals) as an aggregate for data, and perception of conventional data display fit for data displaying goals or contexts.	→	Level 4: Students analyze information presented in statistical graphs to identify the relationships among data values.
Level 3: Students draw statistical graphs associated with individual cases (i.e., with a case-oriented view of data).	←	**Level 3 Connection:** Case-oriented view of data by attending to individual cases in a data display, perception of conventions of/about data display and attention to data displaying goals or contexts.	→	Level 3: Students translate graphical features shown on statistical graphs to find the relationships between and among data values.
Level 2: Students draw informal, unconventional statistical graphs with partial recognition of scale usage on a graph; namely, students are consistent even though they are not using standard scales/visuals.	←	**Level 2 Connection:** Informal perception of data display, not appropriate to data display conventions, and attention to specific detail in data display with insufficient awareness of a given graphing goal or context.	→	Level 2: Students read statistical graphs to locate specific data values with insufficient attention to a given graphing goal or context.
Level 1: Students draw idiosyncratic statistical graphs.	←	**Level 1 Connection:** Idiosyncratic thinking or perception of data display and no awareness to the relevance of data display to a given context or goal.	→	Level 1: Students read statistical graphs idiosyncratically.

Figure 4 A simplified version of the constructing and interpreting data display progress variables with connections.

> the research base cited earlier, together with our own observations of the middle school students in our study, provides ample evidence that being able to display data in a boxplot does not guarantee that one can correctly interpret a boxplot for which there is no access to the raw data. (p. 27)

Since we found no empirical literature suggesting that constructing precedes interpreting, or vice versa, we placed the two progress variables parallel to one another within the data display LP. This visual representation is meant to signify that this relationship is likely symbiotic, and it allows us to test in the future hypotheses about the order of development along the two progress variables at any level of the progression. We aimed to incorporate similar cognitive demands at each level of both the two progress variables. Enhanced knowledge and understanding about the relationship between students' development of competencies in graph construction and interpretation would contribute to advancing curriculum and assessment development in the domain of data display and fields of mathematics and statistics education. After developing the constructing and interpreting data display progress variables, we specified the connections between these two progress variables along the six levels to illustrate the developmental commonalities of students' conceptual understandings in constructing and interpreting statistical graphs (see Figure 4).

In both progress variables, Level 1 is distinguished by idiosyncratic thinking about or perceiving data display. It is also possible that students do not attend to data display or do not see its relevance within a given context and for a graphing goal. A distinctive aspect of Level 2 shared by the two progress variables is students' informal perception of data display revealed through unconventional means (e.g., using nonstandard visuals for statistical graphs or using an unconventionally graded scale on the graphs) or attention to specific detail in data display at the expense of noticing larger trends. Compared to students at Level 1, Level 2 students demonstrate greater engagement with a given data display context or goal, but it is still insufficient for understanding or producing graphs for a particular goal or context of data display.

In both progress variables, we identify Level 3 with a case-oriented view of data (i.e., attending to individual cases in a data display); Level 4 with the transition from a case-oriented view to an aggregate view of data; and Level 5 with an aggregated view of data, perceiving data values as a whole representing a trend. At Level 3, students begin to show their awareness of conventional data displays appropriate for a given context and goal, and this awareness continues to mature into Levels 4, 5, and 6 of both progress variables. As the most sophisticated level of understanding data display, Level 6 is characterized by students' integrated perspective on data, moving back and forth between case-oriented and aggregated views of data depending on the purpose and context of statistical inquiry. As mentioned previously, these proposed connections between the progress variables would need to be studied empirically to reach a holistic developmental progression of student competencies in data display.

Panel Review

In Spring 2017, the hypothesized data display LP was reviewed by an expert advisory panel composed of two external experts and two internal ETS experts in the domain of data display and in the fields of mathematics and statistics education. Reviews of the LP were performed individually, beginning with a virtual meeting to introduce the panel to the LP. We then provided the panel the full version of the data display LP with supporting documentation detailing our rationale for developing the LP, our definition of data display, the conceptual framework, and the organizational structure of the LP. We asked the panel for their feedback on (a) the construct effectiveness of the data display LP and the constructing and interpreting data display progress variables, (b) content accuracy and adequacy of level and transition descriptions of the two progress variables, and (c) usability of the constructing and interpreting data display progress variables in task design for informing future assessment development and research. Once we gathered the panel's individual feedback, we held a virtual debrief meeting in which we discussed the given feedback, categorized by the review criteria, with the panel to get more input and/or clarification (when needed) and also for confirmation of the LP revisions that were made according to the comments and/or suggestions.

Construct Effectiveness

We received positive feedback from the panel regarding the organizational structure of the data display LP, specifically on the two distinct but connected constructing and interpreting data display progress variables. The panel also supported our parallel construct of and the connections between the two progress variables (see Figure 4). A reviewer mentioned that it is theoretically difficult to argue that one progress variable would emerge prior to the other; another reviewer commented that students may reveal a higher level of understanding in one progress variable than the other depending on their prior learning experience. It was also suggested that classroom instruction should attend to the interaction between the activities of graph construction and interpretation to enhance students' development of competence in data display.

Content Accuracy and Adequacy

From the panel, we received overall agreement on the accuracy and adequacy of the increasing sophistication in the levels of the constructing and interpreting data display progress variables. Reviewers mentioned that examples of student work selected from the literature for each level of the two progress variables are helpful in communicating the level descriptions. We also received detailed feedback on the level descriptions and selected examples for each progress variable and incorporated them into the revised version of the LP:

- Moving the selected example of the case value graph of silkworm larvae down from Level 3 to Level 2 of the constructing data display progress variable because the case value graph reflects students' view of ordered values with no acknowledgment of case values of the same size/magnitude in length.
- Including some basic ideas about reading graphs (e.g., finding the highest data value on the graph) in the earlier levels of the interpreting data display progress variable.
- Including the emergence (and progress) of students' understanding of/about the shape of data, regarding the frequency, order, interval, and/or scale of data (values), into Level 4 of the interpreting data display progress variable.

