STUDENT UNDERSTANDING OF SYSTEM OF EQUATIONS AND INEQUALITIES: A COMPARISON BETWEEN ONLINE AND FACE-TO-FACE LEARNING

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ABSTRACT

This study compared the outcomes of student learning between an online Pre-Calculus course and a face-to-face Pre-Calculus course. Participants for this study included nine online and 14 face-to-face students from an urban community college in the Southeastern region of the United States. The study data were written responses from the subjects to a collection of problems focusing on solving systems of equations and inequalities. Adopting a mixed method design, the study revealed limited differences between the online and face-to-face group in their overall score, their problem-solving capacities, and their common errors. The purpose of this study was to see if students taking courses through different modalities made similar or different errors in an effort to begin formulating plans for improving mathematics learning opportunities for both online and face-to-face mathematics learners.

Keywords: Online teaching, Learning outcomes, Pre-Calculus, Systems of equations and inequalities

INTRODUCTION

In an ongoing study of the growth and perception of online education in the United States, Allen and Seaman (2014) have been tracking online enrollments and academic leaders' perceptions of online educational opportunities yearly since 2002. Their study shows that over 1.6 million students were enrolled in an online course in the fall of 2002, and the number increased, with a 16.1 percent compounded annual growth rate, to over 7.1 million in the fall of 2012 (Allen & Seaman, 2014). During the same period, higher education enrollments grew from 16.6 million to 21.3 million, with a 2.5 percent annual growth rate. Approximately 33.5 percent of all higher education students in 2012, compared to 9.6 percent in 2002, were enrolled in at least one online course (Allen & Seaman, 2014; see Figure 1).

Making learning available in a flexible format is a central draw of online education opportunities (Sitzmann, Kraiger, Stewart, & Wisher 2006). An online course can bring learning to students regardless of time, situation, location, and circumstance, and thus allows all types of learners to study at an individualized pace (Johnson, Aragon, Shaik, & Palma-Rivas, 2000). With more and more students turning to online delivery for educational opportunities, it is critical for the educational research community to examine and understand the extent to which these technologies will transform expectations for, and approaches to,

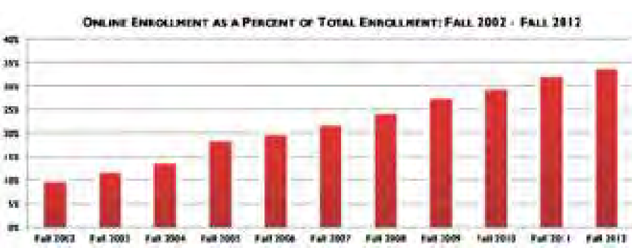

Figure 1. Online enrollment between Fall 2002 and Fall 2012 (Allen & Seaman, 2014).

learning and to explore how the quality of learning achieved in an online setting compares to that of traditional venues of instruction (Allen & Seaman, 2003; Garrison, 2011; Sitzmann et al., 2006).

LITERATURE REVIEW

Sitzmann, Kraiger, Stewart, and Wisher (2006) conducted a meta-analysis study regarding the effectiveness of web-based and traditional classroom learning opportunities. In their analysis, they reviewed a meta-analysis by Zhao, Lei, Yan, Lai, and Tan (2005), which concluded that no difference was present in the effectiveness of the two delivery methods. An additional meta-analysis reviewed by Sitzmann et al. indicated that online instruction is more effective than face-to-face instruction. The meta-analyses reviewed did not focus on specific content but rather included training, procedural knowledge transmission, and declarative teaching. After reviewing 96 studies regarding training courses, Sitzmann et al. found online teaching to be more effective than face-to-face instruction for declarative knowledge presented in training courses in that individuals exhibited greater learning gains and knowledge retention through the online course. Johnson et al. (2000) examined two groups of graduate students enrolled in an instructional design course at a large public university. The study found that students in the traditional face-to-face learning environment tend to be more satisfied with their learning experience, offer a slightly more positive rating for instructor quality, and exhibit stronger personal connections to their instructor, while face-to-face students reported more positive perspectives on their learning environments and higher levels of support from their instructors. The study also found that online students performed equally well as their face-to-face peers regarding meeting learning outcomes. While acknowledging that the online and face-to-face learning environments contain much dissimilarity, Johnson et al. claimed that comparing online education to face-to-face education is like "comparing apples to oranges" and contended that the intent of their examination was not to prove "one fruit is better than the other" but rather that "different fruits can be equal in terms of taste and nutritional value" (p. 31). Upon concluding their study, Johnson et al. determined that optimizing online instructional design to maximize learning opportunities is instrumental in the propulsion of online learning to the equivalence of face-to-face experiences.

