What's the math in Minecraft? A Design-Based Study of Students' Perspectives and Mathematical Experiences Across game and School Domains

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Abstract: This paper presents empirical findings from a qualitative study on Minecraft as a mathematical tool and learning environment. Even though Minecraft has been used for several years in classrooms around the world, there is a lack of detailed empirical studies of how students learn subject-related content by working with the game. This study is based on a design experiment with an inquiry-based teaching unit for fifth graders, which focused on using the coordinate system embedded in Minecraft, as a means to navigate and explore the game in order to solve mathematical problems. Based on student interviews, we explore how the students experienced and switched to new perspectives on mathematical knowledge through their participation in the teaching unit. Using thematic analysis, we explore data from six group interviews. The theoretical framework is based on domain theory, dialogical theory and notions of students' mathematical agency. The key analytical findings regard the students' experience of the coordinate system as part of both the academic domain of mathematics and their everyday domain of playing Minecraft, how they actively use the coordinate system to improve play in Minecraft and how they experience new ways of participating in mathematics. The article concludes by offering design principles for the future use of computer games in mathematics education.

Keywords: Minecraft, game-based learning, mathematics education, domain theory, coordinate system

1. Introduction

Games have long existed as part of mathematics education and have been investigated for several years (Bright, 1983; Oldfield, 1991). According to one review, games are used more frequently for teaching mathematics than for any other subject (Hainey et al., 2016). Thus, there is a widespread belief that games have tremendous potential in mathematics education. Although some scholars have pointed to video games as an ideal medium for teaching mathematics in middle school (Devlin, 2011), two recent meta-analyses showed only small and marginally significant positive learning effects from using games in mathematics education (Byun and Joung, 2018; Tokac, Novak and Thompson, 2019). This indicates that there is only limited research evidence to support the assumed potential of game use in mathematics learning.

A prevailing problem for mathematics education is that many students do not come to see mathematics as a constructive endeavor (Boaler, 2015). Similarly, student interest in mathematics is reported to be one of the most significant predictors in determining mathematical performance and perseverance (Hannula et al., 2016). However, the use of games in mathematics education is often based on game elements serving as rewards or aspects of extrinsic motivation, which do not support learning as much as games, where the learning activities are driven by students' inner motivation (Habgood and Ainsworth, 2011). Cobb (2007) argues that classroom activities being worthy of student engagement in its own right is an important part of the cultivation of students' interest in mathematics and should be considered an important goal for mathematics educators. As such, the aim of the current study is to explore these two research questions:

- How can Minecraft be used in a teaching unit to engage students in mathematics education by enabling different forms of participation?
- How do students experience new perspectives on mathematical knowledge across in-school and out-of-school domains?

2. Learning mathematics with Minecraft

Minecraft is one of the most played video games in the world, and it appeals widely to both boys and girls, especially around of the ages of 9–11 (Mavoa, Carter and Gibbs, 2018). At the same time, *Minecraft* is also increasingly being used as an educational tool in classrooms (Kipnis, 2018), and research has been conducted on its use in promoting learning within a wide variety of school subjects (Nebel, Schneider and Rey, 2016). Several

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studies have explored the envisioned potential for using *Minecraft* as a component of mathematics education (Tromba, 2013; Bos et al., 2014; Ellison and Evans, 2016; Winter, Love and Corritore, 2016). For example, Kim and Park (2018) explored how preservice teachers identify potential mathematics learning benefits when using *Minecraft* as a learning tool. Other studies have described how *Minecraft* can model different mathematical concepts (Short, 2012) or how player behavior can be considered mathematical (Kipnis, 2018), with some proposing teaching material that employ elements from *Minecraft* as a context for mathematical exploration (Moore, 2018). However, these studies share a general lack of empirical evidence to support their claims about student learning, as the focus is primarily speculative explorations of possible learning potentials. Other studies have reported on local teaching experiments using *Minecraft* in mathematical contexts (Al-Washmi et al., 2014; Foerster, 2017; Freina et al., 2017) but provide limited descriptions of students' mathematical outcomes. Another study found no significant correlations between students' *Minecraft* habits and their perceived abilities to solve mathematical problems (Griffin and Griffin, 2018). None of these studies provide detailed descriptions of students' learning in relation to *Minecraft.* An exception, however, is the ethnomathematical qualitative study by Køhrsen and Misfeldt (2015), focusing on mathematical activity in *Minecraft* in an after-school programme. This study used empirical evidence with children, combined with a theoretically founded approach to understanding learning.

Overall, the studies point to learning potentials, promising teaching designs and innovative approaches regarding the use of *Minecraft* in mathematics education. However, most studies are based on anecdotal evidence, and very few articles focus empirically on students' learning or use a theoretically grounded approach to mathematical learning. This article seeks to address this research gap by presenting an empirical qualitative study of how *Minecraft* can help students learn mathematics.

3. Case: Teaching unit with *Minecraft*

The current study is based on a design experiment with a teaching unit using a *Minecraft* map in a fifth grade class comprising 22 students. *Minecraft* worlds are randomly generated, so a key element in the game is the exploration of the specific virtual world in which you are playing (Lane et al., 2017). However, it can be difficult to successfully navigate *Minecraft* and locate specific objects or other players. This can result in problems for players – e.g. getting lost on the map after building a structure and being unable to locate the structure again.
The idea of the intervention originated from the fact that the mathematical concept of Cartesian coordinates used in the mathematics educational curriculum was accessible in the *Minecraft* user interface and that three-dimensional (3D) navigation is a key challenge in the game. More specifically, the player can access the x, y and z coordinates of his avatar in the game. The x-axis indicates the position on an east–west axis, the z-axis shows the position on a south–north axis, and the y-axis indicates elevation (see figure 1 below). One whole number on the axis is equal to the length/height of one block in the game, which is equal to one meter in the real world.

