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Mathematical Literacy from the Perspective of Solving Contextual Problems

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Abstract: The article deals with mathematical literacy in relation to mathematical knowledge and mathematical problems, and presents the Slovenian project NA-MA POTI, which aims to develop mathematical literacy at the national level, from kindergarten to secondary education. All of the topics treated represent starting points for our research, in which we were interested in how sixth-grade primary school students solve non-contextual and contextual problems involving the same mathematical content (in the contextual problems this content still needs to be recognised, whereas in the non-contextual problems it is obvious). The main guideline in the research was to discover the relationship between mathematical knowledge, which is the starting point for solving problems from mathematical literacy (contextual problems), and mathematical literacy. The empirical study was based on the descriptive, causal and non-experimental methods of pedagogical research. We used both quantitative and qualitative research based on the grounded theory method to process the data gathered from how the participants solved the problems. The results were quantitatively analysed in order to compare the success at solving problems from different perspectives. Analysis of the students' success in solving the contextual and non-contextual tasks, as well as the strategies used, showed that the relationship between mathematical knowledge and mathematical literacy is complex: in most cases, students solve non-contextual tasks more successfuly; in solving contextual tasks, students can use completely different strategies from those used in solving non-contextual tasks; and students who recognise the mathematical content in contextual tasks and apply mathematical knowledge and procedures are more successful in solving such tasks. Our research opens up new issues that need to be considered when developing mathematical literacy competencies: which contexts to choose, how to empower students to identify mathematical content in contextual problems, and how to systematically ensure – including through projects such as NA-MA POTI – that changes to the mathematics curriculum are introduced thoughtfully, with regard to which appropriate teacher training is crucial.

Keywords: *Contextual problem, mathematical literacy, NA-MA POTI project, non-contextual problem, sixth-grade students, mathematical knowledge.*

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Introduction

Our introduction is organised into four parts. The first part examines the relationship between mathematical literacy and mathematical knowledge. In the second part, we present a Slovenian project on the development of mathematical literacy entitled Science and Mathematical Literacy, the Development of Critical Thinking and Problem Solving (NA-MA POTI), while the third part addresses the issue of solving mathematical problems in connection with contexts from everyday life. We conclude by substantiating why the issues researched in the empirical part are topical.

Mathematical literacy, mathematical knowledge

In order to respond to the rapidly changing world as successfully as possible, questions are repeatedly raised about the key competencies that a student should acquire during schooling. In the present paper, we are interested in the development of some of these competencies in mathematics lessons, which we most often refer to as mathematical literacy. The process of changing the understanding of school mathematics in response to changes in the world is increasingly present. As noted by Goos and Kaya (2019, p. 8), in many countries "curriculum reforms have initiated reconsideration of the nature of school mathematics, leading to changes in the selection and organization of mathematical content and increasing emphasis on mathematical thinking processes, practices and ways of working". If

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mathematical literacy is thought to be reflected in the competent response of an individual to the world, then it is quite understandable that due to the complexity of the challenges of today, there are several definitions of these competencies. These competencies correlate with the so-called competencies of school mathematics. A precise definition of mathematical literacy represents a standard for curriculum design, an analytical tool for assessing the relevance of the curriculum, and a guideline for lesson planning.

Let us examine some definitions of mathematical literacy. Niss and Hojgaard (2019, p. 12) define mathematical literacy as "someone's insightful readiness to act appropriately and respond to all kinds of mathematical challenges pertaining to given situations". It should be emphasised that these situations are not necessarily mathematical; it is only important that they provoke mathematical thinking. They reflect the fact that "a wide variety of mathematical contexts and situations actually or potentially call for the activation of mathematics in order to solve problems, answer questions, and so on" (Niss & Hojgaard, 2019, p. 12). Suciati et al. (2020) define a mathematically literate person as someone who is sensitive towards determining the mathematical concepts that are relevant to problems. Such a person has the ability to understand, analyse, interpret, evaluate and synthesise the information obtained from the problem at hand, and then model it into a mathematical model and determine the solution with the effective use of mathematical concepts.

According to Stacey and Turner (2015), mathematical literacy is the ability to formulate, use and interpret mathematics in various contexts, including mathematical reasoning, using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena in order to assist individuals in making constructive and reflective decisions. Suciati et al. (2020) add that mathematical literacy is seen as mastering the use of reasoning, concepts, facts and mathematical tools in solving everyday problems. This is in line with Kula et al. (2018), who describe problems in mathematical literacy as problems in interesting real-world contexts that require the use of real-life data in modelling problems. It can be concluded that the presented definitions of mathematical literacy are similar in their definition of mathematical thinking, and allow both mathematical and non-mathematical contexts in terms of the problem-solving context.

There is another important reference to consider when discussing mathematical literacy; namely, the Organisation for Economic Co-operation and Development (OECD). According to Umbara and Suryadi (2019), who provide a very clear presentation of the development of the definition of mathematical literacy within the framework of the OECD, the third period, OECD 2013 and 2017, best corresponds to the current understanding of mathematical literacy, as it defines mathematical literacy as the activity of an individual capable of formulating, employing and interpreting mathematics in various contexts. This definition emphasises the meaning of literacy not only in terms of the individual's ability to recognise and understand the role of mathematics, but more in terms of his/her ability to interpret and articulate mathematics in more complex contexts. On the one hand, this definition forms the basis of various definitions of the concept of mathematical literacy in research on mathematics teaching and learning, while, on the other hand, it is loose enough to allow adjustments in the development of mathematical literacy in different education policies.

Before presenting the project NA-MA POTI, which is based on the definition, just presented and is adapted for the Slovenian school space, we will briefly focus on the interdependence between acquiring mathematical knowledge and mathematical literacy. In so doing, we will certainly not answer the question of what mathematical knowledge is; our intention is merely to provide a few definitions. We will focus in more detail on the definition of mathematical knowledge by van den Heuvel-Panhuizen (2003), which we will refer to when interpreting the research results. Umbara and Suryadi (2019) define the process of acquiring mathematical knowledge as a structured sequence of dealing with mathematical content and procedures, in which students are first expected to interpret mathematical tasks that are directly related to concepts, with the aim of developing their ability to recall internalised concepts and processes in appropriate situations, that is, in solving tasks and problems. Dubinsky (2001), for example, stated that mathematical knowledge is an individual tendency to describe the various contexts of mathematical problems by developing analysis and synthesis actions regarding problems involving mathematical processes and objects in order to find solutions to the mathematical problems they deal with.

Since the mid-1980s, the most prevalent framework of mathematical knowledge has comprised two major kinds of knowledge: conceptual knowledge and procedural knowledge (Hiebert, 1986). Later, in line with deeper reflection on the importance of mathematics in life, these categories of knowledge were joined by others, such as contextual knowledge, which is related to everyday life problems in the real world and enters the school setting through the presentation of the problem in a context with its own story (Rittle-Johnson & Koedinger, 2005). Niss and Hojgaard (2019) revised the conceptualisation of the basic notions of mathematical competencies created and came up with two key categories of mathematical competencies: posing and answering questions by means of mathematics, and handling the language, constructs and tools of mathematics. The authors emphasise the fact that these competencies are complementary and overlapping.

Especially interesting is the definition of the criteria that determine the degree to which the student possesses particular competencies. These criteria are: degree of coverage, radius of action, and technical level (Niss & Hojgaard, 2019). Kilpatrick (2002) presented an elaborated view of mathematical knowledge according to five strands: conceptual understanding, procedural knowledge, strategic competence, adaptive reasoning, and productive disposition (i.e., viewing mathematics as a sensible, useful and worthwhile subject to be learned). For the purpose of our research, in order to relate mathematical literacy and mathematical knowledge, we will refer to the division of mathematical knowledge into four levels (van den Heuvel-Panhuizen, 2003): 1) intuitive knowledge (also identified as informal knowledge, the solver is confronted with personally relevant problems to solve, associated with the out-of-school knowledge of students); 2) the concrete level of knowledge (representations and manipulatives, the use of different symbols and manipulatives that play a crucial role in assisting students to build their initial understanding of the concept); 3) computational or procedural knowledge (students' skills in applying procedures for solving problems, consisting of the formal language or symbol representation system, the algorithms or rules for completing mathematical tasks); and 4) principled-conceptual knowledge (students are able to invent procedures that are mathematically appropriate and to recognise that their knowledge can be applied in a variety of different contexts).

