

Improving Proof Ability of Prospective Teachers with a Contextual Model on Trigonometry Materials

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ABSTRACT

Proof ability of prospective teachers on Trigonometry material is still lacking. It can be seen when they carry out trigonometry proof that does not meet the proof indicators to conduct the research. This study aimed at improving proof ability of prospective teachers with a contextual model on Trigonometry materials. The research method used was classroom action research with the subjects of 30 students of Mathematics Education Study Program, State University of Malang, 2019. The research was conducted in two cycles by providing scaffolding action regarding proof steps, trigonometry identity scaffolding, and conflict cognitive of proof results. The research results showed that an increase of material indicator was 4%, an increase of completeness indicator was 24%, an increase of reason indicator was 59%, an increase of clarity indicator was 31%, an increase of conclusion indicator was 2%, and an increase of application capability indicator was 4% with a total increase of student's ability was 23%.

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1. INTRODUCTION

Mathematics is axiomatic deductive science. It means that formulas and theorems are derived from axioms, definitions and from the previous theorems that have been proven. Definition and theorem proof becomes an important component in learning as well as application in mathematics (Sukoharjo, 2016). Someone who understands the method of proof is not necessarily proficient in applying it to a case. However, understanding on proof method is the first step in carrying out mathematical proof. Mathematical proof ability is needed to be able to think logically and systematically (Susiatiy, 2018).

Proof ability is very necessary in learning because proof ability is important in deep mathematical learning, and proficiency in proof increases broad mathematical ability (Stylianides, *et al*, 2007). According to NCTM (2000) in an effort to change school education, an important component in high-quality mathematics programs is preparing teachers who are able to explore concepts in understanding mathematical proof. Thus, good abilities of prospective teachers must be prepared in proving a statement.

Mathematical proof offers a way to lead students to understand the importance of deductive reasoning and logical inference (Brown, 2017). Proof as a main feature of mathematics has an important role in producing understanding of mathematical propositions and mathematical contexts. In a classroom, students must be led to use justification, proof strategies and techniques and various forms of evidence. An ability in proving mathematics also involves learning strategic knowledge in certain fields related to the problems faced, knowledge, and special rules in proof and reasoning (Pinto & Karsenty, 2017).

From the results of observations and data collection about proof ability, the researcher found that the level of proof ability

of prospective teachers in the class of 2019 offering D in trigonometry class was still low. This is reinforced by an interview with one of the students in the class with initial A. The material note of A in trigonometry learning is shown in Figure 1 below.

39. $\cos 150^\circ = -\sin (90+60)^\circ$
 $= -\sin 60^\circ$
 $= -\frac{1}{2}$

41. $\tan (-135^\circ) = \tan (180-45)^\circ$
 $= \tan 45^\circ$
 $= 1$

43. $\sin 7560^\circ = \sin (84.90+0)^\circ$
 $= \sin 0^\circ$
 $= 0$

45. $\cos 495^\circ = -(\sin (5.90+45)^\circ)$
 $= -\sin 45^\circ$
 $= -\frac{1}{\sqrt{2}}$

Fig. 1. Trigonometry Material Note of Student A

From Figure 1 it can be seen that material note of A only contains routine question exercises and there is no proof solution. A also said that he did not understand what proof was. A also told the way to prove a definition or theorem. A said that there was no a subject that improved proof ability. Even though A said that when he was in high school, he had been taught proof, he had never been taught proof of definition and theorems that should have been mastered by prospective teachers as provisions to study mathematics material in further subjects. Based on the results obtained during the observation

and interview process, a model that could be applied in learning to improve mathematical proof ability of students was needed. One learning model that can be used is CTL.

Contextual Teaching and Learning (CTL) is a learning method considering that the learning process takes place if students are able to process or construct their own information or knowledge so that it is meaningful in accordance with their thinking framework (Suwandy & Mangkurat, 2017). This such learning method assumes that naturally the thinking process in finding meaning is something that is contextual, in the sense that it has something to do with the environment, knowledge, and experience they already have (treasury of memory, experience, response). Therefore, thinking is a process of finding relationships to find meaning and benefits of the knowledge. According to Gafur (2003) contextual learning has principles and learning strategies that encourage the creation of five forms of learning: (1) Interrelatedness, namely learning from existing knowledge (2) Direct experience, namely the learning process obtained from exploration, discovery, investigation, etc. (3) Application, namely the learning process applying what is learned in a real context (4) Cooperation, namely the process of exchanging ideas, asking and answering questions, communicative interactively to all elements of learning (5) knowledge transfer, namely the ability of students to transfer knowledge, skills, and attitudes owned in other situations.

According to Smith's study (2010) the application of contextual learning increases the conceptual understanding of students. According to Surya's research (2017) the application of contextual learning can improve the conceptual understanding of set material. According to the research of Mulhamah & Susilahudin (2016) the application of contextual learning can improve the problem solving ability of SPLDV. According to Susanto's research (2014) the application of contextual learning can improve students' communicative abilities. Based on research that have been carried out by some researchers, contextual learning is expected to improve the students' proof abilities. Six levels of Bloom's Taxonomy rubric with performance descriptions are made on the proof ability indicators. The purpose of this study is to improve the mathematical proof ability of prospective teachers with a contextual model on Trigonometry materials (Hyder & Bhamani, 2017).