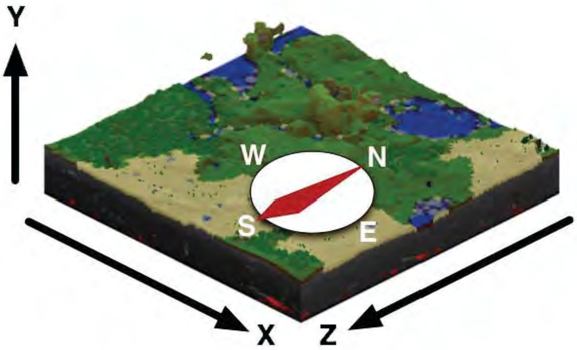

Figure 1: Screendump from the game map and an illustration of the x, y and z coordinates

When the players move their avatar in the virtual world, the values of the axes change according to the position. Moving directly up or down will affect the y-axis and moving directly towards the east will increase the value of the x-axis. Looking at figure 2, we see a player avatar standing on the first, second and third steps of a staircase in *Minecraft*. For each picture, the avatar coordinates are in the upper-right corner.

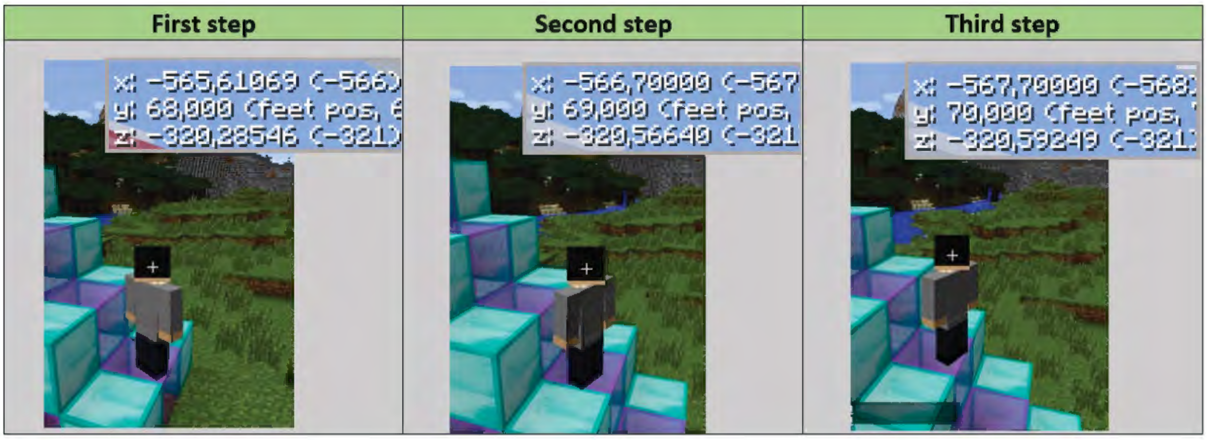

Figure 2: Change in coordinates

Following the avatar going up the stairs, the y-axis changes from 68 to 69 to 70. The x-axis also changes because the avatar is also moving west as it climbs the stairs. The change in the numbers after the decimal point indicates that the avatar can be placed on different locations within one square block.

The accessibility of such an underlying mathematical dynamic is not commonplace in commercial games, where the mathematical rules are often hidden from players, as they may disturb the players' immediate experience of the game (Lowrie and Jorgensen, 2015). Our hypothesis or humble theory (Prediger, 2019) for the design experiment was that the mathematical concept of Cartesian coordinates could be introduced as a means to solving the real player problem of locating objects in *Minecraft* in order to help students master and influence their understanding of the game. Therefore, instead of using the game as an instrumental tool for teaching mathematics, our starting point was to understand how we could design a teaching unit that created meaningful links between in-game challenges and the mathematical aspects of the game. Thus, the coordinate system in *Minecraft* should optimally serve as a useful resource for learning mathematics in order to master the navigation in the game and for playing the game in order to explore the mathematical concept of coordinates.

The teaching unit with the game map consisted of 15 lessons distributed over five days in one week. One of the researchers held meetings with the teacher, who contributed with feedback and ideas for improving the unit. The students' activities were mainly inquiry-based and framed around open questions. The initial task for the students was to understand what the numbers at x:, y: and z: indicated. The staircase shown in figure 2 was used by the students to examine how the coordinate numbers changed when their avatar moved up and down the stairs. This was not explained directly to the students in the way we have done above. Instead, the students were asked to build a staircase and reason about how the coordinate numbers changed. This would help them identify y: as an indication of their height level. In another activity, the students were asked to move in a way so that either the x or z coordinate remained constant while they moved. This was challenging for them, as it required them to move directly towards east, west, north or south. One solution was to build a wall and move in parallel with it. These assignments helped the students to reason about what the coordinate numbers indicated. Later in the unit, the students had to use their knowledge of the coordinate system to solve tasks. One task was to build a railway of exactly 100 blocks long, in pairs, starting from opposite ends and meeting in the middle. Another similar task was to build a tunnel through a mountain from each side. Finally, the students used the coordinate system to create treasure hunts, with new coordinates written on each post. In one instance, a student-created treasure hunt was played by the entire class in pairs. One student in each pair would control the avatar, while the other student would act as the navigator, looking at the coordinate numbers and guiding the controlling student.

