

# Teacher Noticing and Reasoning about Student Thinking in Classrooms as a Result of Participating in a Combined Professional Development Intervention

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Received January 2018 Accepted May 2020

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The study reported in this article addresses the question of how professional development can increase teacher noticing of student thinking over time. The data come from five teachers' participation in a two-year professional development intervention that combined lesson study, video clubs, and animation discussions. The analysis focuses on whether and how teachers attended to student thinking during study group discussions and when implementing problem-based lessons that they designed collaboratively. The study applies a mixed methods approach to the examination of transcriptions from the video clubs and the classroom implementation of the lessons. The findings reveal that the teachers restated students' ideas and provided examples of how to inquire into student thinking upon the facilitators' request during the video clubs. There was a statistically significant difference in the teachers' use of students' ideas for generalising and synthesising showing that teachers used higher levels of reasoning in the 14 lessons taught over two consecutive years. The intervention supported growth in the teachers' abilities to reason using students' ideas and to make the lessons' goals explicit.

## Introduction

Teacher noticing of student thinking is fundamental to teaching. When teachers notice students' ideas, they are acknowledging students as sources of knowledge. Some studies of teacher noticing have focused on how this process occurs outside the classroom, for example, when analysing student thinking using student worksheets and videos of instruction (Goldsmith & Seago, 2011). Studies on professional development have demonstrated how engaging teachers in discussions about student thinking outside the classroom can increase teacher noticing of student thinking in the classroom (e.g., Sherin & van Es, 2009). Our intention in this study was to investigate the ways in which a professional development intervention focused on increasing teachers' attention to student thinking effected teachers' study group discussions and teachers' classroom practices. We expected that teachers' discussions of videos from a lesson that they planned and implemented in their own classrooms would provide opportunities to analyse students' ideas when re-teaching the lesson. As a result of analysing students' work, the teachers could propose teaching moves for reasoning with students' ideas. The study serves as a case study for understanding the links between teachers' engagement in collective discussions of student work and their classroom practices.

We use mathematics teacher noticing and refer to Jacobs, Lamb, and Philipp (2010), who use the term *professional noticing of children's mathematical thinking*, to discuss teachers' ability to attend to students' problem-solving strategies, interpret their mathematical understanding, and respond to students. The intervention in the current study integrated discussions of animated stories of classroom instruction (Chazan & Herbst, 2012), hereafter referred to as "animations," and video clubs (Sherin & Han, 2004) within the steps of lesson study (Fernandez, 2002). The goal was to centre teachers' discussions of student thinking by using teaching and learning artifacts stemming from the teachers' classrooms (Ball & Cohen, 1999). Professional development where teachers can engage in collective interpretation of students' ideas in connection to future work provides rich teacher learning opportunities (Horn, Garner, Kane, & Brasel, 2017). The overarching question in this article is how teachers' collective interpretation of students' problem-solving strategies during video clubs translates to attention to student thinking in the classroom.

## Increasing Teacher Noticing of Student Thinking through Lesson Study, Video Clubs, and Discussion of Animations

### *Increasing Teacher Noticing of Student Thinking in Lesson Study*

Lesson study originated in Japan for improving instruction by increasing attention to student thinking (Fernandez, 2002). Accounts of lesson study in Japan show that lesson study is a fundamental activity through which teachers share and develop knowledge of teaching and of their students (Fernandez & Yoshida, 2004). In a lesson study cycle, a team of teachers and other educators engage in four main steps: (1) establishing instructional objectives through the examination of curricular materials, (2) elaborating a research lesson in relation to the instructional objectives, (3) teaching the research lesson while other members of the group observe the lesson, and (4) reflecting about the lesson using evidence from observations (Lewis, Perry, & Murata, 2006). A possible fifth step is that of revising the lesson to re-teach a new version. The research lesson enables teachers to focus their observations of student thinking in relation to specific learning goals established by the team.

The implementation of lesson study in places other than Japan has prompted researchers' examination of lesson study adaptations. The adaptations are meant to make lesson study viable in novel contexts. An important question in teacher education research is whether and how changes to the traditional lesson study model promote teacher learning. Some adaptations of lesson study in the U.S. have proved to be challenging due to some contextual factors, such as the lack of a centralised curriculum, the lack of institutional support to provide time for teachers to observe each other's classes, an emphasis on teachers working in isolation rather than through collaboration, worries about teaching in front of colleagues, and limited support from administrators (Fernandez, 2002; Olson, White, & Sparrow, 2011). Despite these challenges, lesson study can prompt teachers to focus on student thinking (Lewis & Perry, 2017).

Within the context of the U.S., there are examples of successful lesson study implementation that have increased teachers' attention to student thinking. For example, Perry and Lewis (2009) discussed various changes to the implementation of lesson study in a K-8 district. The district designed opportunities for lesson study leaders to learn more about how to facilitate lesson study. The leaders made a timeline with details about the lesson study steps, designed a template to guide the planning process, and identified exemplar lessons from which the teachers could get ideas for the research lesson. These resources addressed the limited understanding of how to address the intricacies of the lesson study cycle in the U.S. context and the challenges associated

with not having a centralised curriculum upon which to anchor instruction. Through their engagement in this lesson study adaptation, the teachers increased their attention to student thinking and learned to anticipate students' ideas during the lesson planning process. The teachers also demonstrated changes in instructional practices toward allowing students to reason mathematically and using feedback from students' work in their instruction.

Another example of changes to lesson study was reported by Alson, Pedrick, Morris, and Basu (2011). As part of a graduate program that provided opportunities for school-based professional development in an urban middle school (grades 5 to 8), ten mathematics teachers participated in one lesson study cycle. They adapted lesson study by videotaping the research lessons and required teachers to write post-lesson reflections. In the reflections, the teachers referred to evidence from the videos in analysing students' mathematical reasoning during the lesson. The authors report that teachers' capabilities to attend to students' mathematical thinking increased through their participation in lesson study. Moreover, the teachers' written reflections using references from the videos extended their opportunities to analyse students' mathematical thinking during the lessons. With the implementation of the Common Core State Standards for Mathematics in the U.S. (Common Core State Standards Initiative [CCSSI], 2010), there is an opportunity for teachers to centre their attention on student thinking in relation to specific learning goals. In addition, adding technology such as animations and videos may increase opportunities for teachers to notice student thinking during lesson study.

### *Increasing Teacher Noticing of Student Thinking through Video Clubs*

Video clubs support the goal of increasing teacher noticing of student thinking. In a video club, teachers discuss student thinking using videos from their own classroom under the lead of a facilitator (Sherin & Han, 2004). Sherin and colleagues documented that teachers who participated in video clubs increased their noticing of students' thinking during the video club discussions (Sherin & Han, 2004; Sherin & van Es, 2005). In sessions from the end of the year in a two-year video club, mathematics teachers showed shifts in their attention from the initial discussions that focused on other instructional issues to interpretation of students' mathematical thinking (Sherin & van Es, 2009). In the same study, the teachers demonstrated more sophisticated ways to reason with students' ideas as a result of participating in the video club (Sherin & van Es, 2009). In observations of teachers' instruction during the second year, the teachers went beyond restating student ideas; there was more evidence of instances where the teachers investigated the meaning of students' ideas and used students' ideas to make generalisations in observations of teachers' instruction.

The role of facilitating video clubs so that teachers use evidence from student thinking in the video and engage in a deep analysis of students' ideas is crucial (van Es, Tunney, Goldsmith, & Seago, 2014). A case documenting teacher learning through video clubs in a rural school shows that the teachers increased their noticing of student thinking by engaging in more sophisticated ways to interpret and connect students' ideas with teaching and learning principles during the video club discussions as well as by increasing their attention to students' ideas in the classroom (Wallin & Amador, 2019). Recent research on video clubs with preservice teacher education also highlights the facilitator's role for developing preservice teacher noticing of students' mathematical thinking in ways that involve a deep analysis of students' ideas (Castro Superfine, Amador, & Bragelman, 2010). Overall, research on video clubs demonstrates that this professional development strategy enables teachers to increase their attention to student thinking both during discussions of video and in subsequent classroom instruction.

