



## TRAINING MATHEMATICS TEACHERS FOR DEVELOPING CRITICAL THINKING SKILLS IN VI-GRADE PUPILS

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**Abstract:** Critical thinking and creativity along with problem-solving skills have become basic competencies in an increasingly complex and technological world, in which an individual needs to be able to process information in a critical way, find alternative solutions, analyse complex problems, and formulate innovative solutions. As teachers, we train the children for occupations that have not been invented yet – an aspect that thus becomes clear is that it is no longer sufficient for the young students to perfectly execute given tasks, but they need to be able to solve new problems. To achieve this, they need to be able to consider the problem from multiple angles, to weight alternatives, to look critically at what needs to be solved, and to make well-thought and well-argued decisions. With this perspective in mind, developing such skills in pupils should be an essential goal for the teachers. Yet, unfortunately, training teachers for developing critical thinking skills in pupils comes short, in the Romanian educational system. This article presents the results of a training course whose goal was to prepare mathematics schoolteachers to teach mathematics based on questioning and discovery, to help develop pupils' critical thinking skills. Analyzing the short- and long-term impact of our training course confirms the results obtained in OECD (2019) studies, namely that once teachers have been qualified to work in this way, they become more open to acting differently to stimulate the developing of critical thinking and creative skills in pupils.

**Key words:** critical thinking, learning by discovering, questioning, training course, teachers' training

### 1. Introduction

Out of 79 participating countries (OECD and partner countries), Romania was placed on the 47<sup>th</sup> position following the 2018 PISA tests and recorded a decrease in academic performance relative to the year 2015 – a score of 430 (relative to 444 in 2015) in mathematics and a score of 426 (relative to 435) in sciences.

More concerning is the fact that 47% of the pupils prove to be functionally illiterate in mathematics (below the second difficulty level in the PISA test), as they are unable to employ basic arithmetic operations. For instance, they did not manage to compare the total distance of two alternative routes or to convert prices from RON to alternative currencies (OECD, 2019). In fact, the PISA tests are not so much about resolving the mathematical operations correctly, but more about identifying critical thinking and real-life problem-solving skills, as well as identifying the ability to apply the learned knowledge in real-life situations. Testing the science-related skills involves a similar approach, as sciences and mathematics belong to the same curricular area, namely the basic competencies identified by the European Council in mathematics, sciences, technology, and engineering (EU, 2018). Mathematical competence relates to “developing and applying mathematical thinking in order to solve a series of every-day problems”, while the scientific one relates to „identifying questions and drawing conclusions based on evidence” (EU, 2018). Regarding the sciences element in the 2018 PISA test, 44% of the Romanian respondents (relative to 39% in 2015) are unable to realize basic correlations between facts (OECD, 2019).

Following the 2015 PISA testing, a series of measures have been taken for the Romanian educational system i.e., the school curriculum for classes V-VIII has been restructured (MEC, 2017). Also, the OECD recommendation regarding the teacher-pupil interaction referred to interactive teaching and

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evaluation processes which should also be relevant and based on dialogue, the teachers needing to be trained in this sense (Kitchen et al., 2017).

As mathematics teachers for the classes V-VIII and as trainer and evaluator of teachers, we have tried to identify the problem and to ameliorate it, respectively. During the various class inspections that we have been involved with, we have observed along the years the methodological practices of our colleagues and their didactic views, which were limited to memorizing the problem-solving algorithms.

As part of a doctoral dissertation, we have conducted a large experiment involving multiple classes of VI<sup>th</sup> grade pupils, which involved stimulating their critical thinking and creative skills through teaching mathematics. A part of the experimental intervention thus concerned training mathematics teachers from the south-west of Romania who were teaching at these classes to teach differently, namely based on questioning and learning by discovery.

The results of this experiment regarding the impact of the intervention on the pupils' progress has been presented previously (Bădescu & Stan, 2019). Here, we focus on the impact of the intervention program on the teachers. While we used the Watson-Glaser test (Goodwin & Glaser, 2002) to evaluate the pupils' performance, the impact of the intervention on the teachers was evaluated with questionnaires applied repeatedly at certain time intervals (2016, 2018). These measured the short-, medium-, and long-term impact on the potential changes in the teachers' attitudes, concepts, and ways of acting when it comes to teaching mathematics and stimulating pupils' critical thinking. As follows, we bring forward the obtained results and answer the following question: *what kind of didactic strategies do V-VIII grade teachers report for stimulating pupils' critical thinking through teaching mathematics?* At the same time, we have tried to identify how mathematics teachers perceive the importance of developing critical thinking skills via mathematics-related activities.

