Students' difficulties in similar triangle questions

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Abstract

Similar triangles in questions are usually given as separate, adjacent or overlapped. Furthermore, similarity types such as Side-Angle-Side (S.A.S.), Side-Side-Side (S.S.S.) and Angle-Angle (A.A.) are requested in the questions. Students have more trouble in other types of questions. The purpose of this study is to investigate the difficulties of students about similar triangles and the reasons for these difficulties. This research was carried out with the case study method, which is one of the qualitative research approaches. The study was conducted with 55 Science High School 9th grade students and 9 open-ended questions were used to examine students' knowledge about "similarity in triangles". Furthermore, 5 students were interviewed to find out the reasons for their solutions. Descriptive analysis method was used to analyze the data. As a result, it can be concluded that students have difficulties mostly in overlapped triangles and Angle-Angle type questions. On the other hand, it can be concluded that students are quite successful where similar triangles are given separately. In the light of the findings obtained in this study, it can be advised for lecturers to focus on the questions where similar triangles are overlapped while explaining the similarity in the triangle.

Keywords: Similarity, Triangles, Difficulties, High School Students.

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1. Introduction

Geometry is an important branch of mathematics and its teaching is therefore an area to be emphasized. Geometry contributes to develop visualization, critical thinking, intuition, perspective, problem solving, hypothesis, reasoning, logical argument and proofing skills for students (Faggiano, 2012; Jones, 2002). Geometry also provides powerful tools to represent and solve problems in all areas of mathematics, other school subjects, and real-life applications (National Council of Teachers of Mathematics [NCTM], 2001). The knowledge level of students about geometry is generally at comprehension level and there are very few students who have reached the application and above knowledge levels (Arslan & Yıldız, 2010; Athanasopoulou, 2008; Stylianides, 2008; Fujita & Jones, 2007; Thirumurthy, 2003; Prescott, Mitchelmore & White, 2002; Reiss, Klieme, & Heinz, 2001). In the studies with regard to students' knowledge of triangles and quadrilaterals (Athanasopoulou, 2008), polygon definition knowledge (Carreño, Ribeiro & Climent, 2013), concept images related to diagonal in polygons (Cunningham & Roberts, 2010), field information related to hierarchy of quadrilaterals (Erdoğan & Dur, 2014; Fujita & Jones, 2007), researchers stated that students' geometry knowledge were incomplete and not well structured. In international exams such as PISA and TIMSS, the field where students are at the level of lower proficiency appears to be geometry. This brings to the minds the question of "Why are there so many difficulties?". Factors such as focusing on memorization instead of understanding the concepts, using teachers' classical materials, insufficient knowledge of teachers' knowledge in the field, insufficient examples in mathematics textbooks, and students' memorizing the question styles are some of the reasons for the difficulties encountered in geometry (Burns, 2007; Toptaş, 2007). One of the objectives of teaching geometry in schools is to develop students' spatial awareness. It is estimated that this benefit provided by teaching geometry will increase the performance of students in activities which require spatial skills in daily life (French, 2017).

Triangles are one of the most basic planar shapes in geometry and are frequently encountered in daily life, especially in architectural structures. The concept of triangle, which is one of the concepts that can be accepted as the basis for teaching geometry, is frequently used in the teaching of more complex geometric concepts (Kaplan & Hızarcı, 2005). In order to learn completely the concept of triangle, the elements of triangles and the properties of these elements must be learnt well. Concepts such as angles, edges, and heights of a triangle and their properties are included in the school curricula as auxiliary elements of the triangle. (Ministry of Education [MoE], 2017). The concept of the triangle is expressed as a special form of the polygons which is frequently studied and having many important properties. By examining the triangles, it is possible to reach information about other polygons (Argün, Arıkan, Bulut & Halıcıoğlu, 2014). Although it contributes to geometric thinking, the subject of similarity is one of the concepts which students have difficulty in understanding and learning. The equality and similarity of the triangles is one of the most important subjects of geometry teaching since the emergence of geometry and we often encounter with examples of these in daily life (Baykul, 2009). Students get acquainted with the concept of "similarity" in primary school in Turkey, but the concept of "similar triangles" is particularly the subjects of 8th and 9th grades ([MoE], 2018). In this subject, the minimum conditions required for the two triangles to be similar, Side-Angle-Side (S.A.S.), Side-Side-Side (S.S.S.) and Angle-Angle (A.A.) similarity rules and similarity ratio are emphasized. According to Mason (1989), students are not very successful in discovering similarity types.

