

Common Errors in Algebraic Expressions: A Quantitative-Qualitative Analysis

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Abstract: Majority of the students regarded algebra as one of the difficult areas in mathematics. They even find difficulties in algebraic expressions. Thus, this investigation was conducted to identify common errors in algebraic expressions of the preservice teachers. A descriptive method of research was used to address the problems. The data were gathered using the developed test administered to the 79 preservice teachers. Statistical tools such as frequency, percent, and mean percentage error were utilized to address the present problems. Results show that performance in algebraic expressions of the preservice teachers is average. However, performance in classifying polynomials and translating mathematical phrases into symbols was very low and low, respectively. Furthermore, the study indicates that preservice teachers were unable to classify polynomials in parenthesis. Likewise, they are confused of the signs in adding and subtracting polynomials. Results disclosed that preservice teachers have difficulties in classifying polynomials according to the degree and polynomials in parenthesis. They also have difficulties in translating mathematical phrases into symbols. Thus, mathematics teachers should strengthen instruction on these topics. Mathematics teachers handling the subject are also encouraged to develop learning exercises that will develop the mastery level of the students.

Keywords: Algebraic expressions, Analysis of common errors, Education students, Performance

Introduction

One of the main objectives of teaching mathematics is to prepare students for practical life. However, students sometimes find mathematics, especially algebra impractical because they could not find any reason or a justification of using the x s and the y s in everyday life. They question the significance of algebra in buying things in the market. Thus, they sometimes find it boring and irrelevant. On the other hand, Ncube (2016) expressed that in the school curriculum, algebra is considered as an influential mathematics topic. It is considered as a gateway subject because of its application in the different branches of mathematics and science. McIntyre (2005) on the other hand supported this statement by stating that success in mathematics largely depends on the concepts of algebra. But students believed that mathematics is not just as easy for them. The abstract idea of algebra makes mathematics difficult for them.

Furthermore, students considered algebra as a difficult subject in mathematics. Many of the difficulties according to them have sources in the poor understanding of two important concepts – the variable and the algebraic expression (Subramaniam and Banerjee, 2014). In relation to this statement, Mamba (2012) in his analysis of South Africa's Grade 12 Mathematics Paper disclosed that algebraic expressions posed many problems to the learners. The skills of the learners in algebra are very poor as reported by Barry (2014). His report also stressed that students' difficulties in previous mathematics classes posed challenges in their future mathematics classes. Along this line, it can be conceived that students' poor performance in higher mathematics can be influenced by their performance in algebra.

Likewise, Sfard and Tall as cited in Subramaniam and Banerjee (2014) stated that one of the difficulties experienced by students in learning algebra was on the understanding of the process-product duality of algebraic expressions, which encode both operational instructions as well as denote a number that is the product of these operations. The difficulty in understanding the multiple meanings encoded by expressions may underlie the inability of many students to operate with enclosed expressions. However, Koirala (2005) regarded algebra as one of the most important areas of school mathematics. But despite its importance, students find it difficult to understand simple algebraic concepts such as variables, expressions, and equivalence. Although basic algebraic concepts are introduced at the elementary and high school levels, some students and even college students have difficulties in understanding algebra because they find it more abstract than any other field of mathematics.

Along this line, many studies have documented that algebra is one of the most difficult areas of mathematics. Vlassis (2002) and Warren (2003) for instance continue to show that achievement rates in algebra are poor.

Likewise, Subramaniam (2014) disclosed that algebra has been an area of difficulty for most high school and even college students. Thus, Herscovics and Linchevski as cited in Warren (2003) pointed to the „cognitive gap“ between arithmetic and algebras as one of the major reasons of students difficulties in algebra and algebraic expression. In addition, studies conducted during the period preceding the 1990s focused especially on the transitions required by students as they moved from arithmetic to algebra (Kieran, 2007). However, rarely are studies conducted on the analysis of errors and performance in algebra of the preservice teachers, thus this study was conducted.

Literature Review

Error and Error Patterns in Algebra and Algebraic Expressions

Generally, an error is a simple lapse of care or concentration which almost everyone makes at least occasionally. An error is the deviation from a correct solution of a problem as contextualize in mathematics. Errors could be found in wrongly answered problems which have flaws in the process that generated the answers (Young & O'Shea, 2007). On the other hand, error pattern analysis is an assessment approach that allows students to determine whether they are making consistent mistakes when performing basic computations. By identifying the pattern of student's error, mathematics teachers can then directly teach the correct procedure for solving and doing mathematics correctly. While there are common errors that students with performing algebraic expressions, students may demonstrate errors that are individual specific. Along this line, Lim (2012) identified twelve types of errors in simplifying algebraic expressions and concluded that all errors are the results of interference from new learning; difficulty in operating with the negative integers; misconceptions of algebraic expressions; and misapplication of rules.

Furthermore, Bush (2011) indicated that (1) numerous misconceptions and errors identified in the review of literature were present on both the sixth- and eighth-grade open-responses; (2) basic computational errors with whole numbers (a secondary skill), were found consistently throughout the sixth- and eighth-grade open-responses; (3) a greater number of misconceptions and errors identified in the review of the literature were present on the eighth-grade items than were found on the sixth-grade items; (4) students often lost points for reasons other than mathematical misconceptions or errors; and (5) some refinement and reorganization of Welder's (2010) framework could prove beneficial when using the framework for data analytic purposes.