## 2. Theoretical grounding

Critical thinking skills can be thought and learned. The experts mention that the most effective way to teach them is in an explicit and direct manner. The pupils learn how to evaluate the credibility of a given information source and how to make decisions independently of the content material – these abilities may therefore be transferred to other topics and contexts as well (Butterworth & Thwaites, 2013; Moon, 2007). Like any other ability, these skills also need to be practiced, once acquired; in this context, the educational environment is above all meant to form, consolidate, and perfect these skills.

Having such skills represents an asset not only during one's educational path, but also later. The short- and long-term benefits of developing critical thinking skills among pupils and/or students have been repeatedly brought forward in the literature (Vincent-Lancrin et al., 2019; Mevarech & Kramarski, 2014). One study conducted on 1100 pupils indicates the existence of a significant positive correlation between the critical thinking skills and the general academic performance, measured as the grades' average (Facione, 1998). In other words, the better the pupils' critical thinking skills, the better their academic achievements.

Due to the speed at which the professional domains are developing, the need for continuous education/training is more and more often mentioned. In this context, the experience from the educational system is vital. The school experience must teach the pupil to learn, meaning "how to reason for oneself alone or in collaboration with others" (McGregor, 2007); essentially, the school needs to help the student become independent from the teacher, at the end.

According to Halpern (2014), critical thinking skills need to be developed in a persistent manner, in different contexts and integrated across domains, but should also be explored in specific ways. Halpern and his team proposed a training program in which pupils should: (1) learn critical thinking skills in an explicit manner; (2) develop the ability to think and learn in a self-conscious, voluntary way, because of invested effort; (3) learn through activities which facilitate the transfer of the developed skills; (4) explicitly monitor their own actions by using metacognitive strategies.

The scope of developing critical thinking skills is to achieve high-quality thinking abilities not just in school as demonstrated through tests i.e., in contexts specifically designed for that purpose, but also

beyond school, where they should be able to easily transfer, demonstrate, and apply the critical, investigative, and analytical thinking style in concrete life situations (McGregor, 2007; Vincent-Lancrin et al., 2019). Therefore, the pupils need to understand not only the mathematics' content, but also the mathematical thinking process, so that they are able to process the information at an advanced level and transfer it to life situations (Mevarech & Kramarski, 2014; Schoenfeld & Sloane, 2016). Many educators support the idea that critical thinking skills can be learned, and they should be thought explicitly, while the pupils should be informed about the types of thinking skills that they are though (Swartz, 2001; McGregor, 2007). Research shows that pupils' thinking skills can be developed if teachers create a classroom environment which supports thinking activities and base their didactical approach on dialogue and integrating information, for instance based on the metacognitive approach, project-based learning, or research-based learning (Vincent-Lancrin et al., 2019; Smith & Mancy, 2018). The teachers' role is not necessarily to dominate and control the learning activities, but they should encourage the pupils to play an active role and they should also ensure a good multilateral interaction between teacher and pupils (Morgan, 2016; Wright, 2017; Smith & Mancy, 2018).

At a national level, the efforts towards developing critical thinking skills have become the main agenda for the mathematical discipline (Høgheim & Reber, 2017; Malara & Navarra, 2018; Mevarech & Kramarski, 2014). Many researchers have demonstrated that the development of critical thinking skills can improve the mathematical performance (Posamentier & Krulik, 2009; Zsoldos-Marchiş, 2014; Store, 2018; Yong, 2020). Similarly, critical thinking skills will encourage pupils to think independently and to solve school-related problems, or problems in the daily-life context (Mevarech & Kramarski, 2014; Schoenfeld & Sloane, 2016; Moon, 2007).

Through questioning, the pupil is taken out of the situation in which he or she receives the information that is already structured by the teacher and introduced into the situation in which he or she can find the solution to a problem, alone. As also stressed by Cergit (2006), questioning does not aim towards the accumulation of more and more information, but towards forming an individualized working style when under pressure and stimulating a spirit of investigation and courage when making arguments and supporting some personal opinions. This method develops the pupil's ability to identify, analyse, and find suitable solutions to problems by using various thinking strategies such as induction, deduction, analogy, hypothesis-based, etc. Given that such skills and abilities are domain-general, they facilitate the pupil's adaptation not just to the school environment, but also to the one existing beyond the school.