This framework allows us to differentiate students' achievements in the field of mathematical literacy, as it presupposes both the importance of the student's informal knowledge (e.g., his/her knowledge of the contexts of the situations present in problems), concrete knowledge (primarily the student's own representations, which s/he develops in a particular context or transfers from other contexts), procedural knowledge and principled-conceptual knowledge. In the case of principled-conceptual knowledge, the main thing is that in a particular context, the learner recognises and applies the procedural and conceptual knowledge that is structured in the school setting. This kind of demonstration of knowledge is defined as the level of knowledge that most effectively connects structured school mathematics and the field of mathematical literacy. In a certain way, we can say that it is the desired level of knowledge for solving contextual problems. This is not to say that other solutions offered by students are unimportant or irrelevant (the learner can approach contextual tasks in various ways, using a range of knowledge); we simply want to emphasise that identifying the mathematical concept in the selected situation and then applying the mathematical knowledge that is optimal for the given situation is one of the essential goals of developing mathematical knowledge. This represents an important starting point for mathematical literacy, which in turn influences the progress of mathematical knowledge.

To summarise, mathematical literacy abilities are dependent on mathematical knowledge and differ in certain contexts. On the other hand, in solving tasks from real-life contexts, which we would define as an activity from the field of mathematical literacy, the student gains new mathematical knowledge through information processing concepts in a specific way (Spangenberg, 2012) and therefore strengthens his/her mathematical knowledge. The interdependence of mathematical literacy and mathematical knowledge therefore presents itself in mutual influence: the strengthening of one component contributes to the development of the other.

The project NA-MA POTI

As mentioned above, since 2018, the systematic development of mathematical literacy on the national level has become topical in Slovenia within the framework of the project NA-MA POTI. As part of the project, thorough reflection is being undertaken on the development of mathematical literacy among students. It should first be emphasised that such projects for assessing and supplementing the functioning of the school system are welcome, especially if they represent certain long-term shifts in school practice. Projects that, in addition to researchers, also involve teachers and preschool educators can represent an important lever for quality shifts in the teaching and learning of mathematics. Education policymakers and practitioners, as well as experts in the field, are jointly responsible for providing definitions of the key concepts and competencies that will be developed among students, thus empowering them to work in various areas of personal and professional life, both domestically and internationally.

In the project NA-MA POTI, we want to approach the definition of mathematical literacy in a way that is both understandable to teachers and sufficiently applicable. It is precisely for this reason that presentations of specific goals in the project are quite concrete and thus more accessible to teachers, who transfer ideas to their practice. In general, we regard mathematics as a subject that can greatly support students' perseverance and develop their reflectiveness (Krek, 2015), even if these goals are becoming more demanding for teachers in a permissive society and in permissive education, which leads to disruptive behaviour of pupils in the classroom (Krek & Klopčič, 2019). Despite the support given to teachers in transferring didactic approaches to developing mathematical literacy to school practice, we encounter a number of problems (virtually none of which, we believe, are systemic obstacles, such as the curriculum, working conditions, etc.).

The primary purpose of the project NA-MA POTI is to develop mathematical literacy among students and to train teachers in didactic approaches to developing these competencies. The initial phase of the project was based on the development of criteria for mathematical literacy. Simply copying and pasting the aforementioned criteria and competencies, which have arisen in other environments or projects, into the Slovenian education system is not possible; nor is there any point in doing so, as criteria and competencies must be based on the reflection of researchers and practitioners with a good knowledge of the functioning of the Slovenian school system and its development, and of the problems and key documents that define the system. In the project NA-MA POTI, we first identified two cornerstones of mathematical literacy:

1. *mathematical thinking*, that is, understanding and using mathematical concepts, procedures, strategies and communication as a basis for mathematical literacy; and
2. *problem solving* in different contexts (personal, social, professional, scientific) that enable a mathematical approach.

The outcomes of the cornerstone *mathematical thinking* that represent prerequisites for mathematical literacy are defined as: understanding texts with mathematical content; knowing and using mathematical terminology and symbols; representing, justifying and evaluating one's own thinking process; recognising, understanding and using mathematical concepts in different situations; knowing and using appropriate procedures and tools in different situations; predicting and evaluating results; justifying statements, procedures and decisions; and using different strategies for solving problems. In the discourse described above, these outcomes can be understood as mathematical knowledge. The second main cornerstone, *problem solving*, is determined by the following three outcomes: solving simple everyday problems (problems that do not demand mathematical modelling), handling situations with mathematical modelling, and understanding mathematical praxes in different contexts. It is obvious that the second main cornerstone focuses on solving problems that are not structured in the same way as typical school mathematics problems. We call these contextual problems.

We briefly define contextual problems below. In this definition, we refer to the background of tasks designed for PISA international testing. In order to assess the level of mathematical literacy of students, PISA tasks are based on seven specific competencies: "thinking and reasoning, argumentation, communication, modelling, problem posing and solving, representation and using symbolic, and formal and technical language and operations" (OECD, as cited in Saenz, 2009, p. 127). These competencies are not task variables but subject variables, and the design of PISA tasks therefore involves a task variable that measures the complexity of the task. The variable considers three levels (Saenz, 2009): *reproduction*, where the tasks have familiar contexts and consist of knowledge that has already been used, the application of standard algorithms; *connection*, which refers to less familiar contexts, interpreting and explaining, selecting and using strategies for solving non-standard problems; and *reflection*, where tasks require understanding, reflection, creativity, complex problems and generalisation of the results obtained. In our research, the design of tasks and problems for determining mathematical literacy is based on this definition of the three levels of tasks.

Problem solving

Many researchers who deal with various ideas associated with problem solving (Cañadas & Castro, 2007; Manfreda Kolar et al., 2012; Mason et al., 2010; Pólya, 1945; Radford, 2008; Reid, 2002; Schoenfeld, 1985) are in favour of encouraging problem solving among students. Despite this fact, problem solving is not accepted by teachers and developers of teaching materials in the same way as other mathematics topics are (e.g., mastering written algorithms, solving equations, etc.). A brief analysis of mathematics curricula in most countries of the world shows that students should solve problems and reason mathematically. The individual's ability to solve problems is related to the structure of his/her mental scheme for problem solving (Hodnik Čadež & Manfreda Kolar, 2015), while the strength and compactness of this scheme depends on the connectivity of components between schema groups (Eli et al., 2013). According to Islami et al. (2018), mathematical connections can be grouped into two categories: 1) internal connections, i.e., connections between topics and mathematical elements; and 2) external connections, i.e., connections between mathematics and other subjects, as well as between mathematics and everyday life. This means that students demonstrate an understanding of mathematics by establishing connections between mathematical concepts, facts and procedures (Hiebert & Carpenter, 1992). Connections also help students to remember skills and concepts, and to utilise them appropriately in problem solving (Eli et al., 2013). Students who have good connections can solve mathematical problems better, while those with poor connection are less successful (Baiduri et al., 2020).

Our research problem aims to provide an inside view of some of the issues mentioned above: (1) the ability of students to treat mathematical concepts in different contexts, and (2) providing teachers with some ideas for selecting tasks and problems to connect mathematical concepts in contextual and no-contextual tasks. The focus of our research is therefore to determine how mathematical knowledge and abilities are used in a variety of contexts. In other words, we want to analyse the process of mathematical literacy as the relationship between the internalisation of mathematical concepts and the externalisation of contexts that play a role in solving mathematical problems (Umbara & Suryadi, 2019). We are aware that through our analysis of students' solutions to the tasks and problems presented in our research, we cannot conceptualise mathematical literacy in terms of the individual capacity to use and apply mathematical knowledge generally; mathematical literacy only applies to situations where this knowledge is used by the individual (Jablonka, 2003). We also share the opinion of Steen (2001), who stated that literacy cannot have a permanent and constant meaning throughout time and place. This can serve as a benchmark in respecting any differences that arise in describing literacy. We would also like to emphasise that when selecting particular tasks and contextual problems, we do not seek to suggest specific contexts. The choice of these contexts must be left to the thorough consideration of the teacher who seeks to enable his/her students to connect mathematical concepts and procedures in contextual and non-contextual tasks. In the research, we selected the mathematical concepts common multiples, powers and fractions, and placed them in specific contexts. It is true that contexts are crucial in the development of mathematical literacy, but they should not dominate the teaching and learning of mathematics, that is, they must relate meaningfully to the mathematical concepts and procedures being treated (Umbara & Suryadi, 2019).