All of the revisions were shared with and confirmed by the panel at the online debriefing meeting (see Appendix A, Table A1, and Table A2 for the revised, full version of the data display LP).

Usability

Our expert advisory panel suggested particular focal points for task design and further research, such as (a) differences among levels of student understanding revealed in constructing and reading different graph types (e.g., bar graphs and line graphs versus circle graphs), (b) students' selection of appropriate statistical graphs for a given data set and context, (c) possible ways or contexts to examine Levels 5 and 6 understandings of the constructing and interpreting data display progress variables, (d) relationships between the two progress variables, and (e) potential interaction between the data display LP and other LPs in statistics learning (e.g., the probability LP). Following the ideas suggested by the panel, we began to design data display tasks targeting the data display LP.

Learning Progression–Associated Task Design

A set of three data display tasks was designed related to the levels of the constructing and interpreting data display progress variables in line with procedures given in other research and development documents (Graf & van Rijn, 2016; Smith et al., 2006; Wilson, 2009). These tasks are given in Appendix B. Each data display task consists of two subtasks, one assessing constructing and the other assessing interpreting statistical graphs. The two subtasks of each data display task target the same levels of the constructing and interpreting data display progress variables to examine possible connections between individual students' understandings of constructing and interpreting statistical graphs. We designed the tasks to capture students' competence in constructing appropriate graph forms for a given data type and graphing goal and interpreting the information presented in different statistical graphs. The tasks examine students' competence in constructing and interpreting statistical graphs for categorical data (Task 1), numerical data (Task 2), and data/information presented graphically (Task 3).

Data Display Task for Categorical Data

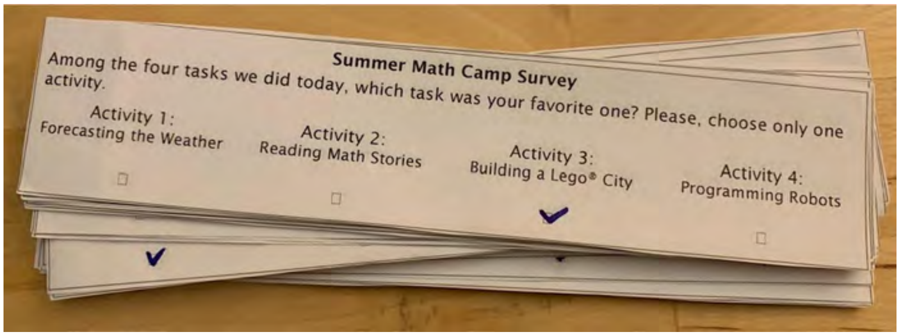
In Task 1, to provide categorical data to the participants, we present mock data from 45 Summer Math Camp surveys in which hypothetical respondents marked a favorite activity of the day among four camp activities: Activity 1, Forecasting the Weather; Activity 2, Reading Math Stories; Activity 3, Building a LEGO® City; and Activity 4, Programming Robots (see Figure 5).

Part 1 of Task 1 asks participants to organize the surveys to determine how popular each activity was for the students who responded and to draw a graph on given grid paper to represent the survey results. This graphing activity targets Levels 2–4 of the constructing data display progress variable regarding participants' selection of the appropriate graph form for the given categorical data (e.g., pictograph or bar graph) and their usage of graphical components (e.g., axes, scales, legends, or bars of bar graphs) in graphing the given data.

In Part 2 of Task 1, participants need to read data values presented on the graph drawn in Part 1 to find the most popular activity among the four camp activities, to determine the number of students who chose Activity 4 as their favorite, to aggregate the data (values) represented on the graph to find the difference between the number of students who chose Activities 1 and 3 and the number of students who chose Activities 2 and 4, and to interpret the shape of the data distribution and/or the pattern of the data values represented on the graph. This graph interpretation part targets Levels 2–4 of the interpreting data display progress variable regarding participants' understanding of data relationships, such as numerical or relative differences or proportions presented on graphs and their descriptions of particular graphical components or the shape of the data distribution graphically represented on their own graphs.

Data Display Task for Numerical Data

In Task 2, we used a context of ordering baseball caps for a student baseball team to present mock data of 25 individual head measures (a set of numerical data) along with a chart of baseball cap sizes in small (S), medium (M), and large (L; see Figure 6). Part 1 of this task asks participants to categorize the 25 head measures of the baseball team students by each baseball cap size and then draw a graph to represent the total number of baseball caps needed in each size. The graphing activity targets Levels 2–4 of the constructing data display progress variable in terms of participants' selection of an appropriate graph form for the given numerical data (e.g., line plots or histograms) and their usage of graphical components (e.g., axes, scales, legends, or dot/x marks of line plots) in graphing the given data. In addition, this graphing



Summer Math Camp Survey

Among the four tasks we did today, which task was your favorite one? Please, choose only one activity.

Activity 1: Forecasting the Weather ☐

Activity 2: Reading Math Stories ☐

Activity 3: Building a Lego® City ☑

Activity 4: Programming Robots ☐

Figure 5 Mock data from 45 Summer Math Camp surveys used in Task 1.

Baseball Cap Size Chart

Head: 21¼ - < 22 inches

Head: 22 - < 22¾ inches

Head: 22¾ - < 23½ inches

Figure 6 Twenty-five individual head measures in card form alongside a baseball cap size chart for use in Task 2.

task can be used to elicit Level 5 understanding by assessing whether students can draw an additional graph for categorical data by reading and interpreting the numerical data values on their first graphs. For example, students might draw a bar graph representing the number of students per cap size by aggregating the number of students per head measure represented on a line plot.

Part 2 of Task 2 asks participants to find the second most frequent head measure among the given head measures and the number of medium-sized baseball caps to be ordered by reading data values on the graph(s) drawn in Part 1. Then, participants are asked to find the difference between the total number of small- and large-sized baseball caps to be ordered and the number of medium-sized baseball caps to be ordered and to describe the shape/pattern of the graph(s) by interpreting the data presented in graph. This graph-interpreting activity targets Levels 2–4 of the interpreting data display progress variable, attending to participants' descriptions about particular graphical components of the shape of the data and their responses to data relationships, such as numerical or relative differences or proportions. We expected that interpretation of an additional graph in this task could reveal participants' Level 5 understandings of the interpreting data display progress variable.