The learning by discovery method is closely related with the questioning method. If in the latter, the emphasis falls on creating the problem-situation and on the reflexive approach towards solving it (e.g., by formulating and testing hypotheses, etc.), in the former, the emphasis falls on finding the solution. One may think of the questioning and learning by discovery as representing two distinct moments in time of the same heuristic process: what is to be solved must first start from a problem-situation (Swartz, 2001; Posamentier & Krulik, 2009, Pólya, 1971). Using the learning by discovery method in the classroom comes with a series of advantages. Because the pupil is actively involved in the learning act, it leads to knowledge that is deeper and more robust, and to a faster consolidation. The path to finding a solution requires knowledge-transfer skills, perseverance, and an independent spirit. Due to the relatively large effort that is required from the student, learning by discovery supports the pupils' intellectual development and even an increased self-confidence. This method offers the possibility for self-discovery and self-control, while also stimulating an interest in research and learning (Ardelean & Secelean, 2007, p. 111).

The two methods presented above, due to their qualities, represent a must-have in the methodological practices of the mathematics teachers who are preoccupied about forming and developing critical thinking skills in pupils. Even though the two methods are known at a theoretical level, teachers are often less inclined to use them. This is the reason why as teachers, but also as trainers and evaluators of teachers, we have developed a training program for the mathematics teachers. This program is described and analysed as follows.

Our research aim was to identify to what extent the teachers of mathematics can change their teaching practices and attitudes towards fostering critical thinking during their classes, once they attend a

dedicated further training program, meant to make them aware about the importance of developing critical thinking and enable them in this regard.

Thus, starting from research questions like: 1) How the teachers rank the importance of the teaching methods they are using; 2) Are there perceived difficulties in using methods meant to foster critical thinking, like questioning and learning by discovery; 3) How familiar are the teachers with critical thinking and what is their opinion about it, we tried to identify, by repeated questioning, in pre-intervention, post-intervention phases, and on long term, how their attitude and answers have changed. The perceived changes were considered as being influenced by the training intervention the teachers were exposed to, training meant to make them more familiar with the way critical thinking can be developed through mathematics classes in the 6<sup>th</sup> grade. We assumed that both the attitude towards critical thinking and the ability to foster it can be very much changed and improved, once the teachers become aware about its importance, and familiar with the ways it can be developed.

### 3. Methodology

Given the ability to develop critical thinking skills through mathematics and based on the evidence presented above indicating that critical thinking skills may be developed through questioning, interactive, and reflexive methods which facilitate learning by discovery, we developed a training program for the V-VIII grade mathematics teachers. The scope of this program was to help the teachers become familiar with these methods when planning their mathematics teaching-related activities.

Research shows that once familiarized with such procedures, teachers become open to adopting them into the classroom (Vincent-Lenvin et al., 2019) and using them constantly. Once the intervention program was realized, we wanted to investigate the manner in which training teachers for developing critical thinking skills in VI-grade pupils leads to a change in the way the mathematics classes take place, and in the way mathematics teachers view the importance of developing pupils' critical thinking skills.

In agreement with the experimental study's methodology (Mujis, 2004), but also with the methodology for evaluating the impact of educational programs (Harty & Newcomer, 2004), the training program was developed and conducted in 2016. We conducted the initial testing of the two samples (including 42 teachers in the experimental group and 41 teachers in the control one) at the start of the educational program in May 2016, while a re-testing took place at the end of the program, in September 2016. To identify the long-term impact of the educational program and the stability of the observed changes, the questionnaire was re-applied in September 2018.

The division of the sample of teachers in the experimental and the control one considered the balanced distribution into the two groups, to keep up the comparability of them, as well, as the compatibility with the general traits of the mathematics teachers in the respective county. Thus, giving the overall proportion at county level of urban-rural (76%), as well as of the teaching experience (12% till 10 years as teachers, 27% between 10 and 25 years of experience, and 61% with more than 25 years of experience, with comparable career advance, with the years of experience), out of the 42 teachers in the experimental group, 32 come from urban area, and 10 from rural area. The same, in the control group, 31 come from urban area, and 10 from rural area.