The intervention reported in this study has similarities with The Problem-Solving Cycle model, a professional development strategy within which video clubs are embedded (Borko,

Jacobs, Eiteljorg, & Pittman, 2008). In this model, all the teachers, including the facilitator, study a mathematical problem that they will use in their classrooms. In follow-up meetings, the participants engage in the video club to discuss video clips illustrating students' problem-solving strategies. The results from a two-year intervention using this model show that over time, the teachers had more sophisticated discussions of students' thinking, unpacked the meaning of students' unanticipated answers, and considered how to extend students' mathematical ideas based upon the students' proposed solutions.

### *Increasing Teacher Noticing of Student Thinking Discussions of Animations*

Animations are stories with cartoon characters using a voiceover or a printed dialogue (Chazan & Herbst, 2012). In contrast with videos, animations portray fictional scenarios, with characters and events that may or may not be based on reality. Similar to video clubs, professional developers can use animations to support teachers' attention to student thinking. While originally conceived as tools for research about teachers' instructional perspectives (Herbst & Chazan, 2011), teachers' discussions of animations create opportunities to discuss students' mathematical thinking (Herbst, Nachlieli, & Chazan, 2011). At the same time, teacher educators can use animations to target specific teacher learning goals. For example, Moore-Russo, and Viglietti (2011) used animations in a graduate geometry course, finding that they enabled discussion of teaching and learning issues particularly related to geometry instruction. Chieu, Herbst, and Weiss (2011) found that preservice and in-service teachers who viewed and discussed animations in an online environment through a chat or a forum were able to notice important aspects of instruction.

In their study, Wieman and Webel (2019) showed animations to preservice and in-service teachers. The animations illustrated various students' reactions after the teacher in the animation had launched a task. The participants had to interpret student thinking and decide how the teacher should respond, two important actions in relation to noticing student thinking according to the framework proposed by Jacobs, Lamb, and Philipp (2010). They found that participants' choices about subsequent teaching actions were connected to what the participants noticed about students' mathematical thinking, which supports the notion that noticing is a fundamental practice that informs teachers' instructional decisions. Animations have also been used as a tool for preservice teachers to create scenarios depicting what they notice when viewing classroom videos (de Araujo, Amador, Estapa, Weston, Aming-Attai, & Kosko, 2015). All this new research with animations shows the potential of using this type of resource to increase teacher noticing of student thinking. Teacher education researchers have evidenced understanding on how to promote teacher knowledge through discussion of animations, but there is a limited understanding of how to use animations for teachers to plan the research lesson in lesson study and for preparing teachers to notice student thinking in their classrooms.

### *Combining Three Professional Development Strategies*

Three main considerations motivated the first author to design an intervention that combined lesson study, video clubs, and animation discussions for increasing teacher noticing of student thinking. First, the use of animations was included to support the implementation of the first lesson study step, wherein teachers engage in instructional materials research for identifying the instructional objectives of the research lesson. This step frames the planning of the research lesson (Watanabe, Takahashi, & Yoshida, 2008). A challenge encountered was the lack of curricular materials with examples of problem-based geometry curricula drawing upon students' prior knowledge, a main focus of the project. This was resolved by the research team creating

animations that showed examples of problem-based lessons that use students' prior knowledge in meaningful ways. The intention was to prime teacher noticing of students' thinking and establish the norms for engaging in discussions about student thinking that would be applicable during the lesson reflection stage. A second intention of using video clubs was to make the teaching of the research lesson more viable and to support the reflection step.

The study included geometry teachers from various schools and districts. Herbst and Chazan (2003) refer to the *practical rationality of mathematics teaching* as shared dispositions that are connected to the instructional demands of specific subjects, such as algebra and geometry. The intervention leans on this idea as it applies to geometry teachers, who can identify student thinking by drawing upon their knowledge of the high school geometry curriculum and their students. By integrating video clubs and lesson study, we did not have to coordinate classroom visits; instead, we identified footage from classrooms to focus the post-lesson discussions, and we developed protocols for the facilitators to lead the teachers to focus on student thinking when analysing the videos.

Finally, the intervention required all of the teachers to teach the research lesson in their classrooms, in contrast with the typical lesson study model where only one teacher teaches the lesson while other members of the research team conduct live observations (Lewis & Hurd, 2011). With this modification to lesson study, all of the teachers had the opportunity to elicit and use student thinking during the implementation of the research lessons. This modification to lesson study is similar to the Problem-Solving Cycle in that the videos discussed come from the teachers' implementation of a lesson around a problem previously discussed by the teachers. We expected that the teachers' engagement in the process of planning the research lesson would help them to set up the opportunities to observe student thinking in relation to specific learning goals.

Overall, the first author's intention in creating the intervention was to take advantage of individual activities that promote teacher noticing of student thinking. According to Jacobs and Spangler (2018), teacher noticing is a "learnable practice" (p. 772). The three professional development strategies—lesson study, video clubs, and discussions of animations—can increase teacher noticing of student thinking when implemented in isolation. By strategies, we mean the structures and activities used to implement the professional development goals (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2009). The combination of the three strategies can help overcome some of the challenges that the individual strategies present. To our knowledge, the combination of lesson study, video clubs, and animation discussions has not been previously implemented. The present study addresses the need to explore whether the intervention increases teacher noticing of student thinking in teachers' discussions of student thinking during video clubs and, also, in teachers' implementation of the research lessons that they designed collaboratively and taught individually.

## Theoretical Underpinnings

The intervention and the analysis of teacher learning that resulted from the intervention draws upon the notion of teacher noticing of student thinking as a core teaching practice (see Jacobs & Spangler, 2018). Sherin and van Es (2009) identified ways to operationalise teacher noticing of student thinking by analysing teachers' comments during a video club and teachers' actions in the classroom. During video club discussions, teachers can *restate* students' ideas in reference to the problem-solving strategies used by the students in the video. According to Sherin and van Es (2009), restating students' ideas allows for reasoning about students' mathematical thinking because teachers are using as evidence what they notice about students' discussion. Horn (2010) has called attention to ways in which teachers' discussion of classroom episodes prompt them to

provide descriptions of specific actions in what she calls *re-visioning practice*. In these statements, teachers enact what they would do in a lesson, at times applying projected speech (Eggins, 2004). In our case, we are interested in understanding whether re-visioning practice statements contain evidence of the three strategies for reasoning with students' ideas identified by Sherin and van Es (2005):

- (1) *restating* students' ideas,
- (2) *investigating* the meaning of student ideas, and
- (3) *generalising* and *synthesising* across students' ideas.

Generalising and synthesising involve a higher level of reasoning about student thinking than restating and investigating.

We use Sherin and van Es' (2005) identification of strategies for reasoning with students' ideas to investigate teachers' attention to student thinking in the study group sessions and in their classrooms. We acknowledge that it is difficult to establish a direct relationship between teachers' actions in professional development and classroom practices. Nevertheless, we assume that teachers' opportunities to re-vision practice allowed teachers to engage in an *approximation of practice* in the study group prior to their enactment of strategies for noticing student thinking in the classroom. An approximation of practice involves performing actions that are characteristic of a practice, in this case teaching practice, in a safe environment (Grossman, Compton, Igra, Ronfeldt, Shahan, & Williamson, 2009).

## Research Questions

As stated earlier, the overarching goal of the study is to investigate whether teachers' participation in the intervention effect teacher learning in their noticing of student thinking in the video club, and, consequently, in their classroom implementation of the research lessons. Four research questions guide our inquiry into the effects of this intervention on the teachers' classroom practices. The first two questions look for evidence of teacher noticing of student thinking in the video club discussions:

- (1) What evidence from the video club discussion suggests that the teachers reasoned about students' ideas?
- (2) What evidence from the video club discussion suggests that the teachers proposed strategies for reasoning with students' ideas in the classroom?