Similar results were found by Larson and Sung (2009) when they studied student performance in three introductory Management Information Systems courses. No significant difference was found among student assessments or course grades between the three learning modalities: online, face-to-face and, blended. Students reported higher ratings for utilization of critical thinking and motivation to work at their highest level in online and blended course settings. Larson and Sung concluded that a significant difference in student performance could not be determined.

This study focuses on students' approaches to solving systems of equations, which is a critical topic that spans over Algebra 1, Algebra 2, and Pre-Calculus courses. Typically, students learn to solve systems of equations using substitution, elimination, graphs, or matrices (Carley, 2014). Different curricula may place different emphases on which method to use. For example, Yang and Lin (2015) examined the differences between Finnish and Taiwanese textbooks for grades 7 to 9 on how to solve systems of linear equations. They found that the main difference was the approach used to solve the systems: graphical techniques were emphasized in Finnish textbooks while Taiwanese textbooks encouraged Algebraic approaches. Traditionally in the United States, students are introduced to the process of solving systems of equations through the use of graphs before being introduced to algebraic procedures (Proulx, Beisiegel, Miranda, & Simmt, 2009). However, greater focus is placed on algebraic solution techniques over graphing techniques. Sfard and Linchevski (1994) contended that students who solely depend on algebraic solution methods understand how to manipulate the algebraic process but lack comprehension of their solution meaning.

RESEARCH QUESTIONS

This study aims to evaluate student learning through a detailed examination of their written responses to questions related to solving systems of equations and inequalities to determine if differences are present between online and face-to-face achievement in a Pre-Calculus course offered at the community college level. Two research questions will be analyzed to serve for this purpose:

1. Is there a statistically significant difference in student assessment scores between online and face-to-face groups?
2. Are there differences in the solution techniques or errors made by students as revealed through student work comparisons?

METHODOLOGY

For this study, 23 Pre-Calculus assessments were analyzed to compare students' problem-solving tendencies relative to their modality of course facilitation. This analysis was conducted to see if there is a difference in the effectiveness of online and face-to-face instructions, as demonstrated through the work students provided in their responses to an assessment on solving systems of equations and inequalities.

Instrument

The face-to-face and online versions of the assessment evaluated similar content topics with minor question differences, as illustrated in Table 1. Multiple versions of the assessment were utilized to deter students from cheating and discussing their answers because students were able to complete their assessment at various times. Although variations were present, the questions are closely aligned and provide a snapshot of student understanding of solving systems of equations and inequalities.

Both the online and face-to-face assessments contained four questions evaluating the student's ability to solve systems of equations and inequalities, as shown in Table 1. The four questions assess students' ability to solve a system of two linear equations involving two variables, to use substitution to solve a system comprised of a linear equation and a quadratic equation, to graph the solution of a system of inequalities, and to apply knowledge of matrices to solve a system of three equations involving three variables.

Participants

The participants for this study were students who attend an urban community college in the Southeastern region of the United States. Each student self-enrolled in either the online or the face-to-face section of the Pre-Calculus course.