The training program entitled *Developing critical thinking skills through teaching mathematics for the VI-grade students* was conducted over a time period of two weeks and concerned the familiarizing of 42 teachers (i.e., the experimental sample) with teaching VI-grade mathematics in a non-conventional manner (based on heuristics) i.e., with a focus on using the questioning method in various contexts, along with the learning by discovery method. More specifically, for every topic of the VI-grade curriculum, we discussed the way in which it can be thought by using questioning and learning by discovery. Based on research and on the methodological recommendations concerning an active, interactive, and collaborative way of teaching based on connections and investigating algebra and geometry in a critically-investigative manner for this age group (Schoenfeld & Sloane, 2016; Magdaş, 2015; Mevarech & Kramarski, 2014; Zsoldos-Marchiş, 2014; Bocos, 2013; Posamentier & Krulik,

2009; McGregor, 2007; Swartz, 2001; Pólya, 1971), a methodological system entitled „criticmath” was developed, with which the teachers were familiarized during the training program.

The psycho-pedagogical experiment method represented the main research instrument utilized here; this referred to introducing a change in the educational practice and represented the independent variable (i.e., training teachers to develop critical thinking skills through teaching mathematics, program conducted through the district’s Teacher Training Institution called „Casa Corpului Didactic”), whose impact was then measured on the dependent variable, namely the measure of how important do teachers perceive pupils’ critical thinking skills to be.

The same questionnaire was applied to the teachers in the two samples in September 2016 and 2018 before the school year started, the comparisons being based on the collected responses at these two moments in time. The choice for these moments in time was not by chance but had the scope of reminding and creating awareness among the teachers as concerns the importance of developing critical thinking skills via teaching mathematics.

The questionnaire consisted out of 20 questions, out of which 6 questions were with closed answers, and 14 questions with multiple choices. The multiple choices were related to ranking preferences for different methods, or to the ways they run the classes, the teaching and evaluation, or the differentiated instruction. It was also questioned the motivation of the teachers for their career, their attitude towards critical thinking. There were questions quite sharp and simple, looking for yes/no/not know answers, without additional requests or proves for their answers. Even we have asked only if they can define the critical thinking, for instance, but not actually to define it, the answers of no and non answers were surprisingly high, as it is shown later.

The obtained results which are illustrated as follows emphasize the training’s direct impact, as brought forward by the change in teachers’ opinions regarding the conceptualization and development of mathematics lessons. The training’s indirect impact that was operationalized by the comparison of results coming from 542 VI-grade pupils who worked with teachers who either received or did not receive the training, was discussed in a previous publication (Bădescu & Stan, 2019).

#### 4. Data analysis

The auto-administered questionnaires that were repeatedly addressed to the teachers participating into the training program (for illustration purposes, the opinions of the teachers included into the control sample are comparatively presented) revealed the following aspects related to the opinions and teaching practices of the mathematics teachers:

To the item “List the following teaching methods in the order of their importance” (here, only the answers listed as most important i.e., listed on the first position are shown in Table 1).

**Table 1.** *The importance of the teaching methods*

Moment in time	Respondents’ sample	Exercise	Questioning	Demonstration	Learning by discovery	Project method
May 2016	Experimental sample	87%	3%	0%	0%	10%
	Control sample	83%	1%	6%	2%	8%
September 2016	Experimental sample	23%	31%	12%	19%	15%
	Control sample	81%	2%	7%	5%	5%
September 2018	Experimental sample	61%	12%	8%	7%	12%
	Control sample	72%	2%	8%	4%	14%

One can notice that for the experimental sample, as illustrated in Table 1, in the answers from May 2016 referring to the pre-test, 87% of the teachers were considering the exercise as the most important method, followed by the project’s method, which was intensively advertised as being an active and integrative method of teaching, but especially of evaluating pupils. Surprising are the 0% results for

the demonstrations and learning by discovery methods, but also the very low percentage for the questioning method.

The immediate impact of the training may be observed in the answers given once the training course was finalized, by comparison to the answers given in September 2016: the questioning method rises from the third to the first position in terms of importance (31%, by comparison to the initial 3%), while the learning by discovery method rises from 0% to 19%, occupying the third position in terms of importance. Even though in the long run, the exercise method comes back to the first position as indicated by the results from 2018, one may continue to observe the long-term impact of the training, by comparing the experimental and control samples. That is, the teachers from the experimental group largely prefer the questioning method relative to those in the control group (12% versus 2%). After two years, the percentages for the questioning and learning by discovery methods are reduced, but importantly, they are still larger relative to the pre-testing moment in time. By comparison, the exercise method is always on the first position for the control sample, while all other methods remain rated similarly.