Performance in Algebra and Algebraic Expression

Research in the area of teaching and learning of algebra has indicated the importance of understanding the structure of arithmetic expressions to make sense of algebraic expressions and their manipulation. However, Linchevski and Livneh as cited by Subramaniam and Banerjee (2014) have raised doubts about whether structure oriented arithmetic teaching is really appropriate as a preparation for algebra. Furthermore, Stafard and Tall as cited by Banerjee (2014) contend that another reason for students' difficulty is that they cannot easily grasp the process-product duality inherent in algebraic expressions, that is, the fact that the expression stands for a number as well as for instructions to perform operations on the number or letter.

Furthermore, researchers have attributed students' difficulties in algebra to the lack of understanding of the letter/variable (Stacey cited in Egodawatte (2011) and algebraic expression. Algebra is a branch of mathematics, which turns relations examined by using symbols and numbers to generalized equations. Not only does it represent letters and quantities, it also allows making calculations using these symbols at the same time (Kieran, as cited in Mason and Sutherland, 2002). In conjunction with this, a large body of research, which continues to grow, has studied the learning of the concepts that underpin students' success in algebra. These concepts include unknowns and variables, expressions and equations, and the expansion of the meaning given to the equal and minus signs.

Egodawatte (2011) expressed that some errors of the students in algebra emanated from misconceptions of a variable. The main reason for this misconception was the lack of understanding of the basic concept of the variable in different contexts. The abstract structure of algebraic expressions posed many problems to students such as understanding or manipulating them according to accepted rules, procedures, or algorithms.

Similarly, Dede et al. (2007) put forth reasons of the hardship students undergo in learning algebra as not knowing about different uses of variables, not knowing about the role of variables in making generalizations,

not being able to interpret variables, and failure to perform operations with variables. On the other hand, Baki (1998) listed students' misconceptions as errors included in parentheses and using operators, carelessness, and turning nonnumerical expressions into algebraic expressions. Perso as cited by Chow (2011) grouped the misconceptions in algebra under three main headings as the location of the letters, use of variables, and algebraic rules.

These scenarios are not healthy anymore to the teachers and students in particular. And reducing these misconceptions or errors, Baki and Kartal (2004) disclosed that students need to understand concepts like variable, equation, and have preliminary knowledge on arithmetic and its operation. Algebraic comprehension depends not on knowledge of the students of the formulas and understanding the calculations right, but instead understanding of the concepts and operations, and development of mathematical thinking. Therefore, concepts and relations should indicate importance, instead of procedural means of solution, and learning should be realized through conceptual learning that involves the knowledge of operations and concepts in a balanced manner.

Statement of the Problem

The main purpose of this research was to determine performance and common errors in algebraic expressions of the preservice teachers of Philippine Normal University Visayas. Specifically, this study aimed to answer the following questions: (1) what is the level of performance in an algebraic expression of the preservice teachers? and (2) what are preservice teachers' common errors in algebraic expressions related to the classification of polynomials according to the number of terms, classification of polynomials according to the degree, addition and subtraction of polynomials, translation of mathematical phrases and sentences into mathematical symbols and equations, multiplication, and division of polynomials?

Method

Research Design

This study employed a mixed-method research, more specifically sequential explanatory design which is characterized by the collection and analysis of quantitative data followed by the collection and analysis of qualitative data (Creswell, 1998). In the quantitative phase, the researcher used a test instrument to identify and classify preservice teachers' common errors in algebraic expressions. The researcher also used Focus Group Discussion (FGD) to expose preservice teachers' reasons for committing errors in the qualitative part of the study.

Typically, the purpose of a sequential explanatory design is to use qualitative results to assist in explaining and interpreting the findings of a primarily quantitative design. The initial quantitative phase of the study was used to characterize individuals along certain traits of interest related to the research questions. These quantitative results were used to guide the purposeful sampling of participants for a primarily qualitative study. The findings of the quantitative study determine the type of data collected in the qualitative phase (Gay, Mills & Airasian, 2006). There are four main stages in the sequential explanatory study. The schematic diagram presented below illustrates these stages.

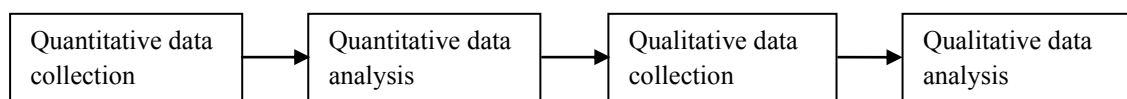


Figure 1. Schematic diagram representing the various stages of the design

Participants of the Study

The participants of the study were the second year education students of Philippine Normal University Visayas. Second year education students were purposely chosen as participants of the study because these are the only students with CEC MO2 College Algebra as a subject. Since the study is a mix-method research, the researcher included all of them as participants of the study to justify the need for a quantitative method of research. Thirty seven of them belong to section one while the rest were from section 2. On the other hand, only 13 of them were

selected as participants for the conduct of the FGD. In selecting participants for the FGD, the researcher considered students who have obtained score below the median and have accumulated 50% and more errors in the test. Two FGDs were conducted by the researcher, one for section one and the other one was for section two.