Data Display Tasks for Data Presented in a Graph

In Task 3, we present two bar graphs (Part 1) representing July and August camp enrollment for basketball, baseball, swimming, and soccer camps and two scatterplots (Part 2) representing swimming camp enrollment in 2016 and 2017. In Part 1 of this task, participants are asked to draw a new graph representing both July and August camp enrollment by reading and interpreting the aggregated monthly camp enrollment data for four sports camps: basketball, baseball, swimming, and soccer. Then, using the information presented in the bar graph, participants are asked to find the least popular camp in July, the number of students enrolled in August, and the difference between the number of students enrolled in the basketball and baseball camps in July and August, and finally, they are asked to describe the shape/pattern of camp enrollment in July and August that is displayed in the graph(s).

In Part 2 of Task 3, participants are asked to draw a new graph representing both 2016 and 2017 swimming camp enrollments by using the data presented in the scatterplots of swimming camp enrollment in 2016 and 2017. Regarding the data displayed in the newly constructed graph, participants are asked to find the third highest enrollment in 2016, the enrollment in January 2017, and the difference between the summer enrollments (i.e., the number of students enrolled in June, July, and August) in 2016 and 2017 and to describe the shape/pattern of swimming camp enrollments for the 2 years presented in the graph. This graphing activity targets Levels 2–6 of the constructing data display progress variable in that the data to be graphed are presented in graph form as aggregated data (values) on the given two graphs. The graph-interpreting activity also targets Levels 2–6 of the interpreting data display progress variable, attending to participants' descriptions about particular graphical components or the shape of the data and their responses to data relationships, such as numerical or relative differences or proportions.

In sum, we designed the three tasks to elicit student understanding at several levels of the constructing and interpreting data display progress variables. We expect that the LP-associated tasks can be used in future research to investigate the progress of students' competence in constructing and interpreting statistical graphs as well as the connections between the two data display activities in terms of the use of graphical components in both graph construction and graph interpretation.

Drawing on the existing empirical literature in this domain and through expert review, we developed a hypothesized progression for the development of students' competence in data display that is structured with two distinct but interconnected progress variables: constructing data displays and interpreting data displays. In this research report, we described the development process for this LP, drawing on the analysis and synthesis of existing literature in a targeted domain (Smith et al., 2006), and we illustrated the design of cognitive tasks targeting levels of the progress variables (Graf & van Rijn, 2016; Wilson, 2009). Following Smith et al.'s (2006) suggestion for developing research-based LPs and associated assessment tasks, we first identified the core concepts in data display to construct a conceptual framework for the data display LP. We then analyzed the research literature for how student understanding progresses in the core concepts for data display to posit the levels of the data display LP. In particular, for each level of the data display LP, we drew from the empirical research on student learning of data display to write descriptions of student thinking and reasoning at each level, including selected examples of student work from the literature as evidentiary support for the level descriptions. The constructed data display LP, as a provisional model (Educational Testing Service, n.d.), was then reviewed by experts in the domain of data display and the fields of mathematics and statistics education; we subsequently revised the LP, according to experts' suggestions. Finally, we designed a set of data display tasks to examine students' learning and understanding related to the levels of the data display LP (Graf & van Rijn, 2016; see also Wilson, 2009). We anticipate that the current work advances the field by providing a set of hypothesized paths of students' development of data display knowledge that can be used for assessment development and future research on potential interactions between constructing and interpreting statistical graphs.

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Appendix A

Data Display Learning Progression

Table A1 Constructing Data Display Progress Variable

Level	Description of student competence in constructing data display	Selected example
6	Students construct statistical graphs with an integrated view of case-oriented and aggregate views of data[a] 1 Students construct statistical graphs with an integrated view of case-oriented and aggregate views of data (Wilson, n.d.; cf. Casey, 2015), namely, moving back and forth between the two perspectives according to a given graphing goal and context 2 Students construct statistical graphs using proportional reasoning skills (Watson & Callingham, 2003) 3 With regard to the selection and usage of the appropriate graph form for the nature of a given data set, students understand the purpose, use, and limitless of different forms of statistical graphs to represent different types/natures of data (Angra & Gardner, 2016)	
↑	Transition from Level 5 to Level 6: Integration of case-oriented and aggregated views of data has developed and data reduction, in connection with a graphing goal and contexts, has developed	
5	Students construct statistical graphs with an aggregate view of individual cases[b] 1 In construction of statistical graphs, students continue to transit to an aggregate view of individual cases through quantification of aggregate properties (like ratios, portion, or percentage) of data values in statistical graphs (Wilson, n.d.) 2 Students are able to construct statistical graphs associated with a bivariate data set (Casey, 2015; Garcia-Mila et al., 2014) 3 With regard to the selection and usage of the appropriate graph form for the nature of a given data set, students develop additional statistical graphs to represent a (univariate or bivariate) numerical data set (Franklin et al., 2007; cf. Baker et al., 2001)	Bar graph: In drawing a bar graph for a given bivariate data set (a list of 25 students who indicated gender and height), a student draws bars with frequencies of student height intervals on the y-axis (dependent variable) by student gender types on gender on the x-axis (two independent variables; Garcia-Mila et al., 2014)
↑	Transition from Level 4 to Level 5: Sufficient data aggregation (and data reduction) emerges and continues to mature into the upper levels of this progression	
4	Students construct statistical graphs with an attempt to aggregate individual cases to address graphing goals[c] 1 In constructing statistical graphs, students start to consider aggregation of individual cases, like grouping similar data values within an interval/range (Lehrer & Schauble, 2000; Wilson, n.d.) 2 Students begin to construct statistical graphs with a conventionally graded scale for major divisions (e.g., fives or tens; Åberg-Bengtsson, 2006) with regard to a given graphing goal or context 3 With regard to the selection and usage of the appropriate graph form for the nature of a given data set, students are able to identify appropriate types of statistical graphs to represent numerical or categorical data (e.g., drawing a bar graph for categorical data and a histogram for numerical data; Franklin et al., 2007)	Bar graph: A student draws bar graphs for major divisions of the data unit scale, like 10s or 50s (Åberg-Bengtsson, 2006) Scatterplot: Students place the informal line of best fit through the most points, or through the first and last points among all the points on a scatterplot, revealing the transition from case-oriented view to aggregate view of data; they attend to a subset of collinear points on the scatterplot, while thinking of the points that were not collinear as outliers (Casey, 2015)

