THE EFFECT ON STUDENTS' ARITHMETIC SKILLS OF TEACHING TWO DIFFERENTLY STRUCTURED CALCULATION METHODS

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Abstract

Mastering traditional algorithms has formed mathematics teaching in primary education. Educational reforms have emphasized variation and creativity in teaching and using computational strategies. These changes have recently been criticized for lack of empirical support. This research examines the effect of teaching two differently structured written calculation methods on teaching arithmetic skills (addition) in grade 2 in Sweden with respect to students' procedural, conceptual and factual knowledge. A total of 390 students (188 females, 179 males, gender not indicated for 23) were included. The students attended 20 classes in grade 2 and were randomly assigned to one of two methods. During the intervention, students who were taught and had practiced traditional algorithms developed their arithmetic skills significantly more than students who worked with the decomposition method with respect to procedural knowledge and factual knowledge. These results provided no evidence that the development of students' conceptual knowledge would benefit more from learning the decomposition method compared to traditional algorithm.
Keywords: *arithmetic skills, decomposition method, intervention study, mathematics education, traditional algorithm, written calculation.*

Introduction

A central theme in mathematics is arithmetic; a key part in school mathematics is therefore how to teach and learn arithmetic. The acquisition of arithmetic proficiency requires the development of sufficient conceptual, procedural and factual knowledge (Baroody & Dowker, 2003; Delazer, 2003; Dowker, 2005; Geary, 1993; Goldman, Hasselbring et al., 1997; Kilpatrick, Swafford & Findell, 2001). *Conceptual knowledge* denotes the understanding of important mathematical concepts such as the base-ten number system and the relationships within and between arithmetic operations (Dowker, 2005; Goldman et al., 1997; Hiebert & Lefevre, 1986; Kilpatrick et al., 2001). *Procedural knowledge* and *skills* refer to knowledge of calculation strategies (Dowker, 2005; Jordan, Hanich & Uberti, 2003; Kilpatrick et al., 2001; Siegler, 1988). Finally, *factual knowledge*, refers to the semantic memory representations concerning the connections between problems and answers to simple arithmetic problems stored in long-term memory (Ashcraft, 1992; Geary, 1993). A student with factual knowledge can manage simple arithmetic problems effortlessly. It has also been stated that multi-digit arithmetic is directly dependent on a conceptual understanding of place value, the base-ten number system, and the relationships within and between arithmetic operations (Hiebert & Wearne, 1996). The present research examined the effect of teaching two differently structured written calculation methods on teaching arithmetic skills (addition) in grade 2 in Sweden with respect to students' procedural, conceptual and factual knowledge.

Traditional Algorithm for Addition and Decomposition Method

For many decades, the dominant aspect in school mathematics was the traditional algorithms for multi-digit addition, subtraction, multiplication and division (Clarke, 2004). In this paper, an algorithm is defined in accordance with Usiskin's (1998) frequently used explanation: "a finite, step-by-step procedure for accomplishing a task that we wish to complete" (p.7). The traditional algorithm we refer to represents one of the two written calculation methods that was used in this present research. It is a vertical algorithm in which the calculation is performed step by step, as illustrated by an example of the traditional algorithm for addition as taught in Sweden (Figure 1). The calculation starts with adding the digits in the ones column, at far right, followed by adding the digits in the tens column and so on.

Figure 1
Traditional written algorithm for multi-digit addition as used in Swedish classrooms

	2	5	3
+	1	2	6
	3	7	9

Besides the traditional algorithms for multi-digit calculation, there are several other written methods that usually originate from mental computational methods in which it has become necessary to jot down notes during the calculation process instead of keeping the numbers in memory (see e.g. Norton, 2012). In Swedish school mathematics, these are often referred to as methods for "written mental calculation" (Rockström, 1991). One such method is the decomposition method, which like the traditional algorithm is building on the idea of the base- ten system (Buys, 2008; Fuson et al., 1997; Verschaffel et al., 2007). The calculation is accomplished by splitting off the hundreds, tens, and units in both integers and then adding them separately, always starting with the highest unit (e.g., 153 + 241 = __; 100 + 200 = 300, 50 + 40 = 90, 3 + 1= 4, 300 + 90 + 4=394). This procedure is usually handled in accordance with Usiskin's (1998) definition of an algorithm. Therefore, we here define the written decomposition method as a horizontal algorithm.

Discussion about Algorithms in School Mathematics

Mastering the traditional algorithms has long formed the core of mathematics teaching, although there have been critical voices. Colburn, one of the early critics, was advocating new ways in teaching arithmetic as early as the 19th century to call attention not only to procedural knowledge but also to conceptual understanding. Colburn (1830, quoted in Monroe & Colburn, 1912, p. 476) claimed that the students would benefit more from using their own strategies instead of being told what to do.