4. Theoretical framework

Our research question was explored by analysing student interviews through a theoretical framework consisting of two complementary theories. The first theory was *scenario-based domain theory* (Hanghøj et al., 2018), which was used to map the interplay of different practices in the game-based teaching unit, with a particular emphasis on students' mathematical obligations (Cobb, Gresalfi and Hodge, 2009). The second theory assumed that students would explore the game-based teaching unit through different *voices* and *perspectives* in relation to the *dialogic space* created in and around the game scenario (Wegerif, 2006). By combining these perspectives in our exploration of *Minecraft* in mathematics, we described the students' perspectives on mathematical learning across in-school and out-of-school domains. We shall now outline the two complementary theories.

4.1 Domain theory

When the students played Minecraft in the mathematics classroom, they took part in a specific educational scenario (Hanghøj et al., 2018), which required them to imagine and perform domain-specific game practices, such as navigating the 3D game space, finding locations, hiding items from other players or building structures. In order to overcome in-game challenges, the students had to develop and use their knowledge of the coordinate system as a mathematical concept. As fifth graders, most of them were quite familiar with the game, having played it at home. In this way they participated in an open-ended inquiry process, which involved an interplay of practices across in-school and out-of-school domains.

A practice involves recognizable ways of doing things for shared social purposes, such as being able to navigate the game world of Minecraft or solving a mathematical task in the classroom (Hanghøj et al., 2018). Domains represent clusters or families of different practices and involve different validity criteria for what counts as legitimate knowledge and what does not. As such, students' exploration of Minecraft in mathematics involves a transformation of experiences and practices across four domains: the domain of everyday life, the pedagogical domain of schooling, the disciplinary domain of mathematics and the scenario-based domain of overcoming challenges in Minecraft. We shall now unpack these domains.

The domain of everyday life concerns students' lifeworlds, such as their life at home with family or friends. In this study, we were particularly interested in how the students were able to link the teaching unit with their out-of-school experience of playing Minecraft and their experience of using mathematics in their everyday life.

The second domain involved the pedagogical domain of schooling, which relates to the asymmetrical relationship between the teacher and student and the norms and expectations of what it means to participate in classroom teaching. These institutional and communicative practices are "school only", as they only occur in school contexts, but are always locally defined at the classroom level. In this study, we looked at how the game-based teaching unit, which differed from their regular teaching, allowed the students to collaborate and change their relationships.

Third, the disciplinary domain of mathematics concerned how the students participated in and experience mathematics as a school subject. In order to describe this aspect, we used concepts from the interpretive framework of Cobb, Gresalfi and Hodge (2009) for understanding students' mathematical identities. The authors argue that being a doer of mathematics implies a normative identity with certain obligations that define and constitutes the role of a good mathematical student in a specific classroom. These obligations concern three aspects. The first concerns the ways in which students legitimately express agency, which relates to both their use of established mathematical solution methods (disciplinary agency) and their ability to choose mathematical methods and develop meanings and relations between concepts and principles (conceptual agency). The second aspect relates to the distribution of authority, i.e. to whom the students are accountable when working with mathematics. The third aspect encompasses specific mathematical obligations that the students are accountable for, that is, what counts as competence in terms of mathematical reasoning and argumentation. Drawing on these concepts, we were able to show how the students, in working with Minecraft, expressed different forms of agency, authority and competence in the disciplinary domain of mathematics compared to their experience of normal classroom practices.

Finally, the fourth domain was the scenario-based domain of Minecraft, that is, the students' exploration of the specific game map used in the teaching unit. We took a closer look at the students' different interests in the game and how these influenced their experience of playing the game in a formal educational setting. Even though nearly all the students had tried Minecraft prior to the teaching unit, there were important differences in their experience and competence with playing the game.

The dynamic relationship amongst the knowledge practices of the four domains involved in the students' game activities is illustrated in Figure 3:

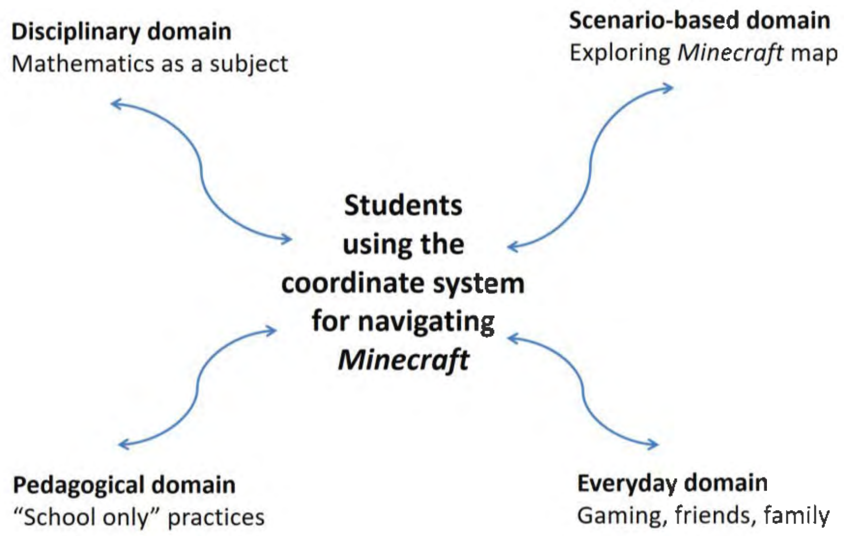

Figure 3: Interplay of practices across domains

By describing this interplay of knowledge practices across the in-school and out-of-school domains, it became possible to explore what counts as valid knowledge and the valid ways of doing math when playing *Minecraft* in the mathematics classroom.