The third and fourth questions investigate teacher noticing of student thinking in the classroom:

- (3) Are there differences in the strategies that teachers use to reason with students' ideas when comparing the teachers' implementation of the same lesson between two consecutive years? If so, what are those differences?
- (4) Are there differences in an individual teacher's enactment of the lesson between two consecutive years? If so, what are those differences?

Overall, the research questions address a need to investigate connections between teacher noticing in professional development sessions and during their classroom instruction. Specifically, because the teachers' video club discussions were about a lesson that the teachers designed and implemented, we wanted to see whether study group discussions supported changes in noticing practices from the first year to the second year. To answer the research questions, we embraced a mixed-methods approach (Greene, 2007) by using qualitative data from videos and transcriptions and quantifying instances according to codes established in prior research on teacher noticing.

### *Research Design: A Combined Professional Development Model*

The data for this study come from a two-year intervention that focused on geometry instruction and included 20 three-hour study group sessions after school hours. Although the focus of the professional development was on increasing teacher noticing of students' prior knowledge, this study pertains to teacher noticing of student thinking. For teachers to pay attention to students' prior knowledge, they must first consider how to attend to students' ideas. Other publications from the project include analysis of teachers' attention to students' prior knowledge (González & Skultety, 2018).

The teachers engaged in two lesson study cycles during the two-year period. For each cycle, the teachers designed a different research lesson, each focused on a concept required in the Common Core State Standards for Mathematics (CCSSI, 2010): the perpendicular bisector and dilation. At the beginning of each lesson study cycle, the teachers viewed animations created by the research team. For each concept, there were two problems represented in the animations. For example, for the dilation concept we showed the "shadow puppets" problem and the "one-point perspective" problem. The animations were to initiate conversations for developing a research lesson. Other activities in the initial sessions were related to lesson study such as reviewing instructional materials (Sessions 1 and 2) and creating the problem for the first research lesson (Session 3). The teachers decided to modify two of the problems presented in the animations to design the research lessons. The teachers taught the research lessons in their classrooms while members of the research team video-recorded the lessons.

After the teachers taught the research lesson, they engaged in video clubs that were co-facilitated by two members of the research team and that focused on understanding the students' mathematical thinking. The research team pre-selected video clips for the video clubs using three criteria established by Sherin, Linsenmeier, and van Es (2009) for having productive discussions: whether the clip provides *windows* to student thinking, the *depth* of the mathematical ideas that students discuss, and the *clarity* of student thinking as illustrated in the clip. During the second year, the teachers discussed new animations based upon the research team's observations of the research lesson taught and included examples of the student thinking that surfaced in the lessons. The teachers also revised and taught a modified version of the lessons. In the second year, the video clubs centred on analyzing students' work on the revised lesson. The teachers engaged in this two-year cycle of activities for both the perpendicular bisector and the dilation lessons. Details of the animations were reported previously in other publications (González, 2018; González & DeJarnette, 2018), as well as research on the facilitation of the sessions (González, Deal, & Skultety, 2016) and the process of designing the lessons (González & Deal, 2019). Analyses of the process of revising the lesson can be found in other publications (Skultety, González, & Vargas, 2017; Deal & González, 2017).

The research lessons were essential in the lesson study cycle because they allowed the teachers to examine how the students draw upon their prior knowledge in geometry problem-solving. The lesson for teaching the properties of a perpendicular bisector asked students to find a "fair" location for a new after-school facility that would serve two given schools. The goal of the lesson was for students to discover that any location on the perpendicular bisector would be fair because it would be equidistant to the schools. The dilation lesson used the context of one-point perspective diagrams and drew upon students' experiences with art. The goal of this lesson was for students to discover that two figures are in dilation if lines passing through corresponding points include the centre of dilation, which is the vanishing point in a one-point perspective diagram. In addition, they should learn that lines that connect two corresponding points and do not pass through the centre of dilation are parallel.

### *Participants and Data Sources*

The intervention is situated in a Midwestern state in the United States. The project director (the first author) recruited high school geometry teachers from high-need schools through conversations with the mathematics department chairs and when visiting mathematics department meetings at several schools. The National Science Foundation established that high-need schools serve a high percentage of students coming from families with incomes below the poverty line. Five teachers from four high-need schools in four districts participated in the program. All of the schools were diverse in terms of race and social class. Table 1 provides other information about the participating teachers and schools. Teachers' involvement was voluntary, and they received a modest stipend for their participation in the study group. The schools also received a stipend for participating in the research. As stipulated by the Institutional Review Board's guidelines, the teachers and students could opt to withdraw their participation any time<sup>1,2</sup>. Due to personal commitments, only four teachers taught the second lesson each year.

**Table 1**  
*Participant Information*

| Name     | School        | Years of<br>experience<br>teaching<br>mathematics | Years of<br>experience<br>teaching<br>geometry | Graduate<br>degree |
|----------|---------------|---------------------------------------------------|------------------------------------------------|--------------------|
| Alexa    | Violet HS     | 26                                                | 20                                             | Yes                |
| Clara    | Burgundy HS   | 4                                                 | 3                                              | In progress        |
| Erin     | Tyrian HS     | 8                                                 | 6                                              | Yes                |
| Gian     | Terracotta HS | 9                                                 | 9                                              | Yes                |
| Madeline | Violet HS     | 8                                                 | 8                                              | In progress        |

*Note.* All names of individuals and institutions are pseudonyms. This information was gathered in a survey at the end of the first year of the professional development intervention.

Following the work by Sherin and van Es (2009), our examination of teacher noticing concentrates on the classroom data captured during the video club sessions in the first year and the teachers' enactment of the research lessons in the first and second years. In terms of the study group sessions, we selected the first-year video clubs because they occurred in the middle of the lesson study cycle, after the teachers had taught the lesson for the first time and before they made changes to teach the lesson the second time (Sessions 4, 5, 7, 8, and 9). In terms of the classroom data, we only analysed lessons for which there were data in two successive years, as some teachers did not teach both lessons. In the first year, Gian taught the perpendicular bisector lesson

<sup>2</sup> <https://oprs.research.illinois.edu/rights-consent>

after having introduced the concept in his class because his curriculum required him to do so before the teachers planned the lesson. Because of other commitments, Madeline and Clara did not teach the dilation lesson in the first and second year, respectively. In addition, for consistency, we only used the videos from the first enactment of the entire lesson to consider teacher decision-making around using student ideas that they had possibly not anticipated. All of the teachers, with the exception of Erin, taught the perpendicular bisector lesson in the first semester and the dilation lesson in the second semester. At Erin's school, the curricular sequence required her to teach the dilation lesson in the first semester and the perpendicular bisector lesson in the second semester. Since most of the teachers taught in different school districts, they implemented the lesson when appropriate in their curricular scope and sequence.

### *Analysis of Video Club Discussions*

The 20 study group sessions were video recorded and transcribed by the research team. To answer the first research question, we studied the transcripts from the video clubs in the first year of the study (Sessions 4, 5, 7, 8, and 9), looking for two types of evidence in each turn of speech by the teachers. A turn of speech is an uninterrupted utterance (Eggins & Slade, 1997). One type of evidence was a restatement of comments made by students in the video clip. Teachers typically used projected speech (Eggins, 2004) to share what the students were saying. In systemic functional linguistics, projections could be paratactic or hypotactic (Eggins, 2005, pp. 272-273). Only the first includes direct quotations. We coded both types of projections as examples of re-visioning. These restatements could be direct quotes, such as "So he's saying, 'the height is different' is bad because the height is the same." Alternatively, teachers could have used indirect quotes, which involves rephrasing the students' comments, such as "he's saying that there's an open space here." Once we coded a turn as having evidence of teachers restating students' ideas, we conducted a second layer of coding identifying whether the session facilitators had prompted the restatement or not. We tested the codes by coding the transcription from Session 4 together. We independently coded the other four sessions (Sessions 5, 7, 8, and 9). There was a total of 2,188 teacher-turns, and we reached a reliability rate of 96% for turns coded after restatement and 96% for turns coded as being initiated by the teachers. We calculated the reliability by finding the ratio between the agreements and the total turns coded between two coders. We resolved disagreements through discussion.