In a one-to-one mapping between two triangles, triangles whose mutual angles are equal or whose lengths are proportional to their opposite sides are called "similar triangles". The similarity status is indicated by the symbol "∼" (Üstündağ-Pektaş, 2016). According to Figure 1-a, ΔABC∼ΔDEF equals m(A)=m(D), m(B)=m(E), m(C)=m(F), in which case the opposite sides of the triangles are proportional and this ratio is called "the similarity ratio". In the one-to-one

mapping between two triangles, if the lengths of the mutual two sides are proportional and if the angles between these proportional sides are equal, there is a similarity between these two triangles as Side-Angle-Side (S.A.S.) (Figure 1-b). If the mutual side lengths of the two triangles are proportional, there is the similarity of Side-Side-Side (S.S.S.) in the triangles (Figure 1-c). In a one-to-one mapping between two triangles, if the mutual two angles of the triangles are equal, these two similar triangles are called Angle-Angle (A.A.) similarity (Figure 1-d). The subject of similarity in textbooks is usually explained through two triangles which are separated. Some examples are indicated in Figure 1-a, Figure 1-b, Figure 1-c, Figure 1-d with regard to similar triangles subject which take place in the section of similar triangles of a book taught at Science High Schools in Turkey.
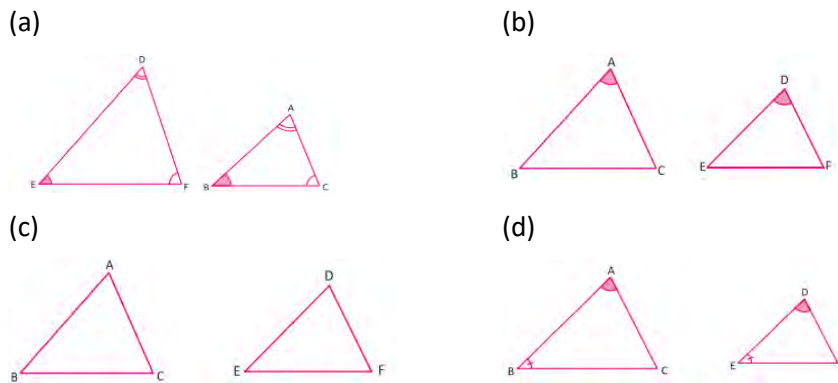

Figure 1. Examples from the textbook for the subject of similar triangles

Even though similar triangles are given separately in the lecture parts of the textbooks, triangles in the exercise questions are given as "separate", "adjacent" or "overlapped"and in the questions Side-Angle-Side (S.A.S.), Side-Side-Side (S.S.S.) and Angle-Angle (A.A.) similarity types are questioned. Students, of course, have more difficulty with some types of questions. However, it is very difficult to determine this. The analyze of the similarity of two triangles more difficult then congruently of two triangles. Errors often occur when analyzing the two triangles are similar (Parastuti, Usodo, & Subanti, 2018). This study aims to explain this situation. According to Poon & Wong (2017), students have frequently difficulties in questions where similar triangles are "overlapped". However, this claim is unfortunately needs to be proved. Therefore, the objective of this study is to determine in which question types students have more difficulties and to examine the causes of these difficulties. In some studies, it is stated that students experience difficulties in questions related to the subject of similar triangles (Parastuti, Usodo, & Subanti, 2018; Gül, 2014; Athanasopoulou, 2008; Aydoğan, 2007; Mayberry, 1983). However, no study has been encountered regarding what types of question types students have difficulties. Therefore, it could be said that this study would be the first research in this respect.