Based on the explanation in the introduction, the researcher would like to conduct a class action research that aims at improving the proof ability of prospective teachers with a contextual model on trigonometry materials.

2. RESEARCH METHOD

The research method used in this research was classroom action research. In this study, the researcher implemented contextual learning in trigonometry identity material. The stages that were carried out in each research cycle were divided into planning, implementing action, observing, and reflecting. Test was also carried out before the implementation as an evaluation. The cycle would be stopped if proof ability criteria of the class were reached, namely with an average score of class was 2, in which score of 2 had fulfilled good category.

The data in this study were obtained from two cycles. The first cycle was model implementation and the second cycle was model application. The data collection phase was carried out on 30 students of class D of 2019 Department of Mathematics, FMIPA, Malang State University on trigonometry identity material. The data obtained were the results of test and

questionnaire. The variables used in this study consisted of one independent variable namely CTL learning model and one dependent variable namely proof ability. The research was focused on evaluating the student solutions written in complex report of mathematical problems. Through written report analysis, some competencies were selected as appropriate components for assessing the level of mathematical solution, namely: mathematical competence, mathematical writing and conclusions.

Indicators of mathematical ability in contextual learning was used for data processing as in Table 1 below.

Table 1. Proof Ability Indicators

Assessed Attributes	Criteria of Assessed Indicators	Score	Indicator
Mathematical Writing	Mathematical Competence	1-3	Material (MT)
	Completeness of Mathematical Writing	1-3	Completeness (KL)
	Mathematical Reasons	1-3	Reason (US)
	Clarity and Easy to Understand	1-3	Clarity (KJ)
Conclusion	Conclusion truth	1-2	Conclusion (KM)
	Proof Application and Completion Process of Learning	1-2	Application (AK)

The score of mathematical competence indicator is 1 for low material ability, 2 for medium material ability, and 3 for high material ability. The score of completeness indicator is 1 for incomplete answers, 2 for less complete answers, and 3 for complete answers. The score of mathematical reason indicator is 1 for mathematical reasons that are not appropriate, 2 for mathematical reasons that are less appropriate, and 3 for mathematical reasons that are appropriate. The score of clarity indicator is 1 for writing that has unclear meaning, 2 for reasons that have unclear meaning, and 3 for reasons that have clear meaning. The score of the conclusion indicator is 1 for inappropriate conclusions, 2 for appropriate conclusions. The application indicator is 1 if the student cannot apply it to the problem, 2 if the student can apply it to the problem.

3. RESULTS AND DISCUSSION

The study was conducted in 2 cycles. The first cycle was carried out on September 21st, 2019 and the second cycle was carried out on September 28th, 2019. The materials presented in the first cycle were Pythagoras identity, trigonometry function identity of addition, subtraction and co-function while at the second meeting the materials presented were the double angles identity, half angle, and trigonometry function of multiplication.

At the first meeting, the researcher applied contextual learning by 30% of the learning duration that was carried out during 3 hours of learning while 70% of learning was still conventional. After opening and giving an explanation of the prerequisite material, learning activities were carried out using conventional method. Towards the end of learning, students were directed to do contextual learning by discussing real problems regarding the application and the implementation of trigonometry identity. Students conducted group discussions. Then they had a presentation session with other groups that responded by asking questions, comments, and giving suggestions. At the end of the lesson, a test was conducted regarding the ability of student's proof on trigonometry identity.

At the first meeting, students had an error. Students were wrong in proving trigonometry identity. Students also had an error in solving real problems regarding trigonometry identity.

In addition, only a few students responded to the group presentation because they lacked of understanding on proof and application of trigonometry identity. According to Maharani & Widhiasih (2016) students who understand tend to respond to feedback when learning takes place. The proof steps error of students is shown in Figure 2 below.

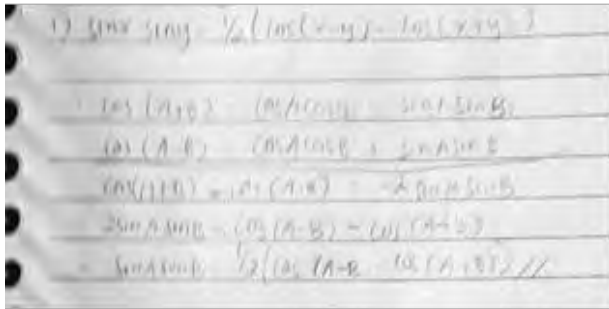


Fig. 2. Student's Proof Result at the First Meeting

At the first meeting, the researcher conducted a *scaffolding* in the form of mathematical proof steps (Beecher, Penna, & Bittinger, 2012). It aimed at making students understand the correct and appropriate proof steps. According to Mills (2016) the process error caused by a lack of information processing is due to students who do not understand the prerequisite material. The researcher provided scaffolding in the form of trigonometry identity. It was intended that students recalled the material on trigonometry identity. According to Coggin (1998) the act of recalling the material that has been studied aims to reactivate the knowledge owned by students so that it can be used to solve problems. The researcher provided demonstrations to students in applying trigonometry identity to problems. According to Hurst & Cordes (2017) giving learning experience is a form of assistance to students that can change their mathematical perspective.

