CONCEPTION AND DEVELOPMENT OF INDUCTIVE REASONING AND MATHEMATICAL INDUCTION IN THE CONTEXT OF WRITTEN ARGUMENTATIONS

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Abstract: Nowadays, mathematical reasoning and making proof have taken importance for all students from the grade level of elementary education to university. More specifically, mathematical induction (MI) is a kind of proof and reasoning strategy taking place nearly all grade levels. Moreover, teachers are important factors affecting student learning and they can acquire necessary knowledge and skills developmentally in their teacher education programs. This paper makes contributions to domain of research by investigating the development of PMT's conception of MI in the context of written argumentations encouraging MI. In other words, the purpose of this multiple case study is to explore how PMT's conception of mathematical induction develop through their written argumentations. These cases show that there exist improvements in PMT's written argumentations, conception of MI and proof construction activities related to MI. In other words, the more organized and structured they produced written argumentation, the more successfully they use and make mathematical induction.

Key words: argumentation, conception, mathematical induction, proof construction.

1. Introduction

Although there exist many research examining the relationship between the teaching and learning of proof and justification of people in mathematics education in different contexts such as by argumentation (Krummheuer, 2000; Yackel, Cobb, & Wood, 1991; Zack & Graves, 2001), by conception of proof (Harel & Sowder, 1998) and by the construction of proofs (e.g., Weber, 2001; Martin & McCrone, 2003; Weber & Alcock, 2004), there are limited research examining mathematical induction (MI) as a strategy of reasoning and proof especially for basic mathematics concepts (Stylianides, Stylianides, & Philippou, 2007). Hence, differently in the current research, the development of conception of MI and construction of proof about MI especially in the context of written argumentation was focused on. Moreover, although the skills related to teaching and learning proof, writing argumentation and forming the conception of proof can be developed efficiently with the help of the lessons designed and implemented effectively by providing necessary knowledge, it is needed to investigate developmentally. With this motivation, this paper explains the written argumentation schemes, conceptions of MI of preservice mathematics teachers (PMTs) through engaging in proof construction activities related to MI proof strategy and presents evidence of a developmental process of these skills with treatments in teacher education programs. In other words, the development of skills of argumentation and conception of MI in the PMT's proof construction activities related to MI through the treatment providing opportunities to develop the skills related to MI strategy and writing argumentation.

2. Theoretical Framework

2.1. Inductive Reasoning and Mathematical Induction in Mathematics Education

As a specific form of the mathematical reasoning and proving as one of the mathematical process skills, inductive reasoning can be characterized by three properties; "inductive reasoning proceeds

from specific cases to conclude general rules, inductive reasoning uses what is known to conclude something previously unknown, inductive reasoning is only probable not certain" (Reid & Knipping, 2010, p. 88). Moreover, NCTM (1989) emphasize inductive reasoning by stating "a mathematician or a student who is doing mathematics often makes a conjecture by generalizing from a pattern of observations made in particular cases" (p. 143). MI as a specific proof method based on inductive reasoning is used to prove the truth of a statement for every natural number by proceeding through two steps: base step and inductive step. The statement is validated for n = n0 for an initial value of P(n), which is an open sentence, in the base step and proved the implication of P(k) => P(k+1) and k is an arbitrary value in the domain of discourse in the inductive step (Stylianides, Stylianides & Philippou, 2007). In other words, MI includes steps such as illustrating the generalization for a base case and then showing that if the generalization is valid for an arbitrary number k, it is also valid for k+1 (Word, 1988). Moreover, Johnson (1960) stated that MI is highly recommended by all mathematicians, since it is used to prove the fundamental theorem of arithmetic, the division algorithm, and the Euclidean algorithm having critical importance for PMTs to teach mathematics effectively in their classrooms in the future. In other words, MI is essential in education since it is used to prove and understand very basic concepts in mathematics of college and pedagogically appropriate subject matter (Reeves, 1972). In the general term, induction is to estimate general propositions based on a number of observed facts (Blank, 1963). From this perspective, it can be stated that MI is deductively proved mathematical statements derived by general inferences from a pattern (Word, 1988).

MI is taught in secondary grade level of students' lessons however it is not placed in elementary grade level lessons explicitly (Stylianides, Stylianides & Philippou, 2007). Its basic characteristic and main property is taught by connecting with algebra and geometry lessons to elementary grade students. For example, an elementary teacher can prove the statement such as "Is n/0 equal to infinity?" that may be used in a sixth grade level classroom, and discuss the reasoning to their students by using MI (Cambridge Conference on Teacher Training, 1967, p. 64). Moreover, the mathematical concepts in elementary grade levels such as natural numbers and patterns are related to MI and MI is necessitated for problem solving, algebraic and geometric thinking (e.g. identifying geometric patterns and symbolizing its order) (MoNe, 2013). Although MI takes different amount of importance in different grade levels, it has common benefits such as ability to cope with certain mathematical issues that may take place in the classrooms, identify basic versions of MI formed by students, promote student understanding of the arguments related to MI and think logically (Stylianides, Stylianides & Philippou, 2007).

2.2. Conception of Proof in the Context of Argumentation

In the curriculum of mathematics, proving and reasoning are essential in mathematics education because it helps students learn mathematics deeply (Hanna, 2000). Proving is a complex mathematical activity searching for a proof and formulating an argument with its dimensions of logical, conceptual, social, and problem-solving by reasoning (Weber, 2005; Bieda, 2010; Stylianides & Ball, 2008). These activities can be exemplified as identification of patterns, making conjectures, testing examples and supporting arguments which are not proof (Weber, 2005). In this respect, it can be stated that proof activities are beneficial in mathematics education to help students improve their mathematical reasoning because it provides students opportunities to realize the mathematical truths on their own being free from their teachers and books (Knuth, 1999, 2002). Furthermore, students are also expected to recognize reasoning and proof as the basic property of mathematics, do and search for mathematical conjectures, write and evaluate proofs and know how to prove and make reasoning appropriately in mathematics before they begin college education (Knuth, 2002).