Table A1 Continued

Level	Description of student competence in constructing data display	Selected example
↑	Transition from Level 3 to Level 4: Aggregation of individual data cases, with regard to a given graphing goal and context, begins to emerge and continues to develop in the upper levels of this progression	
3	Students draw statistical graphs associated with individual cases (i.e., with a case-oriented view of data)[d] 1 In drawing a given form of statistical graphs, students construct graphical components that are appropriate for the given graph form and use scaling and labeling correctly with proper data format (Baker et al., 2001; Garcia-Mila et al., 2014; Lehrer, 2011, as cited in National Research Council, 2014), but this depends on the complexity of given data. For example, in bar graphing, the x- and y-axes are appropriately constructed and labeled; individual cases of categorical variables are structured on the x-axis and quantitative values are structured on the y-axis; the magnitude of the quantitative values on the y-axis are represented by the heights of bars with a conventionally graded scale 2 Students draw statistical graphs associated with individual cases, thus their graphs show insufficient data reduction (Casey, 2015; Franklin et al., 2007; Garcia-Mila et al., 2014) 3 With regard to the selection and usage of the appropriate graph form for the nature of a given data set, students draw inappropriate statistical graphs for the types of data, like drawing a bar graph to display numerical data values (Franklin et al., 2007)	Bar graph: In drawing a bar graph for a given bivariate data set (a list of 25 students who indicated gender and height), a student lists all the student names by gender on the x-axis and puts the student height intervals on the y-axis; however, the differentiated genders (the second independent variable) are not integrated with the student heights (the dependent variable; Garcia-Mila et al., 2014)
↑	Transition from Level 2 to Level 3: Sufficient data unit scale/scaling usage (i.e., the use of a conventionally graded scale) for a given data set and appropriate construction of graphical components in graphing (by different types of statistical graphs) emerge and continue to mature into the upper levels of this progression	
2	Students draw informal, unconventional statistical graphs with partial recognition of scale usage on a graph; namely, students are at least consistent in their representations even though they are not using standard scales/visuals[e] 1 Students engage in graphing data values to capture the "shape" of data that they perceive in a data set (Lehrer et al., 2014) 2 Students draw informal, unconventional statistical graphs of labeled or unlabeled (Åberg-Bengtsson, 2006; Lehrer et al., 2014; Watson & Callingham, 2003), like statistical graphs using an unconventionally graded scale (Åberg-Bengtsson, 2006) or using differently sized visuals/graphical components for one type of independent, categorical variable (e.g., using differently sized "boxes" for the bar of frequency in a bar graph; Lehrer & Schauble, 2000)	Unconventional bar graph: A student draws the tallest bar first and then approximates the heights of the other bars in comparing to the tallest one; the student makes an unconventionally graded scale, and the student labels numbers on the left edge, but the student does not provide a legend for the scale (Åberg-Bengtsson, 2006) Case value graph: A student draws a case value graph, having 261 measurements of silkworms ordered, with no acknowledgment of case values of the same magnitude/size (Lehrer, 2011, as cited in National Research Council, 2014, p. 69)
↑	Transition from Level 1 to Level 2: Early data unit scale/scaling usage for graphing (i.e., the use of an unconventionally graded scale on a statistical graph), in connection with the noticing of any appearance or trend of data values (Lehrer et al., 2014), begins to emerge and continues to develop in the upper levels of this progression	

Table A1 Continued

Level	Description of student competence in constructing data display	Selected example
1	Students draw idiosyncratic statistical graphs[f] 1 Students perceive that data values can be displayed graphically (Lehrer et al., 2014) 2 Students draw idiosyncratic statistical graphs without attention to a given graphing goal (Lehrer et al., 2014; Wilson, n.d.) or engagement within a given graphing context (Watson & Callingham, 2003, 2005) 3 In drawing idiosyncratic statistical graphs, students reveal their personal beliefs and experience (Lehrer et al., 2014; Watson & Callingham, 2003, 2005; Wilson, n.d.) 4 Students are motivated to draw statistical graphs when graphing context is related to their personal interests (e.g., birthdays) [hypothesized]	Idiosyncratic pictograph: Students draw idiosyncratic pictographs with the idea of “we grouped even and odd numbers because we like even and odd numbers” (Wilson, n.d.; see National Research Council, 2014, p. 68)

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[a] This level might be consistent with Watson and Callingham’s (2003) Task-step Level 6 of Critical Mathematical and Wilson’s (n.d.) DaD6. [b] This level might be consistent with Watson and Callingham’s (2003) Task-step Level 5 of Critical, Wilson’s (n.d.) DaD4 and DaD5, and Franklin et al.’s (2007) Levels B and C. [c] This level might be consistent with Watson and Callingham’s (2003) Task-step Level 4 of Consistent Non-Critical, Wilson’s (n.d.) DaD3, and Franklin et al.’s (2007) Level B. [d] This level might be consistent with Watson and Callingham’s (2003) Task-step Level 3 of Inconsistent, Wilson’s (n.d.) DaD2, and Franklin et al.’s (2007) Level A. [e] This level might be consistent with Watson and Callingham’s (2003) Task-step Level 2 of Informal. [f] This level might be consistent with Watson and Callingham’s (2003) Task-step Level 1 of Idiosyncratic and Wilson’s (n.d.) DaD1 and DaD2.