To answer the second research question, we identified evidence of re-visioning teaching in each turn. Re-visioning teaching involves teachers' explicit statements about possible teaching actions. For example, teachers may state how they would change the worksheet or a specific question that they would ask students based upon their observations of students' work in a video clip. A second, later stage of coding involved identifying whether teachers described strategies for reasoning with students' ideas through re-visioning. Specifically, we examined each turn coded as having evidence of re-visioning to see whether teachers said that they would *restate*, *investigate*, or *generalise or synthesise* students' ideas based upon their observations in the video clip. For example, when discussing a video where it was difficult for the teachers to understand the meaning of students' work with the diagram, a teacher said that she would ask, "Show me what you mean by, 'You think it goes down'." This statement is an example of a teacher re-visioning how to investigate students' ideas. In contrast, teachers' comments about changes to the worksheet were coded as re-visioning but not as reasoning with students' ideas. After coding Session 4 together, we coded all the transcripts for re-visioning, with 92% reliability. We also coded whether the session facilitators prompted teachers' re-visioning, achieving 92% reliability. The coding pertaining to the type of reasoning about student thinking attained 92% reliability. This reliability was calculated by taking the ratio of the agreements in coding of the type of

student thinking over the total turns coded for each code. In analysing examples of re-visioning, we were interested in whether the teachers' anticipation of student thinking and identification of strategies to handle students' ideas in the classroom related to the actions that could be observed in the enactment of the lessons.

### *Analysis of Classroom Videos*

We placed cameras in various locations of the classroom during the lesson. One camera, typically located in the back of the classroom, focused on the teacher while the other cameras focused on student groups, with permission having been obtained to include them in the study. Using the software iMovie, the second author created a composite video of each lesson with the video footage that we collected from the various cameras placed in each classroom. The main source was the video footage from the camera that captured the teacher throughout the entire lesson. Then, video clips from the camera filming a group that had given permission to participate in the study were replaced when a teacher approached a group. The composite video allowed us to focus on the teachers' actions and to listen to the teachers' interaction with small groups.

The first author used the composite video to make a timeline of the lesson, noting changes in the activity structure of the lesson following the typical identification of three phases in a problem-based lesson: the launch of the problem, the exploration of the problem, and the summary (Lappan, Fitzgerald, Friel, & Phillips, 1998). In addition, we relied on Lemke's (1990) identification of activities in science lessons for general guidelines about other activities such as taking attendance, classroom business, and going over the homework.

Our data reduction decisions intended to provide consistency across the lessons we coded. We focused on the public discussions of the lesson, which pertained to the launch, the partial summary, and the summary of the lessons. We excluded teachers' interventions during group work from the study because these varied depending upon the teachers' decision to assist a group and the availability of video records in relation to the number of groups that agreed to participate in the study in each class. We also excluded from the analysis segments that focused on classroom management issues such as taking attendance or making school announcements. We noted the codeable segments in the timeline, rounding to the nearest 5 seconds. Then, we identified 2-minute segments following procedures established by prior work that also used this unit of analysis to code for teacher moves (e.g., Borko, Jacobs, Eiteljorg, & Pittman, 2008; Sherin & van Es, 2009). Public discussions were not continuously sequenced. For example, a lesson could start with public discussion during the launch and then be followed by group work (not public discussion), a partial summary (public discussion), group work, and a lesson summary. In the example, there are three public discussion activities that are disconnected: the launch, a partial summary, and the summary. Each of these activities was segmented into 2-minute chunks. It is possible that the last segment was less than 2 minutes, but we still counted it as a segment. The total time for the public discussion was 184 minutes 13 seconds ( $Mdn = 31.75$ ) in year 1 and 196 minutes 46 seconds ( $Mdn = 28.38$ ) in year 2.

We followed the methods established by Sherin and van Es (2009) to identify teachers' attention to students' ideas in the classroom and the type of reasoning involved when attending to students' ideas. Each of us independently coded the 2-minute segments looking for *confirming* or *disconfirming* evidence of teachers' attention to student thinking. Confirming evidence means that the teacher attends to a student's idea. For example, a teacher would confirm a student's response to a question by restating his or her answer or by following up with a question such as "Are you thinking about real life, like on a map?" In contrast, disconfirming evidence means that the teacher does not attend to a student's idea even if the student offers it during class discussions. For example, the teacher may look for a specific answer and dismiss what one student says, while

calling on another student or may ask students a question without engaging them in a discussion of their responses.

The second layer of coding involved identifying the reasoning in the segments where there was confirming evidence that the teachers paid attention to students' ideas. Sherin and van Es (2009) proposed three types of reasoning: (a) *restating*, where the teacher repeats what the student has said, possibly with some variation, (b) *investigating*, where the teacher probes into student thinking by asking questions that can help to clarify an idea, and (c) *generalising and synthesising*, referring to actions that involve connecting ideas between students or extending these ideas in relation to the mathematical goal of the lesson.

We coded the data independently, with meetings to calibrate our coding during the initial stages. Our reliability on the number of segments that had confirming or disconfirming evidence was 91%. We calculated the reliability considering that we agreed on 291 of the codes from the total possible confirming or disconfirming instances in the segments across all lessons ( $n = 320$ ). There was a total of 160 public discussion segments across all lessons. Since there can be disconfirming and confirming evidence in a segment, we multiplied  $160 \times 2$  to obtain the total number of possible codes. The reliability for the number of instances of reasoning within the segments was 90%. We calculated the reliability considering that there was a total of  $n = 640$  possible instances, considering four types of possible reasoning for each segment and acknowledging there could be more than one type of reasoning in the same segment. We agreed on 574 instances. The researchers met to resolve disagreements at each level of coding.

We conducted statistical analyses to answer the last two research questions. These analyses allowed us to determine whether the differences in the results are accounted for by an influence other than chance. For the third research question, we conducted the Wilcoxon-Mann-Whitney test to examine whether there was a statistically significant difference in the strategies that the teachers performed in the first compared to the second year. This test is appropriate because of the small sample size ( $n = 14$  lessons), as we are comparing 7 lessons taught each year: the Wilcoxon-Mann-Whitney test is better than the  $t$ -test for small sample sizes (Siegel & Castellan, 1988). More importantly, the  $t$ -test assumes a normal distribution (Brase & Brase, 1999) and the Wilcoxon-Mann-Whitney test does not make that assumption (Gibbons, 1993). The Wilcoxon-Mann-Whitney test assumes homogeneity of variance. For that purpose, we tested the variance of the lessons in the first year and in the second year using Levene's test of homogeneity of variance.<sup>3</sup> If the variation of the means was statistically significant, we would be unable to conduct the Wilcoxon-Mann-Whitney test, and we would conduct the  $t$ -test instead. The assumption of homogeneity of variance failed for all of the data categories. Therefore, the use of the Wilcoxon-Mann-Whitney test was warranted.

For the fourth research question, we applied Fisher's exact test, which allows comparing the proportions of two categorical variables and is better than the chi-square test for small values (Siegel & Castellan, 1988). The Fisher's exact test is also more appropriate than the chi-square test when expected sizes are small ( $< 5$ ) or zero (Siegel & Castellan, 1988). The test requires making  $2 \times 2$  contingency tables comparing the raw values for the number of 2-minute segments coded for one move to that for the segments not coded for that move for year 1 and year 2 in their respective columns. The Fisher's exact test results in a  $p$ -value that can be verified for its significance. We used *GraphPad* data analysis software to conduct the Fisher's exact tests (<https://graphpad.com>).