To the item “Do you think that you would reach difficulties, now or in the future by using the questioning method in teaching and learning mathematics?” (here, the analysis only considers the answers listed on the first position) in Table 2 are listed their answers:

**Table 2.** Difficulties to using the questioning method

Moment in time	Respondents' sample	Yes	No	I don't know/ Not answering
May 2016	Experimental sample	62%	17%	21%
	Control sample	47%	21%	32%
September 2016	Experimental sample	8%	87%	5%
	Control sample	50%	28%	22%
September 2018	Experimental sample	12%	72%	16%
	Control sample	45%	31%	24%

In the table 2, one may notice that in May 2016, most of the teachers in experimental sample estimated difficulties related to using this method, as indicated by 62%, relative to only 17% estimating that they do not expect any difficulties. The immediate impact of the training program becomes evident, as once the training program was completed, only 8% of the participants (relative to the initial 62%) still respond that they expect difficulties by using the questioning method. This percentage remains relatively stable in time, as suggested by the 12% result in 2018. By comparison, in the control sample, the 50% result from the pre-testing period remains close to the 47% one obtained in May 2016 and to the 45% one obtained in 2018, respectively. If the percentage of the teachers in the experimental group stating that they do not expect difficulties in using the questioning method raises from 17% in the pre-test moment in time to 87% once the training was finalized (72% of them maintaining this position in 2018), the percentage of the teachers in the control group who do not expect difficulties in using this method is also somewhat increasing, from 21% in May 2016, to 28% in September 2016 and 31% in September 2018, respectively. One can thus identify not only the training's impact, but also the need to train teachers to be able to correctly use this method in their teaching activities.

To the item “Do you think that you will encounter difficulties, now or in the future by using the learning by discovery method?” the answers of the teachers, from both groups, before and after the training, can be seen in the Table 3.

For the experimental sample, the 58% of those answering that they might have difficulties by using this method in May 2016, decreased to 2% by September 2016. From this perspective, the data are illustrative for the training's impact. The percentage of those answering that they do not expect difficulties by using this method raises from 19% in the pre-training moment in time to 81% once the training was completed, while these results remained relatively stable after two years (82% in September 2018). By comparison, in the control sample, the 69% of those answering that they expect

difficulties with this method in May 2016 remains relatively constant in time (71% in September 2016 and 61% in September 2018, respectively).

**Table 3.** *Difficulties to using the learning by discovery method*

Moment in time	Respondents' sample	Yes	No	I don't know/ Not answering
May 2016	Experimental sample	58%	19%	23%
	Control sample	69%	12%	19%
September 2016	Experimental sample	2%	81%	17%
	Control sample	71%	16%	13%
September 2018	Experimental sample	11%	82%	7%
	Control sample	61%	28%	11%

As is also the case with the previous item, the 58% result referring to the teachers who expect difficulties by using this method might reflect the fact that for many teachers, not being familiar with the method itself or the fact that the method might be more difficult to implement, creates some fear to using it into the classroom. The impact that the training has here is thus indicative for the necessity of such intervention programs.

If from a methodological perspective, the presented data speak about the teachers' perception on how confident they feel about the methods meant to help develop pupils' critical thinking skills. Yet, it is equally useful to bring forward the teachers' understanding of what critical thinking represents in itself.

To the item "Can you define critical thinking?" it can be seen the the Table 4 below the answers of the teachers responding:

**Table 4.** *Ability to define critical thinking*

Moment in time	Respondents' sample	Yes	No
May 2016	Experimental sample	23%	77%
	Control sample	32%	68%
September 2016	Experimental sample	93%	7%
	Control sample	39%	61%
September 2018	Experimental sample	84%	16%
	Control sample	42%	58%

One may notice based on the table above that a large percentage of 77% of the respondents cannot define what critical thinking is, initially. This drops to 7% once the training was completed, and stabilizes at 16% after two years, in 2018. One can thus easily conclude the training's positive impact on this topic. It remains nevertheless concerning that a large percentage of the mathematics teachers were not even able to define what critical thinking represents, even less so to develop it in students and follow and evaluate its development. Such a large percentage was also identified in the control group (68% in May 2016, 61% in September 2016, and 58% in September 2018, respectively).

Additionally, we remain somewhat sceptical also about those teachers who responded that they are able to define what critical thinking is. That is due to our practical experience, which indicates that many teachers are confounding the action of critically thinking, with that of criticising. We suspect this occurs due to the fact that most of the teachers' training programs focus on teaching content and teaching strategies, but not on the types of abilities that may be developed in pupils.