4.2 Dialogic perspectives on Minecraft and mathematics education

The scenario-based domain theory is a generic theoretical framework, which can be used to describe the interplay of domains in a range of educational contexts. In order to provide a more detailed understanding of the bridging of practices and student experiences across the four domains, we found inspiration in the theory on dialogic education (Wegerif, 2006), which has been further developed in relation to mathematics education (Kazak, Wegerif and Fujita, 2015) and game-based teaching (Arnseth, Hanghøj and Silseth, 2018). Dialogues may be recorded as talk and interaction in external space and time. However, according to Wegerif (2006), dialogues are not simply external phenomena; they also involve an internal aspect, which invokes different times and spaces and a range of voices that reflect the participants' perspectives. The term voice, originally developed by Bakhtin (1981), refers to a first person perspective in a dialogue. In this sense, a dialogic space "is the space that emerges between voices, but that is also a shared space within which voices relate to each other" (Kazak, Wegerif and Fujita, 2015, p. 107).

Playing games at school with others may create dialogic spaces in which participants can explore and exchange a range of voices and perspectives (Arnseth, Hanghøj and Silseth, 2018). In our study, the students explored the use of *Minecraft* in mathematics through different voices, reflecting an interplay of experiences across the four domains. As we shall see in the analysis, some students related the *Minecraft* teaching unit to their everyday experience with the game outside of school or their everyday experience of "doing math class". Some students focused on the mathematical aspects of the game, while others emphasized how the teaching unit allowed them to collaborate and learn from their peers. It can thus be argued that the educational use of *Minecraft* opens up a multi-voiced dialogic space, where students are able to explore several interests and experiences across in-school and out-of-school domains.

According to Kazak, Wegerif and Fujita (2015), participation in a dialogic space enables students to *switch* between different perspectives, which is valuable in terms of extending and developing their understanding of a given problem or concept. However, switching between perspectives is not a mechanical process and requires an active choice, where students are willing and able to *step back* from their own point of view in order to explore different perspectives within a dialogue. The current study is based on group interviews with the students following the completion of the teaching unit with *Minecraft*. Through the interviews, we aimed to explore how the students experienced perspectives on mathematical knowledge in relation to the *Minecraft* teaching unit. We were not interested in documenting specific learning outcomes or how the students' learning processes took place during the teaching unit. Rather, we were interested in describing how the students' exploration of *Minecraft* allowed the emergence of new perspectives on mathematical concepts.

5. Methodological approach

This pilot study is part of an on-going design-based investigation (Barab and Squire, 2004) of how commercial games can be linked to curriculum aims. The data were collected in an urban public school with a large percentage of bi-lingual students in a socially and economically challenged area in Copenhagen. The school had participated in a previous project using commercial games (Hanghøj, Lieberoth and Misfeldt, 2018), where several of the teachers had developed a positive interest in using *Minecraft* for mathematics teaching. Based on this interest, teachers at the school decided to allow two fifth-grade classes – not part of the previous project – to participate in a game-based teaching unit for mathematics. One of the authors of this study conducted video observations and took field notes in one of the classes, where he had on-going dialogue with the teacher about how to facilitate the teaching unit. After the teaching unit was completed, six semi-structured group interviews (Brinkmann and Kvale, 2015) were completed with two fifth-grade students in each group. These interview data formed the empirical focus for this study.

Six student groups of two students each were selected prior to the intervention and in dialogue with the teacher in order to represent a broad spectrum of mathematical performance with students who demonstrated high-, low- and medium-level mathematical skills. The groups were mixed gender, with three male-only groups, two male–female groups and one female-only group. There were mainly male students in the class, which was reflected in the division of gender. The teachers described the class as disruptive and difficult to manage, with an overall low performance in mathematics and only a few mathematically skilled students. In order to ensure a contextual link with the gameplay activities, the students were interviewed once after the teaching unit, in the same pairs described above. The interview guide involved questions regarding their everyday experiences of math class and the mathematics the students used during the intervention with *Minecraft*. Each interview lasted between 45 and 60 minutes. Thereafter, all the collected interview data were subjected to a thematic analysis (Braun and Clarke, 2006), which involved transcription and coding using open categories, which were used to establish themes relating to the students' experiences of learning mathematics and participation in the mathematical activities. Following this, we identified overall analytical themes by linking the categories with concepts from domain theory and the students' mathematical agency (Cobb, Gresalfi and Hodge, 2009).

6. Analysis

In this section, we unfold how *Minecraft* can be used in a teaching unit to engage students in mathematics education through different forms of participation and how the students experienced new perspectives on mathematical knowledge across in-school and out-of-school domains.

Based on the analysis of the six interviews, we identified four analytical themes on different student perspectives on mathematical knowledge construction. The first concerned the students' experience of what it meant to participate as a student in regular math class. The second described how the students experienced learning mathematics through *Minecraft*. The third theme involved students' experiences about using mathematics to learn how to become better players. Finally, the fourth theme had to do with how the students experienced collaborative work in mathematics in relation to *Minecraft*. The four themes show a wide range of connections of student experiences across the different domains, all illustrating how the game-based intervention changed the students' experience of learning mathematics. We describe below the students' perspectives of mathematical learning. To illustrate how the use of *Minecraft* generated experiences that differed from those of the students' everyday math class, we describe the students' perspectives on math class with and without *Minecraft.*

6.1 Participation as a student in "math class"

This theme explored the *students' perspectives on their everyday participation in "math class"* before the intervention. We intentionally used the term math class to underline that this theme did not reflect the students' perspectives on mathematics as such but, instead, how they perceived participation in the local pedagogical context of their everyday mathematics classroom. Hence, this theme focuses less on exploring *Minecraft* and more on the students' established perspective on what it means to participate in math class.