The efforts to make proof central to school mathematics is strictly connected to teachers since they are responsible for being the intersection point of the learners and the curriculum and filtering the curriculum through to the students (Du Plooy, 1998; Graham & Fennell, 2001). In this respect, it is important what teachers know about proof and how they prove. This fact can be explained by using the findings of the previous research (Steiner, 1989; Brousseau & Otte, 1991; Movshovitz-Hadar, 1993). They have claimed that when teachers do not have sufficient knowledge of proof, they hesitate to teach proof or do not teach the proof at all (Stylianides, Stylianides, & Philippou, 2007). Therefore,

it is important to make investigations about preservice mathematics teachers (PMTs) since they attain necessary knowledge about proof and how to teach and integrate it in the lessons in the future in teacher education programs.

Conception of proof can be defined as an individual's ability of proving, perception of the function of proof and need for it in mathematics. In other words, conception of proof is a construct about the ability of proving, analyzing proofs and the beliefs related to proof (Conner, 2007). It is also related to an individual's beliefs about the essence of proving actions in mathematical practices (Knuth, 2002). Conception is important since it is related to problem solving performance of students and their proof writing performance (Moore, 1994). Also, proof is connected to argumentation since it is a right convincing argument. With the motivation of these interrelated terms, they should be searched together. In the literature, there are previous research including all of these variables (Arzarello & Sabena, 2011; Boero, Douek, Morselli & Pedemonte, Pedemonte, 2007/2008). In this respect, differentiating from these research, the present study can provide contribution to the related literature by investigating proof and argumentation by specifying in the context of MI as the basic and important proving strategy necessary for mathematics in schools and written argumentations.

3. Method

The present study was designed with respect to multiple case study since case study enables to understand processes involved in the study thoroughly (Merriam, 1998). It is also defined as to select one or multiple cases related to the actions or phenomenon within real life in order to gather an essential amount of data to understand various procedures related to the research problem (Merriam, 1998). Also, multiple case study is used in this research because it is more beneficial than single case study in terms of generalizability of the findings (Meriam, 1998; Yin, 2003). In other words, using multiple cases is "…a common strategy for enhancing the external validity or generalizability of your findings…" (Meriam, 1998, p.40). It can be stated more specifically that multiple case-holistic design is used in the study. Therefore, the multiple cases were investigated by using standard instruments to collect data separately from each case with respect to the same research problem, and the results of the findings, obtained from these cases were compared (Yin, 2003).

3.1. Participants and Treatment

30 preservice mathematics teachers (PMTs) who were senior and enrolled in an undergraduate course, and volunteered to participate in the present study. 40% of these PMTs were the department from secondary mathematics education and 60% were from the department of elementary mathematics education. Ten (out of a possible 30) PMTs selected for the present study and all of them completed the data collection dealing with argumentation, proof and conception of MI. They were determined by keeping on the ratio between the participants from different departments. These chosen volunteer participants were also interviewed three times in once per two weeks.

In order to provide the participants the opportunities to engage in proof construction activities about MI by written argumentation, instructional sequence taking place through six weeks and two hours in each week was designed. The six-week period of the present study can be explained in three stages. In the first stage, the participants were given information about what argumentation and its components such as claim, data and warrant are and how to write argumentation including these parts with respect to Toulmin's model (1964). Also, participants were provided opportunities to write argumentations as practices. At the end of the stage, the participants were wanted to write argumentations related to the provided proof construction activities of MI. In the second stage, some additional information related to MI was provided to the PMTs so that they could improve their knowledge about the concept. There existed discussions in the classroom. These discussions and information were related to what proof, proving, inductive reasoning and mathematical induction were and what the basic steps to solve any question related to MI were. Moreover, written argumentations formed at the end of the first stage were discussed and feedbacks related to them were explained. Then, they were asked to form written argumentations related to MI activities. In other words, the participants were provided opportunities to attain procedural knowledge of MI. In the last stage, the feedbacks related to PMTs' written

argumentations belonged to previous stage were explained. Next, opportunities were provided to the PMTs in order to obtain more detailed information about MI and deeper understanding related to the concept of MI, and be more experienced about it. They were helped to acquired conceptual knowledge of MI. Then, they formed their written argumentation related to supported proof construction activities MI.

3.2. Data collection and instruments

The PMTs were encouraged to prove given questions by using the principles of MI so that their action could be investigated through their constructions of proof in the second and third stages. Also, they were asked to form the written argumentations three times per two-week sequence related to MI problems through engaging in the problem-solving processes. The PMTs wrote their argumentations based upon Toulmin's (1964) Argumentation Pattern (TAP) framework. Moreover, these participants were conducted to semi-structured interviews three times in order to attain information about the development of their conception of MI. Typical interview questions can be exemplified: What does the notion of induction mean to you? (This question also revised concerning with inductive reasoning and MI) What constitutes MI in school mathematics? Why is MI important in school mathematics?

At the end of the first two-week stage and completed the training about written argumentation, two problems related to MI were asked to the students in order to identify their previous knowledge and conception about MI and inductive reasoning. These problems can be exemplified as follows:

Problem. $\forall n \in N^+$, prove $1^2+2^2+3^2+\ldots+n^2= \frac{n(n+1)(2n+1)}{6}$.

