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GEOMETRICAL PROBLEM-SOLVING PERFORMANCE OF PRESERVICE TEACHERS: EXPLORING THE EFFECTIVENESS OF METACOGNITIVE STRATEGIES

Sevda Göktepe Yıldız
Department of Mathematics and Science Education,
Education Faculty, Biruni University,
Topkapı/İstanbul, Turkey
sgoktepe@biruni.edu.tr

ABSTRACT

This study aimed to investigate the effect of instruction based on metacognitive strategies on geometrical problem-solving performance of mathematics preservice teachers. Geometrical problem-solving tests developed by the researchers were applied as pre-test and post-test before and after the experimental instruction. Preservice teachers predicted the number of correct questions they would solve, and the difference between predicted scores and geometrical problem-solving test scores were assessed as their metacognitive skills. Training implementation was carried out in four sessions, and finally, metacognitive knowledge test was applied. Both qualitative and quantitative research methods were used for collecting and analyzing the data. The sample consisted of forty-one senior elementary school mathematics preservice teachers of a public university in Turkey. Results indicated that metacognitive instruction increased the preservice teachers' geometrical problem-solving performance. A moderate-level correlation was detected between preservice teachers' geometrical problem- solving performance and metacognitive skills. Besides, metacognitive strategies instruction improved the awareness level of the preservice teachers about their ways of learning. Implications for method and practice were further described.

Keywords: *Metacognition; Geometrical Problem-Solving; Preservice Teacher Education; Mathematics Education*

INTRODUCTION

Brown, Flavell, and their colleagues introduced metacognitive development to education literature for the first time in the early 1970s and expanded their research in the coming years (Schneider, 2008). Flavell published his article describing metacognition in 1979 in detail. Various studies were carried out on this subject for about 40 years. This article includes definitions and studies on metacognition that are particularly up-to-date. According to Schunk (2009), metacognition is a high-level cognition. Selçuk (2000) explains metacognition as the knowledge and awareness of the individual about the cognitive process. Metacognition, in its shortest definition, means the awareness of one's own thinking processes and control of these processes (Jager, Jansen, & Reezigt, 2005; Özsoy, 2008). Metacognition, in its broadest sense, is the perception, remembering and awareness of the mental activities involved in one's thinking and the control over these (Hacker & Dunlosky, 2003). Papaleontiou-Louca (2008), who takes metacognition with the dimension of thought, defines metacognition as the person's opinion about perception, understanding and remembering. From a theoretical perspective, the metacognitive approach reveals a more comprehensive perspective on the human mind by examining how the individual's thoughts about their own thoughts are formed. From the perspective of assessment methods, this approach is to rearrange the individual and his/her own mistakes by reorganizing the introspection method (Karakelle & Saraç, 2010).

Various models exist concerned with dimensions of metacognition (e. g. Dunlosky & Metcalfe, 2009; Özsoy, 2007; Schraw & Moshman, 1995; Schunk, 2009; Wells, 2002). Schraw and Moshman (1995) classify metacognition as Metacognitive Knowledge (MK) and Metacognitive Skills (MS), which was used in this study. Wells (2002) states that metacognition has three dimensions: metacognitive knowledge, metacognitive experiences and metacognitive control strategies. Metacognitive knowledge is about our cognition regarding our beliefs. Metacognitive experience is about making sense of special mental events such as ideas, and is related to our emotions. Metacognitive control strategies are related to controlling the activities of one's own cognitive system. Schunk (2009), on the other hand, explains three types of metacognition: declarative, procedural and conditional (both declarative and procedural). Recently, metacognitive knowledge, metacognitive monitoring and metacognitive control are considered as a three-faced structure within the framework of accumulated research and modelling in metacognition (Dunlosky & Metcalfe, 2009).