Later researchers (e.g. Marshall, 2003; Plunkett, 1979) have criticized traditional algorithms as outmoded. Additionally, Marshall (2003) argued – similarly to Colburn, although almost 200 years later – that "rote memorization has to be out, and teaching for understanding has to be in" (Marshall, 2003, p. 194). Some researchers (e.g. Carpenter et al., 1998; Kamii & Dominick, 1997) have advocated that students should invent their own strategies, or at least that learning written calculations should start with students' informal strategies. In addition, Carpenter et al. (1998) stated that a strong justification could be made for not teaching the algorithm at all, though they did not go as far as Kamii and Dominick, who claimed that algorithms should not be taught at all since they "hinder children's development of number sense" (Kamii & Dominick, 1997, p. 51). As can be noticed, these authors blame the algorithms as being the actual problem when students' conceptual knowledge seems to become invisible. On the other hand, they did not express anything about the teaching of algorithms.

In addition, the dominance of traditional algorithm in primary mathematics has met with other critical arguments. Plunkett (1979) claimed that the algorithms should be discarded, not least because they cause "frustration, unhappiness and a deteriorating attitude to mathematics" (Plunkett, 1979, p.4). Others have criticized the teaching of traditional algorithms simply because many children fail to master them (Anghileri et al., 2002). The presence of systematic errors in students' application of algorithms is well-known (Brown & VanLehn, 1980; Fuson, 1990b; Träff & Samuelsson, 2013) and sometimes justified as a result of students relying solely on rote manipulation of symbols (Fuson, 1992).

Other researchers have argued that the knowledge towards which the teaching is directed is not an effect of which calculation method the students will learn (Ma, 1999). More important is how the teacher talks and directs the students' attention. Ma (1999) showed that teachers in the United States and China who taught the same traditional algorithms differed with respect to what knowledge (conceptual or procedural) they displayed in their teaching.

The criticism of traditional algorithms has paved the way for reforms emphasizing variation and creativity in teaching and using computational strategies across the western world (Torbeyns & Verschaffel, 2016). In many countries (e.g. the Netherlands, Germany, the UK and Australia), pedagogical reforms have strongly supported the acquisition of informal, computational strategies in primary mathematics for achievers on all levels (Anghileri, 2004; Norton, 2012). Prevalent in these reform documents was a basic belief in the feasibility and value of striving for variety, flexibility and adaptivity in strategies, which means an approach where conceptual knowledge becomes more visible than it does in the teaching of traditional algorithms (Anghileri, 2004). Researchers have pointed out that in these reform-based documents this viewpoint is sometimes expressed as strong pleas for teaching children mental computation strategies before and alongside the standard algorithm (Beishuizen & Anghileri, 1998; Norton, 2012).

Despite the critique, there are numerous reasons why the traditional algorithm is so frequently used in primary classrooms. Plunkett (1979), Thompson (1997) and Usiskin (1998) provide several examples: the algorithms are powerful tools when solving problems involving calculation with many numbers, they are automatic, they can be instructive, and they provide a written record of the students' solution, which enables teachers to locate errors.

According to Torbeyns and Verschaffel, (2016) and Verschaffel, Luwel, Torbeyns, and Doreen, (2009), the idea of promoting strategy variety and flexibility as a reasonable and valuable goal on all levels of mathematical achievement is an assumption which is less grounded in forceful evidence-based research than in rhetoric. The lack of basis in research is also the overall impression from Brown's (2010) historical overview of the developments in the teaching of number skills in the English National Curriculum. Similarly, Norton (2012) made a remark as concerns the lack of empirical support behind the shift regarding mental computation as an activity performed inside one's head instead of on paper. Further, Thompson (2010) has presented a critical look at the English educational view of written calculation. It is noteworthy that he also complained about the National Numeracy Strategy not setting up a research project in the early stages of its development to ascertain which of the written algorithms incorporated into the framework were the most "child-friendly". Although our research only touches upon two methods of written calculation, Thompson (2010) gives additional reasons for investigating which method is the one most suitable for students to learn in early primary mathematics.

As mentioned earlier, the different views on written calculation are also a matter of contrasting teaching approaches, e. g. reform-based teaching vs explicit teaching (Anghileri, Beishuizen & van Putten, 2002). Other researchers (e. g. Rittle-Johnson, Schneider & Star, 2015) have been discussing whether the teaching should support the development of conceptual knowledge or procedural knowledge and in which order this teaching would occur to get the best effect on the students' learning.

The changed perspective on teaching and learning multi-digit computation has also been evident in Swedish primary-school mathematics education, and the traditional algorithms have been gradually abandoned in favour of written mental calculation methods, , Nowadays, traditional vertical algorithms for multi-digit calculation, as well as other written calculation methods – mainly represented by the decomposition method – are common in Swedish students' textbooks (Johansson, 2011). Depending on the school or on the individual teacher, the traditional algorithms are usually introduced in the second or third grade. According to the curriculum in mathematics (Skolverket, 2018), Swedish teachers are supposed to pay attention to both conceptual and procedural knowledge, independently of which calculation method that is taught. Nevertheless, a central issue is which written calculation method is the most appropriate to introduce in early primary mathematics.