Our research primarily aims to answer the question of how to bring the solution of contextual problems closer to students. At the same time, we seek to help teachers to develop their didactic approaches to developing mathematical literacy in students. The results of our research will contribute to the development of didactic approaches in the project NA-MA POTI, as the reflections and results have the potential to encourage teachers to establish learning situations in which they meaningfully connect contextual and non-contextual tasks or problems without neglecting the development of basic mathematical knowledge. In this light, we view our research problem as relevant to the field of mathematical literacy research.

Methodology

Research model

The empirical study was based on the descriptive, causal and non-experimental methods of pedagogical research (Hartas, 2010; Sagadin, 1991). We used mixed method of quantitative and qualitative research, namely the sequential explanatory strategy which collects and analyzes quantitative data followed by collection and analysis of qualitative data (Creswell, 2003). The data were collected through a mathematical test consisting of six different problems. The results were quantitatively analysed in order to compare the success at solving problems from different perspectives. According to Yin (2017), qualitative research design is used for in-depth investigation of the current situation in real-life contexts. In our case, it was used to obtain an insight into students' strategies and most typical mistakes when solving selected contextual mathematical problems.

Problem definition

Solving contextual problems that reflect a specific real situation and enable the learner to develop strategies and integrate diverse mathematical concepts must be a guiding thread in developing mathematical literacy, with regard to which it is crucial to equip the student with appropriate mathematical knowledge. This begs the question of what is the appropriate mathematical knowledge that equips the student to solve problems. In what case can we say that the student's possession of conceptual and procedural knowledge of the selected concept is such that s/he is, for instance, able to competently solve contextual problems, that is, to recognise the mathematical concept in context? Is demonstrating "school" knowledge always a prerequisite for successfully solving a task/problem that is less structured than a school task/problem? What, then, are the mechanisms that direct or determine the success of the student in dealing with situations in which the mathematical idea is not transparent and the learner must recognise the idea in order to engage with the solution? We are interested in such questions, with a focus on students' solutions to contextual problems (in comparison with non-contextual problems).

The aim of the study, which was conducted with sixth-grade students, was to explore the relationship between students' knowledge of mathematics (referred to as the cornerstone *mathematical thinking* in the project NA-MA POTI) and their success in solving contextual problems (referred to as the cornerstone *problem solving* in the project NA-MA POTI). The basic guideline of our interest was to explore students' understanding and their strategies for solving selected contextual mathematical problems. Quantitative research was used in order to compare students' success in solving contextual problems with corresponding non-contextual problems, where correspondence means that the same mathematical idea was present in the contextual problems as in the non-contextual problems, but was not directly visible. In addition, we were interested in whether the students would recognise that pairs of tasks were related to the same mathematical context, and that the process used in the mathematical tasks could be transferred to solving the corresponding contextual problems.

Furthermore, qualitative research was used to analyse the students' strategies for solving the contextual problems. We wanted to explore the influence of school mathematics on the choice of strategies when solving contextual problems from real-life situations, and to identify the most typical misconceptions that occur among students when solving a contextual problem. An analysis of the errors revealed patterns that allowed us to relate the main type of knowledge and the subject competencies in more depth.

Research questions

The following research questions were defined:

1. How is success in solving a mathematical problem related to the context of the problem (contextual vs. non-contextual)?
2. What type of strategy is used by students when solving contextual problems?
3. What kind of mistakes do students make in solving a particular contextual problem?
4. Do the students recognise the same mathematical content in the non-contextual problem and the contextual problem?

Sample description

A non-probability purposive sampling technique was used for the purpose of our study. The study was conducted in three public primary schools located in different urban areas in Slovenia. The sample of students consisted of 72 sixth-graders, between eleven and twelve years of age. All of the participating students were from standard classes. In all of the classes that participated in our sample, teachers were following the general goals of the national curriculum for mathematics.

Data processing procedure

The data and sources in this study were obtained from students' problem-solving tests, which were analysed according to the solution process and the final solution. We created a test for sixth-grade students that consisted of six mathematical problems from selected arithmetic content:

1. Problems 1 and 2 were related to the mathematical content of common multiples;
2. Problems 3 and 4 were related to the mathematical content of powers;
3. Problems 5 and 6 were related to the mathematical content of fractions.

In all of the pairs of problems, the first item corresponded to a non-contextual problem and the second to a contextual problem.

For each problem, the student had to present:

1. the solution process;
2. the final solution.

Each student solved the problems individually with no additional explanation by the teacher or researcher. The students' reasoning was analysed on the basis of their written work, which included solving the problems and writing notes about the process. The time for solving the list of problems was not limited.

Table 1. List of problems used in the study

Non-Contextual Problem	Contextual Problem
Problem 1 Find all of the numbers up to 100 that are multiples of eight and six at the same time. What is the smallest such number?	**Problem 2** Andrea became ill and her doctor prescribed her medication. One tablet should be taken every four hours and an antibiotic should be taken every six hours. As soon as she gets home, she takes both medicines at once. After how many hours will she take both medicines at once again? How many times will she take both medicines at once in the next 60 hours?
Problem 3 Calculate what is more: $2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9 + 2^{10}$ or 100×20?	**Problem 4** Matic is facing the following problem: His parents offered him two possible plans to pay out pocket money for the next ten weeks, and he has to decide which he will choose: Plan 1: He receives €2 each week. Plan 2: The first week he gets two cents, then each week the amount from the previous week doubles. Try to find out which plan is more profitable for Matic.
Problem 5 Guess My Number: I have a number and divide it into thirds. I keep two-thirds and divide it into thirds again. Again, I take two-thirds and divide it into thirds. Two-thirds of the last division represents the number eight. What was my initial number?	**Problem 6** Mina, Kaja and Lara have just returned from school. Mina comes into the kitchen and finds a full tray of freshly baked biscuits on the table, along with a note with a message from her mother: "You should distribute the cookies fairly among yourselves." Mina eats a third of the biscuits from the tray and leaves. Then her sister Kaja enters. Not knowing that Mina has already been in the kitchen, she reads her mother's message and eats a third of the biscuits from the tray. Finally, Lara enters. Unaware that Mina and Kaja have already been in the kitchen, she eats a third of the cookies from the tray. When their mother returns home, she finds that there are still eight untouched biscuits on the tray. How many biscuits did their mother bake?

Table 1 presents the pairs of problems, with the non-contextual problems (reproduction and connection level according to OECD (2003)) in the left column, and the corresponding contextual problems (connection and reflection level according to OECD (2003)) in the right column. We regarded the problems from the right column as contextual problems because they are taken from real word situations that children are directly or indirectly familiar with: pocket money, fair distribution, and taking medication.

Content validity was estimated by testing the feasibility or relevance of the test content through rational analysis by a competent panel of experts (Tohir et al., 2020). In order to assess the validity of the six problems in our test, consultations were undertaken with experts participating in the project NA-MA POTI and with mathematics teachers of the selected classes, who verified whether the tasks matched the curricular learning outcomes related to fractions, powers and common multiples in the sixth grade. The content validity of the problems for the students was further examined by a pilot study that included two classes of sixth-graders. The aim of the pilot study was to provide feedback regarding the suitability of the selected problems for the objectives of our study. Based on the students' solutions received, it was decided to replace two tasks that proved to be unsuitable for measuring the relationship between non-contextual and contextual problems. We established the objectivity of our research by giving the students of different classes equivalent, precise instructions about the problem-solving process. An inductive process was used to determine how to categorise the responses and to find relationships between the different categories (Tohir et al., 2020). This process consisted of the following three stages:

1. In the first stage, the responses from 20 students were evaluated and those indicating the same idea were grouped together. The categories were created by the researchers and authors of the present paper.
2. In the second stage, the solutions of the remaining 52 students were classified and additional categories were created where necessary.
3. In the final stage, the researchers switched roles and classified the students' solutions according to the created categories. This stage was repeated until there was complete consensus on the categorisation of the results.