Table 1. Questions on Solving Systems of Equations or Inequalities in the Assessments

Assessment Question	Face-to-Face	Online
1. Students are asked to solve a system of linear equations using a method of their choice and notating their final answer as an ordered pair.	1. Solve the system of linear equations. State your final answer as an ordered pair. $3x+2y=2$ $4x-y=-23$	1. Solve the system using the method of your choice. (State the final solution as an ordered pair, or state "no solution," or "infinite solutions," as applicable). $7x+9y=-10$ $3x-y=20$
2. Students are asked to solve a system of equations comprised of one linear equation and one quadratic equation using the substitution method.	2. Solve the system of nonlinear equations by using the substitution method. $3x+y=-4$ $y=x^2-2x-10$	2. Solve the nonlinear system by the substitution method. (State the solution(s) as ordered pair(s).) $x+y=3$ $y=x^2-5x+6$
3. Students are asked to graphically solve a system of inequalities.	3. Graph the system of inequalities. $\begin{cases} y<\frac{1}{2}x+3 \\ y\geq x^2-5 \end{cases}$	3. Graph the following system of inequalities and shade to show the solution set of the system. $\begin{cases} y>x^2-7 \\ x+y\leq 2 \end{cases}$
4. Students are asked to solve a system of equations in three variables using the matrix method, Gaussian elimination.	4. Solve and state the solution as an ordered triple, using the MATRIX method. $x+y-z=-5$ $2x-y+z=-1$ $-x+5y-4z=1$	4. Solve the system using the matrix method of Gaussian elimination. $x-7y-z=-16$ $x+y+7z=24$ $x-y+z=2$

Data for this study are composed of the responses from nine students enrolled in the online section, and 14 students enrolled in the face-to-face section during the fall semester of 2015. The prerequisite required for both sections was identical and could be achieved one of three ways: 1) place into Pre-Calculus through a satisfactory score on the college's placement assessment, 2) successfully complete the course preceding Pre-Calculus in the college's course sequence, or 3) successfully complete an equivalent AP assessment at the high school level.

Both courses were taught by the same instructor, used the same textbook, and had the same learning objectives. All students had equivalent access to the same course resources through Pearson's My Math Lab learning suite: video lectures, calculation examples, worked solutions, and problem-solving guidelines. All assessments were completed on paper by hand for both the online and face-to-face courses. Assessments in the face-to-face course were conducted in class and proctored by the course instructor. Assessments for the online course were conducted at a proctored testing location. Students close to a campus of the community college were able to go to the testing center provided by the community college. Online students who were not close to a campus location had other options (such as using a different educational institution, library, or testing center) through which they could complete their assessment in a proctored setting.

Data Analysis Procedure

Assessing student understanding of mathematical processes is difficult because it requires students to clearly communicate their thought process (Szetela & Nicol, 1992). Szetela and Nicol (1992) contend that the best way to assess a student's problem-solving performance is to review the student's work relative to a devised scale that rates student work and responses. An analytic scale is an easy-to-use ranking system that allows teachers to focus on each stage of problem solving as desired (Szetela, & Nicol, 1992).

A two-tier data analysis procedure was utilized to analyze the student's work. First, a statistical analysis was conducted to establish homogeneity of variance between the groups and compare group means for student scores on each question. The second level of analysis was a close examination of each student's work to extract examples of their understanding and evaluate the solution techniques they utilized.

Statistical analysis. The first level of review pertained to statistical analysis of the scores that students received on each question. SPSS was used to run Levene's test and F-Tests for each question. Levene's test was used to test the null hypothesis that the variance in scores between the online and face-to-face sections was equivalent. Once homogeneity of variances was established, ANOVA F-tests were used to explore between-group differences relative to mean scores.

Student work analysis. The second level of analysis was a close examination of the student's work, which was reviewed to look for patterns and see if differences were present in how online or face-to-face students approached each question. Each question was reviewed and procedural comments were noted for all student work, and patterns of techniques, errors, and processes were analyzed.

FINDINGS

The following analysis will discuss in detail the observed differences between the online and face-to-face courses. The students' assessment scores were statistically analyzed and a detailed examination of the students' work was conducted as well. Initial statistical reviews found a nonstatistically significant difference between student scores on each question. Table 2 shows class averages, in percent, for each question. The final average of all four questions is shown in the last column.