Problem Statement

The examples from research illustrate that the issue about computational methods in school mathematics has long been a subject of discussion although mainly in regard to differences between informal and formal methods or different teaching approaches. The issue concerning computation methods in school mathematics is important for mathematics teachers in their practice. Yet, there seems to be very few studies focusing on the use of the same teaching method for the teaching of two different calculation methods, which is the case in this present study. Therefore, this research aimed to answer a question, with respect to what calculation method teachers should teach, the traditional algorithm or the decomposition method.

Research Focus

This research focuses on the effect of teaching two differently structured written calculation methods - traditional algorithm and decomposition – on children's arithmetic skills (procedural, conceptual and factual knowledge) with respect to addition in grade 2 (age 8 years) in Sweden. The same teaching method, explicit teaching, is used regardless of which written computation method that is taught. The research was based on the following research questions:

(1) To what extent do students develop procedural knowledge, with respect to arithmetic, when they are taught traditional algorithm or decomposition method?
(2) To what extent do students develop conceptual knowledge, with respect to arithmetic when they are taught traditional algorithm or decomposition method?
(3) To what extent do students develop factual knowledge, when they are taught traditional algorithm or decomposition method?

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 78, No. 2, 2020

Research Methodology

Research Design

In the following table, the design of this research is clarified with respect to an independent variable (teaching method used for teaching two calculation methods) and a dependent variable (arithmetic skills).

Table 1
Research design with respect to independent and dependent variables

Teaching method used	Explicit instruction making conceptual understanding visible	
Calculation methods taught	Traditional algorithm (TA)	Decomposition method (DC)
Arithmetic skills learned	Procedural, conceptual and factual knowledge	

Participants

The participants were recruited from two cities. Principals and teachers decided of participation in the project. All the caregivers were informed that the study was a part of the ordinary teaching of the children and gave consent to the use of the data in research.

A total of 390 students attending 20 classes in grade 2 were included in this study. There were 188 female students and 179 male students (indication of gender for 23 of the students was missing). All students were at the individual level randomly assigned one of the two calculation methods. During the intervention period, all students were taught in teaching groups other than the regular groups. Some students had their ordinary teacher while others had another teacher at the same school; 192 students were taught the traditional algorithm (the TA-group) and 197 students were taught the decomposition method (the DC-group). All teachers involved in the teaching procedure were experienced teachers who had taught both calculation methods.

Interventions

The intervention period was 3 weeks (in March 2016) and contained of 12 lessons of 30 minutes each. Two researchers and four teachers planned all lessons. Four main activities that could occur in a lesson were identified:

(a) Repetition (The teacher gives feedback and listens to how the students have understood earlier presented content.)
(b) Instruction (New content is presented. This could be done on the chalkboard, using manipulatives, or with other representations.)
(c) Practice (Students practice their skills.)
(d) Review and closing (The teacher summarizes what the students have practiced and what they would do next time.)

Both interventions (the TA-intervention and the DC-intervention) were planned to be as equal as possible with the intention of making conceptual understanding visible. The intention was that the only difference between affordances occurring in the lesson concerned the calculation methods. All lesson plans contained the following headlines: Activities, Time, Material and Language. Activities were related to main activities (A) Repetition, (B) Instruction, (C) Students' independent work/practice and (D) Review and closing, as presented above. The heading Time described how long the specific activity should last; Material described the materials needed in the activity. Finally, Language described how the teacher would speak in order to make conceptual understanding visible. Table 2 illustrates an example of a Lesson Plan (lesson number 2 of 12).

Table 2
Lesson plan example

Activities	Time	Material	Language
Ask your students to say a number between 1 and 9999. Write four of these examples on the board. Ask/discuss with the students which units of place value (ones, tens…) that are represented in each of the four numbers. Illustrate with base ten blocks and write the units on the board.	5 min	Magnetic base ten blocks for board/ base ten blocks for whiteboard	Besides using the notion of ones, tens, and hundreds etc. we also talk about the value, for example, of. tens, and that one ten is the same as ten. How much is then five tens? Etc.
Press 10+8= on the calculator on the board and ask your students: If we have 10 and add 8, where do 1 and 0 and 8 go? Why do we write the number like this? Discuss with the class, especially the function of zero to mark an empty space for a unit. Give some more examples to discuss, 90+6, 105+3, 207+30.	10 min	Calculator for demonstration in class/on smartboard	E.g. What happens with the (tens and ones) units in the number 90 when we add 6 (add 6 six ones)?
The students work together (in pairs). Each pair get five cards with 2-4 -digit numbers written in word form. Ask the students to make the numbers in numeric form, using the place value cards and to represent each number by place ten blocks.	10 min	Whole number place value cards and base ten blocks for each pair of students.	
Summarize the workshop. Write an example on the board, like one of the previous in this lesson. Ask the students to describe what they have learnt or what they have been practicing.	5 min	Whiteboard	40 + 6 And why do we write the sum as 46?

Once the researchers and the four teachers had planned all the lessons, they met with all the participating teachers and discussed the lesson plans in order to calibrate and thereby improve reliability among all teaching groups. All teachers thought the lessons were reasonable, but some of the teachers were concerned that the pace might be too fast for some of their students. The following twelve lessons were planned (Table 3).