The following section presents an analysis of the results, as well as various observations.

Results

In the continuation, we will present the results in the same order as proposed by the research questions:

1. from the perspective of the students' success in solving non-contextual and contextual problems;
2. from the perspective of the students' strategies used for solving contextual problems;
3. from the perspective of the students' mistakes in solving a particular contextual problem;
4. from the perspective of the relationship between the strategies used for solving non-contextual and contextual problems.

After presenting the results, we will interpret them with regard to our research questions.

Success in solving non-contextual and contextual problems

Let us first explore the results according to the success in solving non-contextual versus contextual problems.

Table 2. Success in solving non-contextual and contextual problems

Non-contextual problem	Frequency (%)	Contextual problem	Frequency (%)
Problem 1	32 (44%)	Problem 2	23 (32%)
Problem 3	17 (24%)	Problem 4	12 (17%)
Problem 5	5 (7%)	Problem 6	11 (15%)

The results show that in the case of problems with common multiples and powers, the students were better able to solve the non-contextual problem than the contextual problem with the same mathematical content. Exactly the opposite was found for the problems with fractions: the students were more successful in solving the contextual problem.

Analysis of students' strategies for solving contextual problems

The analysis of the students' strategies for solving contextual problems will be presented only with regard to Problems 2 and 6. The decision to narrow the field of studying strategies for the purpose of the second research question stems from the fact that the mathematical contexts of common multiples and calculating with fractions are not strictly reduced to one possible procedure, and we therefore expected a wider range of approaches among the students. In both problems (2 and 6), we were firstly interested in whether the students would recognise the mathematical context in solving a real-life problem and use standard, school-learned procedures, and, secondly, whether they would develop new strategies for solving the contextual problem.

Table 3. Students' strategies for Problem 2

	Strategy	Frequency (%)	Correct solutions
p2_1	writing down multiples for both numbers	15 (20.8%)	11
p2_2	only multiples of the first number and underlining the common multiples	1 (1.4%)	1
p2_3	adding numbers, starting from a certain common point	7 (9.7%)	7
p2_4	number line	5 (6.9%)	4
p2_5	clock	1 (1.4%)	0
p2_6	meaningless records	20 (27.8%)	0
p2_7	multiplying both numbers together	5 (6.9%)	0
p2_8	no solution	18 (25.0%)	0
	total	72 (100.0%)	23

In Problem 2, the standard school-learned solution process is represented by the strategies "writing down multiples for both numbers" and "only multiples of the first number and underlining the common multiples". Only 22.2% of the students used these strategies. In the comparable, non-contextual Problem 1, as many as 57% of the students employed the strategy used in school for such types of tasks. We can see, therefore, that the proportion of students who recognised the mathematical context in Problem 2 and were able to apply the learned school procedures to situations with a different, non-mathematical context, declined sharply. On the other hand, the context of Problem 2 encouraged students to explore new strategies that did not occur in Problem 1. These are: "adding numbers, starting from a certain common point" (Figure 1c), "number line" (Figure 1b) and "clock" (Figure 1a).

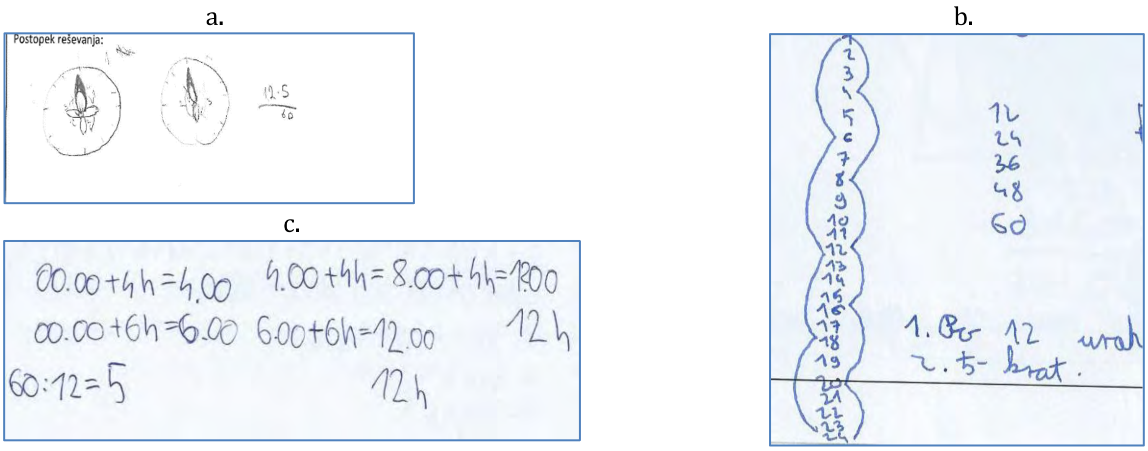

Figure 1. Examples of students' new strategies: a. "clock", b. "number line", c. "adding numbers, starting from a certain common point"

The common property of these three strategies is that they retain a connection with the context of the problem: the strategies of line number and clock are based on a graphical representation, while "adding numbers, starting from a certain common point" is a symbolic representation that reflects the temporal flow of the two medicine-taking events: the mapping contains the common starting time, to which the temporal intervals of four hours and six hours are then added, until the events coincide again at the same moment. In terms of effectiveness, we note that not all of the three new strategies were equally effective. All of the students who used the strategy "adding numbers, starting from a certain common point" were successful in solving the problem. Among the students who used the number line, four out of five were successful, while the student who used the clock was not successful (Table 3, column 3).

Let us further explore the strategies that emerged when solving the fraction problem, Problem 6. The strategies used by the students with correct solutions and incorrect solutions are presented separately. The erroneous strategies are presented in the next subsection, which refers to students' mistakes in solving a particular contextual problem.

Table 4. Correct strategies for Problem 6

	Strategy	Frequency
C_1	record of partial results without intermediate procedures (e.g., 8, 12, 18)	5
C_2	broken-down calculations with intermediate steps (e.g., 8 : 2 = 4, 4 x 3 = 12, 12 : 2 = 6, 6 x 3 = 18, etc.)	2
C_3	graphical representation	1
C_4	equations	1
C_5	solution in a forward direction	1
C_6	typical school strategy	1
	total	11

a. b. c.

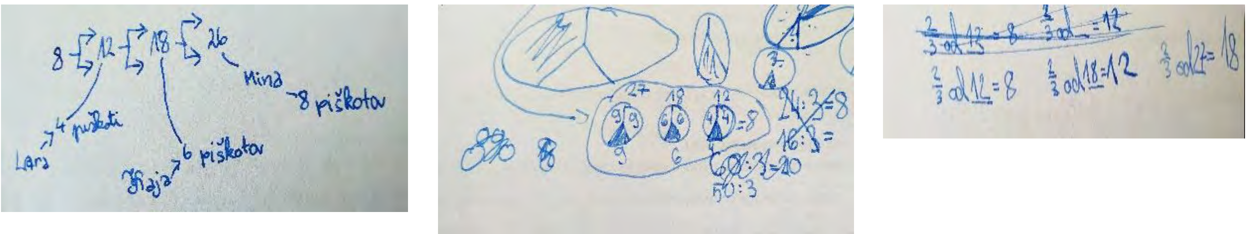

Figure 2. Examples of the students' strategies for Problem 6: a. example C_1, b. example C_3, c. example C_6

The majority of the correctly used strategies were based on the backward solution of the problem, i.e., the students derived from the known data of the number of cookies remaining at the end, then gradually moved towards the initial situation. The only student who solved the problem in the forward direction was the student who used a "trial and error" strategy, but his solution record does not show how he discovered the initial number 27 that led to the final residual 8. Again, we can see that among the few students who solved this problem correctly, only one student strictly used the school procedure that is prescribed for solving such types of tasks (Strategy C_6, Figure 2a). This was followed by students who simplified school-learned procedures (Strategy C_2), and then students who performed most of the process in their minds and recorded only significant intermediate results (C_1). It is worth highlighting one student who provided a very clear graphical representation of solving the problem (Figure 2b), and another student who used procedures not taught until upper primary school (Strategy C_4) to solve the problem.