Table 2. Averages in percent

Test	1	2	3	4	Final Avg
Online	81.48	72.60	65.19	61.48	70.1875
Face-to-Face	92.86	72.32	61.61	54.46	70.3125

Looking at the average scores on each question, the online student's average scores were higher on questions 2, 3, and 4 but lower on Question 1. Averaging overall scores on questions 1 through 4 reveal almost identical scores overall for the online and face-to-face courses with the online average being only slightly less. F-tests were conducted to investigate if the between-group differences of the average scores for each question is statistically significant. In addition, an examination of solutions and common errors made by the students was conducted. The following offers detailed findings in analyzing the students' scores and responses for each question.

Question 1

Question 1 asked students to solve a system of linear equations using a method of their choice. The students elected to use methods of substitution or elimination and needed to carefully execute their solution strategy to identify the solution point.

Question 1 statistical analysis. The class averages for Question 1 differed by 11.38%. The online class average for Question 1 was 81.48%

while the face-to-face class average was 92.86%. As shown in Table 3, Levene's test does not reflect a statistically significant difference in the variances garnered by these scores, suggesting that the assumption of homogeneity of variances is not rejected. An F-test was used to further explore between-group differences. The F-test, shown in Table 4, conveys a nonstatistically significant difference at the level with a conclusion of $F(1, 21) = 1.192$, $p > .05$, supporting no between-group differences relative to Question 1 scores.

Table 3. Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
3.986	1	21	0.059

Table 4. ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	708.826	1	708.826	1.192	0.287
Within Groups	12483.82	21	594.468		
Total	13192.65	22			

Question 1 student work analysis. In the online group, six students correctly answered this question, one student left this question blank, and two made algebraic mistakes when multiplying by a fraction or evaluating with a negative sign. Three students elected to solve through the substitution method, and five students used the elimination method. All students who used substitution correctly solved the system of equations by solving the bottom equation for y = and substituted into the top equation. Each student then distributed, combined like terms, simplified, solved for x, solved for y, and notated their solution point, as shown in Figure 2.

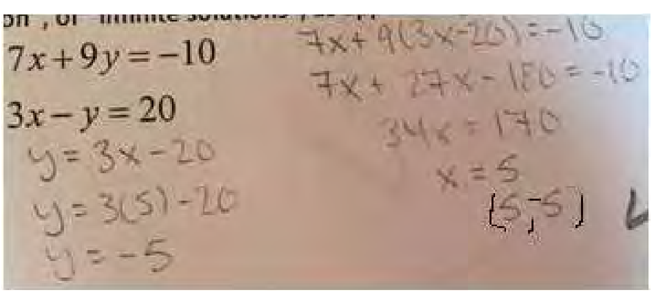

Figure 2. Online student substitution work for Question 1

The five students who solved by elimination each multiplied the bottom equation by 9 and subtracted the equations to eliminate the y variable and solve for x. After calculating x, the students then solved for y and listed their ordered pair solution, as shown in Figure 3.

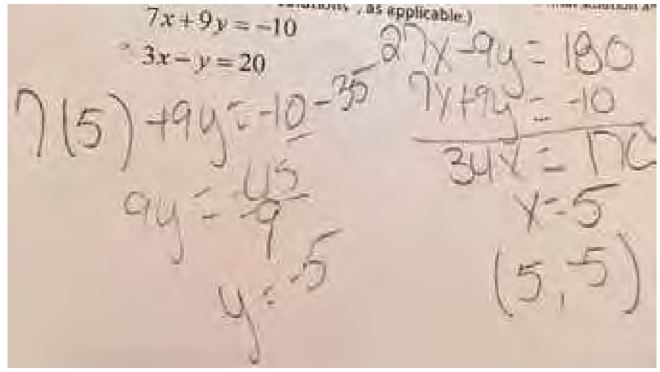

Figure 3. Online student elimination work for Question 1

Like the online students, most face-to-face students elected to solve Question 1 through substitution. Eleven face-to-face students utilized substitution while three used elimination. Of the students who used substitution, eight elected to solve for y = in the bottom equation and substitute into the top equation. Three students tried to solve the top equation for y = and substitute into the bottom, but this left a negative fractional coefficient of x and lead one student to calculation errors. As shown in Figure 4, like their online peers, the students who solved the bottom equation for y = substituted into the top equation, distributed, combined like terms, simplified, and solved for x, then solved for y and wrote their solution as an ordered pair.