One of the categories that emerged around this theme was the students' perception that it was important to calculate quickly. This finding relates back to the notion by Cobb et al. (2009) of disciplinary agency, i.e. being able to follow established methods in order to be considered mathematically competent. For instance, when Melanie was asked what it meant to be a good mathematics student, she replied:

> *When I know something, or if I have listened and understood it, then I can be fast and answer quickly.*

Other students also linked being a good math student to the speed at which answers are provided, knowing something or being able to understand the teacher's explanation. In Henrik's words: "*You have to be good at calculating fast*." Henrik further explained that, sometimes, it is best to know new concepts and answers before they are introduced by the teacher, as this gives an advantage in terms of answering quickly and correctly.

Another category indicated that the students rarely experienced links between the domain-specific activities and concepts from math class with experiences from their everyday domain. In reflecting on his experience of the intervention, Mads replied:

> *I didn't know that you could use the coordinate system in games or in Mine… [craft] or in reality.… No, I just thought it was a… a thing you had to learn.*

Earlier in the interview, Mads recalled that he had worked with the coordinate system before the Minecraft intervention. However, as the quote shows, he had not considered whether this mathematical concept had any relevance outside of the subject domain. From his perspective, what you do in mathematics is not necessarily useful in "reality". Rather, the coordinate system is just "a thing you had to learn", which suggests that the students did not see the purpose of what they had to learn. Thus, Mads did not anticipate that the content in math class was of relevance to his everyday life.

Summing up, the students' general perspective on mathematical classroom activities was that "math class" was an isolated domain and that the legitimate exercise of agency in everyday math class was disciplinary. The students experienced that in order to participate competently, they had to listen closely to what the teacher was saying and answer quickly, indicating that authority in the classroom was largely distributed to the teacher.

6.2 Learning mathematics through Minecraft

The second analytical theme concerned the *students' perspectives on learning mathematics through Minecraft*. The interviews involved several examples of students reflecting on how the *Minecraft* teaching unit created new perspectives on what constitutes mathematical knowledge. One example was Henrik's reflections on the intervention:

> Henrik*: It was the first, almost.*
> Interviewer*: It was the first?*
> Henrik*: Mm... approximately.*

> Interviewer: *Uhm… I'm just trying here to understand. The first what?*
> Henrik*: I could use, in all subjects, from mathematics.*
> Interviewer*: Ahh. It was the first [time] you have experienced in mathematics…*
> Henrik*: That you can use in all subjects. Yes.*

Henrik's experience of using the coordinate system in the intervention was a first in terms of understanding that mathematical concepts can be used in subjects other than mathematics. This indicates that he experienced that what he learnt in mathematics could have connections to the world. The use of the word "first" also suggests that he expected more connections to be made. The word "almost" indicates that such an experience might have been present, though not to any lasting degree, or that he, in some way, thought that mathematics should be related to the world but that he had not experienced it before. Henrik further reflected on how he switched his perspectives on mathematics in relation to the *Minecraft* teaching unit. He was asked whether he used mathematics in the teaching unit, and he replied that he did not see it as mathematics at first. The interviewer asked him to elaborate. He replied as follows:

> *Look… in the beginning, I thought the coordinate system was entirely mathematics, just something like, that you can just do in mathematics. But no, you can use it in all subjects. You can also use it in physical education and in umm… and arts class because there is a coordinate system on a picture, there and in the upper left and the upper right and all sorts of different things.*

Here, Henrik described how he switched his perspective from seeing the coordinate system as exclusively linked with math class to seeing possibilities for applying it in other subjects, thereby creating connections from the mathematics subject domain to a more general school setting. Henrik expressed conceptual agency by developing the meaning of the concept of the coordinate system, explaining that it is "*where you can be*", which refers to the notation of placement. Additionally, he made connections between the coordinate system and new possible areas of use, which is also a way of expressing conceptual agency across different domains.

Similarly, other students created new connections between using the coordinate system in *Minecraft* and the everyday domain in a more general sense. This can be illustrated in Maja's description of how she perceived the coordinate system:

> *it was new and you could… you could use it to find places. Like a compass but, where it says... a place… uhm. You can kind of call it an address of some sort.*

This quote shows that Maja was able to exercise conceptual agency in two ways. First, she described how the coordinate system could be meaningfully used to find places. Moreover, she used her own words to explain the meaning of coordinates in a system by creating connections to similar phenomena such as "compass" and "address". Both the description of what the coordinate system can be used for and the comparison with other concepts are indicative of conceptual agency. The connection made by Maya is not limited to the subject domain of mathematics; it is linked to the scenario of the teaching unit, which required the students to locate specific coordinates in order to create or overcome in-game challenges. In this way, she made linkages between using the coordinate system to find places in *Minecraft* and its use in the everyday domain by comparing coordinates to an address.

As the examples suggest, the students generally experienced a higher degree of conceptual agency when engaging with the mathematics in *Minecraft* than in their everyday math class. Moreover, they experienced several connections across the in-school and out-of-school domains, which they did not mention in relation to regular math class.