2. Methodology

2.1 *Research design*

This research was carried out with the case study method, which is one of the qualitative research approaches. The purpose of case studies can be said as evaluating a situation, seeing and identifying the factors that cause a situation to occur, and developing possible explanations about a situation (Yin, 2014; Hancock ve Algozzine, 2006).

2.2 *Participants*

This study was conducted in Turkey in 2017-2018 academic year. Fifty-five 9th grade students attending to the Science High School participated in the study. Participants were selected according to the purposive sampling method (Plano Clark & Creswell, 2015; Patton, 2002). In purposeful sampling, suitable persons are included in the study group to find answers

to the research problem (Gay, Mills & Airasian, 2006). In order to discover which type of question the students have more difficulty, firstly the basic geometric knowledge of students selected for this study should be very good. For this reason, Science High School was preferred especially for this study because students are admitted to this school with a selection exam where Mathematics and Science questions are mainly asked. Students who are admitted to Science High School must have done all the math questions correctly or only a few wrong. For this reason, it was assumed that students who participated in the study had strong math and geometry background. In addition, since "similarity in triangles" is a subject in the 9th grade curriculum, especially 9th grade students are preferred. All students voluntarily participated in the study. The purpose of the research was explained to the participants and it was stated to them that the information obtained during the study would be used only for a scientific study and their personal information would be kept confidential.

2.3 *Data Collection Tool*

In this study, 9 open-ended questions were used to examine students' learning about "similarity in triangles". The questions were prepared with the support of the literature and an expert mathematics educator. Regarding the "similarity in triangles" included in the 9th grade mathematics curriculum, students are expected to use their skills to determine the similarity ratio between the two triangles and use this similarity in the solutions of the questions.

In the examinations carried out, it was observed that two similar triangles were given in a separate, adjacent or overlapped manner in the triangle problems given in the exercise and homework questions related to this subject in the textbooks. Therefore, 9 questions have been created for the data collection tool of this research which includes both the S.A.S., S.S.S. and A.A. similarities and where the two triangles are separate, adjacent or overlapped. It is determined according to the "expert opinion" whether the problems in the data collection tool are suitable for the purpose of measurement and whether they represent the area to be measured (Karasar, 2004). To do this, firstly the measurement objectives and content analyzes required by these objectives were set by two academicians and a mathematics teacher. Accordingly, a consensus was reached when all questions were appropriate for the purpose of measurement. A data collection tool with multiple choice questions was not preferred for this research because there is only one answer for multiple choice questions and it is not known in which items the student is busy with. Therefore, it is possible to see the solutions of students through open-ended questions while it is also possible to see how students reflect their knowledge and skills about the subject to the solution.

In addition, in this study, 5 students were interviewed to find out the reasons for their solutions. In the interview, students were asked to explain their solutions for each question. With the interviews, it is aimed to deepen, enrich and increase the reliability of this research which has a qualitative pattern. The examination of the answers given by the interviewed students to the questions is given in Table 1.

Table 1. Solutions of Students Attending the Interview

No	Question Type	Feyyaz (S_{12})	Fidan (S_{23})	Aytuğ (S_{27})	Sibel (S_{35})	Mehmet (S_{48})
1	Separate Triangles and Side-Angle-Side (S.A.S.) Similarity	Right	Right	Right	Right	Right
2	Overlapped Triangles and Side-Angle-Side (S.A.S.) Similarity	Right	False	Right	False	Right

3	**Adjacent Triangles and Angle-Angle (A.A.) Similarity**	Right	Right	Right	Right	Right
4	**Overlapped Triangles and Angle-Angle (A.A.) Similarity**	False	False	Right	False	False
5	**Separate Triangles and Angle-Angle (A.A.) Similarity**	Right	Right	Right	Right	Right
6	**Separate Triangles and Side-Side-Side (S.S.S.) Similarity**	Right	Right	Right	Right	Right
7	**Adjacent Triangles and Side-Side-Side (S.S.S.) Similarity**	Right	False	Right	Right	Right
8	**Adjacent Triangles and Side-Angle-Side (S.A.S.) Similarity**	Right	Right	Right	False	Right
9	**Overlapped Triangles and Side-Side-Side (S.S.S.) Similarity**	Right	False	False	False	False