<sup>3</sup> We used SPSS® Statistics software to conduct these tests.

## Findings

We start this section by introducing the findings for the first two research questions about teachers' video club discussions. Then, we show the findings pertaining to the teachers' implementation of the lessons.

### *Teacher noticing in the video club*

The teachers applied all of the strategies for reasoning with students' ideas identified during the video club discussions (see Table 2). The teachers restated what the students said in the video. The teachers also engaged in reasoning with students' ideas through re-visioning. The teachers extended the discussion of students' solution strategies to consider pedagogical issues. The session facilitators promoted these discussions to bridge the analysis of student thinking and pedagogical moves. We found that in 8% of all the teacher-turns, the teachers practiced restating students' ideas in the video clubs. Sessions 5 and 8 included more evidence of the facilitators prompting teachers' restatements of students' ideas, thus promoting an inquiry stance. We did not find examples of teachers restating students' ideas through re-visioning practice. In contrast, the teachers engaged in re-visions of practice where they investigated, generalised, or synthesised students' ideas as they would do in the classroom, using the video as a starting point. Re-visioning constituted only 1.6% of the teacher-turns (35 out of 2,188 turns). Most of these re-visioning moves were prompted by the facilitators and happened in the sessions at the end of the first year.

Table 2  
*Teachers' attention to student thinking during video clubs*

| Session | No. of teacher turns | No. of turns with restating | %  | % prompted | No. of turns with re-visioning | %   | Type of Reasoning when Re-visioning |               |    |                              |    |
|---------|----------------------|-----------------------------|----|------------|--------------------------------|-----|-------------------------------------|---------------|----|------------------------------|----|
|         |                      |                             |    |            |                                |     | Restating                           | Investigating | %  | Generalising<br>Synthesising | %  |
| 4       | 784                  | 60                          | 8  | 23         | 2                              | 0   | 0                                   | 0             | 0  | 0                            | 0  |
| 5       | 306                  | 14                          | 5  | 43         | 10                             | 0   | 0                                   | 0             | 0  | 1                            | 10 |
| 7       | 514                  | 16                          | 3  | 31         | 3                              | 0   | 0                                   | 1             | 33 | 1                            | 33 |
| 8       | 438                  | 72                          | 16 | 42         | 16                             | 75  | 0                                   | 13            | 81 | 3                            | 19 |
| 9       | 146                  | 18                          | 12 | 17         | 4                              | 100 | 0                                   | 2             | 50 | 2                            | 50 |
| Total   | 2188                 | 180                         | 8  | 3          | 35                             | 0.7 | 0                                   | 16            | 46 | 7                            | 20 |

Table 3 shows an excerpt of the transcript to illustrate the discussions undertaken. In this case, how to generalise students' solutions of the problem in the perpendicular bisector lesson, drawing from the context of finding a fair location.

Table 3  
*Discussion of generalising in study group Session 9*

| Turn No. | Speaker       | Turn                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|----------|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 151      | Facilitator 1 | So, yeah, Alexa: you had said before that she just looked for places where things crossed. Is that what everybody kind of thinks, or are there any other ideas?                                                                                                                                                                                                                                                                                                                              |
| 152      | Erin          | That's what I got. She was just looking for where things were intersecting.                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 153      | Facilitator 1 | So, I guess the next – you know, if you're working with a student like this, <u>where could you go, I guess, to try to help her think about –</u>                                                                                                                                                                                                                                                                                                                                            |
| 154      | Erin          | I feel like she needs like to be reminded of the problem, " <b>Hey, what are we doing?</b> " And, " <b>Oh, what was your thought process in drawing the circles?</b> " Like, " <b>why did you do this? What did you want?</b> " Like, I would try to get her to explain her thinking, and then from there you'd kind of have to guide her whatever direction her thought process is.                                                                                                         |
| 155      | Madeline      | I think even if you said like, " <b>Is it fair?</b> " Sorry, that it would bring her back to –                                                                                                                                                                                                                                                                                                                                                                                               |
| 156      | Clara         | Right, so I would say like, " <b>Okay, if one of the questions was you have to explain why this location is fair, can you prove to me using a ruler or prove somehow that these are fair?</b> " And the point that is not on the perpendicular bisector is very clearly not going to be fair. You can just look at the measurements. So, just trying to guide her in that direction, I think, would give her like a concrete reason why that wouldn't work and maybe why the other ones did. |

*Note.* The turn number pertains to the session. There were two facilitators in the session, and "Facilitator 1" denotes the person who typically led the video clubs. Examples of the facilitator prompting re-visioning are underlined. Examples of teacher re-visioning are in bold.

In Turn 153, the facilitator asked the teachers to consider the next step by asking, "where could you go?" The use of "could" signals a possible teaching action following the observation of students' work on the problem. Three teachers, Erin, Madeline, and Clara, proposed specific ideas for generalising using the context of finding a fair location. Clara purposefully selected a location that was unfair for students to discover the importance of choosing points equidistant to the two schools. The example illustrates what Horn (2010) calls "re-visions of practice" because the teachers established what they would say when dealing with a problem's solution in their classrooms.

The video clubs were grounded in the teachers' common experience of planning and teaching the research lessons. Analysing students' solution strategies enabled the teachers to brainstorm various alternatives that were specific to the problem and to the solution strategies observed in the video. The video clubs were specifically connected to the research lessons and supported teachers in re-visioning practice. Discussing with other colleagues how to investigate and generalise students' ideas prepared the teachers to enact these moves in the classroom.

### *Teacher noticing in the classroom*

Overall, according to the Wilcoxon-Mann-Whitney test, there was a statistically significant difference in the teachers' performance of moves to generalise or synthesise students' ideas between the first and second years of implementing the lessons ( $U=7.0, p=.026$ ). The Wilcoxon-Mann-Whitney test did not yield a statistically significant difference when comparing the performance of other moves in the first and second year. Three pairs of lessons had evidence of changes in the individual teacher's performance of moves for attending to students' ideas according to Fisher's exact test. All of the moves coded showed evidence of statistically significant changes in consecutive years for at least one teacher except for "no reasoning about students' ideas." Gian's dilation lesson included statistically significant changes in three different categories: confirming, investigating, and generalising or synthesising. The dilation lessons included more evidence of statistically significant changes in the performance of moves than the perpendicular bisector lessons during the second year of implementation. Three of the five teachers showed significant changes in the performance of moves in at least one of the second enactments of the lessons (Alexa, Gian, and Madeline).

**Confirming or disconfirming.** Table 4 shows the results of the coding for confirming and disconfirming evidence for using students' ideas. The Wilcoxon-Mann-Whitney test did not demonstrate statistically significant changes from year 1 to year 2. All of the teachers, with the exception of Clara, decreased the number of segments showing disconfirming evidence about attention to students' ideas in the second year. In Clara's first year lesson, there were no segments with disconfirming evidence, and there was one segment with disconfirming evidence in the second year. The disconfirming occurred during a partial summary in which a student was sharing the strategy they used to find fair points, but Clara did not follow up with the student. The only statistically significant difference pertains to Alexa's perpendicular bisector lesson, which showed a decrease in the number of 2-minute segments, with disconfirming evidence changing from 74% to 33% ( $p=.0139$ , based on Fisher's exact test). In terms of segments with confirming evidence of attention to students' ideas, Gian's dilation lesson demonstrated the most growth in the number of segments with confirming evidence, changing from 55% in year 1 to 100% in year 2 ( $p=.0351$ , based on Fisher's exact test).