Table A2 Interpreting Data Display Progress Variable

Level	Description of student competence in interpreting data display	Selected example
6	Students interpret statistical graphs from the integrated view of data relations to make predictions about general trends of data values[a] 1 Students interpret statistical graphs with an integrated view of case-oriented and aggregate views of data (Wilson, n.d.; cf. Casey, 2015), namely, moving back and forth between the two perspectives according to a given graphing goal and context 2 Students interpret statistical graphs using proportional reasoning skills (Watson & Callingham, 2003) 3 With consideration of the central tendency and variability of data values represented in statistical graphs (Lehrer et al., 2014), students make a prediction drawn on their interpretation of/about statistical graphs (Watson & Callingham, 2003, 2005) 4 Students provide summary comments on the data values represented in statistical graphs (Watson & Callingham, 2003)	Scatterplot: To an item of reading a scatterplot (Watson & Callingham, 2005) of Grades 7 and 8 students' math scores and the hours per day they spent on math homework, asking "Give some reasons why the graph has the shape it does?" (p. 149), a student creates a hypothesis on the relation between two variables (math scores and math homework hours) and hypothesizes possible contribution factors, responding "The struggling students take more time to do their homework" (p. 149), and "The drop in scores is due to stress and frustration" (p. 149), respectively
↑	Transition from Level 5 to Level 6: Integration of case-oriented and aggregate views of data to make predictions associated with the overall distribution of data values has developed	
5	Students interpret data relationships displayed in statistical graphs to infer the relationships implicit in data values[b] 1 Students describe the shapes of the data distributions presented in the statistical graphs through quantification of aggregate properties (like ratios, portion, or percentage) of data values in statistical graphs (Watson & Moritz, 1999; Wilson, n.d.) 2 Students identify the trends of data values (e.g., increases, decreases, or fluctuations) through visual comparison across specific graphical features represented in statistical graphs, and then make statistical inferences about data relationships that are not directly presented on the graphs (Friel et al., 2001) 3 In comparison of two groups of different sizes presented on statistical graphs, students take proportional strategy with visual comparison (Watson & Callingham, 2005; Watson & Moritz, 1999)	Scatterplot: (a) A student quantifies the percentage of an interval of data values to describe the shape of the data distribution, stating "I found out that measurements between 45 and 55 were 70% of our measurements. So, I guess the true height is somewhere between 45 and 55" (Wilson, n.d.; see National Research Council, 2014, p. 68); and (b) to an item of reading a scatterplot (Watson & Callingham, 2005) of Grades 7 and 8 students' math scores and the hours per day they spent on math homework, asking "What does the graph tell you about maths homework time and maths scores?" (p. 149), students respond like "You are more likely to get good grades if you do an hour" (p. 149) and "Those better at maths finished early" (p. 149)
↑	Transition from Level 4 to Level 5: Sufficient integration of explicit and implicit information presented in statistical graphs, regarding the shapes that the distributions of data values take, emerge and continue to develop in the upper levels of this progression	Bar graph: In comparing two groups of different sizes presented on bar graphs (each cell represents the score of one person on a test; Watson & Moritz, 1999), asking "Which is better at quick recall of 9 maths facts" (p. 150), a student responds like "I think that Black would have done better. They have got, for the amount of people in their class they have got a higher number, a highest percentage or something" (p. 156)

Table A2 Continued

Level	Description of student competence in interpreting data display	Selected example
4	Students analyze information presented in statistical graphs to identify the relationships among data values[c] 1 Students find data relationships through integrating information across two or more data values shown on statistical graphs (Friel et al., 2001) 2 Students begin to see the shape of data with regard to frequency, order, interval, and/or scale of data (values) represented on statistical graphs [hypothesized]; they attend to the shapes that the distributions of data values take (Watson & Moritz, 1999; e.g., symmetric or lopsided; Franklin et al., 2007) with consideration of the interval of having most data values on statistical graphs (Wilson, n.d.) 3 Students are able to provide an appropriate summary of the data presented on statistical graphs (Watson & Callingham, 2005), involving commenting on the symmetry of the graphs (Watson & Moritz, 1999) 4 In comparison of two groups of equal size presented on statistical graphs, students identify data values that are different between the two graphs, and then take numerical comparison strategy, like calculating the sum of the data values of each group (Watson & Moritz, 1999)	Bar graph: In comparing two groups of equal size presented on bar graphs (each cell represents the score of one person on a test; Watson & Moritz, 1999), asking “Which is better at quick recall of 9 maths facts” (p. 150), a student makes visual and numerical comparisons between the two graphs, responding “Well by looking at it, you can sort of see that it’s kind of even, because it’s kind of the same. There’s those there and those there … they add up to the same as those two … ” (p. 155)
↑	Transition from Level 3 to Level 4: Early integration of information (i.e., graphical features, data trends, and graphing goal and context) presented on statistical graphs emerges. Recognition of the shapes that the distributions of data values take begins to emerge and continues to mature into the upper levels of this progression	
3	Students translate graphical features shown on statistical graphs to find the relationships between/among data values[d] 1 Students translate particular graphical components shown on statistical graphs (Lowrie et al., 2011) with regard to a given graph goal or context 2 For data values/quantities directly presented on a statistical graph, students figure out data relationships of numerical difference, relative difference, and proportions through multiple strategies, like numerical comparison of sum, mean, or ratio of the data values or visual comparison of part-to-part or part-to-whole (Friel et al., 2001; Watson & Moritz, 1999) 3 In comparison of two groups of equal sizes presented on statistical graphs, students attend to individual data values/quantities presented on the graphs	Dotplot: To an item of reading a dotplot (Watson & Callingham, 2005) of the number of years that the families of a class of students had lived in their town, asking “What can you tell by looking at Graph 1?” (p. 160), a student responds like “The numbers along the bottom tell you how many years” (p. 160) or “3 and 12 have the most” (p. 160) Bar graph: In comparing two groups of equal size presented on bar graphs (each cell represents the score of one person on a test; Watson & Moritz, 1999), asking “Which is better at quick recall of 9 maths facts” (p. 150), a student takes visual comparison strategy between the two groups with attention to the numbers of individual scores on the graphs: “It could be the Brown, or it could be the Yellow because the Yellow has lots of people got 5, and one person in the Brown class got 7. So it could really be either” (p. 154)