Metacognitive Knowledge (MK) and Metacognitive Skills (MS)

Metacognitive knowledge is the descriptive knowledge about the interaction between the individual's own characteristics, task features, and appropriate strategies in the learning situation (Schraw & Moshman, 1995). It is categorized into three components: procedural knowledge, declarative knowledge, and conditional knowledge (Flavell, 1979; Schraw & Moshman, 1995). Procedural knowledge is knowing how to achieve success in a work and how to finalize it. For example, knowing how to calculate the volume of a cylinder is about procedural knowledge. Declarative knowledge is expressed as an individual's thoughts regarding whether s/he could do a task given, in other words, it is the knowledge about an individual's own proficiencies (Flavell, 1979). Since metacognitive knowledge is stored in long-term memory, they are stationary in nature and are declarative knowledge (Schneider & Lockl, 2002). Metacognitive knowledge includes human cognitive characteristics (personal knowledge), knowledge of the nature of different cognitive tasks (task knowledge) and knowledge of possible strategies (strategy knowledge) that enable the individual to overcome different cognitive tasks (Flavell, 2000).

Researches are mostly focused on four MK components: predicting, planning, monitoring and evaluation (Desoete & Roeyers, 2002; Desoete, Roeyers, & Buysse, 2001; Schraw & Moshman, 1995). According to Lucangeli and Cornoldi (1997), prediction made before starting a task affects cognition. The ability to predict allows students to anticipate the difficulties of the tasks they face and allows them to adjust the way they work quickly or slowly (Desoete & Roeyers, 2002). In the planning phase, "Which knowledge can help me about this topic?", "What should I do firstly?", "Why am I reading this?" questions; in the monitoring phase "Am I proceeding correctly?", "What should I do next?", "What should I change?" questions; in the evaluation phase "Have I done everything right?", "What did I learn from the work I performed?" questions are asked (NCREL, 1995). Similar to the process in the planning stage of metacognition, Yayan (2010) stated that students use the methods of controlling and evaluating what they do in problem-solving processes. In summary, the metacognition knowledge is relevant procedural knowledge that is required to individuals for editing and controlling their own learning activities. Planning the learning, monitoring, checking, and editing are reflections of this thinking skill (Schraw & Moshman, 1995).

It is extremely difficult to measure metacognition (Desoete & Roeyers, 2006) and the student's self-assessment is the most widely used technique for the evaluation of metacognition. These techniques are listed by Gay (2006) as verbal statements about the past, simultaneous verbal notifications, written statements, and personal predictions. By favour of personal predictions, it is asked to evaluate a person's own performance before or after a given task. This method is considered more successful than verbal or written report since it compares their predictions with their actual performance. Metacognitive prediction is considered as a cognitive ability (Desoete & Royers, 2006). This ability can be defined as whether the estimated value matches with the actual performance. The difference between the estimated values and the actual number of correct question is assessed as metacognition skills. There is little difference between them; namely, lower scores correspond to higher MS (Pennequin, Sorel, Nanty, & Fontaine, 2010).

In fact, when all the factors mentioned above are examined, we can say that metacognition plays an active role in the problem-solving process. Because metacognition is an important factor in understanding the problem in the problem-solving process, developing strategies for the solution of the problem, and solving the problem with the developed strategies. Montague (2008) stated that expressing the problem appropriately is the basis for understanding the problem. Also, Naser (2008) stated that it is easier for students to solve the problem shaped by their words. Besides, interactive problem-solving (Kramarski, Mevarech, & Liberman, 2001) is one of the metacognition strategies used. Problem-solving process includes analyzing the information given in the problem, organizing the information that is planned to be used, preparing a plan and evaluating the whole process. These processes in problem-solving require organizing each step and making decisions at the same time. These processes carried out throughout the problem solving-process are called metacognition (Yimer, 2004). In this way, individuals with high metacognition ability perform better during problem-solving (Yıldız, Baltacı, & Güven, 2011). Besides, there is a positive relationship between metacognition and critical thinking (Arslan, 2018). Research results show that there is a relationship between MS and students' achievement levels (Desoete & Roeyers, 2002). Research has been reported that metacognition instruction has a positive and significant effect on students' academic achievement (Naglieri & Johnson, 2000; Teong, 2002). In their research, which examined students' metacognition knowledge in the process of geometry problem-solving, Danilovic and Rukavina (2015) observed some weaknesses on how students associated and transferred their conceptual and procedural knowledge with problem situations.