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 78, No. 2, 2020

Table 3
Content and objectives in each lesson

The mathematical content	Learning goal
Ones, tens, hundreds, thousands	Understand Place value
Ones, tens, hundreds, thousands	Understand Place value
Addition: two-digit numbers without regrouping	Understand the procedure Carry out the calculations Communication skills
Addition: one- and two-digit numbers without regrouping	Understand the procedure Carry out the calculations Communication skills
Addition: one- and two-digit numbers with regrouping	Understand the procedure Carry out the calculations Communication skills
Addition: one- and two-digit numbers with regrouping (practice)/ skill training	Understand the procedure Carry out the calculations
Addition: Mixed one- to four-digit numbers without regrouping	Understand the procedure Carry out the calculations Communication skills
Addition: Mixed one- to four-digit numbers without regrouping (practice)/skill training	Understand the procedure Carry out the calculations
Addition: Three-digit numbers with regrouping, ones to tens and tens to hundreds, only one per item (practice)/skill training	Understand the procedure Carry out the calculations
Addition: Three-digit numbers with regrouping, ones to tens and tens to hundreds, only one per item. Focus on the digit 0 and place value.	Understand the procedure Carry out the calculations Communication skills
Addition: Four-digit numbers with more than one regrouping	Understand the procedure Carry out the calculations Communication skills
Addition: Four-digit numbers with more than one regrouping (practice) /skill training	Understand the procedure Carry out the calculations

To make teaching as equal as possible in all groups teaching the same calculation method several activities have occurred, a) we made lesson plans for each lesson where we described how the teacher would talk to make conceptual understanding visible, b) we encouraged the teachers to discuss the lesson with each other before and after the lesson in order to calibrate, c) teachers and researchers have discussed the lesson plans in order to avoid misunderstanding in the instruction, d) the teachers have estimated to what extent in per cent they followed the lesson plan after each lesson. Fidelity to the lesson plans was estimated by all teachers as between 85 and 95 per cent for each lesson. Most estimates were at the 95 per cent level. When the teachers did not follow the lesson plan exactly and estimated 85 per cent accordance, something extra ordinary had occurred during the lesson, for instance, two students irritated each other, and the teacher needed to help them solve the problems. When teachers estimated 95 per cent accordance, they didn't exactly follow what they were supposed to say.

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Test Procedure and Measurement of Arithmetic Skills

Four different tests were distributed: a speed test, a factual knowledge test, a conceptual knowledge test, and a procedural knowledge test. The students conducted all the tests at four different times, before the intervention (week 0), in the middle of the intervention (week 1,5), just after the intervention (week 3), and at a later follow-up test session, four weeks after the intervention (week 7).

The following measurements of arithmetic skills were used (see Appendix 1 for complete test battery).

Speed. The task was to copy in writing as many numbers, both single- and double-digit numbers, as possible in 1 minute. The purpose of the speed test was to control for the writing speed of the students. The average test-retest correlation over the four measurement points was in the DM-group $\bar{r}$ = .63 and in the TA-group $\bar{r}$ = .65.

Factual knowledge. The task was to answer as many one-digit addition problems, or number combinations, as possible in one minute; the total correct score was used as the dependent variable. The average test-retest correlation over the four measurement points was $\bar{r}$ = .59 in the DM-group and in the TA-group $\bar{r}$ = .79.

Conceptual knowledge. The test contained 22 tasks (number pattern, compose multi-digit numbers from ones, tens and hundreds, place value tasks); one task was to select which mathematical expression that best fit the presented simple word problem, and four of the tasks were number line estimations. The average test-retest correlation over the four measurement points was in the DM-group $\bar{r}$ = .79 and in the TA-group $\bar{r}$ = .81.

Procedural knowledge. The test contained 30 multi-digit addition tasks from adding a one-digit to a two-digit number to adding a four-digit to another four-digit number. All the numbers were expressed horizontally (e.g. 26+84 =). The average test-retest correlation over the four measurement points was in the DM-group $\bar{r}$ = .61 and in the TA-group $\bar{r}$ = .60.

Analysis

A mixed model approach with a piecewise model was used to analyse the result and an intervention phase with three time points 0, 1.5, and 3 weeks. The follow-up phase was two time points, week 3 (which was the same as the third point above, just after the intervention) and week 7 (four weeks after the intervention). The statistical software used was SPSS 23. Furthermore, an autoregressive covariance structure (AR1H), which allowed the variance to differ at each time point at level one (the effect of time, repeated measurement) was used and an unstructured covariance structure at level two (the time effect is nested within each individual). One school was excluded from the analysis due to lack of randomization. Potential outliers were not excluded from the analysis; there were, however, few such cases, and no score was out of bounds from the min-max value of the different tests. Anything missing was assumed to be unrelated to the different outcomes, fulfilling the assumption of missing at random. The analysis approach was straightforward: we wanted to test the interaction between group and growth on four different outcomes, and to be able to model possible natural heterogeneity (random intercept and slopes). The effect sizes of the significant fixed interaction effect were calculated according to the formula presented in Feingold (2013):

$$d = \frac{(time * group) * no\ of\ timepoints}{SD\ raw\ value\ of\ controlgroup}$$

Research Results

Results based on descriptive statistics are presented in Table 4, which shows the means and standard deviations of the different groups and time points for the four dependent measures (procedural, conceptual and factual knowledge and speed). Table 5 displays the results of a mixed model analysis of the different dependent measures.