Table A2 Continued

Level	Description of student competence in interpreting data display	Selected example
↑	Transition from Level 2 to Level 3: Decoding and translation of graphical features shown on statistical graphs emerge and continue to mature into the upper levels of this progression. Early recognition of the relationships between/among data values through reading of the data values shown emerges	
2	Students read statistical graphs to locate specific data values with insufficient attention to a given graphing goal or context[e] 1 Students begin to connect data values to graphical components (e.g., the bars of a bar graph) shown on statistical graphs (Åberg-Bengtsson, 2006) 2 Students locate the most salient data values (like maximum, minimum, or middle value) shown on statistical graphs (Casey, 2015; Friel et al., 2001; Lehrer & Schauble, 2000; Lowrie et al., 2011; Watson & Moritz, 1999; Wilson, n.d.), with no attention to the central tendency (like mean, median, or mode) and variability of the data values represented on the graph 3 In reading a statistical graph, students misread some aspects of the statistical graph (Lehrer et al., 2014), since they only attend to salient data values shown on the graph with intuitive contextual idea to a given graph context (Åberg-Bengtsson, 2006; Lowrie et al., 2011; Watson & Moritz, 1999) 4 In explaining the trends of data values presented on statistical graphs, students give nonstatistical reasons, like giving a prediction based on an occurrence pattern of data values (Watson & Callingham, 2005)	Bar graph: (a) Students connect the data value of "a group of people" to individual bars on a bar graph (Åberg-Bengtsson, 2006); (b) in reading bar graphs, students locate the tallest bar automatically without attention to a given graph context (Lowrie et al., 2011); and (c) in comparing two groups of equal size presented on bar graphs (each cell represents the score of one person on a test; Watson & Moritz, 1999), asking "Which is better at quick recall of 9 maths facts" (p. 150), a student responds like "Brown got a 7 and no one else did" (p. 154), attending to the highest score of one person Pictograph: To an item of reading a pictograph (Watson & Callingham, 2005), asking "A new student came to school by car. Is the new student a boy or a girl? Explain your answer" (p. 159), a student gives a nonstatistical prediction based on pattern "Boy, the graph goes—girl, girl, boy, girl, girl … " (p. 159)
↑	Transition from Level 1 to Level 2: Locating of the most perceptually salient data values shown on statistical graphs begins to emerge and continues to develop in upper levels of this progression	
1	Students read statistical graphs idiosyncratically[f] 1 Students perceive that statistical graphs show (a series of) data values (Lehrer et al., 2014; Wilson, n.d.) 2 Students read statistical graphs without attention to a given graph goal (Wilson, n.d.) or engagement with a given graph context (Watson & Callingham, 2003, 2005) 3 In reading statistical graphs, students reveal their idiosyncratic idea, like personal beliefs and experience (Lehrer et al., 2014; Watson & Callingham, 2003, 2005; Wilson, n.d.)	Pictograph: To an item of reading a pictograph (Watson & Callingham, 2005), asking "A new student came to school by car. Is the new student a boy or a girl? Explain your answer" (p. 159), a student responds like "Girl, because she is nervous about her first day at school" (p. 159)

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[a] This level might be consistent with Watson and Callingham's (2003) Task-step Level 6 of Critical Mathematical, Wilson's (n.d.) DaD6, and Lehrer et al.'s (2014) Informal Inference LP Level 7 and Meta Representational Competence LP Level 5, and also with Watson and Moritz's (1999) Second Cycle Relational Responses (R_2). [b] This level might be consistent with Watson and Callingham's (2003) Task-step Level 5 of Critical, Wilson's (n.d.) DaD4 and DaD5, and Lehrer et al.'s (2014) Informal Inference LP Level 4 and Meta Representational Competence LP Level 3, and also with Watson and Moritz's (1999) Second Cycle Unistructural Responses (U_2) and Second Cycle Multistructural Responses (M_2) and with Franklin et al.'s (2007) Levels B and C. [c] This level might be consistent with Watson and Callingham's (2003) Task-step Level 4 of Consistent Non-Critical, Wilson's (n.d.) DaD3, Watson and Moritz's (1999) First Cycle Relational Responses (R_1), and Franklin et al.'s (2007) Level B. [d] This level might be consistent with Watson and Callingham's (2003) Task-step Level 3 of Inconsistent, Watson and Moritz's (1999) First Cycle Multistructural Responses (M_1), and Franklin et al.'s (2007) Level A. [e] This level might be consistent with Watson and Callingham's (2003) Task-step Level 2 of Informal, Wilson's (n.d.) DaD2, Lehrer et al.'s (2014) Meta Representational Competence LP Level 1, and Watson and Moritz's (1999) First Cycle Unistructural Responses (U_1). [f] This level might be consistent with Watson and Callingham's (2003) Task-step Level 1 of Idiosyncratic and Wilson's (n.d.) DaD1, and also with Lehrer et al.'s (2014) Informal Inference LP Level 1 and Meta Representational Competence LP Level 1.

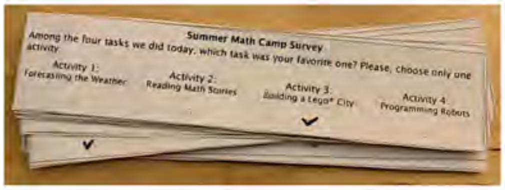
Appendix B

Data Display Cognitive Interview Tasks

Task 1: Description for Interviewers

Task 1 Presentation

- Present the participant with blank, nonruled paper, grid paper, two pens of different colors, and a ruler (with both centimeter and inch sides) along with the color printed task instructions.
- Present the mock data of 45 Summer Math Camp surveys to the participants, which students might organize in a table and display in a graph as below:



Summer Math Camp Survey

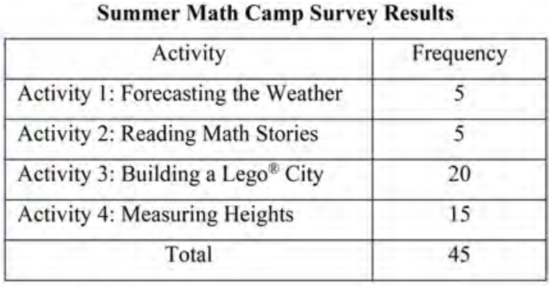
Among the four tasks we did today, which task was your favorite one? Please, choose only one activity