In the 2000s, most researchers suggested that metacognition is an important issue in children and adults education (Kapa, 2001; Kramarski, Mavarec, & Arami, 2002; Teong, 2002). What mathematics teachers do to prompt their students' metacognition in problem-solving environments is a question that needs to be answered in this respect (Yıldız & Güven, 2016). It is widely welcome in educational studies where it is important to develop metacognitive strategies and problem-solving skills to increase students' mathematics achievement (Danilovic & Rukavina, 2015). It is important that preservice teachers receive training on metacognition during their undergraduate education. Because in a few years, they will start their professional lives and train students according to the education they receive. Therefore, in the present study, metacognition knowledge and metacognition skills of mathematics teacher candidates in the process of geometry problem-solving were examined.

In recent years, it has been observed that researches on how metacognition skills affect problem-solving process and how metacognition skills are used in this process have increased (Aydurmuş, 2013; Azak, 2015; Demir, 2016; Kanadlı & Sağlam, 2013; Kramarski et al., 2002; Oğraş, 2011; Özsoy, 2007; Pilten & Yener, 2010; Yıldız & Güven, 2016). We can say that the use of problem-solving in metacognition studies arises from the fact that the cognitive and metacognitive structure is a nested complex process. On the other hand, a considerable body of research related to metacognition in mathematics education has been carried out in recent years (Pennequin et al., 2010; Sevgi & Çağlıköse, 2019). However, especially the studies that examine the effects of metacognition instruction on geometrical problem-solving performance of preservice mathematics teachers are limited (Danilovic & Rukavina, 2015). In this respect, this research is important for this bulk of studies.

Having established these facts mentioned above, the main aim of this study was to investigate the effect of instruction based on metacognitive strategies on elementary mathematics preservice teachers' geometrical problem-solving performance. The following research questions guided the study:

1. Is there any effect of metacognitive instruction on the development of the preservice teachers' MS?
2. Is there any effect of metacognitive instruction on the development of preservice teachers' geometrical problem-solving performance?

3. Is there a correlation between MS of preservice teachers and their problem-solving skills?
4. Which MK is used by preservice teachers during problem-solving process?

METHODOLOGY

Research Design

This study used the mixed method in the phase of collecting data. The quantitative instruments examined the effect of metacognitive instruction on elementary mathematics preservice teachers' geometrical problem-solving performance; and participants' MK was assessed qualitatively through metacognitive knowledge test. The study used experimental design with one group and pre-test and post-test application as described by Fraenkel, Wallen, and Hyun (1993) in the quantitative part of the study. There was one selected group. A measurement was done before and after applying the experimental instruction. Content analysis was employed for qualitative data. The content analysis brings similar data together under specific codes (Creswell, 2012).

Research Group

The study group consisted of forty-one undergraduates from a public university in Turkey. Students were fourth-year (senior) preservice teachers who were attending Department of Elementary Mathematics Education in Faculty of Education. The participants had sufficient content knowledge required for this study, because they attended the "Geometry" course. They took part in this study voluntarily. While conducting the study, necessary permissions were obtained from the students, the lecturer and the management. Preservice teachers' names were not given even when direct quotations were made from their sentences.

Research Instruments

To assess preservice teachers' geometrical problem-solving skills and to determine their MS, geometrical problem-solving tests were developed by the researchers. Besides that, a form consisting of open-ended questions was used for evaluating preservice teachers' MK.

Geometrical Problem-Solving Tests

Geometrical problem-solving tests were prepared to determine preservice teachers' MS and geometrical problem-solving performance. In both pre-test and post-test, there were eight problems. Problem-solving tests required using these strategies: making a systematic list, predicting and checking, drawing a diagram, prediction, correlating/searching relationship, benefiting from similar simple problems, generating tables, and reasoning strategies. Moreover, problems were open-ended and constituted verbal and visual items.

Firstly, students were asked to look at the problem-solving tests and then they predicted how many questions they would answer correctly. Then, they solved the problems. The difference between the predicted values and the number of their correct answers was used to measure their MS and this score was determined as metacognitive prediction skills.