**Table 4**  
*Teachers' attention to student thinking*

| Teacher  | Lesson | Year 1                                        |                                             |    | Year 2                                   |    |                                      |                                             |     |                                          |      |
|----------|--------|-----------------------------------------------|---------------------------------------------|----|------------------------------------------|----|--------------------------------------|---------------------------------------------|-----|------------------------------------------|------|
|          |        | No. of 2-minute segments of public discussion | No. of segments with disconfirming evidence | %  | No. of segments with confirming evidence | %  | No. of segments of public discussion | No. of segments with disconfirming evidence | %   | No. of segments with confirming evidence | %    |
| Alexa    | PB     | 19                                            | 14                                          | 74 | 16                                       | 84 | 24                                   | 8                                           | 33* | 17                                       | 70   |
| Clara    | PB     | 7                                             | 0                                           | 0  | 6                                        | 86 | 10                                   | 1                                           | 10  | 7                                        | 70   |
| Erin     | PB     | 4                                             | 1                                           | 25 | 1                                        | 25 | 6                                    | 0                                           | 0   | 3                                        | 50   |
| Madeline | PB     | 16                                            | 4                                           | 25 | 9                                        | 56 | 8                                    | 1                                           | 13  | 4                                        | 50   |
| Alexa    | DI     | 16                                            | 5                                           | 31 | 15                                       | 93 | 18                                   | 4                                           | 22  | 14                                       | 78   |
| Erin     | DI     | 5                                             | 1                                           | 20 | 3                                        | 60 | 6                                    | 1                                           | 17  | 5                                        | 83   |
| Gian     | DI     | 11                                            | 3                                           | 27 | 6                                        | 55 | 10                                   | 0                                           | 0   | 10                                       | 100* |

*Note:* "PB" refers to the perpendicular bisector lesson and "DI" refers to the dilation lesson. The highest percentages when comparing years 1 and 2 per teacher and lesson are in bold.

\* $p < .05$ .

**Attention to Student Thinking.** Table 5 shows the results for teachers' engagement with students' ideas in relation to the type of reasoning used in discussions. Examples of interactions coded as "no reasoning" included those in which a teacher acknowledged a student's answer by responding "okay" or "right" without further discussing the student's rationale for the answer. Since the results of the Wilcoxon-Mann-Whitney test were insignificant, we can conclude that the teachers did not change in their "no reasoning" engagement with students' ideas when comparing the lesson implementation between the two years.

**Table 5**  
*Teaching actions for reasoning with students' ideas*

| Teacher  | Lesson | Year 1                                        |                                   |    |                                |     |                                    |      |                                                |
|----------|--------|-----------------------------------------------|-----------------------------------|----|--------------------------------|-----|------------------------------------|------|------------------------------------------------|
|          |        | No. of 2-minute segments in public discussion | No. of segments with no reasoning | %  | No. of segments with restating | %   | No. of segments with investigating | %    | No. of segments with generalising/synthesising |
| Alexa    | PB     | 19                                            | 12                                | 75 | 14                             | 88  | 9                                  | 56   | 1                                              |
| Clara    | PB     | 7                                             | 0                                 | 0  | 5                              | 71  | 3                                  | 43   | 4                                              |
| Erin     | PB     | 4                                             | 1                                 | 25 | 1                              | 25  | 0                                  | 0    | 0                                              |
| Madeline | PB     | 16                                            | 4                                 | 25 | 4                              | 25  | 7                                  | 44   | 3                                              |
| Alexa    | DI     | 16                                            | 7                                 | 43 | 9                              | 56  | 5                                  | 31   | 4                                              |
| Erin     | DI     | 5                                             | 1                                 | 20 | 2                              | 40  | 1                                  | 20   | 1                                              |
| Gian     | DI     | 11                                            | 3                                 | 27 | 9                              | 82  | 5                                  | 45   | 1                                              |
| Year 2   |        |                                               |                                   |    |                                |     |                                    |      |                                                |
| Alexa    | PB     | 24                                            | 9                                 | 37 | 14                             | 58  | 8                                  | 33   | 7                                              |
| Clara    | PB     | 10                                            | 1                                 | 10 | 7                              | 70  | 4                                  | 40   | 6                                              |
| Erin     | PB     | 6                                             | 0                                 | 0  | 3                              | 50  | 1                                  | 17   | 2                                              |
| Madeline | PB     | 8                                             | 3                                 | 38 | 6                              | 75* | 6                                  | 75   | 3                                              |
| Alexa    | DI     | 18                                            | 5                                 | 28 | 13                             | 72  | 10                                 | 56   | 3                                              |
| Erin     | DI     | 6                                             | 1                                 | 17 | 6                              | 100 | 5                                  | 83   | 4                                              |
| Gian     | DI     | 10                                            | 4                                 | 40 | 9                              | 90  | 10                                 | 100* | 6                                              |

*Note:* "PB" refers to the perpendicular bisector lesson and "DI" refers to the dilation lesson. The highest percentages when comparing years 1 and 2 per teacher and lesson are in bold.

\* $p < .05$ .

**Restating.** Five of the seven pairs of lessons included more evidence of restating in the second year, but the differences between the first and the second years were statistically insignificant. Madeline's perpendicular bisector lesson demonstrated a statistically significant increase in segments coded with evidence of restating, from 25% to 75% ( $p = .0324$ , based on Fisher's exact test). Table 6 shows an example of how Madeline used restatement.

Table 6

*Example of restatement from Madeline's perpendicular bisector lesson in year 1*

| Turn No. | Speaker  | Turn                                                                                                                                                                                                                                                                                                                                                                                                       |
|----------|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1        | Madeline | What was your train of thought here when you were deciding where to put the three points? What did you think?                                                                                                                                                                                                                                                                                              |
| 2        | Destiny  | What did I think?                                                                                                                                                                                                                                                                                                                                                                                          |
| 3        | Madeline | What was going through your head?                                                                                                                                                                                                                                                                                                                                                                          |
| 4        | Destiny  | That they all have to be the same distance from like each one. You couldn't have one closer than the other.                                                                                                                                                                                                                                                                                                |
| 5        | Madeline | Okay, so you can't have the centre closer to one school than the other school. Why not?                                                                                                                                                                                                                                                                                                                    |
| 6        | Destiny  | Cause then one school is closer to it, and it's not fair.                                                                                                                                                                                                                                                                                                                                                  |
| 7        | Madeline | It's not fair and someone is going to cry about it, right? If we put it closer to Violet high school, then all the Redwood junior high kids are going to cry about it. Right? So, we are trying to find somewhere where it's in the middle where it's equal. So, what did you do? What was the first point you decided, and actually we can kick it to Terell. Terell, what was the first point you chose? |
| 8        | Terell   | The middle between the two.                                                                                                                                                                                                                                                                                                                                                                                |
| 9        | Madeline | The middle of the two.                                                                                                                                                                                                                                                                                                                                                                                     |

In Turns 5 and 9, Madeline restated what the students said. In Turn 5, she added a question after the restatement, making a scaffolding move for *requesting an explanation*. The basis for requesting the explanation was repeating the student's answer that the location of an after-school centre should not be closer to one of the given schools. In Turn 7, Madeline also restated when she said, "It's not fair." She repeated the rationale that the student had provided in Turn 4 about the after-school centre being closer to one school than the other and added a humorous comment about some students crying if the after-school centre is far from their school. She used restatement before repeating the goal of the task, "trying to find somewhere where it's in the middle." Then, Madeline asked the student to explain the procedure. Madeline's restatement was fundamental to setting up the next question. Through restatements, a teacher can accomplish two objectives of revoicing: acknowledging a student's answer and making the answer public for the class to consider (O'Connor & Michaels, 1996). By restating an answer, Madeline used the students' ideas and their conjectures about the diagram to promote discussion.

**Investigate.** Four of the seven pairs of lessons included more evidence of teachers investigating students' ideas in the second year, but the differences were statistically insignificant. The difference in Gian's investigation of students' ideas between year 1 and year 2 was statistically significant, from 45% to 100% of the 2-minute segments coded ( $p = .0124$ , based on Fisher's exact test). Table 7 shows an example of investigating from Gian's dilation lesson in year two.