Through the second stage, they learned the definition, meaning and procedures of MI. Then, at the end of this stage, the participants were asked to prove one of these given problems and form their written argumentations. After they had completed these tasks, they were interviewed about their conception of MI for this stage. Moreover, some feedbacks related to their written argumentations were supported. In the second two-week stage, two problems were explained for the PMTs' proving and forming written argumentations. These problems can be exemplified as follows:

Problem .   Prove $\cos\alpha \times \cos2\alpha \times \cos3\alpha \times \cos4\alpha \times \ldots \times \cos2^n\alpha = \frac{sin2^{n+1}}{2^{n+1} \times sin\alpha}$.

After the lesson for the second stage had been completed, the PMTs were interviewed to be followed in the consideration of the development of their conceptualization of MI. In the last two-week stage, they were asked problems as follows:

Problem.   Prove that the sum of the cubes of 3 successive positive integers is divisible by 9.

Then, the PMTs wrote their argumentations and then, they were interviewed for the last time with the same purposes to the previous two stages.

3.3. Data analysis

In the process of analysis of qualitative data, six steps suggested by Creswell (1998) were considered to analyze qualitative data. These six steps are exploring the data through the process of coding it; using the codes to illustrate descriptions and themes; representing the findings; interpreting the meaning of the obtained results by reflecting personally on the impact of the findings and on the literature; and conducting strategies to satisfy validation and the accuracy of the findings (Creswell, 1998). The analysis process of the present study includes two analysis processes such as the analysis of the data collected through interviews and written argumentations. In both processes, six steps suggested by Creswell (1998) were followed. Initially, all of the interviews were transcribed. Next, all of data were read and it was decided to use existing codes and themes in the literature. The codes for written argumentations were the argumentation logs identified through data analysis by using Toulmin's (1964) Argumentation Pattern (TAP) framework and the five codes emerged the

interviews: nature, function, meaning, importance and necessity of MI for the analysis of MI. Three themes were determined and represented as the titles for findings. In this framework, an argumentation log is composed of clam (idea to be justified or ideas stated as true), data (evidence, procedures and methods for the truth of claim) and warrant (connection among claim and data). Afterwards, the findings obtained through this analysis process were reported by adding the interpretations of the meaning of the results. Then, the lists of categories and themes and descriptions were formed by the researcher and an academician having mathematics education background independently, and then compared by discussing to provide validity and reliability. With the aim of providing reliability and validity of the analysis of all of the data in the present study, a multi-case design applied to compare and contrast the obtained data. Also, all the processes, procedures and steps of the analyzing part of the study were explained as clear as possible for the reliability. Moreover, the validity of analysis of the data was supported by the investigator triangulation. The analyzers formed their list of categories independently and then compare their lists to provide a common list with approximately 85% inter-rater reliability. At the end of the analysis, a different researcher read and assessed the analysis part of the study considering the properties of consistency and coherence. By doing investigator triangulation and peer debriefing, the validity of the present study was satisfied successfully (Lincoln & Guba, 1985). Moreover, the data of each participant collected through written argumentation and interviews were analyzed separately and comparatively to make with-in and cross-case analysis.

4. Findings

4.1. Reasoning about the Procedural Proof Production

The first application was made at the end of the initial two-week stage. In this stage, the PMTs' proof constructions by written argumentations were analyzed. The participants' written documents on which they wrote their proofs of MI. It was found that the PMTs solved the proof construction problems in this stage by considering proof writing that they did in previous lessons or looking the solutions of similar solved problems. The interview data made clear this situation in a way that:

> "I remembered doing this kind of proving in a lesson of previous semester. Initially, I examined the book that we used in the lesson of Calculus since we learned the proof strategy of MI in this lesson in the past. There were the parts including problem solutions. I searched the chapter of MI and problem solutions of this chapter. I found a similar problem solved by MI. Then, I solved the problem of this lesson by looking this book and following similar steps."

Another PMT explained "…initially, I try out the rule or formula for some numbers such as 1, 2, 3, 4 and 5 successively. Then, when I see that it works, I can say that it is true". All of the participants made similar explanations to the question of how to prove the problems related to MI in the interviews. It was observed that the PMTs solved the proof construction activity related to MI by exploring and using similar problems and their solutions.

4.1.1. Written Argumentations. By the first stage of the training, written argumentations and conception of the PMTs towards MI were analyzed in proof construction activities related to MI. Ten written argumentations were analyzed in this stage. All of them could not write the warrant effectively. Three of the participants write the data and the claim by benefiting from the steps and information used by similar proofs in the textbooks. The typical examples are as follows (Figure 1):

DATA. We determine the cubes of the successive numbers and make subtractions between cube values of two successive numbers. When we make this kind of subtraction three times, third of the subtraction values are same.