Metacognitive Knowledge Test

The 8-item open-ended meta-cognitive knowledge test was designed to reflect preservice teachers' MK. Since open-ended items provide detailed information about a problem situation for researchers (Bridgeman, 1992), this data collection tool was formed by the researchers themselves. "How many times have you read before you start solving a problem?", "How do you check that you understand what the problem asks from you?", "How do you evaluate how much time you need to solve a problem?", "What are the strategies that you benefit frequently for the solution of a problem?", "How do you determine the best strategy for the solution of a problem?", "What kind of challenges do you experience while you are solving a problem in general?", "Do you check the accuracy of your calculations and whether your answers are significant or not? Tell us briefly about how to control.", "What do you pay attention to solve a problem?" were the questions in the meta-cognitive knowledge test. In the phase of preparation of the metacognitive knowledge test, firstly, preservice teachers were asked a general question "Which process do you follow while you are solving a problem. Please give information in detail." Besides, they were asked to solve a given example problem systematically. The form was finalized from the students' answers to this question, and the test recommended by Pennequin et al. (2010) was used in the study. Metacognitive knowledge test was given after the training implementation, and the data were collected in written form.

Procedure

To improve MK, Schraw (1998) stated a check list that consisted of three steps: Planning, monitoring, and evaluation. In the present study, these three steps were employed in training implementation in four sessions.

The first session included the planning process and related to the understanding of the problem. At the planning stage, training was provided for preservice teachers by asking the following questions "Which information on this subject helps me?" "What should I do firstly?", "Why am I reading this?". Besides these, drawing appropriate figures and finding relationships in the problem, dividing the problem into sub-problems, answering "What is given, what is also desired?" questions, controlling whether missing or more information was available parts were highlighted. Information was given about some topics such as reading the question several times, thinking aloud and taking notes.

The second and third step contained monitoring. "Am I correct in this process?", "Then, what should I do?", "What should I change?" questions were answered. After the given problem was understood, information regarding the use of different strategies to facilitate the resolution steps was introduced. Moreover, brief information was given about using strategies; and in this respect, reasoning strategies were introduced, and the second researcher solved example problems.

The fourth stage was the evaluation part of the training. The solution of the problem was assessed by asking "Have I done everything right?", "What have I learned from this problem?" questions.

Before metacognitive training implementation, geometrical problem-solving pre-test was given to preservice teachers. Firstly, preservice teachers predicted the number of problems they would solve correctly; and then, each participant solved the problems individually. Later, training implementation was carried out in four sessions. Geometrical problem-solving post-test was performed after the training practice. Finally, metacognitive knowledge test was applied.

Data Analysis

Pre-test and post-test metacognitive prediction scores were compared to determine whether metacognition training implementation affected elementary mathematics preservice teachers' MS. Problem solving scores were calculated by the number of questions correctly solved. Furthermore, it was examined whether there was a relationship between the success of problem-solving and MS of preservice teachers.

Quantitative evaluation was applied for these three parts, and collected data were analysed statistically. The normality of pre-test and post-test scores was examined, and the results revealed that the data did not have a normal distribution. For this reason, non-parametric test statistics were used.

The data obtained in writing from the Metacognition Knowledge Test were analysed qualitatively. Each question in the test was evaluated separately and the answers given by the preservice teachers were presented. The percentage-frequency values corresponding to the themes were included in the tables. Besides, direct quotations from the sentences of the participants were presented.

RESULTS

The results were presented based on the study questions. Firstly, the pre-test and post-test metacognitive prediction scores of preservice teachers were compared with Wilcoxon Signed Rank Test.

Table 1
Results of metacognitive prediction scores

Metacognitive prediction skills		n	Mean Rank	Sum of Ranks	z	p
Post-test – pre-test	Negative Ranks	14	13.61	190.50	-1.631	0.103
	Positive Ranks	19	19.50	370.50		
	Ties	8				
	Total	41				

*based on negative ranks

When Table 1 was examined, it was seen that there was no significant difference between the pre-test and post-test metacognitive prediction scores of preservice teachers ($z=-1.631$, $p>.05$).