To the item "Do you think that developing critical thinking skills is important?", the respondents provided the answers listed in the Table 5. Even not asking for arguments, just for their yes or no answer, the percentages of answers by each category are as follows:

**Table 5.** *The importance of critical thinking from teachers' perspective*

Moment in time	Respondents' sample	Yes	No	I don't know/ Not answering
May 2016	Experimental sample	23%	13%	64%
	Control sample	21%	37%	42%
September 2016	Experimental sample	92%	0%	8%
	Control sample	29%	12%	49%
September 2018	Experimental sample	64%	9%	27%
	Control sample	28%	36%	36%

The data above indicates that 77% of the respondents included in the experimental sample did not think that developing critical thinking skills is important or they preferred not to give an answer in this sense, initially. This situation changes dramatically once the training program was completed, as 92% of the experimental sample teachers responded that developing critical thinking skills is necessary (September 2016). The percentage of those initially answering that critical thinking skills are not important or who prefer not to answer is very similar, also in the control sample (79%). This is of course very concerning, especially since this large percentage appears to remain relatively stable in 2018, namely 72%. These data might finally explain the pupils' poor performance in the PISA tests.

For brevity reasons, we cannot include all the collected data. However, we briefly name a few additional data obtained by means of questioning teachers, which are also quite worrying. In this context, overall, 94% of the mathematics teachers do not consider that they help develop pupils' critical thinking skills. 87% of them prefer the algorithm method by comparison to those preferring the learning by discovery method (13%). Furthermore, 60% of the teachers think that overall, school education is not useful to the pupils.

Of course, these data need further corroboration – more detailed reflections are necessary, along with solutions to ameliorate the identified problems.

## 5. Discussion and conclusions

In this article, we approached the important issue of developing critical thinking skills in school through mathematics-related activities, focusing on the V-VII-grade pupils. The data obtained from teachers' questionnaire responses indicated a worrying situation as concerns teachers' knowledge about critical thinking, and their views on how such skills can be developed through the teaching-related activities. Teachers overall indicated a preference for using the exercise and algorithm methods over the questioning and the learning by discovery methods. Of course, these data come with the limitation of a small sample size, namely the mathematics teachers from a single district in our country. A similar research project should be conducted on a larger scale, because any measure towards improving the efficiency of the educational system and the pupils' performance should be based on facts indicated by the teachers, as they would be the ones to concretely realize the necessary improvements.

Additionally, we described a possible intervention regarding a training course for mathematics teachers that is meant to improve their skills towards teaching in a creative, interactive, and questioning manner but also based on research and discovery, towards the developing of critical thinking skills. The evaluation conducted at the end of the course and once again, two years later indicates the positive effects of this intervention. In agreement with previous studies, we believe that it is necessary for such interventions to be multiplied so that the teachers may openly engage in such approaches once they have developed the concrete skills for developing the pupils' critical thinking (Vincent-Lancrin et al., 2019; Cai & Leikin, 2020; Malara & Navarra, 2018; Yong et al., 2020).

Of course, contextual factors must be considered in addition to teachers' skills and abilities. One such factor refers to the way pupils are evaluated. Considering that at the end of the VIII<sup>th</sup> grade students go through a standard national evaluation, teachers must choose in between conforming to the typical evaluation practices or teach in a different manner according to their abilities and beliefs (Kitchen et al., 2017; Wright, 2017). A more detailed analysis is certainly needed; nevertheless, the data briefly presented in the previous section may generate deeper reflections and solutions towards improving not



just the pupils' academic performance in (inter)national test scenarios, but also their solid skills and basic competences for the society of the future. Critical thinking skills represent one of those competences, next to the metacognitive and reflexive skills, the ability to be creative, to discover, to question, etc.

We have seen that the systemic measures implemented to change the curriculum (MEC, 2017) are not sufficient if they are not supported by teachers' suitable skills for implementing these changes. Similar solutions relative to the teachers' training course described here are unfortunately lacking. It is necessary that they become available and systematically implemented. Additionally, as indicated by the long-term evaluation, it is often not sufficient to merely apply such an intervention, but follow-up activities are necessary. These may include the possibility that teachers may share and reflect upon the likely improvements, create their own communities based around practical and professional experiences, obtain the necessary means for continuous professional development.

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