Activity 1: Forecasting the Weather

Activity 2: Reading Math Stories

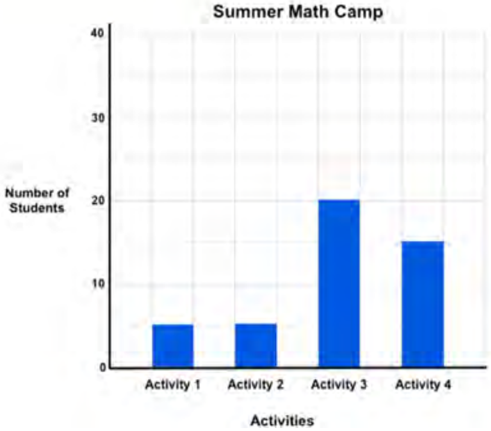
Activity 3: Building a Lego® City

Activity 4: Programming Robots



Summer Math Camp Survey Results

Activity	Frequency
Activity 1: Forecasting the Weather	5
Activity 2: Reading Math Stories	5
Activity 3: Building a Lego® City	20
Activity 4: Measuring Heights	15
Total	45



Task Instruction for Study Participants

Task 1: Favorite Activity of Day

Some students went to a camp for kids to learn and practice math. The Math Camp teacher surveyed 45 students, asking, "Among the four tasks we did today, which task was your favorite one?"

Summer Math Camp Survey			
Among the four tasks we did today, which task was your favorite one? Please, choose only one activity.			
Activity 1: Forecasting the Weather	Activity 2: Reading Math Stories	Activity 3: Building a Lego® City	Activity 4: Programming Robots
☐	☐	☐	☐

Part 1. Create a way to organize the surveys in order to show how popular these activities were, and then draw a graph to represent which activities were chosen by students.

Part 2. Based on the information presented on your graph, answer the questions below:

a Which activity was the most popular activity?

b How many students chose Activity 4 as their favorite activity?

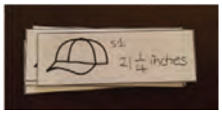
c When compared with the number of students who chose Activities 2 and 4, how many more or how many fewer students chose Activities 1 and 3?

d Describe the shape/pattern of your graph with some possible explanations.

Task 2: Description for Interviewers

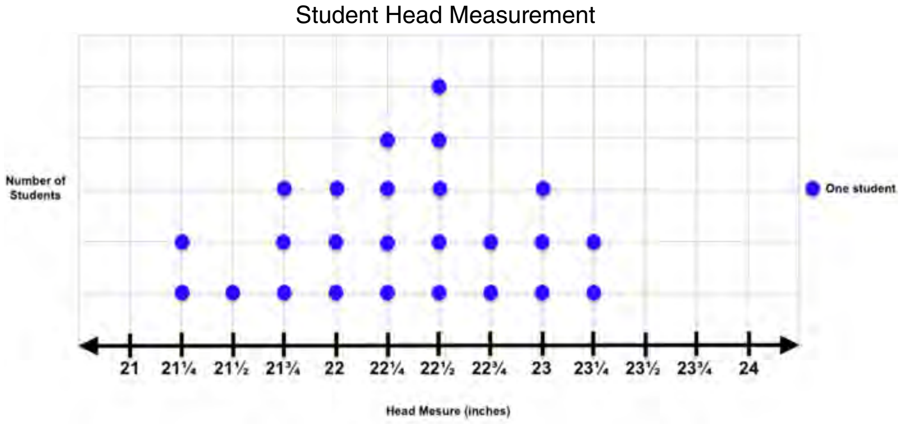
Task 2 Presentation

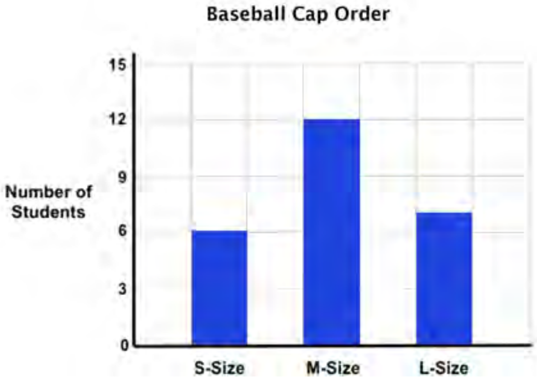
- Present the participants with blank, nonruled paper, grid paper, two pens of different colors, a ruler (with both centimeter and inch sides), and a measuring tape along with the color printed task instructions.
- Present the mock data of 25 individual head measurements to the participants, which students might organize in a table and display in a graph as below:



Head Measurement Results

Interval	Frequency	Note
21¼ - < 22 inches	6	Baseball caps in S-size
22 - < 22¾ inches	12	Baseball caps in M-size
22¾ - < 23½ inches	7	Baseball caps in L-size
Total	25	





Task Instruction for Study Participants

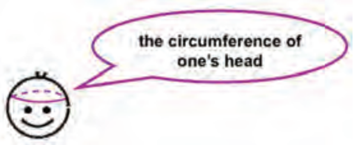
Task 2: Baseball Caps

One baseball team of students was interested in ordering baseball caps for each member of the team. To order these baseball caps, the baseball team coach asked students to list each student's head measurement. [Explain for the participants what a head circumference is by pointing out the figure below and show a measuring tape.]

Individual head measurements in inches are listed below:

List of Students' Head Measures

Student Number (SN)	Head Measure
SN 1	21¼ inches
SN 2	22½ inches
SN 3	22 inches
SN 4	22¼ inches
SN 5	23¼ inches
SN 6	22½ inches
SN 7	22¼ inches
SN 8	23 inches
SN 9	22 inches
SN 10	22½ inches
SN 11	21¼ inches
SN 12	21¾ inches
SN 13	23 inches
SN 14	21½ inches
SN 15	22¼ inches
SN 16	22½ inches
SN 17	22¾ inches
SN 18	23¼ inches
SN 19	23 inches
SN 20	21¾ inches
SN 21	22 inches
SN 22	21¾ inches
SN 23	22¾ inches
SN 24	22½ inches
SN 25	22¼ inches



Baseball Cap Size Chart

S Head: 21¼ - < 22 inches

M Head: 22 - < 22¾ inches

L Head: 22¾ - < 23½ inches

Part 1. Create a way to organize the individual head measurements in order to show the number of head measures in each baseball cap size, and then draw a graph to represent how many to buy of what size.