Table 2
Results of geometrical problem-solving performance

Geometrical problem-solving performance		n	Mean Rank	Sum of Ranks	z	p
Post-test – pre-test	Negative Ranks	18	18.25	328.50	-2.445	0.014
	Positive Ranks	11	9.68	106.50		
	Ties	12				
	Total	41				

*based on positive ranks

The results presented in Table 2 revealed that a significant difference was found between pre-test and post-test scores of preservice teachers regarding geometrical problem-solving performance ($z=-2.445$, $p<.05$).

In the last part of quantitative analysis, it was examined whether there was a relationship between geometrical problem-solving performance and MS of preservice teachers. In this respect, Spearman Rank Correlation Test was applied.

Table 3
Results of Spearman Rank Correlation Test (Pre-test)

			Geometrical Problem-Solving Test Scores	Metacognitive Prediction Scores
Spearman's rho	Geometrical Problem-Solving Test Scores	Correlation Coefficient	1.000	-.569
		Sig. (2-tailed)	.	.000
	Metacognitive Prediction Scores	Correlation Coefficient	-.569	1.000
		Sig. (2-tailed)	.000	.

As can be seen in Table 3, there was a significant relationship between geometrical problem-solving performance and metacognitive prediction skills of preservice teachers. Geometrical problem-solving pre-test scores related negatively with metacognitive prediction scores. Since low metacognitive prediction scores indicate high metacognition skills, and high metacognitive prediction scores indicate low meta-cognition skills, having a negative relationship is possible. Namely, there was an inverse correlation among meta-cognitive prediction scores and metacognitive abilities, thus the correlation was negative. The correlation coefficient was 0.569, and it showed that there was a moderate relationship between the MS and geometrical problem-solving performance of preservice teachers.

Table 4
Results of Spearman Rank Correlation Test (Post-test)

			Geometrical Problem-Solving Test Scores	Metacognitive Prediction Scores
Spearman's rho	Geometrical Problem-Solving Test Scores	Correlation Coefficient	1.000	-.518
		Sig. (2-tailed)	.	.001
	Metacognitive Prediction Scores	Correlation Coefficient	-.518	1.000
		Sig. (2-tailed)	.001	.

According to post-test scores, there was a significant relationship between geometrical problem-solving performance and metacognitive prediction skills of preservice mathematics teachers. This relationship was negative as in the pre-test. The correlation coefficient was 0.518, and it showed that there was a moderate relationship between the MS and geometrical problem-solving performance of preservice teachers.

Responses to the metacognitive knowledge test were analysed qualitatively. Percentage-frequency table determined for each question and direct quotations were presented. Since some of the students' answers took part in both themes, the total number of responses for themes was more than the number of participants.

The findings obtained from the first, second and third questions in the metacognitive knowledge test are given in Table 5.

Table 5
Findings obtained from the questions in the meta-cognitive knowledge test

Question in the form	Themes	Number of participants f	%
1. How many times have you read before you start solving a problem?	Reading one time	6	14.6
	Reading more than one time	26	63.4
	Reading until understanding	7	17.0
	Reading according to the difficulty of the problem	2	5.0
2. How do you check that you understand what the problem you ask?	Reading again	20	43.5
	Writing given and desired information	13	28.3
	According to effect in the mind	8	17.4
	Control after solution	5	10.8
3. How do you evaluate how much time you need to solve a problem?	According to difficulty of the problem	27	51.9
	According to state of understanding of the problem	20	38.4
	By reading the problem	5	9.7

Nearly 63% of preservice teachers stated that they read a problem more than once. A total of 17% of participants read until they thought that they understood the problem. A total of 6 students expressed that they read a problem only once; 2 students also indicated that the number of reading changed depending on the difficulty of the problems.

The statement "Two or three times. It varies depending on the question." emphasized that they were reading a problem more than one time. A statement related to reading according to the difficulty of the problem was "It varies depending on the problem. If it is easy, I read one time; if it is difficult, I read at least three times." On the other hand, one student stated, "It varies according to problems and environments." In the reading according to the difficulty of the problem theme, one student said "I read until I think that I understand" in the reading until understanding theme.