Analysis of the students' mistakes in solving a particular contextual problem

Since the students were the least successful in solving Problem 6 regarding fractions, we decided to undertake a more detailed analysis of the erroneous strategies that students used in their solutions and attempted to identify the causes of the errors. We believe that it is precisely this kind of analysis of the student's mistakes that can enable the teacher to gain an insight into the student's thinking and his/her deficits in the acquisition of certain types of knowledge, as well as indicating a path forward in planning mathematics lessons with the aim of developing mathematical competencies in the areas of deficit.

Table 5. Incorrect strategies for Problem 6

	Strategy	**Frequency**
I_1	The remainder represents a third, the whole is growing: 8 x 3 = 24, 24 x 3 = 72, 72 x 3 = 216	8
I_2	Ignoring the remainder, the whole is fixed: 3 x 8 = 24	6
I_3	The remainder represents a third, the whole is fixed: 4 x 8 = 32	8
I_4	A third is equal to 3: 3 x 3 + 8	5
I_5	A part represents a rational number: 1/3 +1/3 + 1/3 + 8	1
I_7	Graphical representation	1
I_8	Other (each used only by one student)	6
I_9	Meaningless record	5
I_10	No solution	21
	Total	61

Some of the incorrect solutions are presented in more detail and analysed from the perspective of the misconceptions about fractions that are evident in the solution process.

Strategy I_1: The student attributes the wrong meaning to the number 8 and solves the task as if there were one third of the cookies remaining instead of two thirds. On the other hand, the three equations in the solution process indicate that the student is aware that the whole changes during the problem solution process and therefore executes a backward solution process in three consecutive steps.

Strategy I_3: As in strategy I_1, the student assigns the role of one part of the whole to the number 8 instead of two parts of the whole, but at the same time fails to realise that the whole changes during solution. All three girls therefore receive the same part. The solution indicates that the student has divided the whole into 3/3 + 1/3. The remaining 8 therefore actually represents a quarter of the initial total. According to Hackenberg (2007), the student has difficulties with coordinating units: s/he thinks of a third iterated four times as four quarters and each part is considered as being transformed into one quarter.

Strategy I_4: This strategy indicates a misunderstanding of the various aspects of fractions, particularly the fraction as a number: the student does not distinguish between the fraction number 1/3 and the natural number 3.

Strategy I_5: The student converts the part 1/3 of the whole to the rational number 1/3. S/he therefore confuses two different aspects of fractions: fractions as dividing a given whole, and fractions as positions on a number line. This kind of error has been identified by several researchers (Kieran, 1976; Hannula, 2003; Ni, 2001; Tunç-Pekkan, 2015).

Relationships between strategies for solving non-contextual and contextual problems

In determining the students' recognition of the same mathematical content in a non-contextual and a contextual problem, we proceeded from an analysis of the solution strategies for individual pairs of problems (1 and 2; 3 and 4; 5 and 6). Based on a comparison of student strategies, we formed two categories:

1. matching strategies: when the student solved a particular pair of problems in the same way, with the same intermediate and final results, regardless of whether or not s/he solved the problems correctly;
2. mismatching strategies: the method of solution differed in a particular pair of problems.

Table 6. Number and percentage of students with matching strategies in solving particular pairs of problems

	Problems 1 and 2	**Problems 3 and 4**	**Problems 5 and 6**
Matching strategies	10 (14%)	19 (26%)	10 (14%)
Mismatching strategies	62 (86%)	53 (74%)	62 (86%)
Total	72 (100%)	72 (100%)	72 (100%)

We used matching strategies with all of the intermediate solution steps as a measure for assessing whether, while solving the non-contextual and the contextual problem, the student recognised that both examples involve the same mathematical content.

The results in Table 6 show that for all three pairs of problems, only a few students recognised their interconnectedness, with the highest proportion of recognition being in the problems concerning powers (26%).

We were also interested in the proportion of students who correctly solved the contextual problem by relying on the strategy used to solve the non-contextual problem, and the proportion of those who used a different strategy in solving the contextual problem (Table 7).

Table 7. Number and percentage of students with matching strategies in connection to the success of solving the problems

	Problems 1 and 2	**Problems 3 and 4**	**Problems 5 and 6**
Matching strategies	7 (30%)	11 (92%)	4 (36%)
Mismatching strategies	16 (70%)	1 (8%)	7 (64%)
Total	23 (100%)	12 (100%)	11 (100%)

We again see that within the group of those who successfully solved the contextual problems, the majority of the students recognised the connection of the problems about powers. In the case of the problems regarding common multiples and fractions, respectively, a smaller proportion of success was observed, thus indicating non-recognition of the relatedness of the problems.

Discussion

In the continuation, we provide interpretations of our results and answers to the research questions.

Research Question 1: How is success in solving a mathematical problem related to the context of the problem (contextual vs. non-contextual)?

We can observe that the non-contextual problems differ in degree of difficulty (Table 1). Problem 1 is an example of a typical school task of finding a common multiple of two numbers, so students can use the classical procedure learned in school. Problem 3 is also a school example of a task. It is given in the form of a symbolic record, so all the student has to do is follow the algorithm. It is, however, a more complex task, also requiring an understanding of mathematical notation. This is confirmed by the analysis of the students' errors in solving Problem 3, which can be categorised into two major groups: errors of a procedural nature, which occurred mainly due to the complexity of the solution process (32% of the students), and conceptual errors, i.e., failing to understand the notation for powers (28% of the students). In this group, the error of substituting powers with multiplication dominated. Problem 5 requires the student to convert the text into a mathematically structured notation. The solution path is not obvious, and most of the students had difficulty understanding the text: they were unable to transfer it into a proper mathematical record.

Comparing the success in the non-contextual tasks with the related contextual tasks reveals an interesting two-way relation:

1. Higher achievement in the non-contextual tasks for the content common multiplies and powers: the transition from classical, routine and familiar mathematical tasks to new non-mathematical contexts is difficult for students. They are unable to translate the learned mathematical knowledge into a real-life situation in which the mathematical context is hidden (Problems 1 and 2, and Problems 3 and 4).
2. Higher achievement in the contextual task for the content fractions: when solving a more complex task with a core mathematical context, we can clarify students' understanding by assigning a realistic context to the task (Problems 5 and 6).

Relation 1 clearly shows that possessing knowledge and a comprehensive view of mathematics as a discipline does not necessarily suffice to generate mathematical literacy in the individual learner (poorer student achievements in solving contextual problems about common multiples and powers compared to non-contextual tasks). The students' mathematical understanding is therefore questionable. As Afgani et al. (2019) state, an individual with mathematical understanding will be able to use this ability to learn mathematics without any significant difficulties and can think mathematically where this way of thinking is very useful in everyday life. This highlights the importance of the strength and compactness of the individual's scheme for solving problems, which, according to Eli et al. (2013), depends on the connectivity of components between schema groups. Students with good connections can solve mathematical problems better, while those with poor connections are less successful (Baiduri et al., 2020). We can conclude that the decline in the success of solving contextual problems in comparison to non-contextual problems indicates poor connections between the cornerstone *mathematical thinking* and the cornerstone *problem solving* (according to the NA-MA POTI definition).

Relation 2 indicates that although the result shows that the students were more successful in the selected contextual task than in the corresponding non-contextual task, one should not generalise this finding and conclude that lessons should be primarily focused on solving contextual tasks. As North and Christiansen (2015) find, although this can lead to the successful mathematical treatment of individual contexts, it cannot replace the development of abstract mathematical concepts.