2.4 *Data Analysis*

Descriptive analysis method was used to analyze the data. The solution papers of the students were coded as S1, S2, S3,…, S55 and the solutions of the students were examined in two categories as true and false. Two experts from the field of mathematics education coded the data independently. A reliability study was conducted between the coders and the percentage of agreement between the two coders was calculated as 90% according to the formula of Miles and Huberman (1994). The items that were disagreed were reviewed again and consensus was reached. Descriptive statistics techniques (percentage / frequency) were used to analyze the data obtained from the relevant test.

3. Findings

The findings and comments in line with the scope of this study is placed under this section. Each question is examined separately and the data obtained are presented in the form of a matrix table (Table 2). In this matrix, the number of correct answers of students and percentages corresponding to these are given. The results are presented in the form of a matrix so that the reader can easily follow the evaluations.

Table 2. The Evaluation of Student Solutions

Question Type	Side-Angle-Side (S.A.S.)		Side-Side-Side (S.S.S.)		Angle-Angle (A.A.)		Average	
	f	%	f	%	f	%	f	%
Separate Triangles	*52*	95	*46*	84	*53*	96	*50*	92

Adjacent Triangles	*39*	71	*44*	80	*52*	95	*45*	82
Overlapped Triangles	*37*	67	*27*	49	*24*	44	*29*	53
Average	*43*	78	*39*	71	*43*	78	42	76

According to Table 2, it is seen that students are affected almost identically by similarity types. On the Side-Angle-Side (S.A.S.) and Angle-Angle (A.A.) questions, an average of 43 students (78%) found the correct answer. An average of 39 students (71%) were successful in Side-Side-Side (S.S.S.) questions. When Table 2 is analyzed according to the titles of lines, it is seen that the success rate (92%) is very high in questions in which similar triangles are given separately. However, the success rate (53%) seems to be quite low in questions in which similar triangles are overlapped.

According to Table 2, it is observed that the students are frequently successful in the question which has the S.A.S. similarity and when similar triangles are given separately (95%).
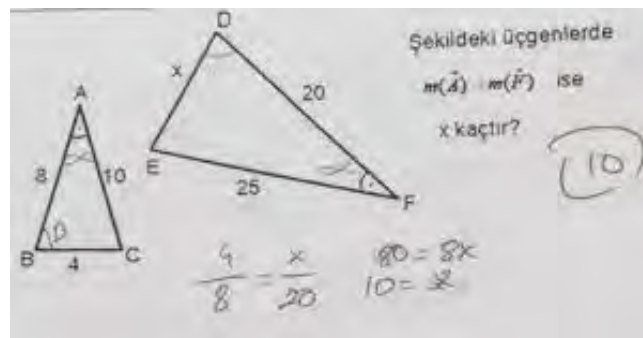

Figure 2. Question on S.A.S. Similarity and Separate Triangles

In Figure 2, there is an example solution for the question on S.A.S. similarity and Separate Triangles where the highest success rate is obtained. This type of question is one of the most easily solved one by students.

Feyyaz, who solved this question correctly, expressed his opinion about this question as follows; "*Everything is clear in this question. I was able to solve it easily because what was given and what was asked was exactly clear.*"

Aytuğ explained his solution as follows; "*Since the angle and the two sides are clear, it was not difficult to find the similarity and thus the similarity ratio.*"

According to Table 2, the correct solution rate is only 49% for the questions in the S.S.S.-type when the triangles are overlapped.
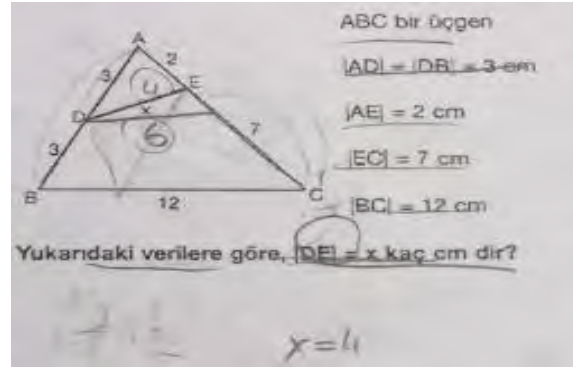
Figure 3. S.A.S. Question on Similarity and Overlapped Triangles

In Figure 3, an example solution where triangles are overlapped and the S.A.S. similarity is questioned is given. In this question, the angle $m(A)$ is common in both triangles and the two opposite lengths are clear. This type of question is one of the most challenging questions for the students.