In the second question, preservice teachers were asked "How do you check that you understand what the problem asks from you?" A total of 44% of preservice teachers expressed that they read again the problem to check it, 28% of participants stated they wrote the given and desired information. While 17% of them decided according to the effects generated in their mind, 5 students controlled after solution.

Example of the direct quotations are as follows: "I read the problem several times, look at the given and desired information in the problem. I comment about it" writing given and desired information and reading again. "If I understand the same thing after reading at least three times, I comprehend the problem" reading again; "If something comes to my mind immediately regarding the solution of the problem, then I say that "it asks that" to myself, I comprehend the problem" according to the effect in the mind. "After writing the given information in the problem, I control the information (given) again" writing the given and desired information.

According to "How do you evaluate how much time you need to solve a problem?" question results, 52% of preservice teachers determined the time they needed according to the difficulty of the problem, 38% of them determined the time according to their understanding the problem status, and 5% of them decided by reading the problem. Elementary mathematics preservice teachers expressed their opinions as follows:

"After writing the given and desired information, if I have difficulty in understanding, it means solution lasts more time." (According to state of understanding of the problem).

"Depending on the difficulty of the problem. After reading the problem, if I think it is difficult, I think 1-2 minutes in advance, then I write the given information slowly one by one." (According to the difficulty of the problem).

"I read the question, and then, the issue of how long it will last become clear." (Reading the problem).

Another question asked to participants was "What are the strategies that you benefit frequently for the solution of a problem?" "Please specify if you use different strategies."

Table 6
Findings obtained from the fourth question in the metacognitive knowledge test

Strategies	Number of participants	
	f	%
Benefiting from similar simple problems	40	37.8
Searching relationship	29	27.3
Prediction	12	11.3
Drawing a diagram	12	11.3
Generating tables	6	5.7
Making a systematic list	3	2.8
Other	4	3.8
Total	106	100

Almost all preservice teachers stated they used more than one strategy. Benefitting from similar problems is the most used strategy they preferred. A total of 40 out of 41 students who participated in the study stated that they used this strategy. Afterwards, about 27% of preservice teachers reported that they used the searching relationship strategy; 12 students chose both prediction and drawing a diagram strategy. Other different strategies they used were:

"To make the problem become simple"
"Associating with everyday life"
"Solving the question according to solution steps"
"I summarize the important information given in a short note."

Table 7
Findings obtained from the meta-cognitive knowledge test

Question in the form	Themes	Number of participants f	%
5. How do you determine the best strategy for the solution of a problem?	According to the desired information in the problem	19	39.6
	By looking similar problem	17	35.4
	By using the most practical way	12	25
6. What kind of challenges do you experience while problem solving in general?	In the understanding of the problem part	17	36.2
	In the process part	14	29.8
	During the determination of strategy	11	23.4
	During the strategy implementation	5	10.6
7. Do you check the accuracy of your calculations and whether your answers are significant or not? Tell us briefly about how to control.	Checking procedures	28	50.9
	Solving from different ways	12	21.8
	Checking the steps of the solution	9	16.4
	No control	6	10.9
8. What do you pay attention to solve a problem?	Better understanding of the problem	16	35.6
	Not do processing errors	15	33.3
	Whether the solution is logical	8	17.8
	Determining the right strategy	6	13.3

According to Table 7, 40% participants stated that they determined their strategies according to desired information in the problem; 35% of them reported they preferred the strategies they had used previously in similar problems; and 12 preservice teachers expressed that the most practical way for them was their own strategies.

The statement of "According to desired information in the problem… For example, I utilize prediction strategy if it requires doing complex operations with large numbers (By rounding)." was assessed in the "according to desired information in the problem" theme. A student's answer was "I look at solved problems, and how my teacher solved" in the "by looking similar problem" theme. "I develop strategies based on problem. If I solve similar problem asked before, I solve the problem by taking advantage of the previous problem. I underline the parts if required to pay attention to diagrams." statement was stated in both according to desired information in the problem and by looking similar problem themes. "I try to choose the shortest and most reliable way from the alternative solutions" statement was also analysed in by using most practical way theme.