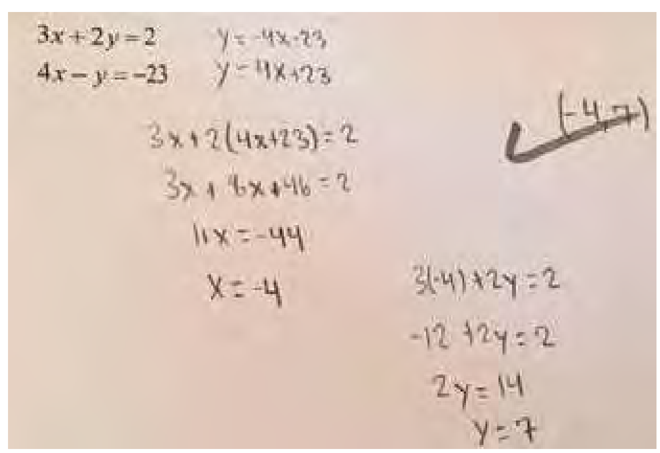

Figure 4. Face-to-face student substitution work for Question 1

In the face-to-face group, three students solved using the elimination method. All three students elected to multiply the bottom equation by two and subtract the equations. Two of the three students were successful with this computation, and one student incorrectly distributed and the remainder of their calculations were misaligned. The students who used elimination followed the process shown in Figure 5. After distributing and subtracting the equations, the students simplified to solve for x and then solved for y before writing their solution as an order pair.

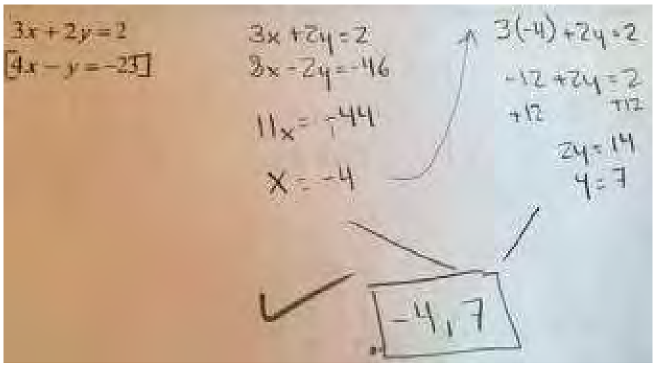

Figure 5. Face-to-face student elimination work for Question 1

The work executed to solve this system of linear equations by substitution or elimination shows little discrepancy between the online and face-to-face courses. As shown in figures 2, 3, 4, and 5, the work provided by the students is comparable in nature and in execution of the systematic solution processes assessed by this question. It was not found that the students taking the face-to-face course were more proficient or chose different strategies than their peers in the online course. Procedural differences could not be identified through this item analysis for solving systems of linear equations using substitution or elimination.

Question 2

The second question reviewed asked students to again solve a system of equations, but this system contained one linear equation and one quadratic equation. To successfully solve this system of equations using substitution students must understand how to set a quadratic equation equal to 0, factor, and look for multiple solution points. After substituting and simplifying, students had to set the resulting quadratic equation equal to 0 and factor to find two x solution values.

Question 2 statistical analysis. Class averages for Question 2 were very close. The online class average was 72.6% and the face-to-face class average was 72.32%. As shown in Table 5, in accordance with the Levene's test, the assumption of homogeneity of variances is accepted. An F-test, shown in Table 6, further reveals there is not a statistically significant difference between the group means at the level with a conclusion of $F(1, 21) = .000$, $p > .05$.