Part 2. Based on the information presented on your graph and the given "Baseball Cap Size Chart," answer the questions below:

a Among the head measures, which is the second most frequent measure? [If the participant does not know the meaning of "frequent," it will be paraphrased as "the second most common head measure"].

b How many baseball caps in M-size are going to be ordered for this team?

c When compared with the number of baseball caps in M-size to be ordered, how many more or fewer baseball caps in S- and L-sizes are going to be ordered?

d Describe the shape/pattern of your graph with some possible explanations.

Task 3: Description for Interviewers

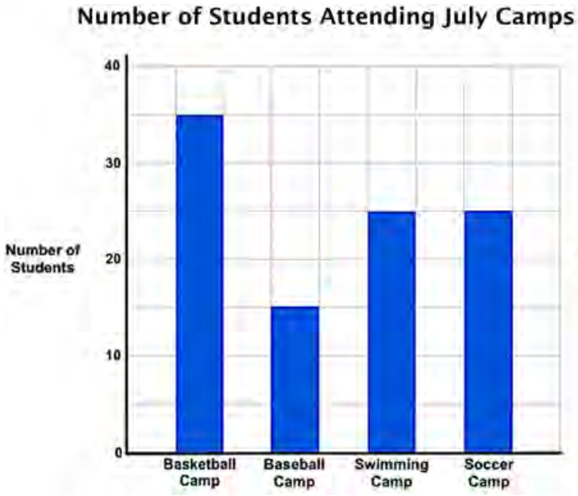
Task 3 Presentation

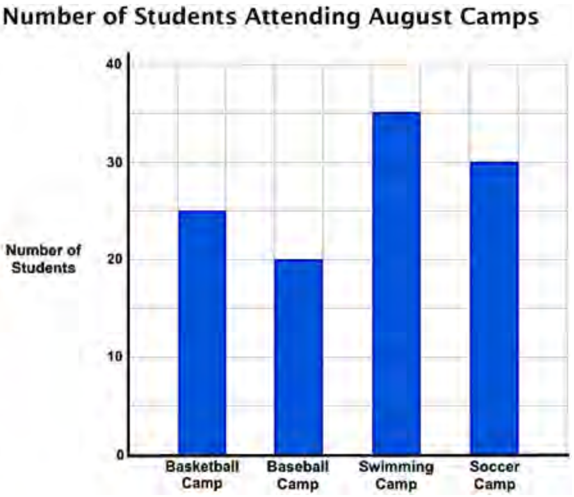
- Present the participant with blank, nonruled paper, grid paper, two pens of different colors, and a ruler (with both centimeter and inch sides) along with the color printed task instructions.
- For Parts 1 and 2 of Task 3, students may draw graphs as shown below:

Task Instruction for Study Participants

Task 3: Sports Camps

Part 1. Some students went to different camps for kids to practice sports. The Director of Sports Camps drew two bar graphs showing how many students went to four different camps in July and August as shown below:





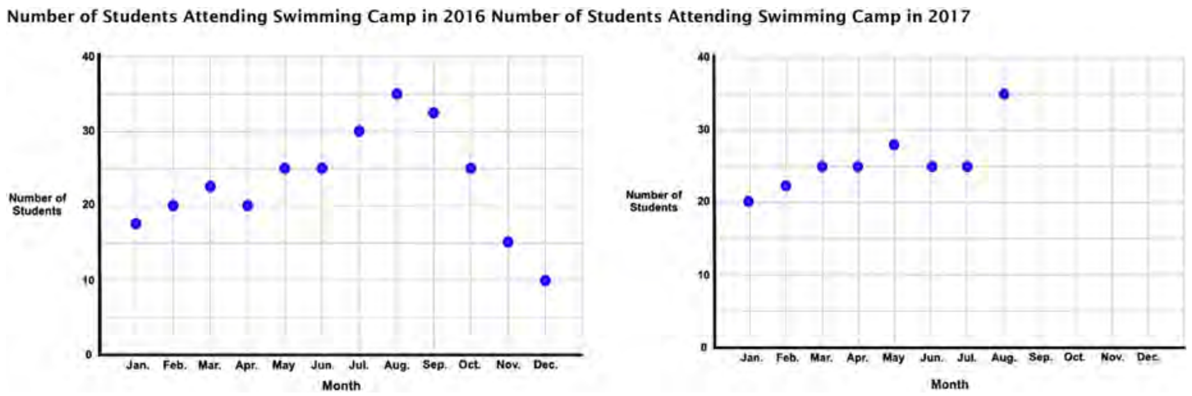
a Draw one graph to represent the number of students who went to the Basketball, Baseball, Swimming, and Soccer Camps in *both* July and August.
b Based on the information presented in your graph, answer the questions below:

  1 In July, which camp was the least popular one?
  2 In August, how many students went to the Swimming Camp?
  3 When compared with the number of students who went to the Basketball and Baseball Camps in July, how many more or fewer students went to these camps in August?
  4 Describe the shape/pattern of your graph with some possible explanations.

Task Instruction for Participants

Task 3: Sports Camps

Part 2. By using the same data as Part 1 (but with more months of data), the Director of Sports Camps drew two scatterplots showing how many students went to Swimming Camp in 2016 and 2017 as shown below:



a Draw one graph to represent the number of students attending the Swimming Camp in *both* 2016 and 2017.
b Based on the information presented on your graph, answer the questions below:

1 In what month did the Swimming Camp have the third highest number of students attending in 2016?
2 How many students went to the Swimming Camp in January 2017?
3 Comparing the number of students who went to the Swimming Camp during the Summer Semester (Jun, July, and August) in 2016 and in 2017, which year had more students attending?
4 Describe the shape/pattern of your graph with some possible explanations.

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