CLAIM. …pattern in mathematical statement.
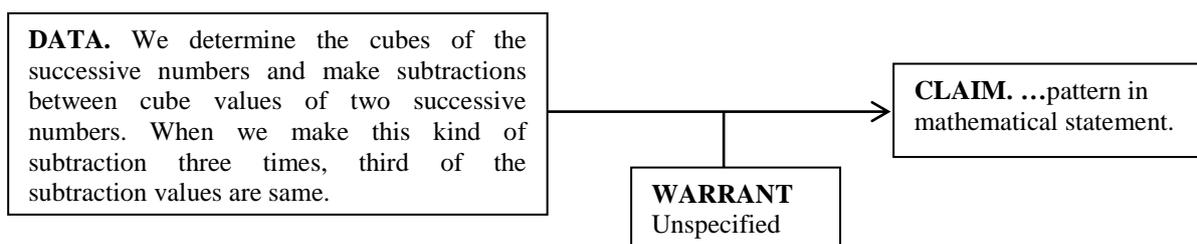
WARRANT Unspecified

Figure 1. *Example of written argumentation*

Moreover, other four of the participants wrote unrelated data and claim without specifying the warrant. The typical example (Figure 2):
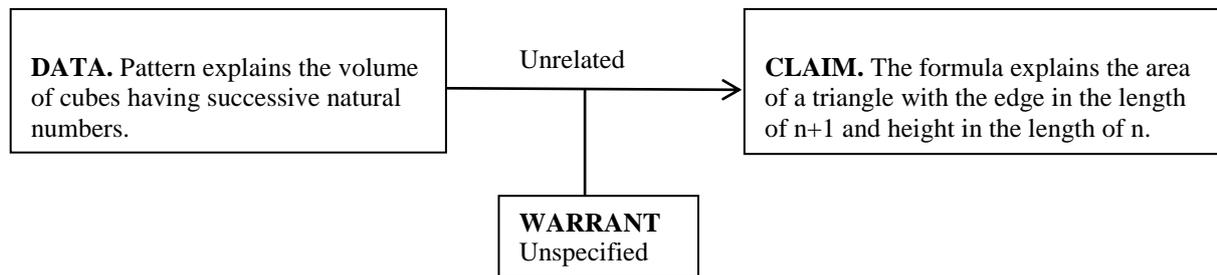
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┌─────────────────────────────┐                        ┌─────────────────────────────┐
│ DATA. Pattern explains the  │      Unrelated         │ CLAIM. The formula explains │
│ volume of cubes having      │ ──────────────────────▶│ the area of a triangle with │
│ successive natural          │          │             │ the edge in the length of   │
│ numbers.                    │          │             │ n+1 and height in the       │
│                             │          │             │ length of n.                │
└─────────────────────────────┘          │             └─────────────────────────────┘
                               ┌──────────┴──────────┐
                               │ WARRANT             │
                               │ Unspecified         │
                               └─────────────────────┘
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Figure 2. *Example of written argumentation*

The other remaining three participants could not write data, warrant and claim. In this stage, the participants could prove the statements since there were similar problems and their solutions to look deeply. They did not know the meanings and the importance of the steps so they could write the data and the claim unrelatedly and separately and they could not write the warrant for them.

4.1.2. Conception of MI. The PMTs' conception of MI was examined through the analysis of the interview data. The PMTs thought that there were important terms and elements in MI. In other words, the nature of MI was stated that there were elements necessary to be included in the proof statements. There were typical examples for the PMTs' explanations for the nature of MI as follows:

PMT_1: When we prove a mathematical statement, it is important to determine the borders of the set in which the statement is identified. Then, we can determine and make the proof in case of conditions and sets.

PMT_3: There exist theorems discovered and identified in the past and explained by using mathematical signs and language. Also, these theorems are stated in a general way as possible as the borders of the statement determine.

PMT_7: The proof statements are able to be checked, examined and confirmed about the truth. They explain the relationship between data and patterns with respect to cause and effect. In addition, they are statements of the results of a situation.

In this category, the roles and functions of the proofs in mathematics are explained. In other words, the questions of what the roles of the proof are and how they perform these roles are examined. There are examples of the PMTs' interview data for the function of MI:

PMT_4: The proofs explain how to form the rules and concepts, why they are true by stating the relationship between concepts and how to show the evidence of them by the related data.

PMT_6: The proofs are the way of communication of the truth of the statements and the relationship between concepts by using logical assumptions through mathematical language.

The proofs are placed in mathematics since they are needed to use. Some questions examining the reality and truth of the mathematical statements are explained with the help of the proofs. There are examples of the PMTs' interview data for the necessity of MI:

PMT_2: We need the proofs to remove the confusion and doubts about the concepts in our minds. Also, we can improve our knowledge by adding new knowledge and supporting new perspectives.

PMT_5: Proof is the base of the mathematics as the ground of a building. The proof is important for mathematics as much as the importance of the ground of a building. Moreover, we can explain the truth and reality of abstract things by proving as the concrete ones are supported by our senses.

The proofs have significant properties for people and mathematics. There are examples of the PMTs' interview data for the importance of MI:

PMT$_8$: With the help of the proofs, we can understand the concepts and control over them in many respects. Hence, we can acquire the knowledge instead of memorizing it. Also, we can think about the concept in different perspectives with the help of the properties of thinking analytically and logically and interpreting of them since we obtain these properties by proving.

PMT$_{10}$: We obtain the skill that we accept the reality or truth of a concept by questioning it instead of accepting without any doubts. Hence, we explain the concepts by making sure about people, convincing them about the concepts and removing probability and estimation about them.

What the proof means in mathematics. There are examples of the PMTs' interview data for the meaning of MI:

PMT$_9$: Proof means showing the truth of a statement through the use of mathematical language with the help of evidences about it and explaining the reality of a statement in a mathematical way. Also, proof is a strategy of forming a mathematical statement which is generalization of the concept in a valid way.

PMT$_4$: I think that proof is a strategy of convincing and persuading someone about the concept by using evidences of it and mathematical language.

In the light of the interview data explained above, we can state that the PMTs have *general perspective* since they define the nature, the function, the meaning, the necessity and the importance of MI based on the definition of the proof and its feature of being a kind of proving strategy. The PMTs defined MI as a proving strategy and then, they insisted on that the properties of the proof were held by MI.