Table 4
Descriptive statistics (Means and Standard Deviation) divided over test and time points

Group	Test	Week 0 *M(SD)*	Week 1.5 *M(SD)*	Week 3 *M(SD)*	Week 7 *M(SD)*
DM	Procedural knowledge	7.49 (3.78) *n*= 179	7.97 (4.08) *n*= 131	8.60 (4.29) *n*= 179	9.03 (4.39) *n*= 148
	Conceptual knowledge	14.55 (5.66) *n*= 186	16.76 (6.06) *n*= 132	18.47 (5.22) *n*= 183	19.26 (5.54) *n*= 172
	Factual knowledge	20.70 (6.91) *n*= 186	24.14 (9.35) *n*= 132	24.91 (8.88) *n*= 186	24.66 (7.50) *n*= 171
	Speed	38.91 (13.49) *n*= 185	47.85 (15.63) *n*= 134	49.79 (13.96) *n*= 182	48.97 (11.28) *n*= 173
TA	Procedural knowledge	7.25 (3.91) *n*= 179	9.44 (4.51) *n*= 126	12.23 (4.69) *n*= 176	12.21 (5.42) *n*= 163
	Conceptual knowledge	14.28 (5.56) *n*= 179	16.84 (6.00) *n*= 129	18.52 (5.08) *n*= 178	19.32 (5.30) *n*= 173
	Factual knowledge	19.22 (7.43) *n*= 178	23.22 (8.00) *n*= 127	26.37 (9.33) *n*= 174	25.42 (7.87) *n*= 174
	Speed	36.66 (13.36) *n*= 177	43.28 (15.46) *n*= 127	46.89 (14.42) *n*= 175	48.13 (13.18) *n*= 173

Note. DM = Decomposition method, TA = Traditional algorithm

Table 5
Results of a mixed model analysis of the different dependent measures

	Procedural knowledge Estimate (SE)	Conceptual knowledge Estimate (SE)	Factual knowledge Estimate (SE)	Speed Estimate (SE)
Fixed effects				
Intercept	7.41 (0.31), *p* < .001	14.56 (0.41), *p* < .001	20.71 (0.53), *p* < .001	39.09 (0.98), *p* < .001
Intervention phase	0.35 (0.10), *p* < .001	1.27 (0.09), *p* < .001	1.43 (0.15), *p* < .001	3.66 (0.31), *p* < .001
Follow up phase	0.41 (0.32), *p* = .205	0.81 (0.12), *p* = .001	-0.50 (0.47), *p* = .290	-1.82 (0.77), *p* = .019
Group	-0.26 (0.44), *p* = .548	-0.16 (0.58), *p* = .784	-1.50 (0.76), *p* = .050	-2.71 (1.39), *p* = .053
Intervention phase x group	1.33 (0.14), *p* < .001	0.14 (0.12), *p* = .247	0.98 (0.22), *p* < .001	0.56 (0.56), *p* = .211
Follow up phase x group	-0.47 (0.45), *p* = .296	-0.12 (0.33), *p* = .705	-0.58 (0.67), *p* = .386	0.46 (1.10), *p* = .677
Random effects				
Intercept	10.56 (1.12), *p* < .001	27.24 (2.62), *p* < .001	42.66 (3.58), *p* < .001	85.53 (12.58), *p* < .001
Covariance intercept slope		-1.12 (0.56), *p* = .046		
Slope		0.39 (0.21), *p* = .063		

Note. Group coding was as follows: Decomposition method (DM) as 0 and Traditional algorithm (TA) as 1. Only conceptual knowledge had a random slope.

The result of the mixed model analysis showed statistically significant interaction effects with respect to procedural knowledge and factual knowledge. On the procedural knowledge measure the TA group gained 1.33 points more every 1.5 week of the intervention phase than the DM group (Figure 2); the effect size was d = 0.7. The factual knowledge test showed that the TA group gained .98 points more every 1.5 week of the intervention than the DM group (Figure 3); the effect size was d = 0.28. Hence, the calculation method that was taught had effect on both procedural knowledge and factual knowledge. No interaction effect was detected between group and time on the conceptual knowledge test or the speed test.