Mehmet, one of the students who solved this question correctly explained that; "*The question is not very difficult, it is important to write the corner points of the triangles in the proper order. Here, if we indicate the small triangle with AED and the big triangle as ABC, we can find the similarity ratio immediately.*"

Sibel, who solved the question incorrectly said that; "*I wrote the triangles as given. I mean as ADE and ABC. Then I found x = 6 from AD / AB = 3/6 = 1/2 and DE / BC = x / 12.*"

Fidan, who left this question empty, thought about the question; "*The question seemed too complicated to me. I left it blank because I understood that I couldn't do it.*"

According to Table 2, it is observed that the success rates of the questions are high and very close to each other for the solutions of A.A. type where similar triangles are given separate (96%) and adjacent (95%). In the answers of question when Angle-Angle type and similar triangles overlapped, the success rate is only 44%.
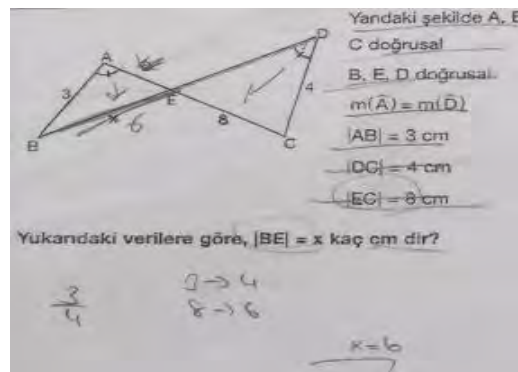

Figure 4. Question on A.A. Similarity and Adjacent Triangles

In Figure 4, an exemplary question-solution could be seen in which triangles are adjacent and angle-angle similarity is questioned. In this type of question, one of the highest success rate was obtained. Since $m(A) = m(D)$ and $m(AED) = m(DEC)$, $m(ABE) = m(CED)$. Therefore, the ΔABE and ΔDCE triangles are similar.

Sibel, one of the students who solved this question correctly explained that; "*I like solving butterfly-shaped questions because the angle would be the same at the point where the triangles meet which is in the middle of the butterfly. When you move from here, the question is solved by itself.*"
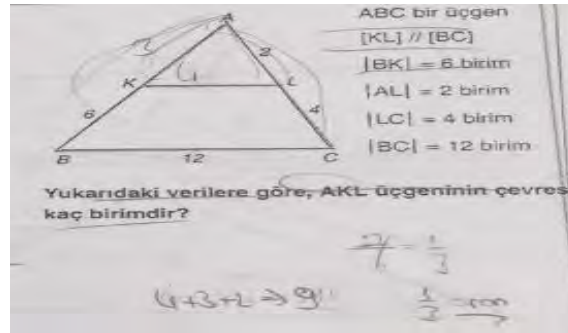
Figure 5. Question on A.A. Similarity and Overlapped Triangles

In Figure 5, an exemplary question-solution could be seen where triangles are overlapped and angle-angle similarity is questioned. In this type of question, one of the lowest success rate (44%) rate was obtained. Here, to solve the question, it is necessary to correctly determine the equal angles of the two triangles. It is given as [KL] // [BC]. Therefore, $m(K) = m(B)$, $m(L) = m(C)$ and since both triangles accept the same point A, m (A) is equal for both. Therefore, ΔAKL and ΔABC are similar triangles and the solution is obtained based on this determination.