However, the realization that the coordinate system could be of relevance across several domains did not occur to all the students. One student, Melanie, did not think that the coordinate system was taught in normal math class at all and described math class as being only about numbers and calculations. Melanie reasoned that the teaching unit concerned mathematics primarily because it was taught by her mathematics teacher. As such, Melanie failed to make a connection between the game domain and the mathematical subject domain. Moreover, Melanie created a dichotomy between the domains by explaining that mathematics is about calculations, in contrast to *Minecraft*, which is a game. These examples show that making connections between the subject domain and other domains relied heavily on the students being able to alter their perception of mathematics as an already established subject domain.

6.3 How Minecraft is given new meaning through mathematics

During the analysis, it became clear that the teaching unit not only gave the students opportunities to develop new perspectives on mathematics. some of the students also valued the teaching unit for providing *perspectives that gave new meaning to playing Minecraft as a game*, as shown in the following example:

> Interviewer*: Have you learned anything new about the coordinate system that you didn't know before. Or can you do something now that you couldn't do before?*
> Hasan*: Yes. Well, I didn't know, even though I have been playing Minecraft a lot, I didn't know where the coordinate system was. Even though I pressed F3 and it [the coordinates] then it appeared, but I didn't know, I didn't know what it was. When I moved, it just changed a lot, and I turned it off again, and I didn't know what it was. But now, now I know what it is. Now I use it often in Minecraft.*
> Interviewer*: Okay, so you have used it afterwards after the course?*
> Hasan*: Yes, because if I have to find something that I have forgotten but I have the coordinates to, then I can just, then I can just go over to them.*

Hasan knew from playing the game at home that he could press F3 and prompt new and changeable information but was unable to translate the information into something meaningful. After learning what the numbers meant, he now frequently used them to locate objects in the game. In terms of domains, Hasan referenced his everyday activities in *Minecraft* (the game domain) to explain the usefulness of the coordinate system from the mathematical domain. As such, he created a strong connection between the different domains. His use of the coordinate system in *Minecraft* became a new way to interact with the game. If we expand the notion of agency in math class to understand the changes in how Hasan now played the game, then his statements validates the use of the coordinate system as a legitimate way of expressing what we could call player agency in *Minecraft* because it can be used to address the actual challenge in the game of how to locate objects by keeping track of their placement via coordinates.

The position of being a mathematical *Minecraft* player refers to the students' experiences of using the coordinate system to remember and find places in *Minecraft*. When the students experience that they can use the coordinate system in the game to solve the mathematical task given by their teacher in math class, they establish new connections between the two already established, though initially separated, domains, that is, mathematics and the everyday domain, which in turn transform both domains.

6.4 Collaborating around Minecraft in math class

The fourth and final analytical theme concerned the *students' perspectives on collaboration in math class in relation to Minecraft*. In addition to affecting the students' view of mathematics as a subject and their ability to navigate within *Minecraft*, the teaching unit also changed the students' perception of participating in math class. To exemplify, we shall focus on two students, Mads and Adam, who the teacher regarded as the highest-performing students in math class. Here, Mads explained that always being the first to finish tasks in regular math classes often put the two friends in an awkward position in relation to their classmates.

> Mads*: And then they [the other students] always get angry and say, you shouldn't always say "we are finished, we are finished"... But we understand why they say it*
> Interviewer*: Okay*
> Mads: *Because it is only... Most of the time, we are the only ones who finish first. It may never… they can't look forward to finishing faster than us.*

As the first theme showed, being mathematically competent in this classroom was highly dependent on speedy task-solving. However, when the same students repeatedly finished first, their peers found themselves in a situation where it was difficult for them to be regarded as mathematically competent. According to Mads and Adam, the other students felt like they had no chance of ever finishing first. Because of this, Mads and Adam experienced the other students' frustration and anger towards them.

These two students had little prior knowledge of *Minecraft* before the teaching unit. However, they had a very positive experience of the teaching unit because it had freed them from their original positions in math class as the fastest to solve tasks. Moreover, they were also able to learn about the game from their classmates, indicating a greater distribution of authority in the classroom. This is exemplified in the following excerpt:

> Adam*: But when we are working in pairs and we are together, then (Mads: Then they all get mad) sometimes. Because we are very fast and they have to keep up, but are slower, but in this [Minecraft unit], it was just fantastic.*
> Interviewer*: Yes, and how can it be that this was fantastic?*
> Adam*: Because. it. We usually don't play Minecraft, but the others know how to play. The others, who need some time to understand mathematics, they have played Minecraft, so they can react faster and know everything you have to do and everything you can build. We had to learn that from them.*
> Interviewer*: You had to learn from them?*
> Adam*: Yes, and then we could see how it is when we do math quickly and they have to keep up with us.*
> Mads*: But with this, with this Minecraft, then we were all equal (Adam: yes); nobody was better or worse.*
> Interviewer*: at playing Minecraft?*
> Mads*: At everything.*

Mads and Adam's positive experience of having to learn from the other students can be seen as a renegotiation of the authority in the classroom towards a more distributed model of teaching and learning than they would normally experience. Drawing on the other students' expertise with *Minecraft*, the game opens up a space for exercising conceptual agency, where Mads and Adam became the novices and had to learn from their classmates. Thus, the game created new connections for knowledge construction across the pedagogical and game domains.