Figure 2
Development of procedural knowledge in DM and TA groups

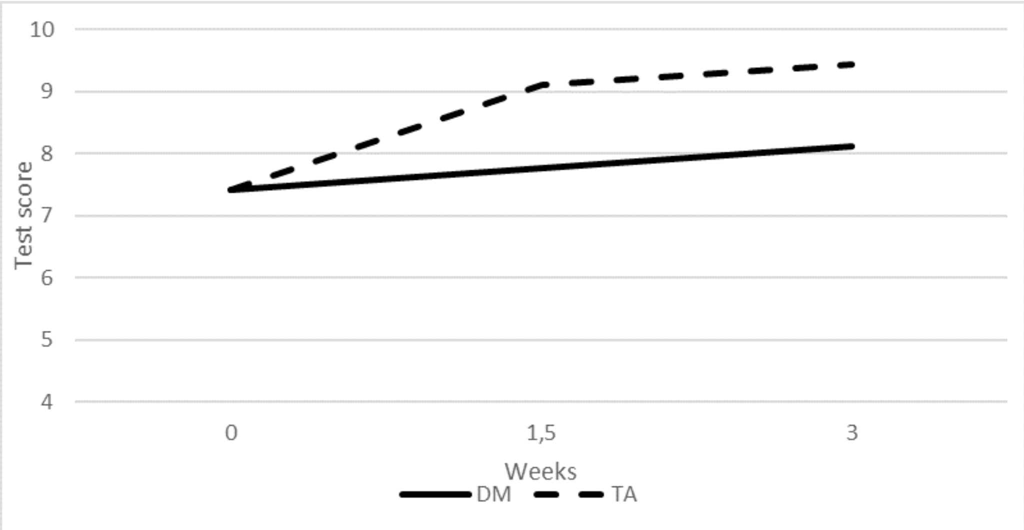
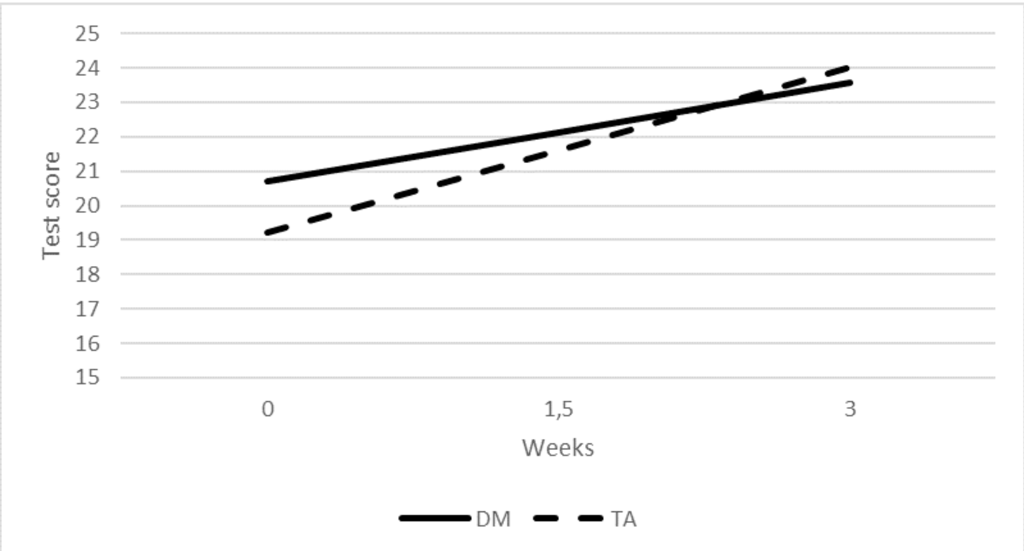


Figure 3
Development of factual knowledge in DM and TA groups



Note. The scales for test score are not the same in both diagrams.

The results also demonstrated that the conceptual knowledge test seemed to have heterogeneity between individuals and how they developed over time. A statistically significant random slope indicates that the growth in conceptual knowledge is different between individuals. All other models had only a random intercept, meaning that the participants within each group had different starting values when the intervention began. Examination of the follow-up phase reveals that students' conceptual knowledge and speed continued to develop positively and that these effects were also statistically significant (see Table 5). No interaction with group was detected in the follow-up phase, which indicates that the two groups did not differ in development after the intervention ended.

Discussion

In many countries (e.g. the Netherlands, Germany, the UK, Australia, and Sweden), pedagogical reforms have strongly supported the acquisition of informal, computational strategies in primary mathematics (Anghileri, 2004; Norton, 2012). One problem with these reforms is that they have not been grounded in evidence-based research (Brown, 2010; Torbeyns & Verschaffel, 2016; Verschaffel, Luwel, Torbeyns & Doreen, 2009). From a Swedish perspective, the traditional algorithms have been gradually abandoned in favour of written mental calculation methods. For several years, decomposition method for mental written computation was the predominant method in Swedish primary mathematics textbooks.

Which computation method students should learn in the early years has also been a subject of discussion in several studies (e.g. Thompson, 2010). One problem with these studies is that arguments for and against different methods are grounded less in forceful evidence-based research than in rhetoric (Torbeyns & Verschaffel, 2016; Verschaffel, Luwel, Torbeyns & Doreen, 2009). In an attempt to fill a small part of this gap in the body of literature and thus answer an important question for the mathematics-teacher profession, we examined the effect of teaching two differently structured calculation methods – traditional algorithm and decomposition – on children's arithmetic skills (procedural, conceptual and factual knowledge) with respect to addition.