4.2. Reasoning about Syntactic Proof Production

The second stage of the written argumentation application was made after the second two-week treatment. Proof construction activities by written argumentations about them were examined. It was found that the PMTs performed the proof construction activities by argumentative writing in this stage by using their knowledge that they obtained through two-week treatment. It becomes clear with the interview data as a typical example as follows:

"There are two parts of a proof statement formed by MI: *base step* and *inductive step*. When I see a mathematical statement proved by MI appropriately, I write the validation of the statement for the first element in the set limiting the statement and I assume that the statement is true for any element in the set and then I try to prove the truth of it for the element which is the consecutive of arbitrary element."

All of the participants made similar explanations in response to the question of how to prove the problems related to MI through the interviews. In the light of the interview data, it can be said that the PMTs proved the mathematical statement by MI based on its definition and general steps to follow rather than its meaning.

4.2.1. Written Argumentations.
Written argumentations of all of the participants were analyzed. Half of the participants wrote the data and the claim by specifying the warrant by benefiting from general steps and information related to MI. The typical example is as follows (Figure 3):
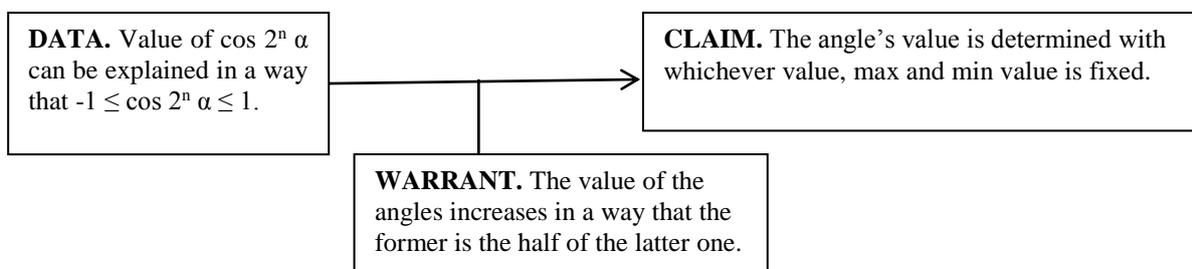
DATA. Value of $\cos 2^n \alpha$ can be explained in a way that $-1 \leq \cos 2^n \alpha \leq 1$.

CLAIM. The angle's value is determined with whichever value, max and min value is fixed.

WARRANT. The value of the angles increases in a way that the former is the half of the latter one.

Figure 3. *Example from written argumentation*

Moreover, other half of the participants wrote different data and claim with the warrant. The typical example is as follows (Figure 4):
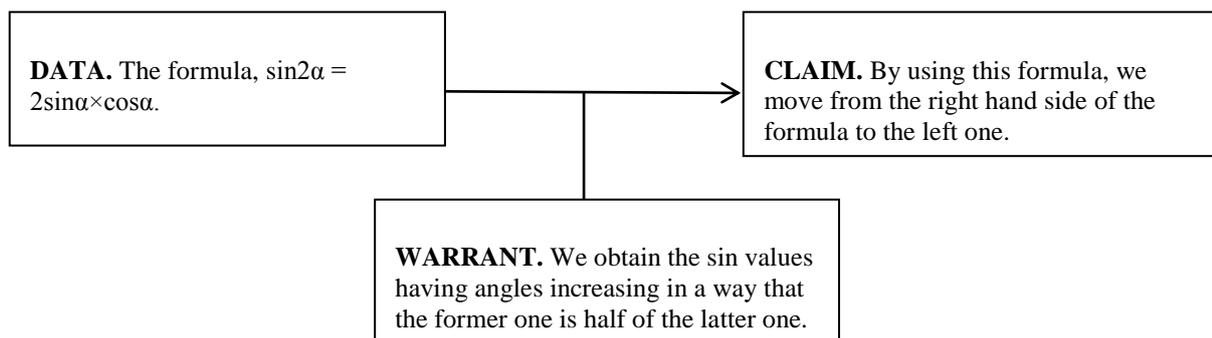

DATA. The formula, $\sin 2\alpha = 2\sin\alpha \times \cos\alpha$.

CLAIM. By using this formula, we move from the right hand side of the formula to the left one.

WARRANT. We obtain the sin values having angles increasing in a way that the former one is half of the latter one.

Figure 4. *Example of written argumentation*

In this stage, the participants were able to prove the statements by using the definition of MI and following general steps of it. They were also able to write the warrant in addition to the data and claim, and the warrants were based on procedural knowledge, definitions and formula related to MI. Hence, it could be concluded that they did not know the reasons of the steps, why these steps are followed and why they used these definitions and formula. In other words, it can be stated that although they could the procedures and limited conceptual knowledge about MI, they could not use inductive reasoning effectively. Therefore, it could be claimed that the participants using inductive reasoning in this way were able to write data, warrant and claim for their proofs but they were not able to explain how and why they use and explain them.

4.2.2. Conception of MI. The PMTs' conception of MI was examined by analyzing through the interview data. There is an example of the PMTs' interview data for the nature of MI as follows:

PMT$_2$: Initially, we determine the situation and borders of it. Then, we observe it in many conditions. If the situation is valid for all conditions without any exception, we can reach a generalization about it. We make the assumption of the truth of the step of n = k and we show the truth of it for n = k+1. Therefore, we can form a generalization about the concept. MI includes the steps starting with the parts of a concept and ending its general.