Feyyaz, one of the students who solved this question incorrectly stated that; "*The question seemed too complicated to me. There is not much information about the triangles. Also here, the perimeter of the AKL triangle was requested. I couldn't find the similarity ratio.*"

Again, Mehmet, who solved this question incorrectly said that; "*I did [AL] / [LC] = 2/4 = 1/2 to find the similarity rate. Then, I noticed that I did wrong but I did not know how to act later.*"

When Table 2 is analyzed according to the titles of lines, it is seen that the accuracy rates are close and high in the questions where similar triangles are given separately. The highest accuracy rate was achieved in the solutions of questions such as Angle-Angle with 96% and Side-Angle-Side with 95%. However, the success rate (84%) in the solutions of questions such as Side-Side-Side type is also quite high in questions where similar triangles are given separately.
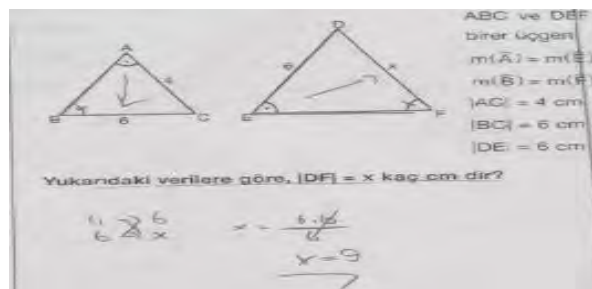

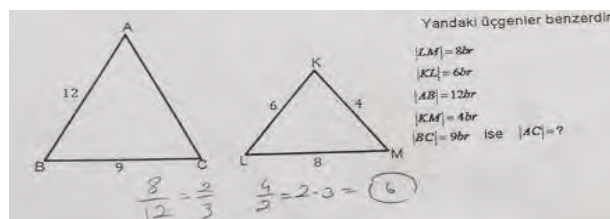Figure 6. Question on A.A. Similarity and Separate Triangles

Figure 7. Question on S.S.S. Similarity and Separate Triangles

In the questions given in Figure 6 and Figure 7, the triangles are given separately. Angle-Angle similarity is asked in Figure 6 while Side-Side-Side similarity is asked in Figure 7. Most of

the students have solved these questions correctly. All of the students interviewed said that the separation of the triangles facilitates the solution of the problem.

When Table 2 is reviewed, it is observed that the accuracy rate (95%) is the highest in the Angle-Angle type of questions in which similar triangles are given adjacent. In addition, it is seen that the accuracy rates are close to each other in the questions such as Side-Side-Side (80%) and Side-Angle-Side (71%).
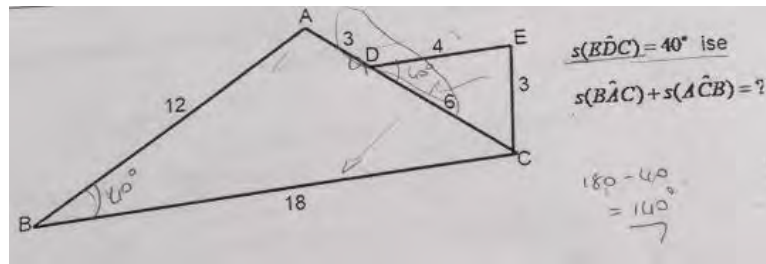

Figure 8. Question on S.S.S. Similarity and Adjacent Triangles

In the question in Figure 8, the sum of the angles $m(A)$ and $m(C)$, i.e. $m(A) + m(C)$, is questioned based on the similarity of Side-Side-Side in the adjacent triangles. It is given as m(D)=40^0 in the question. Based on the side lengths of the triangles, it is found that the triangle ΔDCE and ΔBCA are similar. If these triangles are similar, the ordered angles would be $m(D) = m(B)$, $m(C) = m(C)$ and $m(E) = m(A)$. Since $m(D) = 40^0$, m (E) + m (C) = m (A) + m (C) = 140^0$. Although this question is a relatively difficult question, the success rate is high (80%). A few examples to find out the reason behind this are given below as outputs of the interviews:

Feyyaz: "*In the question, I recognized the side lengths which are given as 6 and 18. I searched and found the ratio between these two in the others. The rest of it is an easy angle question anyway.*"