Moreover, the two students concluded that the reversal of student authority and switch of perspectives proved beneficial to everyone. Valuable knowledge was not tied simply to the first to finish a mathematical task but to how all the students could collaborate about integrating knowledge across the various domains. This challenged Mads and Adam to understand knowledge about *Minecraft*, which became a valid and valuable aspect of competency in the intervention. The students who were skilled at mathematics were not necessarily the same students who were skilled at *Minecraft*, which means that the students' competences were redistributed across different domains. Mads and Adam's experience of being more on the same level as the other students released them from the normal classroom obligation of finishing tasks quickly. What they addressed with this change was not, however, that they were losing an opportunity to display mathematical competence towards the teacher; rather, to be free from this obligation was "fantastic" and was a shift toward positive identification. This underlines the fact that the focus on being the first to finish creates very narrow opportunities for students to live up to their classroom obligations.

Collaboration was far from the norm in the students' experience of everyday math class, where most work was done individually. They described that it was best "*to keep to themselves*" and only check results with the teacher, not with other students, which was regarded as cheating. Even though Mads and Adam had tried to play other learning games in mathematics, such as the mini-games in the Matfessor learning portal, their experience of game-based intervention was highly different. As Adam put it: "*in Matfessor, there is no need to talk to each other at all*", whereas working with *Minecraft* requires "*a lot about collaboration*", where they "*talk to each other all the time*". In summary, this final theme emphasizes how collaboration opens up to various ways of participation relating to the different ways of working with mathematical activities and helping each other navigate and build things in the game.

7. Discussion

The analysis showed how the intervention allowed the students to experience new perspectives as they transformed mathematical knowledge across the different domains of the school subject, the game map and their everyday game experiences. Table 1 presents a summary of our findings in the four analytical themes.

Table 1: Summary of analytical findings

Theme	Focus	Domain crossings	Findings
Participation as a student in "math class"	Students' experience of participating in their regular mathematical lessons	Links between the disciplinary domain and the pedagogical domain	The students stress the importance of being able to calculate and answer quickly and listening closely to the teacher. Mathematical knowledge is disconnected from the everyday domain Experience of mathematics as following established procedures (disciplinary agency)
Learning mathematics through Minecraft	Students' experience of a changed perspective on mathematics as a subject	Links between the disciplinary domain, the everyday domain and the scenario-based domain (game map)	The students describe how the coordinate system in and beyond the game represents (or does not represent) mathematical knowledge Different experiences of the coordinate system as a mathematical concept (conceptual agency)
Minecraft is given new meaning through mathematics	Students' experience of a changed perspective on Minecraft	Links between the scenario-based game domain and the disciplinary domain	The students are able to use their knowledge of the coordinate system as a new way to play Minecraft Experience of how the coordinate system is meaningfully linked to in-game actions (conceptual agency)
Collaboration in math class in relation to Minecraft	Students experience a changed perspective on participation in math class	Links between the pedagogical domain, the disciplinary domain and the scenario-based game domain	The students experience broader possibilities for participation, which involve more ways of being considered competent as well as more distributed authority between the teacher and students Working collaboratively opens up for the emergence of different understandings between the students of the coordinate system (conceptual agency)

Generally, the findings show how, compared with everyday math class, the students experienced a higher degree of freedom as they explored and solved tasks in the *Minecraft* map. This involved a broad distribution of authority, with several ways of seeking and getting help from other students. The game-based learning environment allowed new ways of participation and expression of agency, which, for some students, evened out the hierarchical playing field in their math class. The results also indicate shifts in what counts as legitimate mathematical agency in the classroom, as the intervention created new perspectives on how the mathematical concept of the coordinate system could be used in Minecraft and beyond. Thus, the students experienced several *ways of being competent* – i.e. within the domain of mathematics education, learning to navigate in the game domain and making connections relating to how mathematical knowledge could be relevant in other domains.

As mentioned earlier, much of the research on games in mathematics education is focused on using single-player learning games as somewhat instrumental tools for increasing student motivation and narrowly defined learning outcomes. Using the concepts of Cobb and colleagues (2009), learning games are often used in mathematics education to enforce disciplinary agency through skill and drill exercises, with lesser emphasis on students' conceptual agency. However, the findings of our study suggest that there are good reasons for exploring the use of open game worlds and inquiry-based teaching with games in the mathematics classroom – e.g. to develop students' conceptual understanding of the coordinate system. Based on our design-based approach and findings, we suggest three design principles that could inform future research on games in mathematics education.

The first design principle relates to how the students created meaningful experiences across in-school and out-of-school domains by collaboratively solving tasks in the *Minecraft* map. Based on these findings and borrowing from scenario-based domain theory (Hanghøj et al., 2018), we propose that the educational use of games in mathematics could benefit from designing and teaching with game worlds, where students *are given opportunities for meaningful agency through problem-solving, which relate to domains that go beyond the disciplinary domain of mathematics as a school subject.* Here, we assume that commercial games can be used to create valuable learning experiences in mathematics education, which relate to the students' everyday lifeworlds and challenge the current training paradigm for learning games.

The second design principle concerns the choice and use of specific commercial games for mathematics education. Computer games are, by definition, built on algorithms and an extensive use of mathematical concepts. However, the mathematical aspects are often hidden from the player in order to ensure immersion in the game (Lowrie and Jorgensen, 2015). Based on the results from this study and our other attempts at using commercial games in mathematics education (Hanghøj et al., 2018), we find it highly important *to select and teach with games that provide direct access to specific mathematical aspects that are relevant to the mathematics curriculum.* Access to recognizable mathematical activities in a game allows students and teachers to create their own experimental conditions for inquiry-based teaching. In this study, the students' inquiry of the coordinate system in a *Minecraft* map allowed them to explore a somewhat familiar game world from new perspectives and develop meaning that was valid across various domains. Consequently, the freedom to access and explore underlying mathematical game features is a key precondition for repositioning students from disciplinary agency towards conceptual agency.