Our results are in line with Celik (2019) who concluded that students encounter various mathematics problems in daily life on different levels of difficulty and suggested that activities for students should be organized in a way which would clearly demonstrate the usage of mathematics in daily life, provide basic concepts and skills and help improve their mathematical thinking and problem-solving skills.

<u>Research Question 2: What type of strategy is used by students when solving contextual problems?</u>

We were interested in whether the students would use learned, school strategies specific to the given learning content in solving a contextual problem, or whether they would develop their own, new solution strategies. We found that in common multiples, the share of students who used school solution procedures was quite low (16 students (22.2%), of which 12 were successful), while only one student successfully solved the problem regarding fractions using a school procedure. Among the students who solved the problems correctly, a large proportion of them employed new, non-school solution strategies: in common multiples, 23 of the students solved the problem correctly, of which 11 used a non-school strategy; in the case of the fractions problem, a non-school strategy was used by 10 of the 11 students who provided correct solutions.

The strategies used in solving the selected contextual problems were further analysed from the perspective of types of knowledge according to van den Heuvel-Panhuisen (2003):

1. In the case of Problem 1, all procedures based on looking for a common multiple were defined as procedural knowledge. If the student used the same strategy of looking for a common multiple for solving Problem 2, we categorised his/her knowledge as principled-conceptual knowledge. In this case, the student had to find a relationship between Problems 1 and 2 and apply school mathematical knowledge (learned principles) meaningfully.
2. We defined the new strategies in Problem 2 referred to above as "adding numbers, starting from a certain common point", "number line" and "clock" as presenting concrete knowledge, since the context encouraged students to choose/develop a new strategy.
3. All of the strategies used for solving Problem 6 correspond to principled-conceptual knowledge except C_3, which represents concrete knowledge.

According to van den Heuvel-Panhuisen (2003), it follows that the majority of students who successfully solved contextual Problems 2 and 6 used solution strategies at higher levels.

Developing mathematical literacy through solving contextual problems imposes an important task on the teacher, that is, the development and promotion of various solution strategies in students. Teacher effectiveness research has shown that effective teachers are expected to help students both to use strategies and to develop their own strategies in order to enable them to solve different types of problems (Amit & Portnov-Neeman, 2017; Kyriakides et al., 2002). A backward strategy is a specific type of solution strategy that students need to learn, and it is rarely present in school mathematics assignments. It is precisely such less common strategies that require special attention and should be systematically developed in the classroom. The students' poorer achievements in solving the problem about fractions in general could also be attributed to the solution strategy that must be used: this problem requires a backward strategy, while the problems about common multiples and powers can be solved by the learner "in a forward direction".

Research Question 3: What kind of mistakes do students make in solving a particular contextual problem?

The analysis of the students' mistakes is presented regarding the selected contextual problem about fractions (Problem 6). Erroneous solution strategies (Table 5) and their analysis expose some of the students' misconceptions about fractions:

1. disregarding the changing whole when solving problems, whereby we calculate the part of a part of the whole;
2. misunderstanding the context of the problem: the student misinterprets the given data and assigns the wrong meaning to the rest of the cookies;
3. misunderstanding various aspects of the concept of fractions: the student shifts between different aspects and is not aware that the notation1/3 can have different meanings;
4. non-differentiation between different sets of numbers: natural and rational numbers.

Based on an analysis of the errors in the solution strategies for Problem 6, we determined that the errors can be classified into those related to limitations in school knowledge (misunderstanding the concept of fractions), and those related to difficulties in activating mathematical competencies (the student does not know how to adapt the appropriate mathematical concept to a contextual problem).

The poorer results in Problem 6 compared to Problems 2 and 4 are therefore also a reflection of the latter: difficulties of a contextual nature were compounded by difficulties of a conceptual nature, which were less evident in the mathematical content of common multiples and powers. According to Rittle-Johnson and Koedinger (2005), we can therefore speak of a deficit both in the area of contextual knowledge and in the field of conceptual knowledge. A similar analysis of results can be found in a study by Saenz (2009), which showed the importance of contextual knowledge as a link between school mathematics (expressed as concepts and procedures) and the activation of the competencies required to solve contextual problems.

Research Question 4: Do the students recognise the same mathematical content in the non-contextual problem and the contextual problem?

The results (Table 6) show that only a few of the students recognised the same mathematical content in the pairs of content-related problems. The slightly better recognition of the similarity of the problems related to powers is not surprising, as the structure of the text itself suggests that Problems 3 and 4 are more obviously connected to each other, and we believe that in the process of solving a contextual problem, the student more quickly recognises a mathematical context that is identical to the previously solved problem. In Problem 2, on the other hand, reading the text alone does not indicate a connection with the mathematical context of finding a common multiple. It can be said that the connection between Problems 1 and 2 is not as obvious on the outside as the connection between Problems 3 and 4: it requires the solver to have a high level of understanding of mathematical context (taking several types of medication simultaneously depends on the different intervals between taking each medication) and to create connections between this context and the mathematical context of common multiples.

We believe that an insight into the connection of these two contexts is required, whereas, in the problems with powers, the solver can gradually realise the connection between the two problems through the process of solving the contextual problem. The occurrence of matching strategies for the fraction problems (Problems 5 and 6) was also lower than for the problems concerning powers. Here, this cannot be attributed to the non-recognition of the mathematical context in the contextual problem, as the student can quickly see from the text that this problem is related to the calculation of parts of a whole. The difficulties lie elsewhere, in the complexity of the initial, non-contextual problem, which the students solved more poorly than the rest of the problems, suggesting that it presented an obstacle rather than a bridge to solving the contextual problem.

We again see that within the group of those who successfully solved the contextual problems, the majority of the students recognised the connection of the problems about powers. In the case of the problems about common multiples, a smaller proportion of success was associated with non-recognition of the relatedness of the problems, because in solving Problem 2, the successful students used the new problem-solving strategies that we have shown above. In solving the problems about fractions, a smaller proportion of success is related to the failure to solve Problem 5. In summary:

1. the poorer matching of strategies in the problems concerning common multiples is due to a failure to recognise the relatedness of the two problems and the consequent transition to new solution strategies;
2. the poorer matching of strategies in the problems about fractions is a consequence of the failure to solve the non-contextual problem, which, from the outset, represents a difficulty in solving new contextual examples.

We can only support the conclusions of Umbara and Suryadi (2019), who find that the desired process of mathematical literacy is based on a strong relation between the internalisation of mathematical concepts and the externalisation of contexts that play a role in solving mathematical problems.

Conclusion

The results obtained highlight the need for careful reflection on the appropriate way to develop mathematical literacy in schools and suitable ways to integrate contextual problems into mathematics instruction. We believe that despite the best intentions, the poorly considered selection of problems can hinder rather than encourage the development of principled-conceptual knowledge. The transition from the level of reproduction to the level of connection and reflection (OECD, 2003) can be manifested under different conditions, which determine the level of difficulty of this transition. The transition can be made more difficult by choosing overly demanding tasks that do not belong on the level of reproduction, or by initiating a transition from the level of reproduction to the level of reflection or connection that is not transparent.

Planning mathematical literacy by creating connections between non-contextual and contextual tasks therefore requires consideration of the factors that affect the degree of complexity of the transfer of knowledge to life situations, specifically:

1. the level of difficulty of the initial non-contextual mathematical task;
2. the recognisability of the connection between mathematical contexts in non-contextual and contextual tasks.

Among our pairs of problems, we consider the problems about powers to be the most appropriate pair for developing children's mathematical literacy, as it meets both of the stated conditions to the greatest extent. The problem about common multiples does not meet the first of the proposed factors, as the mathematical context in the contextual problem is concealed to the solver. On the other hand, the problem about fractions does not meet the second of the proposed factors, as the initial, non-contextual task presents too high an obstacle for students to successfully solve it.

We are aware that it is not easy to formulate quality contextual problems, and we agree to some extent with Alsina (2002, p. 244), who writes: "Many mathematics examples presented in frozen textbooks are useless because the applications which are explained are already understood". This part of instruction can undoubtedly be improved, especially through appropriate teacher education.