Table 5. Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
0.234	1	21	0.634

Table 6. ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	0.405	1	0.405	.000	0.985
Within Groups	22854.95	21	1088.331		
Total	22855.36	22			

Question 2 student work analysis. In both the online and face-to-face course, factoring was observed to give students trouble. Figure 6 shows the work of a face-to-face student and Figure 7 shows the work of an online student, both of whom did not successfully complete the factoring step to solve this system of equations.

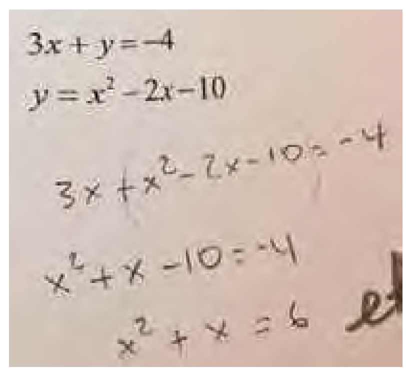

Figure 6. Face-to-face student factoring error

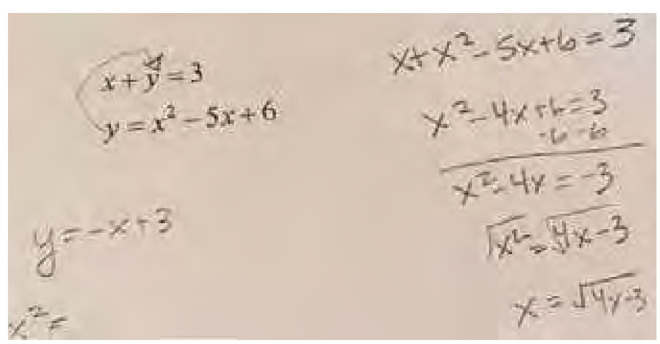

Figure 7. Online student factoring error

The factoring errors shown in Figures 6 and 7 are different in that the online student incorrectly tried to take the square root of each side to progress with solving while the face-to-face student stopped when they arrived at the quadratic equation . But the work displayed by both students indicates a misunderstanding surrounding the process of factoring to finish solving.

Question 3

For both Questions 1 and 2, all students who answered the questions relied on an algebraic calculation process to arrive at their solution. Question 3 required students to move from an algebraic interpretation to a graphical interpretation and required students to graph the solution region generated by a linear inequality and a quadratic inequality.

Question 3 statistical analysis. The class averages for Question 3 differed by 3.58%. The online average was 65.19% and the face-to-face average was 61.61%. Levene's test, as shown in Table 7, does not reflect a statistically significant difference in the variances of these scores, suggesting that the assumption of homogeneity of variances is accepted. The F-Test shown in Table 8 conveys a nonstatistically significant difference at the level with a conclusion of $F(1, 21) = .043$, $p > .05$, supporting no between-group differences relative to Question 3 scores.

Table 7. Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
.000	1	21	0.998

Table 8. ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	70.149	1	70.149	.043	0.837
Within Groups	33949.05	21	1616.621		
Total	34019.2	22			

Question 3 student work analysis. In both groups, the most common error was in shading the appropriate region on the graph. The second most common error was incorrectly graphing the equations. Seven students from the face-to-face course used test points to determine solution regions. One example of this work is shown in Figure 8.

The students in the face-to-face course who

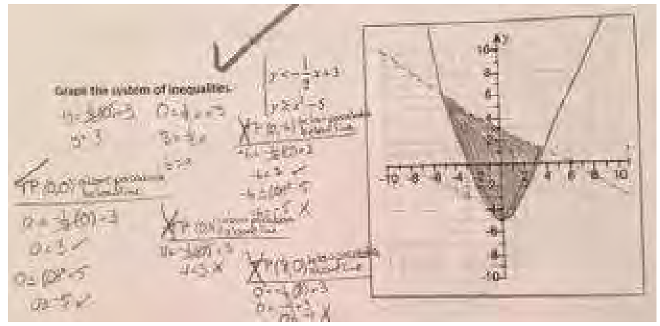

Figure 8. Test points for Question 3

used test points were able to accurately shade the solution region on their graph. In the online course, four students also attempted to use test points, but only one was able to successfully translate their test points into accurate shading. Figure 9 shows an example of the errors discovered while reviewing Question 3 on the online course exams.