Table 7

*Example of investigating from Gian's dilation lesson in year 2*

| Turn No. | Speaker  | Turn                                                                                                                            |
|----------|----------|---------------------------------------------------------------------------------------------------------------------------------|
| 1        | Gian     | So, tell me some strategies you guys used to get that lamppost in front of the other house? What were some things we did there? |
| 2        | Doug     | Construct the main points, points first. Like the big post ones.                                                                |
| 3        | Gian     | Like these points?                                                                                                              |
| 4        | Doug     | Yeah.                                                                                                                           |
| 5        | Gian     | And then, what did you do with those points?                                                                                    |
| 6        | Doug     | Oh, basically I— I just did the perspective lines there.                                                                        |
| 7        | Gian     | Okay, but there is no perspective point on this one, so how did you know where they would go to?                                |
| 8        | Doug     | I did the other stuff from the houses.                                                                                          |
| 9        | Gian     | Okay.                                                                                                                           |
| 10       | Doug     | So, I made one using the sa — the positioning, line positions of the houses.                                                    |
| 11       | Gian     | Okay, so you took corresponding points or parts on the houses —                                                                 |
| 12       | Doug     | Yeah.                                                                                                                           |
| 13       | Gian     | — you drew the lines through and then the perspective point was what? What did the lines look like?                             |
| 14       | Students | They intersect.                                                                                                                 |

In Turn 1, Gian asked the students their procedure for replicating a lamppost in front of the first house so that it was in front of the second house (see Appendix). After Doug's explanation stating that he "constructed the main points first" (Turn 2), Gian followed up with a question (Turn 3). Then, Gian asked how he used the points to construct the second lamppost (Turn 5). After Doug answered that he drew the perspective lines, Gian investigated how he was able to draw the perspective lines given that there was not a "perspective point" in the diagram, referring to the vanishing point. By asking this question, Gian provided an opportunity for Doug to be more explicit about the procedure for finding the vanishing point (Turns 8 and 10). In Turns 11 and 13, Gian generalised Doug's strategy for finding the vanishing point. By investigating Doug's solution, Gian enabled the class to understand Doug's procedure for replicating the lamppost and emphasized that the intersection of the perspective lines is the vanishing point. The example illustrates how investigating a student's idea can be a springboard for synthesising.

**Generalise/Synthesise.** A Wilcoxon-Mann-Whitney test indicated that the number of 2-minute segments with evidence of teachers generalising or synthesising students' ideas was significantly greater in the second year ( $Mdn = 38$ ) than in the first year ( $Mdn = 19$ ),  $U = 7.000$ ,  $p = .026$ . In addition, Gian's changes in statements for generalising and synthesising based upon students' ideas in the dilation lesson was significantly different, from 9% in year 1 to 60% in year 2 ( $p = .0237$ , based on Fisher's exact test).

Table 8 shows an example from Gian's dilation lesson in year two. Using the diagram, the students needed to investigate whether two trees were the same height in real life (see Appendix). Gian discussed the question at the end of the lesson. A student offered a solution, stating that he drew a line connecting the bottoms of the trees and the vanishing point (the centre of dilation). The student said that in contrast, when he drew a line to connect the two treetops, the treetops and the vanishing point did not match. In other words, the three points were not collinear. Gian used restatement and investigating to inquire into the student's solution. At the end of the discussion, Gian generalised the

implications of having collinear points in a one-point perspective diagram. Specifically, if corresponding points on two figures are collinear with the vanishing point, then one can infer that the objects in the diagram represent objects that are the same size in real life. Another student, Paige, answered Gian's questions as he generalised the solution.

Table 8

*Example of generalising from Gian's dilation lesson in year 2*

| Turn No. | Speaker | Turn                                                                                                                                                                                                                                                                                                                                                                         |
|----------|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1        | Gian    | So, he drew the second one, and it came out like this. [Draws a line on the board through the two treetops. The line is above the vanishing point.] So, if your line comes out like this, what do you think is happening with the tops of the trees? Which tree is going to be taller, the front or the back one?                                                            |
| 2        | Paige   | Wait –                                                                                                                                                                                                                                                                                                                                                                       |
| 3        | Gian    | If, if he drew the line and it – it hit the point, it would all be the same size. But, if it was ab – if it shot above it, go ahead Paige, what would it be? [2-second pause] Would the front one be taller or the back one be taller if the line was above the perspective point?                                                                                           |
| 4        | Paige   | The back one.                                                                                                                                                                                                                                                                                                                                                                |
| 5        | Gian    | The back one. Okay. And this one is taller. If it was a little bit shorter, then they would all line up. So, when we are saying, “we matched things up,” we gotta be careful when we write stuff down. We are matching up three things: we are matching up the top of the tree, the other top of the tree, and the perspective point. If they all match up, what do we know? |
| 6        | Paige   | They are the same size.                                                                                                                                                                                                                                                                                                                                                      |
| 7        | Gian    | The same size. And if they don't match up, we know they are not the same size.                                                                                                                                                                                                                                                                                               |

Gian generalised the procedure of connecting corresponding points to investigate whether the figures are in dilation. Gian noted that the treetops and the vanishing point were not collinear. He was pondering the implication of this result when he asked, “if your line comes out like this, what do you think is happening with the tops of the trees?” (Turn 1). Then, he called attention to the conclusion, “Which tree is going to be taller, the front or the back one?” (Turn 1). When he stated, “If you drew the line and hit the point, it would all be the same size,” he was generalising the main implication of collinearity with the vanishing point (Turn 3). Gian asked which tree was taller, and Paige concluded correctly that the back tree was taller (Turn 4). Gian restated Paige's answer and explained that the treetops would line up with the vanishing point in the diagram if the second tree were shorter in the diagram (Turn 5). In response to Gian's question about what to conclude if the three points “match up,” Paige answered correctly that the trees would be the same size (Turn 6). Gian restated Paige's answer and made a generalisation, “if they don't match up, we know they are not the same size” (Turn 7). Some of the questions that Gian asked exemplified the initiation-response-feedback (IRF) pattern of interaction typical of classroom discourse (Cazden, 2001), which uses restatement to confirm that the answers were correct. The use of restatement in tandem with generalisation enabled Gian to extend the students' ideas in relation to the underlying mathematical ideas in the lesson. Overall, the significant changes in the teachers' use of students' ideas for generalising and synthesising suggests

that the teachers made the goal of the lesson more explicit in the second year than in the first year while using the students' ideas as a source of knowledge.

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## Discussion

The study shows a case in which teachers' analysis of videos from their own classrooms, anchored in the lessons that they planned and implemented, promoted changes in instructional practices. Evidence that changes in teachers' implementation of lessons increases attention to student thinking is significant considering research suggesting that analysing classroom videos from teachers' own classrooms is insufficient to promote teacher reflection about their own lessons and hold teachers accountable for instructional changes (Beisiegel, Mitchell, & Hill, 2018). The integration of lesson study and the video clubs provided an opportunity for the teachers to apply their analysis of student thinking to the second implementation of the lessons. More importantly, the teachers increased their engagement in higher levels of reasoning concerning students' ideas in the second year.

During the study group, the teachers had concept-specific discussions in which they considered a specific task and how students would approach the problem. These discussions enabled the teachers to solidify their understanding of how to use the students' ideas to make explicit the underlying mathematical ideas for solving the problem. The reflection step in the lesson study cycle, motivated by the video club discussions, prompted the teachers to modify the lessons to optimise the students' learning opportunities. Through these activities, the teachers developed a better understanding of the connections between the students' solution strategies, the tasks, and the lessons' goals. In turn, the teachers were able to make explicit for their students the mathematical ideas in the lesson by generalising and synthesising the students' contributions. This finding exemplifies how video clubs can inform planning for instruction, a question posed by Sherin and van Es (2009) regarding the connections between video clubs and other aspects of teaching. The discussion of instructional strategies during the study group enabled the teachers to later draw upon a repertoire of actions during instruction. The intervention promoted learning how to summarise a problem-based lesson using students' solutions, based on the work by Smith and Stein (2011), through various activities, including the discussion of animations. The teachers applied the strategies discussed in the study group to

generalise and synthesise students' ideas in their classrooms. The evidence of statistically significant changes when comparing the first to the second year implementing the same lessons demonstrates the teachers' growth in their ability to connect students' ideas with the lesson goals.