The results from this research strengthen earlier discussions (Plunkett, 1979; Thompson, 1997; Usiskin, 1998), in which the authors argued that traditional algorithms are a powerful tool for solving calculation problems. During the intervention period, students who were taught and practiced traditional algorithms developed their arithmetic skills significantly more, with respect to procedural knowledge and factual knowledge, than students who worked with decomposition method. One explanation to why procedural and factual knowledge was developed more in the traditional (TA) group could be the effectiveness of the traditional algorithm (c.f. Usiskin, 1998). The efficiency of the traditional algorithm, maybe, made the students process more tasks and thereby practiced their procedural and factual knowledge more than the students in the decomposition (DM) group.

No effect was found, confirming the notion that learning the traditional algorithm means less support for the development of conceptual knowledge compared to learning a calculation method that focuses on mental calculation. When improving teaching for conceptual understanding, something different than changing the algorithm needs to be done. The design of this research, a randomized field experiment, with a large sample, made this null-finding tenable (cf. Kamii & Dominick, 1997). Several researchers claim that conceptual knowledge becomes more visible if other computational methods than traditional algorithms are used (e.g. Anghileri, 2004). In this study, teachers had to display conceptual knowledge irrespective of the calculation method they were teaching. How teachers talk and direct the students' attention has greater impact on their learning of conceptual knowledge than the calculation method the teacher teaches. This interpretation is similar to the discussion by Ma (1999), who showed that the knowledge the teaching is directed towards, is not an effect of the calculation method taught to the students, but rather how the teacher talks and directs the students' attention. Thus, teaching decomposition method instead of traditional algorithm does not make conceptual knowledge more visible.

Limitations

No study is perfectly designed or without flaws that may limit its interpretations. One limitation in our study is the estimation of fidelity made by the teachers, which is not a very strong measure. To assess whether the intervention procedures were carried out as designed, we could have video-recorded lessons for each of the instructors and analysed whether the lessons were carried out according to the design.

Conclusions

This research focused on development of procedural, conceptual and factual knowledge with respect to addition. The results provide evidence that explicit instruction, where teachers make conceptual understanding visible in teaching the traditional algorithm, has greater effect on students' procedural knowledge and factual knowledge, than explicit instruction where teachers make conceptual understanding visible in teaching the decomposition method. The answer to the research question, "what calculation method teachers should teach?", is that teachers should teach students the traditional algorithm. With respect to this result further research should investigate why the traditional algorithm is superior. Further research also needs to examine the effects of teaching two differently structured calculation methods – traditional algorithm and decomposition – on children's arithmetic skills, for instance, by replicating the present study. Additionally, more studies comparing students' learning of arithmetic when they are taught different calculation methods are needed. Otherwise, teachers will be left to rhetorical discussions instead of support from forceful evidence-based research for their decisions on teaching arithmetic.

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Appendix 1

This appendix consists of the four tests that have been used in the study.
The test labelled Conceptual knowledge originates mainly from test 2 and 3 in McIntosh (2008).

SPEED

Your name:

Write the same number in the box!

4	1	6	7
9	5	2	2
8	3	3	3
5	9	4	9
1	4	7	1
3	6	8	3
6	3	9	8
7	8	1	4
8	1	3	5
2	7	8	9

5	16	23	54
12	13	29	46
56	4	2	71
8	10	94	33
25	14	72	64
38	17	26	92
9	19	43	41
65	13	7	77
23	41	83	12
57	21	44	11

FACTUAL KNOWLEDGE

Your name:			
1+8=	1+1=	2+7=	2+1=
1+7=	3+1=	3+6=	4+1=
2+6=	5+1=	1+6=	6+1=
4+5=	7+1=	3+5=	8+1=
1+5=	1+2=	2+2=	5+4=
3+2=	4+4=	4+2=	3+4=
5+3=	3+3=	6+2=	5+2=
7+2=	4+3=	2+3=	2+4=
1+3=	6+3=	1+4=	2+5=
1+6=	2+2=	6+3=	7+1=
5+4=	2+5=	7+2=	4+2=
4+4=	6+2=	3+5=	1+1=
8+1=	1+7=	4+3=	1+8=
3+3=	2+6=	1+5=	5+3=
1+3=	3+4=	1+2=	1+4=
6+1=	4+1=	3+2=	2+1=
2+4=	5+2=	3+1=	5+1=
2+3=	3+6=	2+7=	4+5=
7+1=	2+2	6+3=	1+6
4+2=	7+2=	5+4=	5+1=

CONCEPTUAL KNOWLEDGE

Your name:____

1 Continue the number pattern.

24, 25, 26, ____, ____, ____

2 Continue the number pattern.

3, 5, 7,____, ____, ____

3 Underline the ones digit 534

4 Underline the hundreds digit 7569

5 Continue the number pattern.

72, 74, 76, ____, ____, ____

6 Continue the number pattern.

96, 97, 98, ____, ____, ____

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PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 78, No. 2, 2020

CONCEPTUAL KNOWLEDGE

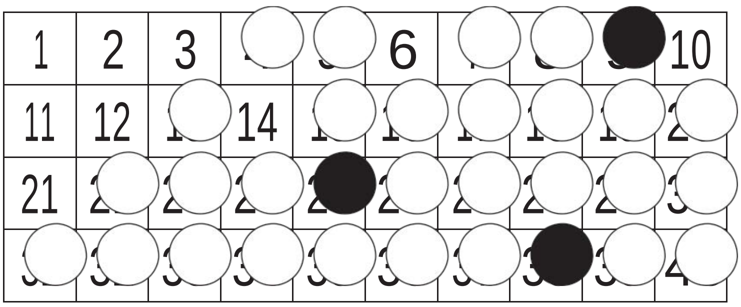
7 Calculate

100+60+5=__________

8 Calculate

200+10+9=__________

9



The numbers are placed in order, but some hide behind black dots.
Which numbers are behind the three black dots?