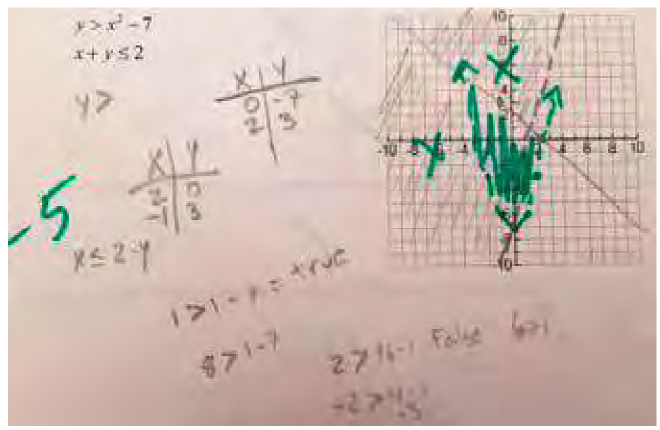

Figure 9. Online errors

In Figure 9, the instructor's corrections are darker than the student's work. The student incorrectly interpreted the parabolic equation as a line, which they have graphed as a dotted line through quadrants 1, 3, and 4. The student did correctly interpret the solid line to denote or equal to for the quadratic inequality, but their test points did not lead them to a correctly shaded solution.

Question 3 also has the least amount of work present as many students elected to draw the graphs without showing any work. Students were permitted to use graphing calculators on this assessment. It is hypothesized that some students used their calculators to generate graphs that they then translated to their assessment papers.

Question 4

Question 4 asked students to use matrix reduction techniques to solve a system of three equations in three variables. The teacher specified that matrix row reduction must be shown for full credit to be awarded. While 19 of the 23 students analyzed were able to convey an understanding of what the question asked for through initially setting up their matrix and embarking on the row reduction process, only six students were able to successfully navigate to reduced echelon form and solve this system of equations

Question 4 statistical analysis. Of the four questions reviewed, Question 4 had the lowest average in both the online and face-to-face groups, 61.48% and 54.46% respectively. As shown by the Levene's test, in Table 9, there is not a statistically significant difference in the variances garnered by these scores, suggesting that the assumption of homogeneity of variances is accepted. Table 10 shows the F-Test, which conveys a nonstatistically significant difference at the level with a conclusion of $F(1, 21) = .171$, $p > .05$, supporting no between-group differences relative to Question 4 scores.

Table 9. Test of Homogeneity of Variances

Levene Statistic	df1	df2	Sig.
1.057	1	21	0.316

Table 10. ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	269.812	1	269.812	.171	0.683
Within Groups	33112.84	21	1576.802		
Total	33382.65	22			

Question 4 student work analysis. Computational errors in the row reduction process caused both online and face-to-face students to stumble and not successfully complete the solution process. No one from the face-to-face course tried to solve using substitution and elimination techniques. Four students from the face-to-face group left this question blank. One example of a properly initiated, but ultimately incorrect, solution is shown in Figure 10. The step circled is where each student made an error, resulting in an incorrect solution for their calculations.

As shown in Figure 10, students were able to correctly set up the matrix and begin the row reduction process, but computational errors caused students to not successfully complete this solution process.