Fidan: "*I was able to find the edge lengths, but I didn't know how to get to the angles from here. I also thought I should proportion them when the AC side and the DC side overlap, but I couldn't.*"

Aytuğ: "*I normally do the angle questions easily. I know that the two triangles would be similar. If the angle of the DCE triangle is 40 degrees, I know that the angle of one corner is 40 degrees in the other triangle. I can say that I recognized the total of the angles requested in the question will be 180-40 = 140 directly.*"
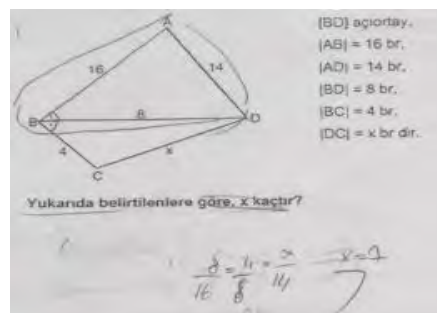

Figure 9. Question on S.A.S. Similarity and Adjacent Triangles

In Figure 9, two adjacent triangles could be seen again, and this question is asked with regard to the Edge-Angle-Edge similarity. Although it is given that [BD] is bisector in the question, it is seen that $m (CBD) = m (DBA)$. Therefore, the ΔBCD and ΔBDA triangles are similar which have an angle and two side lengths. The success rate in this question is 71%. However, it is clear that a significant number of students (29%) had difficulty with this question.

Feyyaz, who participated in the interview and answered this question correctly said that "*When an angle is given in similarity questions, the solution becomes easier. This question is an example of this. The important issue is to identify similar triangles correctly*."

On the other hand, Sibel, who solved this question incorrectly stated that "*The angle is okay, equal. BD sides are also equal. I structured the similarity ratio based on the BD side which is equal. I thought I could solve it from here, but the similarity ratio did not come out*."

According to Table 2, it is seen that success rate is lower in the questions where similar triangles are overlapped compared to other types of questions. Here, it is determined that the accuracy rates are highest with a percentage of 67% in the questions of Side-Angle-Side type and the accuracy rates are close and low with percentages of 49% and 44% respectively in questions for Side-Side-Side and Angle-Angle.
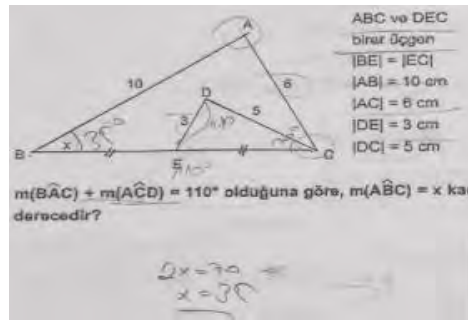

Figure 10. Question on S.S.S. Similarity and Overlapped Triangles

In Figure 10, an example solution where triangles are overlapped and the S.S.S. similarity is given. This question is one of the two questions with the lowest success rate (49%). In this question, the lengths of the sides of both triangles are clear. It is necessary to find the similarity ratio following this way. Therefore, ΔDCE and ΔABC triangles are similar. After determining this, it is completely an angle question. Finding these equal angles requires some skill. However, with proper coding, it is easy to achieve the result. Let $m(A) = y, m(B) = x$ and $m(C) = z$ in the big triangle ABC. In the ΔDCE triangle, $m(D) = y, m(C) = x$ and $m(E) = z$. Also given as $m(BAC) + m(ACD) = 110^0$. If $m(ACD) = t$, then $y + t = 110^0$ and in the big triangle ABC, it will be $x + y + t + x = 180^0$. Thus it will be $2x = 70^0$ and $x = 35^0$. For this solution, it is necessary to establish the S.S.S. similarity in triangles and to place the angles appropriately. The ideas of the students who solve this question incorrectly are given below.

Fidan: "*The question was very complex. I tried to solve it a bit, but I left it because the lengths of sides were given but I didn't know how to find the angle from there*."

Aytuğ: "*I found the similarity ratio but I did not know how to use the angles*."