For "What kind of challenges do you experience in problem-solving in general?", understanding of the problem was the most difficult part for preservice teachers in problem-solving process. Subsequently, they had difficulty in making operations, determining the strategy, and implementing the strategy parts. For these themes, the examples of participants' answers are as follows:

"Once I read the problem and if I have difficulty to understand, then I read again and again. Additionally, it can be hard to determine the solution strategy." (In the understanding of the problem part and during the determination of strategy).

"While solving a problem, I have difficulty in finding the right strategy. If the problem is different what I have seen before, I have great difficulty in solving." (During the determination of strategy).

"I am experiencing some challenges: failure in process in long problems or unable to find the relationship." (In the process part)

"Misunderstanding: I usually interpret questions differently." (In the understanding of the problem part).

"In problem-solving, when I come across complex operations, I become confused what I should do and forget the desired information from me." (During the strategy implementation).

According to the results of the "Do you check the accuracy of your calculations and whether your answers are significant or not? Tell us briefly about how to control." question, for controlling a problem, more than 50% of preservice teachers said that they checked procedures they implemented; 12% of participants stated that they were solving the problems with different ways, 9 of them checked procedures; and 6 preservice teachers expressed that they did not check the problem.

Example of the direct quotations are: "I try to find a different way to solve or I check by resolving, if it is difficult and I am not sure", I try to solve it in different ways; "I repeat the solution from the beginning. I always check the accuracy of my calculations." checking procedures; "I check whether I pass the steps of the solution and do procedural errors or not." checking procedures and checking the steps of solution; "Yes, I check. I review the question and I examine each step, if I reach the same solution, I go to the next questions." checking the steps of solution; "I cannot find time to check the accuracy of the question, I pass other question after previous question" no check.

When "What do you pay attention to solve a problem?" question was examined, approximately 36% of preservice teachers focused on understanding a problem in a better way, 33% of them did not do procedural errors; 18% of participants stated that they paid attention whether the solution was logical or not, 13% of them tried to determine the right strategy.

The statement of "I look at desired information for me in the solution of a problem. I avoid unnecessary research and solutions" was settled in better understanding of the problem theme. A statement related to better understanding of the problem and not do processing errors was "If I cannot understand the problem or I can understand to some extent, I cannot solve that question until the end of it. I focus on whether the solution is consistent; I have a chance if I do a procedural error." For, whether the solution is logical theme, example response was "If the question is simple, I think there is an error. Even in the simplest question, I think it is complicated and I am confused. For example, when I saw "2x2=?" question I thought which mode was asked, mode (10), mode (3)…". A student stated "I care how I can solve the problem in the shortest time and in the most practical way, and I control the solution if I made mistakes in the determining the right strategy theme.

DISCUSSION

Results of the first research problem revealed that the effect of metacognitive strategies on the development of metacognition skills of preservice teachers was not significant statistically. In spite of this result, metacognitive prediction scores of preservice teachers changed. Prior to the training practice, while the difference among metacognition prediction scores was more, after training, this difference decreased. Metacognitive prediction is considered as a cognitive ability (Desoete & Royers, 2006). In order to improve metacognition skills, it may require a longer time.

According to the second study problem results, metacognitive instruction improved geometrical problem-solving performance of preservice teachers. This finding was consistent with the findings of previous research (Teong, 2002; Yıldız et al., 2011), which suggested metacognition had an important role in problem-solving performance of individuals in education. During the training practices, preservice teachers encountered a variety of strategies they knew or not. In the post-test, they reached solution by using these strategies. Once pre-test and post-test scores were compared statistically, it was observed that preservice teachers' geometrical problem-solving success increased. Since the problem-solving test prepared for this study also measures the academic performance of students in geometry in mathematics lesson, this result is also consistent with the result that there is a relationship between academic achievement and MS, as suggested by Naglieri and Johnson (2000), Teong (2002) and Desoete and Roeyers (2002).

In relation to the third study problem, it was determined that there is a moderate-level significant correlation between geometrical problem-solving performance and MS of elementary mathematics preservice teachers. This result was consistent with previous reports (Desoete & Roeyers, 2002) arguing that the level of success and metacognition are associated. Previous research concluded that the level of success of students who attended metacognitive instruction increased positively and significantly (Naglieri & Johnson, 2000; Teong, 2002). In their study, Pennequin et al. (2010) argued that it was determined in the regression analysis that a portion of improvement of participants in problem-solving after instruction was explained by the development of MK and MS. This result was parallel with our result of a relationship between metacognition and mathematical skills.