In conclusion, we would like to draw attention to an important aspect of treating contextual problems: whether the thoughtful selection of contextual problems can stimulate and improve the student's understanding of non-contextual mathematical tasks and the use of typical mathematical strategies. Some examples from our study suggest this possibility; namely, the students who solved the contextual problem more successfully than the corresponding non-contextual problem. These examples prove that on the basis of an unsolved non-contextual mathematical problem, we cannot conclude that the learner could not solve a contextual problem with the same mathematical content. This is in line with the findings of Sfard (1991), who emphasises that identifying the conceptual problems experienced by students in trying to understand knowledge can be a difficult process, and that even when students do not seem to understand mathematical concepts, it is still possible that they have the knowledge in their thinking process.

Developing mathematical literacy enables the student to act in a mathematical way, making use of the mathematics subject areas occurring in various manifestations in different contexts. Mathematical literacy can be expressed in contexts that potentially or actually involve mathematical ideas. The question of how mathematical literacy and school mathematics complement each other remains open. We know that mathematical literacy is not realised without a knowledge of mathematics (this is confirmed by our results), and that mathematical knowledge that, for the student, is not realised in a particular context, or that is not shown in context, is questionable.

We shall mention three issues that are, in our opinion, important for improving students' mathematical literacy: the teacher's role, connecting mathematical literacy with computer literacy, and support of textbooks.

Research on the teacher's role stresses the complex relationship between teachers' knowledge and school practice, results and perspectives (Leikin & Levav-Waynberg, 2007). Sullivan (2011) defines six key principles for effective mathematics teaching: articulating goals, making connections, fostering engagement, differentiating challenges, structuring lessons, and promoting fluency and transfer. These key principles – together with teachers' knowledge of content and teaching, content and students, and the curriculum – can serve as prerequisites for teachers in organising and reflecting on their teaching practice. The basic idea is that learning mathematics involves the development of mental schemas that require students to connect mathematical ideas and help them to become good problem solvers.

In order to achieve this goal, the focus must be on the systematic development of different types of mathematical knowledge, considering that different types of knowledge require different types of approaches. For example, insisting on repetition is crucial to achieving basic procedures and knowledge, whereas for principled-conceptual (and other) skills and knowledge, developing heuristic is necessary.

Connecting mathematical literacy to computer literacy has become very topical at recent times (e.g., Geraniou & Jankvist, 2019). Regarding this connection, Ic and Tutak (2017) concluded that there is a moderate and positive correlation between computer literacy and mathematical literacy levels of sixth-graders, and that computer literacy influences the mathematical literacy of the individual.

School textbooks also play an important role in improving mathematical literacy. Despite remaining the most widespread tool among teachers for pursuing learning mathematics, they have been slow to adapt to the most recent epistemological paradigms, often still conveying distorted views on mathematics. "Textbooks facilitate the teacher's work guarantee mathematical knowledge and exercises for students and are consistent with the school system, but they reduce both freedom and responsibility of the teachers" (Johansson, as cited in Glasnović, 2013, p. 214).

Calado and Bogner (2013) present a theoretical framework specifically intended to highlight the potential of textbooks to promote students' scientific literacy. This framework can also be adopted for the analysis of the treatment of mathematical literacy in mathematics textbooks. The authors argue that often the "misconceptions conveyed by textbooks represent obstacles to the acquisition of a fair image of science and, therefore, to the acquisition of scientific literacy" (Calado & Bogner, 2013, p. 51). This argument could also hold for mathematics textbooks in relation to mathematical literacy. Teachers who are willing to accept the challenge of promoting mathematical literacy are often left to their own devices. Considering all of the issues mentioned above, the key thing is our expectations, which must remain high, as well as our awareness of the responsibility we have in educating students.

Recommendations

We believe the present study can encourage teachers to reflect on their teaching practice when dealing with developing students' competencies in mathematical literacy. Our research can also contribute to diagnosing students' understanding of mathematical concepts – as well as their misunderstandings – and ways of using representations when solving problems that are not necessarily the same as those presented in the school structured environment.

Based on the results of the study, we suggest that one of the possible ways to develop mathematical literacy is to include pairs of problems that are connected by common mathematical content, but differ in the degree of connection with a real situation, that is, pairs of non-contextual and contextual problems. When designing pairs of non-contextual/contextual problems, we can differentiate the entry, non-contextual task and the exit, contextual task. Differentiation of the entry task refers to the level of mathematical difficulty of the task, while differentiation of the exit task refers to the transparency of the mathematical context, which can be more or less concealed.

We believe that in the beginning, it is necessary to ensure that the initial, non-contextual task is one in which the students recognise the mathematical context and can solve the task successfully using appropriate mathematical procedures. If the selected initial task is too mentally demanding for the student, then, at the very beginning, we eliminate the possibility of transferring knowledge to a more demanding, contextual problem. Furthermore, when selecting a contextual problem, it is advisable to initially select contexts in which the learner quickly recognises the connection with a particular mathematical context and then gradually proceed to contextual problems where the mathematical concept is not as easily recognisable. To address the development of mathematical literacy at all levels of schooling, it is necessary to accommodate mathematical literacy in curricula. The high-level thinking activities involved in literacy can be trained by designing functional learning approaches that enable competencies to connect the real world with the world of mathematics. At the same time, the evaluation of students' procedures and mistakes in solving contextual problems must be executed regularly in practice in a planned way.

Limitations

We are aware that our findings are related only to specific arithmetic problems solved by a selected sample of students. Further research is therefore needed to expand the conclusions to different mathematical concepts. The aim of the present paper was primarily to stimulate reflection on the relationship between the cornerstones of *mathematical thinking* and *problem solving*, as defined in the project NA-MA POTI. Our intention was not to generalise our results, but to raise certain issues that can stimulate researchers to investigate this relationship in the direction of identifying criteria for the selection of problems, designing the teaching approach to problems, and understanding students' strategies for solving different types of problems, especially problems connected in concepts but differing in context (mathematical context and everyday life context).

References

Afgani, M. W., Suryadi, D., & Dahlan, J. A. (2019). The enhancement of pre-service mathematics teachers' mathematical understanding ability through ACE teaching cyclic. *Journal of Technology and Science Education*, *9*(2), 153–167. https://doi.org/10.3926/jotse.441

Alsina, C. (2002). Too much is not enough. Teaching math through useful applications with local and global perspectives. *Educational Studies in Mathematics, 50*, 239-250. https://doi.org/10.1023/A:1021114525054

Amit, M., & Portnov-Neeman, Y. (2017). Explicit teaching' as an effective method of acquiring problem solving strategies - the case of 'working backwards. In T. Dooley, & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education (CERME10)*. DCU Institute of Education and ERME. https://hal.archives-ouvertes.fr/hal-01950528

Baiduri, , Utami, O.R., & Alfani, P.I. (2020). Mathematical connection process of students with high mathematics ability in solving PISA problems. *European Journal of Educational Research, 9*(4), 1527-1537. https://doi.org/10.12973/eu-jer.9.4.1527

Calado, F. M., & Bogner, F. X. (2013). A Reflection on distorted views of science and technology in science textbooks as obstacles to the improvement of students' scientific literacy. *European Journal of Educational Research, 2*(2), 51-68. https://doi.org/10.12973/eu-jer.2.2.51

Cañadas, M. C., & Castro, E. (2007). A proposal of categorisation for analysing inductive reasoning. *PNA, 1*(2), 67-78.

Celik, H. C. (2019). Investigating the visual mathematics literacy self-efficacy (VMLSE) perceptions of eighth grade students and their views on this issue. *International Journal of Educational Methodology, 5*(1), 165-176. https://doi.org/10.12973/ijem.5.1.177

Creswell, J. W. (2003). *Research design: Qualitative, quantitative, and mixed methods approach* (2nd ed.). Sage Publications, Inc.

Dubinsky, E. (2001). *Using a theory of learning in college mathematics courses*. University of Warwick.