There is an example of the PMTs' interview data for the function of MI as follows:

PMT$_5$: We reach MI as a strategy of a statement by following steps in a hierarchical way. While making proof of MI, we follow the steps beginning with specific parts and ending with a general statement. The results obtained from these specific parts are generalized to all parts by explaining mathematically by relating the steps and the concepts.

There is an example of the PMTs' interview data for the necessity of MI as follows:

PMT$_4$: With the help of MI, we can make connections between the parts of a concept, the steps of it gradually. In this respect, the concept can be understood and the people can be convinced and persuaded easily.

There is an example of the PMTs' interview data for the importance of MI as follows:

PMT$_7$: We can reach the general result by beginning from specific situations so that we can understand the concept effectively, logically and permanently. Moreover, I think that MI is an easy and striking strategy to apply in mathematics by persuading and convincing the concepts.

There is an example of the PMT's interview data for the meaning of MI as follows:

PMT$_9$: MI is a kind of proving strategy to make generalization by moving from parts to whole and from specific parts to the general one.

In the light of the interview data explained above, it could be claimed that the PMTs had procedural perspective since they defined the nature, the function, the meaning, the necessity and the importance of MI based on its definition and general steps in this stage. The PMTs could acquire procedural

knowledge without the logical meaning of the steps. Through the written argumentations and interviews, the PMTs tended to ignore the importance of base step and make sense of the inductive process.

4.3. Reasoning about Semantic Proof Production

The last written argumentation application was made at the end of the process of the treatment. Proof construction activities by written argumentations of the PMTs were analyzed. It was found that the PMTs performed the proof construction activities by written argumentations in this stage by using their procedural and conceptual knowledge that they acquired with the help of the treatment. It can be examined to understand with the interview data as a typical example as follows:

> "I know what MI means and how and why we use this proving strategy. I think that it is based on the real life and we can use it effectively in the real life. For example, each day lasts twenty four hours and we can generalize this statement with the help of MI. It is similar to form a car by collecting all parts of it together. In my opinion, it is a beneficial strategy to use in elementary and secondary schools and the students for these grade levels are able to understand this strategy easily. Moreover, when I prove the statement, I can easily check my proof for any point that I want. This checking way is easy since I know MI and when and for what values it is valid in any mathematical statement. Also, these students can become motivated by proving a statement."

All of the participants made similar explanations in response to the question of how to prove the problems related to MI in the interviews. Considering the interview data, in the last stage, it could be stated that the PMTs proved the statement in a mathematical way by MI based on inductive reasoning effectively.

4.3.1. Written Argumentations. Written argumentations of all of the participants were analyzed. Three of the participants wrote the data and the claim by specifying the warrant by benefiting from procedural and conceptual knowledge about the mathematical contexts and MI. The typical example is as follows (Figure 5):
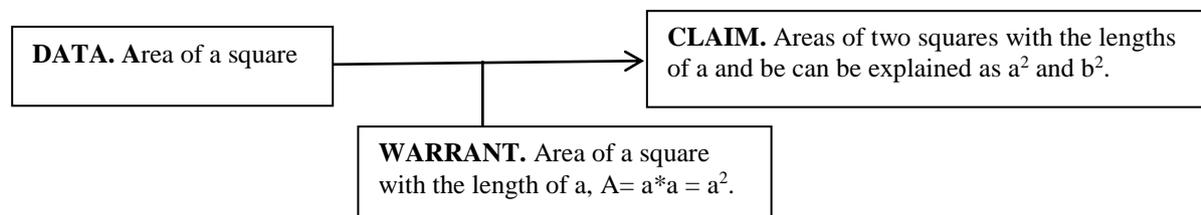

Figure 5. *Example of written argumentation*

Moreover, other three of the participants used different data and claim with the warrant in their written argumentations. The typical example is as follows (Figure 6):
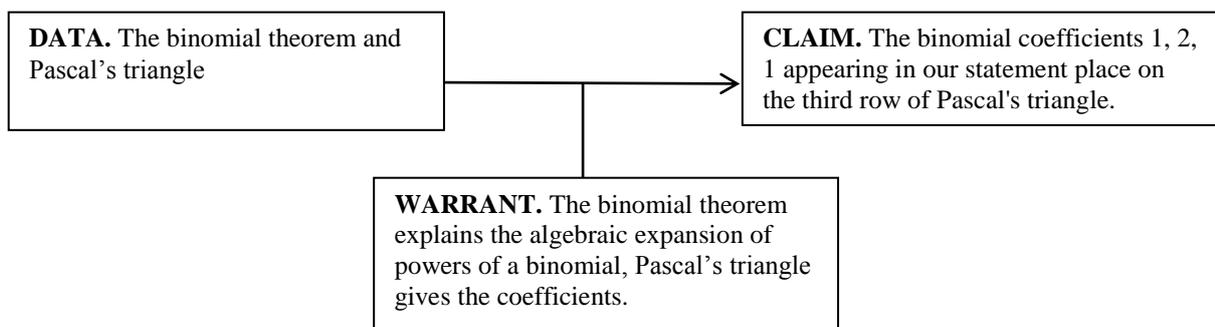