When the results related to MK were reviewed, majority of teachers were not content with themselves by reading one time. They read the problems more than one time and until understanding them. If understanding of the problem is considered one of the most important stages, we can say that metacognitive awareness of the preservice teachers is high. This result supports the conclusion of Montague (2008) and Naser (2008) that expressing the problem in an appropriate way is the basis for understanding it, and it is easier for students to solve the problem when they shape it with their words.

With regard to how to control whether preservice teachers understood the problem, they generally preferred to check by reading the problem repeatedly. They also expressed that they wrote given and desired information based on their previous knowledge in the understanding of the problem part. Likewise, Yayan (2010) also stated that students use various control and evaluation methods in problem-solving processes. Even if after seeing several strategies, they preferred to use the same ways. Responses about performing mental activities, creating mental schemes, and imagining the problem in the mind were settled in "the effect in the mind" theme. Since mental activities are relevant cognition and more advanced of them is about metacognition, we can say that nearly 17% of preservice teachers provide control through metacognitive activities. At the same time, given answers settling into both themes showed that they used more than one way to control. Yıldız and Güven (2016) also stated that, in the light of the results of their research, teachers should insist their students to check every step.

Preservice teachers in determining the time to solve the problem took into account "the difficulty of the problem, understanding the problem" situations. Besides, they stated that by reading problems they determined how much time they needed. If the problem was easy, they reached the solution by reading once; if it was more difficult, they began to solve the problem by reading one more time. Since the level of difficulty changed individually, the time needed for solution varied. To summarize, according to the qualitative answers of the preservice teachers, they are faced with various situations such as the difficulty of the problem, understanding the problem, and the time required to solve the problem. The preservice teachers stated in the Metacognition Knowledge Test that they thought of different possible situations. Similarly, Hacker and Dunlosky (2003) defined metacognition as being aware of all mental activities in one's thinking and control processes.

Preservice teachers indicated that they generally used multiple strategies in solving problems; the most commonly used strategy was to benefit from similar problems. Our education system may also be effective in such a tendency because similar questions have been asked in courses, in schools or in exams. Students try to reach a solution by making use of this solution by looking at previous notes. Therefore, we can say that the participants transfer their old information to the new problem situations. Unlike this result, Danilovic and Rukavina (2015) observed some weaknesses on how students associated and transferred their knowledge to problem situations.

In the section where the preservice teachers' metacognitive knowledge, which is the 4th sub-problem of the study, was investigated, the discussion was made in the paragraphs above based on qualitative findings.

CONCLUSION

In summary, training based on metacognitive strategies positively affected the development of geometric problem-solving skills of preservice mathematics teachers. There was a moderate level significant correlation between geometrical problem-solving performance and MS of participants. It was observed that preservice teachers used various metacognition strategies in solving geometry problems.

When the researches related to metacognition and problem-solving are analysed, it is seen that the researches are progressing not from the field of geometry but from other fields of mathematics (Yıldız & Güven, 2016; Sevgi & Çağlıköse, 2019). Therefore, this research fills the gap in this field, and makes an important contribution to the field of mathematics education at international level, as it relates problem-solving situations in geometry and metacognition. Although it is thought to contribute to the international literature with this dimension of the research, the following suggestions are given for further research.

A total of forty-one preservice teachers participated in the present study. More information can be obtained by increasing the number of participants involved in the study group. Results can be compared by selecting students from different age groups and different grade levels. Educational environments can be prepared where metacognition strategies related to different learning areas are used, such as numbers, geometry, and algebra.

Despite educational practices, expected improvement may not be visible in some students. Research is needed in order to examine the underlying psychological reasons, and it may be taken into consideration in further studies. Lacking teaching experience, preservice teachers may fail to implement meta-cognitive strategies in everyday classroom settings and students' learning. Metacognition issue may take part in preservice education context intensively.

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