Figure 10. Online matrix errors

SUMMARY OF FINDINGS

This study did not reveal any differences between the outcomes of the online and face-to-face learning based on students' responses to the assessment on solving systems of equations and inequalities. Looking at the average scores on each question, the online student's average scores were higher on Questions 2, 3, and 4 but lower on Question 1. Averaging overall scores on Questions 1 through 4 reveal almost identical scores overall for the online and face-to-face courses with the online average only slightly less. Further analysis using Levene's test and F-test revealed that the differences in the online and face-to-face scores are not statistically significant. Analysis of the student work on each question revealed similar characteristics relative to the techniques used and mistakes made.

The only notable difference revealed is that students in the online course demonstrated a more frequent tendency to try a question if they were not fully sure how to complete it and arrive at an accurate solution, while the face-to-face students demonstrated a more frequent tendency to leave a question blank. This observation is quite interesting and will be considered as the topic of future research and investigation. Additionally, students in the online class made common arithmetic errors, struggled with factoring techniques, made graphing mistakes, and were thrown off course by calculation errors while performing row reduction calculations in a matrix. These mistakes are also observed in the face-to-face work. No clear evidence was observed that one group has more of a tendency to make algebraic or computational mistakes.

IMPLICATIONS

Online learning provides unique learning opportunities for students. Through this study, it was observed that online and face-to-face student opportunities and experiences are being approached similarly; courses are structured in similar manners and equivalence between the learning media is sought after as teachers are striving to make their online courses mimic their face-to-face courses. The results found through this study show that online and face-to-face students performed similarly, which inspires the question, how can online and face-to-face courses be structured differently to captivate students in distinct ways unique to each platform? If online courses are being made to mimic face-to-face courses, are potential learning opportunities that are truly unique to the online learning platforms being lost? Would face-to-face students perform better with resources particular to the face-to-face platform? The teacher in this study strived to make their online and face-to-face courses identical, but is that the best practice to consider? Would both online and face-to-face students perform better if less emphasis was placed on congruency across platforms and more emphasis was placed on platform-specific best practices relative to mathematics pedagogy? While not addressed in this study, these questions have been raised and necessitate further research.

CONCLUSION

The way people live, work, learn, and play is impacted by technology. Laptops, tablets, smartphones, and the internet are increasing the rate at which knowledge can be accessed and transferred. With many students turning to online means for their educational opportunities, "we are just beginning to discover and understand the extent to which these technologies will transform expectations for, and approaches to, learning" (Garrison, 2011, p. 5). As students increasingly embark on online learning experiences, it is important to ensure that the quality of learning is not being negatively impacted.

This study joins a limited body of works that compare student acquisition of mathematics knowledge in an online setting to a face-to-face setting. Newlin, Lavooy, and Wang (2005), Larson and Sung (2009), and Johnson et al. (2000), reported that no statistical differences were found between the online and face-to-face groups. Similar to Proulx et al.'s (2009) recommendations, this study looked at students' ability to solve systems of equations using not just algebraic but also graphical and matrix methods. Looking at various ways to solve systems of equations gave a more holistic picture of students' comprehension of this topic rather than looking at only the algebraic manipulation technique.

Additional studies should be conducted to compare the online and face-to-face work of mathematics students in other courses, at other grade levels, and with other instructors and programs, to establish transferability and replicability of findings. Student attrition, the perceptions of their online experiences, and future mathematics course experiences should also be evaluated to better understand a holistic view of students' online mathematics course experiences and how their experience impacts the broad spectrum of their mathematics learning endeavors. To gain a deeper understanding of the students' experiences in their online and face-to-face courses, interviews and round table discussions would shed additional light on various forms of instruction. A snapshot of student work is valuable to examine, but having student voices to further explain their solutions would provide a much deeper level of insight into the students' understanding. Having a face-to-face conversation with students to review their systems of equations and inequalities assessment and talking through the decisions represented by their work would be a valuable means of adding depth to this study. While this study does not suggest any monumental results, it does provide evidence for the need for further exploration of online mathematics education pedagogy and best practices. Online education provides distinct opportunities for students. With these opportunities come particular challenges. If online courses are being taught as replications of face-to-face courses and online students are performing congruently to face-to-face students, are online courses taking full advantage of the opportunities provided through virtual learning platforms?

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