The third and final design principle concerns the complex relationship between game-related mathematical tasks and the actual experience of playing a game. One of the core in-game challenges when playing *Minecraft* is navigating the game world in order to find specific locations and avoid being lost. Our findings suggest that the students could use the game not only to learn *about* the coordinate system but also to understand the concept of coordinates *by* exploring in-game goals, such as building railroads or finding each other. This enabled the students to value not only the ability to learn mathematics by exploring *Minecraft* but also to improve their navigational skills in the game. Several of the students were able to engage in what they deemed a significant game practice from their everyday domain in a mathematically substantial way. Based on these findings, our third proposed design principle is that educators using commercial games in mathematics education should be able *to facilitate teaching that links mathematical aspects in a game to in-game challenges that are meaningful to the players*. This recommendation is in line with the study of Habgood and Ainsworth (2011), which documented the increased value of using games for learning mathematics that are intrinsically motivating in contrast to games, which are mostly based on external rewards.

In summary, the intervention created several positive findings regarding the students' newfound perspectives on and transformation of mathematical experiences, which may inform the future use of games in mathematics education. However, there were also several limitations to our study, one of which was that the analysis was based on interview data and did not incorporate a detailed analysis of actual gameplay. It can be argued that the students' switching to new conceptual perspectives mainly emerged in the interviews as reflective responses to the interviewer and that they would not have realized these conceptualizations without the interview. In this way, the validity of the findings would have benefitted from comparing the interview data more closely with observational data from the students' classroom dialogues in situ. Another limitation is that our analysis did not focus on the role of the teacher in the game-based teaching unit, which is often emphasized as crucial to students' learning experiences in terms of facilitating dialogue before, during and after gameplay (Arnseth, Hanghøj and Silseth, 2018). As our findings show, the students valued relatively different mathematical aspects of the intervention – i.e. the ability to use mathematical concepts in a meaningful way, to improve navigation skills in the game or to collaborate and reposition the social hierarchy of the math class. Arguably, these aspects are all valuable in terms of facilitating engagement and improving learning outcomes. However, the wide range of experiences among the students also suggest that it may be highly demanding for teachers to identify students' individual experiences and provide relevant feedback through dialogue. As we mentioned in the analysis, not all the students were able to clearly identify the mathematical aspects of the teaching unit. Thus, the current study could have benefitted from a closer analysis of the dialogue between the teacher and students as well as between the teacher and us, as researchers, about the perceived possibilities and challenges of using *Minecraft* in mathematics education.

On a final note, we wish to suggest directions for future research methodologies for studying how games can be used to learn mathematics. Despite few convincing results, most researchers working within this field have mainly been interested in quantitative approaches, which measure whether students become more motivated or learn more with games than with other tools or instructional approaches. However, the diversity of student perspectives and experiences in our qualitative data analysis shows that there are good reasons to be critical of instrumental approaches to games and learning, which is currently the dominant research paradigm. Our study underlines the importance of moving away from narrow measurements and asking questions such as *how* and, in particular, *what* are students learning differently when they use games in education. There is a need for researchers to explore the ways in which mathematical concepts become meaningful to students in relation to game-based learning. One future focus for in-depth studies of mathematical learning processes with games could be how dialogical reasoning occurs in students' interaction with games. As we have shown, *Minecraft* can potentially be used to positively affect students' perception of mathematical activities and development of mathematical interest. Moreover, had the game been an instrumental learning tool, it would have been impossible for the students to engage with the learning scenario in a variety of ways or for each of them to represent different meaningful ways of participation. Therefore, researchers working with games and learning should not only focus on students' ability to achieve mathematical standards or future demands. Instead, we should also pay more attention to how students' everyday experiences with games may relate to mathematical concepts and create multiple gateways for participating in and understanding mathematics education as a meaningful practice.

8. Conclusion

The objective of the study was to investigate how *Minecraft* could be used in a teaching unit to engage students in mathematics education through different forms of participation and how students experience new perspectives on mathematical knowledge across in-school and out-of-school domains. The study showed that the *Minecraft* intervention created new perspectives in the students' experience of mathematical knowledge, which they related to different in-school and out-of-school domains. The teaching unit clearly marked a change from the students' everyday experience of "doing mathematics", which they mainly described as the speedy solving of procedural tasks. Our analysis suggests that one of the main reasons for this successful recontextualization of knowledge between the game and school domains was that the students were actually able to use their knowledge of the coordinate system inside *Minecraft* as part of the teaching unit in terms of navigating the game space and overcoming meaningful challenges. Moreover, the analysis shows how the intervention motivated shifts in social relations, conceptual agency and perceptions of competence. As the findings indicate, the students were offered several possibilities for participation, which were sometimes related to understanding mathematics and other times to mastering the game. These findings stand in sharp contrast to the students' everyday experience of complying with classroom norms, e.g. listening to the teacher, following closely what happens on the blackboard and providing answers quickly. We should be careful with generalizing these findings based only on one classroom intervention. Nevertheless, we still believe that these findings call for more in-depth studies of the use of commercial games in mathematics education, where students can use mathematical knowledge to overcome meaningful challenges.

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