Figure 6. *Example of written argumentation*

Furthermore, other remaining four of the participants used different data and claim with the warrant in their written argumentations. The typical example is as follows (Figure 7):
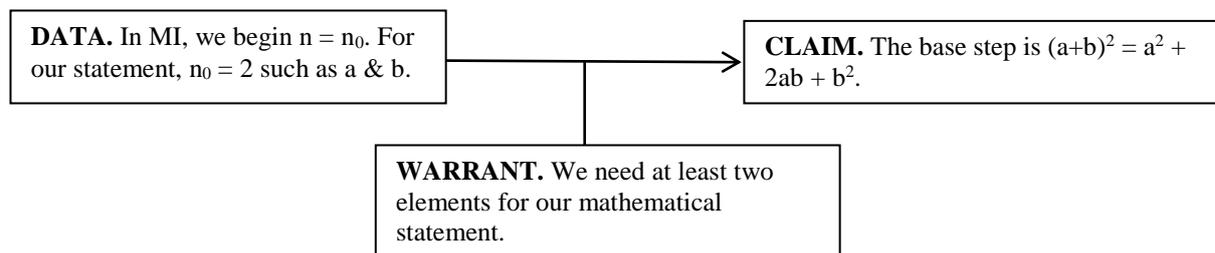
DATA. In MI, we begin n = n_0. For our statement, $n_0 = 2$ such as a & b.	CLAIM. The base step is $(a+b)^2 = a^2 + 2ab + b^2$.

WARRANT. We need at least two elements for our mathematical statement.

Figure 7. *Example of written argumentation*

In this last stage, the PMTs were able to convert written statements into mathematical statements and prove them by using procedures of MI and knowing the reasons and logic in each step. After the participants proved the mathematical statements, they checked for their proving results by some arbitrary values such as n = 6. Also, the participants proving the statement with the help of the area of a square checked their results by drawing two squares with the lengths of a and b and then added their areas. Moreover, they were able to write data, warrant and claims by strictly and logically relating them. They successfully knew the reasons of the steps, why they followed these steps and why they used these definitions and formula. Hence, it could be said that the PMTs using the semantic proof were able to write data, warrant and claim for their proofs and they could explain effectively how and why they used and explained them. Hence, it can be stated that the PMTs could perform MI proving strategy by using inductive reasoning.

4.3.2. Conception of MI. The PMTs' conception of MI was examined by analyzing through the interview data. There are five themes arisen from them: nature, function, meaning, importance and necessity of MI. There is an example of the PMTs' interview data for the nature of MI as follows:

PMT_4: In proof statements formed based on MI, there are concepts related to each other in a systematic way. Also, they are the result or summary statements explaining procedural and conceptual knowledge related to the concepts.

There are examples of the PMTs' interview data for the function of MI as follows:

PMT_3: MI is an important proving strategy to convince and persuade people having ambiguity about the concepts when it is appropriate to use. It is also useful to convince them about the result and generalization about the concepts. For example, there are concepts in elementary education and MI is appropriate to prove them. Hence, MI can be used effectively in elementary education and also secondary education.

PMT_5: There are elements, concepts and steps in proof statements of MI. With the help of the usage of MI, it is possible to relate them to each other. The actions formed through the steps are explained as a whole.

PMT_8: We examine many specific situations having common properties or steps in order to determine the general pattern valid for all of them and then we explain it in a mathematical way. This pattern is a general explanation summarizing and covering all of them. Moreover, we understand whole action and problem by using its parts and elements.

There are examples of the PMTs' interview data for the necessity of MI as follows:

PMT_2: We need MI to explain the truth or reality of the concepts that we question. Also, we summarize the procedural and conceptual properties of them with the help of MI. Hence, we can use MI to convince people about the concepts and provide a valid strategy for the truth of the concept.

PMT$_6$: Whenever I hear the term of MI, I remember the action of reasoning. In reasoning, we form a wall by using bricks connecting them. MI is similar to reasoning. In my opinion, the bricks are the concepts or steps, the wall is the general pattern and reasoning is MI.

There is an example of the PMTs' interview data for the importance of MI as follows:

PMT$_7$: MI improves our skills of thinking in analytical and totalitarian way. We can also improve our problem solving skills since proving the statements by MI can be accepted as solving a problem. In addition, I think that MI can be considered with respect to Gestalt approach. In MI, we handle with the parts and actions of a whole. This whole explanation is more meaningful than its parts as occurred in Gestalt approach.

There is an example of the PMTs' interview data for the meaning of MI as follows:

PMT$_9$: I think that MI is a strategy of obtaining a general explanation by using its parts, steps and specific examples. Also, the truth valid for a specific situation is valid for the truth of the whole explanation.

PMT$_{10}$: In my opinion, MI is similar to making a puzzle since both of them includes gathering the parts to form whole.

In the light of the interview data stated above, the PMTs talked about the nature, the function, the meaning, the necessity and the importance of MI by focusing on formal procedures and making sense of MI. The PMTs used procedural and conceptual knowledge with the logical meaning of it and inductive reasoning.

5. Discussion and Conclusion

In the current study, the development of the PMTs' conception of MI and inductive reasoning through three stages. In the first stage, it was observed that the PMTs tend to focus on the steps of MI by ignoring its conceptual aspects. They were also likely to perform these steps based on memorized procedures or assimilating the steps to a similar problem on the textbooks or resources. Written argumentations showed that the PMTs used these steps without making connection among them and relating procedures to the reasoning about the problem context since although they could produce claim and data, they could not produce the warrant. Hence, this stage can be explained with the term of procedural proof defined by Weber (2005) because of the consistency among the properties of this term and the PMTs' actions through this stage. Weber (2005) called as procedural proof in a way that external sources were found to be proved. Hence, it can be claimed that the PMTs using the procedural proof are not able to relate the data and claim and write the warrant for them. Also, Weber (2005) explained that "when asked to prove a statement, undergraduates sometimes attempt to locate a proof of a statement that is similar in form and use this existing proof as a template for producing a new one" (p. 353). At the first stage, it was observed that the PMTs could not make sense about the usage of base step and inductive steps [progression from P(k) => P(k+1)]. This finding is parallel to the previous research (Pedemonte, 2007; Stylianides, Stylianides & Philippou, 2007).