At the end of the learning, a test regarding the student's proof ability on trigonometry identity was conducted. Based on the completion of 30 students, the results were obtained based on indicators of proof ability as shown in Table 2 below.

Table 2. Proof Indicator Score of Cycle 1

Indicator	Score			Total
	1	2	3	
Material (MT)	2	24	3	59
Completeness (KL)	3	26	0	55
Reason (US)	27	2	0	31
Clarity (KJ)	22	7	0	39
Conclusion (KM)	2	27	-	56
Application (AK)	19	10	-	39
Total number				279

At the second meeting, the researcher applied contextual learning by 70% of the learning duration that was carried out during 3 hours of learning with 30% of conventional learning at the beginning of the activity. After carrying out contextual learning with discussion for 60 minutes, students made presentations in turn.

During the second meeting, students could prove the trigonometry identity and could apply it in solving real problems regarding trigonometric identity. The results of student's proof ability at the second meeting is shown in the Figure 3.

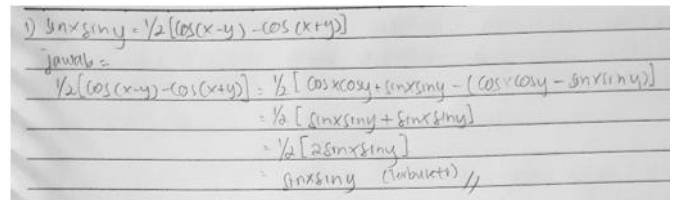


Fig. 3. Student's Proof Result at the Second Meeting

In the second meeting, it could be seen that other groups responded to the presentation results. Students responded to the presentations with various comments, suggestions, and improvements. During the presentation session, a *cognitive conflict* process between the presenter group and other groups regarding the solution result was also found. According to Subanji (2016) *cognitive conflict* can trigger students to do reflection in solving their problems. In addition, other groups also began to show a critical nature in which they gave comments, suggestions, and improvements actively and well. According to Batlolona (2016) when students do activities by responding to learning, one of the ideal learning activities has been achieved.

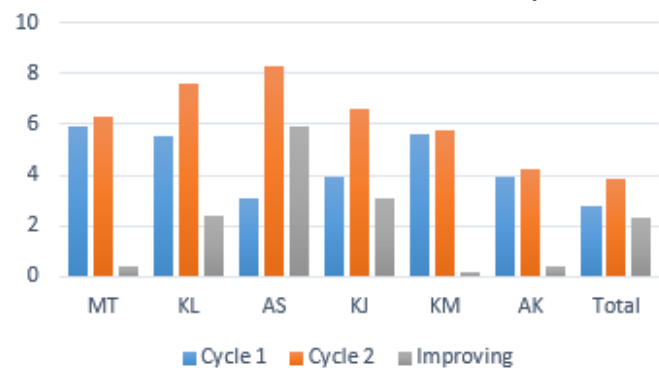
At the end of learning, a test regarding the student's proof ability on trigonometry identity was conducted. From the completion/solution results done by 30 students, the results were obtained based on indicators of proof ability as shown in Table 3 below.

Table 3. Proof Indicator Scores of Cycle 2

Indicator	Score			Total
	1	2	3	
Material (MT)	5	14	10	63
Completeness (KL)	4	3	22	76
Reason (US)	0	4	25	83
Clarity (KJ)	5	11	13	66
Conclusion (KM)	0	29	-	58
Application (AK)	16	13	-	42
Total number				388

An increase of mathematical proof ability based on cycle 1 and cycle 2 is shown as in Table 4 below.

Table 4. The Result of Proof Ability



Based on the learning outcomes it can be seen students had an increase in mathematical proof ability based on indicators. An increase of material indicator was 4%, an increase of completeness indicator was 24%, an increase of reason indicator was 59%, an increase in clarity indicator was 31%, an increase of conclusion indicator was 2%, and an increase of

application ability indicator 4% with a total increase of student's ability was 23%.

4. CONCLUSION

Based on the research results and discussion of the study, it can be concluded that the contextual model in learning trigonometry identity material can improve students' mathematical proof ability with the steps (1) Interrelatedness, by providing *scaffolding* of proof techniques and trigonometry identity (2) Direct experience, by implementing proof in groups. (3) Application, by applying to group problem solving (4) Cooperation, by applying discussion and group learning activities (5) Knowledge transfer, by applying to non-routine problem solving. The study was conducted in two cycles by providing scaffolding actions regarding proof steps, trigonometry identity scaffolding, and conflict cognitive of proof results. The research results show that an increase of material indicator is 4%, an increase of completeness indicator is 24%, an increase of reason indicator is 59%, an increase of clarity indicator is 31%, an increase of conclusion indicator is 2%, and an increase of application ability indicator is 4% with a total increase of student's ability is 23%.

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