Sibel: "*The question seemed to me very difficult. People are even afraid to look at such questions. I couldn't decide where to start. I couldn't go further on the question. Since BE and EC are on the BC side, I could not sort while finding the similarity rate*."

Mehmet: "*When I asked about the angle, I tried to find equal angles. But I couldn't find out what the $m(ACD)$ angle was, so I couldn't continue*."

4. Conclusions and discussions

In this study, it is aimed to reveal the problems which students experience about "similarity in triangles". For this, the answers given by the students to 9 open-ended questions prepared on this subject were examined.

When the answers given by the students to the questions in the study are examined, the most important issue which is noticed firstly is that the correct answer rates are high. This

indicates that students' geometric knowledge is at a good level. This is because the students who participated in the study were selected from Science High School. This was actually a targeted situation in the selection of samples for the study since the aim of the study was to determine where students with very good geometry knowledge had difficulty with similarity in the triangle. Therefore, the objective was achieved for the selection of this sample group. Contrary to this situation, similar studies have shown that students' geometry knowledge is not good (Gül, 2014; Cunningham & Roberts, 2010; Athanasopoulou, 2008; Fujita & Jones, 2007; Mayberry, 1983).

When the questions asked to students in the study are evaluated in terms of similarity types, it can be conluded that students have high levels of success in such questions. This situation does not match with İç and Demirkol's (2008) study results. According to them, students cannot establish a relationship between the concepts of Side-Angle-Side similarity in the triangle. In addition, it was observed that success in student solutions did not change much in similar questions. Therefore, whether the questions asked to students include Side-Angle-Side, Angle-Angle or Side-Side-Side similarity, it can be said that this situation is not very effective in students' achievements. This is also seen in the interviews carried out with students. While the students expressed their views on the questions and solutions, they did not mentioned about the types of similarities much. Therefore, it is not important for students which similarity situation the question requesting. According to Burns (2007), by focusing on geometric shapes, focusing only on the image and naming them is not sufficient in structuring geometric concepts. In order to teach geometric concepts, it is necessary to include exploratory, related-unrelated, inverse examples and different representations into the lessons. In the geometry teaching process, the learning of children should be enriched by going beyond traditional teaching materials (textbook, workbook…).

If an evaluation is done according to the condition of similar triangles included in the questions asked to the students, it is seen that the students are most successful in questions where similar triangles are given separately. In contrary, success rate is quite low in overlapped triangles. In this case, the claim that students had difficulties in questions where similar triangles were overlapped was correct (Poon & Wong, 2017). Therefore, it is very important for students how similar triangles are given in the question. If the questions are given separately, the students do not have difficulty in solving such similarity questions. Likewise, the success of students is also very high in questions where triangles are adjacent. The reason for this may be that in school books where similarity types are handled, examples are usually given based on separate triangles. Perhaps for this reason, it can be said that the students have difficulty in questions where triangles are overlapped.

When the questions are evaluated together according to the similarity types and condition of the similar triangles they include, it can be said that students have difficulties mostly in overlapped triangles and Angle-Angle type questions. On the other hand, it can be concluded that students are quite successful where similar triangles are given separately. Therefore, it can be said that students like Angle-Angle type questions more in the questions where similar triangles are given separate or adjacent. Contrary to this finding, according to Parastuti, Usodo & Subanti (2018), students find it difficult to write corresponding angles on two similar triangles. However, according to Aydoğan (2007), it is important for students to realize the similarities of Side-Angle-Side, Side-Side-Side or Angle-Angle even though students cannot solve the question about similar triangles. Knight (2006) stated that it is important for students to realize the relationships between the properties of geometric shapes and to make judgments about these relationships but most of the students are not at this level.

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5. Recommendations

In the light of the findings obtained in this study, it can be advised to focus on the questions where similar triangles are overlapped while explaining the similarity in the triangle. Students should be told in detail how similar triangles are matched in which order the angle and edge ratios are equal. In addition, for book authors, it is recommended that they prepare book contents focusing on examples on the overlapped triangles.

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