Most changes were evident in the second implementation of the dilation lesson. We offer two possible explanations that consider the placement of the lesson within the curricular sequence and the lesson's content. All of the teachers except Erin taught the dilation lesson in the second semester. The dilation lesson was the last lesson taught in the intervention and possibly demonstrated the teachers' growth in their ability to notice student thinking. It may take two lesson study cycles for teachers to hone the skill of listening to students' ideas and using these ideas in instruction. A second explanation is that dilation is a new topic in the geometry curriculum (CCSSI, 2010). The discussions of the dilation animations promoted growth in the teachers' mathematical knowledge that they could use when teaching and, in particular, a deeper understanding of dilation. It is possible that in the second lesson study cycle, the teachers were more knowledgeable about both the lesson's goal and the topic of dilation, which resulted in increased opportunities for attending to students' ideas.

Three of the five teachers demonstrated changes in the second year of lessons by increasing their moves for reasoning using students' ideas. One teacher, Gian, demonstrated statistically significant changes in the performance of various moves in the second enactment of the dilation lesson. Two teachers, Clara and Erin, did not show statistically significant changes in the performance of strategies for reasoning with students' ideas between the first and second year of lesson implementation. We have three possible explanations for this finding. First, during the first implementation of the lessons these teachers showed that they attended to students' ideas frequently and consistently. Another explanation is that Clara and Erin taught the dilation lesson in the first year, after some teachers had already taught the lesson. Consequently, they were able to consider the other teachers' input about students' solutions and to implement lesson modifications. Last, these two teachers were in schools that supported problem-based curricula, and in our observations of typical lessons, they frequently led discussions about students' problem-solving strategies. Nevertheless, the finding of a statistically significant difference in generalising and synthesising suggests that all of the teachers met the professional development goal of noticing and reasoning about students' ideas in instruction.

One limitation of this study is that there were only five participants. However, the small sample allowed for in-depth analysis of teacher noticing. The methodology employed was consistent with other studies of teacher noticing that utilised small sample sizes (Sherin & Han, 2004). While the results may not be generalisable, we contend that our adaptation makes lesson study viable (Perry & Lewis, 2009) and supports instructional improvement (Lewis, Perry, & Hurd, 2009). Future work should examine whether implementing the combined professional development intervention at scale yields similar results to prior work studying the implementation of lesson study to scale, which resulted in teacher and student learning outcomes for specific mathematical content (Lewis & Perry, 2017).

Methodologically, we replicated and extended the methods of Sherin and van Es (2009) for investigating teachers' attention to students' ideas in three ways. We identified teachers' use of projected speech during the sessions as a linguistic marker for reasoning with students' ideas. We incorporated statistical analyses to the classroom data. In addition, although Sherin and van Es (2009) focused on elementary and middle school teachers, our participants were high school mathematics teachers in the same content area: geometry. The facilitator prompted teachers to shift the discussion to pedagogical issues based on their collective interpretation of students' work. The results comparing the teachers' enactment of the same lesson in two successive years showed that the teachers engaged students in higher levels of reasoning by generalising and synthesising the students' ideas during the second year of lessons. In addition, three of the seven pairs of lessons showed significant differences in the teacher's attention to their students' ideas.

## Conclusion

Professional development that supports teachers in noticing students' ideas is crucial to improving student learning opportunities. The combination of lesson study, animation discussion, and video club had the effect of increasing teachers' attention to student thinking in the classroom. Lesson study enabled the teachers to connect video club discussions and their classroom observations about students' solution strategies. Moreover, the video club discussions allowed the teachers to practice how to reason with students' ideas in a safe space before teaching this lesson in the classroom. The teachers' collective examination of students' ideas allowed them to re-vision practice with specific examples from the lesson that they planned and taught (Horn, Garner, Kane, & Brasel, 2017).

The intervention promoted discussion about students' possible solutions and teaching strategies that went beyond a particular classroom. Discussing strategies for reasoning with students' ideas was crucial for promoting student learning. The combined model supported teachers in achieving the learning goals of the professional development by helping teachers in attending to student thinking during instruction. Future work should continue to examine the viability of this model to promote teacher learning.

## Acknowledgements

The research described in this article was supported by a National Science Foundation grant to Gloriana González for the project entitled "CAREER: Noticing and Using Students' Prior Knowledge in Problem-Based Instruction," Grant No. DRL-1253081. Opinions, findings, conclusions, or recommendations are those of the authors and do not necessarily reflect the views of the National Science Foundation. The authors appreciate valuable feedback from Enrique Galindo and Miriam G. Sherin during the initial stages of the study. Jason T. Deal and Lisa Skultety were members of the research team and assisted in coding the study group data.

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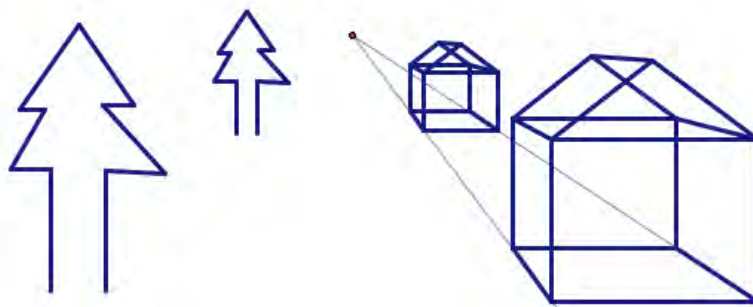
Department of Curriculum and Instruction, University of Illinois at Urbana-Champaign

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## Appendix

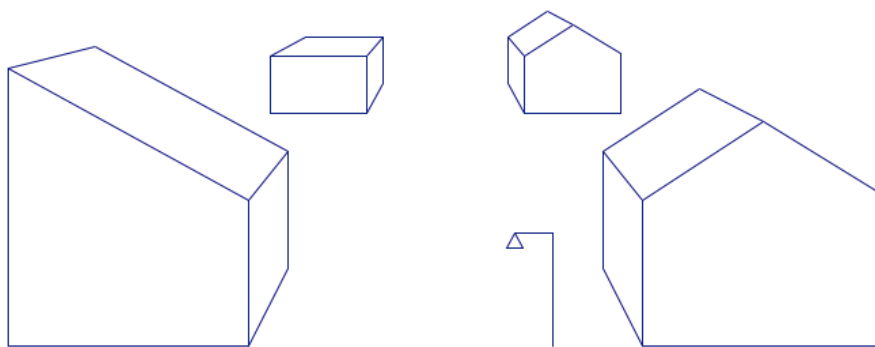
### *Dilation Lesson in Year 2*

Linear perspective is a method designers use to show three dimensions in a picture. The simplest way to use linear perspective is to select one point in the distance, called a “vanishing point”. The objects in the picture below follow lines called “perspective lines” back to the vanishing point.



1. Are the real-life heights of the two houses the same or different? How can you tell?
2. Now investigate the trees. Are they the same height in real-life? How can you tell?

The following picture is a perspective drawing of a neighbourhood that is still being built.



3. The second house back on the left side of the street isn't finished yet. Construct the roof so that it is the same as the front left house.
4. Construct a lamppost in front of the second house on the right. Construct the lamppost so that it is the same as the existing lamppost.
5. Decide on an item to add to the houses on the left. For any item you add to one house, construct the same item on the other house. Make sure you follow the rules of one-point perspective.
6. Colour corresponding pairs of segments in your drawing. Use a different colour for each pair. What do you notice?