In the second stage, the PMTs were able to connect claim and data by producing warrants. They performed this action by benefiting from procedural knowledge and limited conceptual knowledge about MI such as its definition and formula of context. In other words, they used procedural knowledge and limited knowledge without making sense of inductive reasoning effectively. The actions performed in the second stage by the PMTs could be stated as an example of syntactic proof production put forward by Weber (2005). Weber (2005) explained that "when undergraduates attempt to construct a proof, they sometimes begin with a collection of definitions and assumptions, and then draw inferences about these statements by applying established theorems and logical rules" (p. 355) and added "syntactic proof productions possibly the opportunity to develop strategies and heuristics for proof construction" (p. 358). At the second stage, it was observed that the PMTs tended to ignore the base step. Although they used base step and inductive process from P(k) => P(k+1), they could not make sense of these steps. This finding is parallel to the previous research (Dubinsky, 1990; Harel, 2002; Pedemonte, 2007; Stylianides, Stylianides & Philippou, 2007).

In the last stage, the PMTs performed MI using formal mathematical language and procedures by making sense of inductive reasoning. It can be claimed that the PMTs could have conceptual perspective since they could explain the nature, the function, the meaning, the necessity and the importance of MI based on its logical meaning and general steps and the meanings of MI benefiting from inductive reasoning in this stage. The PMTs could have procedural and conceptual knowledge with the logical meaning of it and inductive reasoning. Moreover, in conceptual perspective, the PMTs benefited from semantic proof by making sense of MI and inductive reasoning. The proof construction activities used in the last stage by written argumentations of the PMTs can be an example of semantic proof production put forward by Weber (2005). Weber (2005) explained that "undergraduates use to construct a proof is to consider informal or intuitive representations of relevant concepts to see why the statement to be proven is true" (p. 356) and added "semantic proof productions afford students the opportunity to develop or refine informal representations of mathematical concepts, and use their reasoning with these representations to gain a conviction and understanding of why mathematical theorems are true" (p. 358). The applications based argumentative writing enhanced the PMTs' conception and conceptual knowledge of inductive reasoning and MI. This finding is parallel to the previous research (Chen, Park, and Hand, 2016; Yaman, 2018).

To conclude, in this paper, the PMTs took treatment in which they acquired the knowledge about how to form written argumentations and MI. This treatment lasted six weeks. In the process of the treatment, the PMTs improved their skills about their proof construction activities by written argumentations and conception of MI. First, the PMTs could improve their proof construction activities by written argumentations. In the first application, their proof construction activities by written argumentations can be identified as procedural proof since they could prove the mathematical statements by exploring and using similar problems and their solutions by formed other people. In the second application, the proof construction activities can be identified as syntactic proof because they could prove the concepts with the help of the definition and general steps of MI to follow rather than its meaning. In the last application, their proof construction activities by written argumentations are defined as semantic proofs since they could know it by using the logic and meaning of MI and they could use the representations and check their proofs effectively. These proof construction activities can be explained through the stages of procedural, syntactic and semantic proofs. Second, in six-week period treatment, PMT improved their skills of written argumentations. In the first application, their written argumentations included unrelated and specifying elements. In the second application, they could use all elements of argumentation based on procedural knowledge. In the last stage, they could improve their argumentations with the help of conceptual knowledge and reasoning. Third, the PMTs improved their skills of conception of MI and this might result from the information and knowledge that they obtained through the six-week period treatment. In the first application, the PMT's conception of proof and inductive reasoning is based on general perspective since they could think general properties of the proof and also MI as a proof strategy. In the second one, it is based on procedural perspective because they could know the general definition and steps of MI. It is based on conceptual perspective in the last application since they could know and use all the information and logic of MI.

Most of the studies exploring the proof and argumentation together have focused on the identification and establishing of the reasoning ways and structures through proving with the help of Tolumin's argumentation model or Knipping's (2008) global argumentation structures (Erkek & Işıksal-Bostan, 2019; Strom, Kemeny, Lehrer, & Forman, 2001; Uygun & Akyuz, 2019). There have been limited research examining the development of proof construction and argumentation simultaneously by showing the impact of argumentation on proof construction (Pedemonte, 2007). In the previous research, Pedemonte (2007) stated that argumentation helped the abductive and deductive reasoning while there were obstacles in inductive reasoning. In order to make contribution to this point, in the current study, proof construction activities were supported by written argumentations to make analyze the steps of MI and process pattern generalization. Also, interviews were also made to enhance and analyze their conception of inductive reasoning and MI. In the light of the findings, it was observed that this application process helped the PMTs' overcome their obstacles on inductive reasoning and MI. Hence, it can be stated that written argumentation can enhance proof construction by MI and

inductive reasoning. This conclusion can be encouraged by the previous research emphasizing the positive impacts of argumentation on proof construction (Pedemonte, 2007).

In the current study, the impacts of written argumentation on inductive reasoning and MI were explored. The impacts of written argumentations on other proof production strategies such as abductive and deductive reasoning can be explored in further research. Moreover, the current study was conducted to a public university of Turkey, the ways of the impacts of written argumentations might vary among the university students in other cultures since the argumentation as kind of discourse encouraging critical thinking can be affected by cultural values. Moreover, Reid and Knipping (2010) talk about the necessity of applying argumentations into other cultures and contexts because of the actions of criticizing, convincing, justifying and refuting ideas through interaction needed for argumentation. Also, how the culture reflect on written argumentations can be explored and compared with focusing on different cultures.

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