Eli, J. A., Mohr-Schroeder, M. J., & Lee, C. W. (2013). Mathematical connections and their relationship to mathematics knowledge for teaching geometry. *School Science and Mathematics*, *113*(3), 120–134. https://doi.org/10.1111/ssm.12009

Geraniou, E. & Jankvist, U.T. (2019). Towards a definition of "mathematical digital competency". *Educational Studies in Mathematics, 102*, 29–45. https://doi.org/10.1007/s10649-019-09893-8

Glasnović Gracin, D. (2014). Mathematics textbook as an object of research. *Croatian Journal of Education,16*(3), 211-237.

Goos, M., & Kaya, S. (2019). Understanding and promoting students' mathematical thinking: a review of research published in ESM. *Educational Studies in Mathematics, 103*, 7-25. https://doi.org/10.1007/s10649-019-09921-7

Hackenberg, A. (2007). Units coordination and construction of improper fractions: A revision of the splitting hypothesis. *Journal of Mathematical Behaviour, 26*(1), 27–47. https://doi.org/10.1016/j.jmathb.2007.03.002

Hannula, M. S. (2003). Location fractions on number line. In N. A. Pateman, B. J. Dougherty, & J. T. Zillox (Eds.), *Proceedings of the 27th conference of the international group for the psychology of mathematics education* (pp. 17-24). PME.

Hartas, D. (2010). *Educational research and inquiry, qualitative and quantitative approaches.* Continuum International Publishing Group.

Hiebert, J. (Ed.). (1986). *Conceptual and procedural knowledge: The case of mathematics.* Lawrence Erlbaum.

Hiebert, J., & Carpenter, T. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–97). Macmillan.

Hodnik Čadež, T, & Manfreda Kolar, V. (2015). Comparison of types of generalizations and problem-solving schemas used to solve a mathematical problem. *Educational Studies in Mathematics, 89*(2), 283-306. https://doi.org/*10.1007/s10649-015-9598-y*

Ic, U., & Tutak, T. (2017). Correlation between computer and mathematical literacy levels of 6th grade students. *European Journal of Educational Research 7*(1), 63 - 70. https://doi.org/10.12973/eu-jer.7.1.63

Islami, M. D., Sunardi, S., & Slamin, S. (2018). The mathematical connections process of junior high school students with high and low logical mathematical intelligence in solving geometry problems. *International Journal of Advanced Engineering Research and Science, 5*(4), 10–18. https://doi.org/10.22161/ijaers.5.4.3

Jablonka E. (2003) Mathematical Literacy. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second international handbook of mathematics education*. *Springer international handbooks of education* (pp. 75-102). Springer. https://doi.org/10.1007/978-94-010-0273-8_4

Kieren, T. E. (1976). On the mathematical, cognitive and instructional foundations of rational numbers. In R. A. Lesh (Ed.), *Number and measurement: papers from a research workshop* (pp.101–144). ERIC/SMEAC.

Kilpatrick, J. (2002). Understanding mathematical literacy: The contribution of research. *Educational Studies in Mathematics, 47,* 101-116.

Krek, J. (2015). Two principles of early moral education: a condition for the law, reflection and autonomy. *Studies in Philosophy & Education, 34*(1), 9-29.

Krek, J., Klopčič, L. (2019). Teacher authority and the educational role of the class teacher in the era of permissiveness. *Didactica Slovenica, 34*(3/4), 122-140.

Kula Unver, S., Hidiroglu, C. N., Tekin Dede, A., & Bukova Guzel, E. (2018). Factors revealed while posing mathematical modelling problems by Mathematics student teachers. *European Journal of Educational Research, 7*(4), 941-952. https://doi.org/10.12973/eu-jer.7.4.941

Kyriakides, L. Campbell, R. J., & Christofidou, E. (2002). Generating criteria for measuring teacher effectiveness through a self-evaluation approach: a complementary way of measuring teacher effectiveness. *School Effectiveness and School Improvement, 11*, 501-529.

Leikin, R., & Levav-Waynberg, A. (2007). Exploring mathematics teacher knowldege to explain the gap between theory-based recommendations and school practice in the use of connection task. *Educational Studies in Mathematics, 66*, 349-371. https://doi.org/10.1007/s10649-006-9071-z

Manfreda Kolar, V., Slapar, M., & Hodnik Čadež, T. (2012). Comparison of competences in inductive reasoning between primary teacher students and mathematics teacher students. In B. Maj-Tatsis & K. Tatsis (Eds.), *Generalization in mathematics at all educational levels* (pp. 299-311). Wydawnictwo Uniwersytetu Rzeszowskiego.

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking Mathematically*. Pearson Education Limited.

Ni, Y. (2001). Semantic domains of rational numbers and the acquisition of fraction equivalence. *Contemporary Educational Psychology, 26*, 400-417.

Niss, M., Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics, 102*, 9-28. https://doi.10.1007/s10469-019-09903-9

North, M., & Christiansen, I. M. (2015). Problematizing current forms of legitimized participation in the examination papers for mathematical literacy. *Pythagoras, Journal of the Association for Mathematics Education of South Africa, 36*(1), 1-11. https://doi.10.4102/pythagoras.v36i1.285

Organisation for Economic Co-operation and Development. (2003). *The PISA 2003 assessment framework. Mathematics, reading, science and problem solving knowledge and skills*. OECD.

Pólya, G. (1945). *How to Solve It.* University Press.

Radford, L. (2008). Iconicity and contraction: a semiotic investigation of forms of algebraic generalizations of patterns in different contexts. *ZDM Mathematics Education, 40*, 83-96.

Reid, D. (2002). Conjectures and refutations in grade 5 mathematics. *Journal for Research in Mathematics Education, 33*(1), 5-29.

Rittle-Johnson, B., & Koedinger, K. R. (2005). Designing knowledge scaffolds to support mathematical problem solving. *Cognition and Instruction, 23*(3), 313-349. https://doi.10.1207/s1532690xci2303_1

Sáenz, C. (2009). The role of contextual, conceptual and procedural knowledge in activating mathematical competencies (PISA). *Educational Studies in Mathematics, 71*, 123-143. https://www.jstor.org/stable/40284590

Sagadin, J. (1991). *Razprave iz pedagoške metodologije* [Discussions on pedagogical methodology]. Znanstveni inštitut Filozofske fakultete Univerze v Ljubljani.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Academic Press, Inc.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, *22*(1), 1–36. https://doi.org/10.1007/BF00302715

Spangenberg, E. D. (2012). Thinking styles of mathematics and mathematical literacy learners: Implications for subject choice. *Pythagoras, 33*(3), 1-12. https://doi.10.4102/pythagoras.v33i3.179.

Stacey, K., & Turner, R. (2015). The evolution and key concepts of the PISA mathematics frameworks. In K. Stacey & R. Turner (Eds.), *Assessing mathematical literacy* (pp. 5-33). Springer international Publishing.

Steen, L. A. (Ed.) (2001). *Mathematics and democracy: The case for quantitative literacy*. National Council on Education and the Disciplines.

Suciati, Munadi, S., Sugiman, & Febriyanti, R. W. D. (2020). Design and Validation of Mathematical Literacy Instruments for Assessment for Learning in Indonesia. *European Journal of Educational Research, 9*(2), 865 - 875. https://doi.org/10.12973/eu-jer.9.2.865

Sullivan, P. (2011). *Teaching mathematics: Using research informed strategies*. Australian Council for Educational Research.

Tohir, M., Maswar, M., Atikurrahman, M., Saiful, S., & Pradita, D. A. R. (2020). Prospective teachers' expectations of students' mathematical thinking processes in solving problems. *European Journal of Educational Research, 9*(4), 1735-1748. https://doi.org/10.12973/eu-jer.9.4.1735

Tunç-Pekkan, Z. (2015). An analysis of elementary school children's fractional knowledge depicted with circle, rectangle, and number line representations. *Educational Studies in Mathematics, 89*, 419–441.

Umbara, U., & Suryadi, D. (2019). Re-Interpretation of Mathematical Literacy Based on the Teacher's Perspective. *International Journal of Instruction, 12*(4), 789-806. *https://doi.org/10.29333/iji.2019.12450a*

Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics*, *54*(1), 9–35.

Yin, R. K. (2017). *Case study research and applications: Design and methods*. Sage publications.