_____ _____ _____

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CONCEPTUAL KNOWLEDGE

10 Make a circle around the smallest number.

Underline the largest number.

14 62 9 35

11 Use each of the digits 9, 4 and 1 and write a number as small as possible. ________________

12 Use each of the digits 9, 4 och 1 and write a number as close to 200 as possible.______________

13 Use each of the digits 9, 4 och 1 and write a number as large as possible. ______________

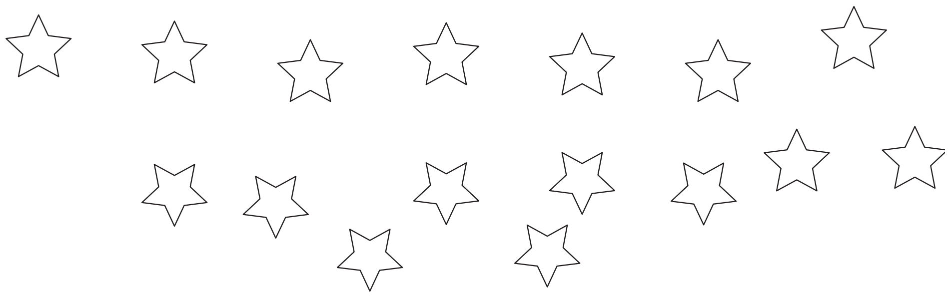
14 Make two numbers between fifty and one hundred.. You have to use all the four digits, 4, 7, 3 och 9.

________________ ________________

PROBLEMS OF EDUCATION IN THE 21st CENTURY
Vol. 78, No. 2, 2020

CONCEPTUAL KNOWLEDGE

15 Here you can see 16 stars..



The digit 6 in 16 stands for ___ stars.

The digit 1 in 16 stands for ___ stars.

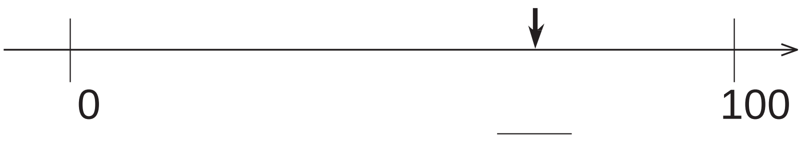
16 Which number is almost before four hundred?

____

17 Leon was born 2005. Which year will he turn one hundred?

____

18 What number is the arrow pointing to, *approximately*?

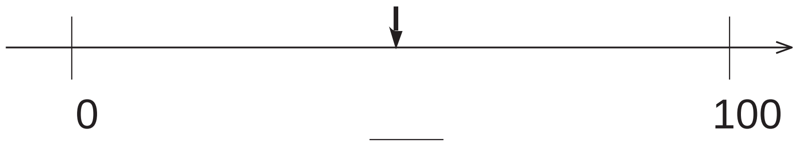


PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 78, No. 2, 2020

CONCEPTUAL KNOWLEDGE

19 What number is the arrow pointing to, *approximately*?

20 What number is the arrow pointing to, *approximately*?



21 What number is the arrow pointing to, *approximately*?

22 Elias has fifteen marbles and gets four marbles from Nora. Which of the following expressions fits/fit the described situation above? Make a circle around each one that fits.

4 + 15   4 – 15   15 + 4   15 – 4

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 78, No. 2, 2020

PROCEDURAL KNOWLEDGE

Your name:	
1) 23+32=	
2) 52+13=	
3) 12+46=	
4) 64+5=	
5) 74+4=	

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PROCEDURAL KNOWLEDGE

6) 23+8=	
7) 45+8=	
8) 56+35=	
9) 18+36=	
10) 538+347=	

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 78, No. 2, 2020

PROCEDURAL KNOWLEDGE

11) 49+34=	
12) 7658+1231=	
13) 362+449=	
14) 56+63=	
15) 132+141=	

PROCEDURAL KNOWLEDGE

16) 326+213=	
17) 876+111=	
18) 27+67=	
19) 643+172=	
20) 123+694=	

PROCEDURAL KNOWLEDGE

21) 6543+2379=	
22) 768+153=	
23) 453+378=	
24) 1264+4523=	
25) 64+55=	

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PROCEDURAL KNOWLEDGE

26) 3765+1234=	
27) 2563+5926=	
28) 4655+2537=	
29) 3421+5654=	
30) 26+84=	

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