Research Instrument

A 30-item test was used to determine the level of performance in an algebraic expression of the participants. This was also used to identify participants' common errors in classifying polynomials according to the degree, addition and subtraction of polynomials, translation of mathematical phrases and sentences into mathematical symbols and equations, multiplication, and division of polynomials. On the other hand, the researcher developed a guide of topics for discussion in the conduct of the FGD and the focus was on topics where the students obtained poor performance and majority of errors. To establish the reliability of the test, pilot testing was conducted. However, Kuder Richardson formula 20 (KR20) was used to determine the reliability of the test. The r -value of 0.91 using KR20 formula indicates that the research instrument has very high reliability.

On the other hand, validity of the test was established through expert validation. Experts in the field of mathematics more specifically in the teaching of algebra were asked to validate the research instrument in terms of its content and appropriateness to the specified areas of concern. The content of the test was discussed with three experts and suggestions provided by them were included prior to the administration of the test. The test developed by the researcher follows the principle of test construction. Along this line, experts' validation of the test is valid to a high degree.

Data Analysis

The pencil-and-paper test comprised the quantitative data of the study. The mean was used to determine the level of performance in algebraic expression and mean percentage error in each of the areas in algebraic expressions. To determine the responses of the participants in the conduct of FGD on the difficulties and common errors, themes were determined from each of the identified areas in algebraic expression such as: (a) classification of polynomials according to the number of terms; (b) classification of polynomials according to the degree; (c) addition and subtraction of polynomials; (d) translation; (e) multiplication of polynomials; and (f) division of polynomials.

Results and Discussion

Performance in Algebraic Expressions

Table 1 reflects that the level of performance in algebraic expressions of the preservice teachers is average ($M = 12.86$, $SD = 6.37$). However, when areas are considered individually, preservice teachers' level of performance in classifying algebraic expressions according to the number of terms ($M = 3.06$, $SD = 1.76$) and division of algebraic expressions ($M = 3.56$, $SD = 1.98$) is high. On the other hand, the level of their performance in classifying algebraic expressions according to the degree ($M = 1.28$, $SD = 1.19$) and in multiplication of algebraic expressions ($M = 1.90$, $SD = 1.79$) is low but very low in the translation of mathematical phrases and sentences into mathematical symbols and equations ($M = 0.73$, $SD = 1.02$).

Results indicate that preservice teachers have difficulties in algebraic expressions more specifically in classification of algebraic expressions according to the degree, multiplication of algebraic expressions, and translation of mathematical phrases and sentences into mathematical symbols and equations. Analysis of the responses of the participant indicates that they were not able to determine the correct degree of the given algebraic expression. What they did is that they count variables in a given term as one and not counting the sum of the exponents in the given term. Another difficulty observed by the researcher is that they find it more complex determining the degree if it's already in an algebraic expressions.

In multiplication of algebraic expressions, results indicate that preservice teachers have difficulties in multiplying algebraic expressions with more than one term more specifically when they are written in a different format like using parenthesis and using the cross product method. According to them during the FGD, *they are used to multiplying algebraic expressions using the cross product method because their math teacher during high school does not explore other methods of doing multiplication*. There are also students who know how to

multiply algebraic expressions; however, they were confused whether they will also add the exponents or just copy the variables and the exponents. Further analysis of their responses shows that some of them added the exponents instead of copying them.

Furthermore, preservice teachers find translation of mathematical phrases or sentences into mathematical symbols or equations very difficult. Result in this regard reveals that they are very poor in this competency. They find this translation very difficult because this requires comprehension and analysis. This is complex because aside from the computational skills, they also need skills in reading comprehension. According to Fletcher (2005) and Leppänen (2006) word problem-solving performance and reading comprehension skills are related to the technical reading skills of the students. For example, technical reading skills have been shown to be connected to reading comprehension skills (Holopainen, 2002 & Leppänen, 2006). In addition, mathematical abilities have been found to be related to technical reading skills. It is very clear that comprehension and reading skills are important elements in word problem solving.

Table 1. Performance in Algebraic Expressions

Competencies	M	SD
Classification of algebraic expressions according to the number of terms	3.06	1.76
Classification of algebraic expressions according to the degree	1.28	1.19
Addition and subtraction of algebraic expressions	2.33	1.62
Translations	0.73	1.02
Multiplication of algebraic expressions	1.90	1.79
Division of algebraic expressions	3.56	1.98
Overall Mean	12.86	6.37

Mean Percentage Errors for Each Area

As reflected in Table 2, translation of mathematical phrases or sentences into mathematical symbols or equations had the highest percentage error of 84.82, followed by classification of algebraic expressions according to the degree with the mean percentage error of 74.18. As a mathematics teacher for quite a long time, I have observed that translation of mathematical phrases or sentences into mathematical symbol or equation was considered by the students as the most difficult. The process on how to translate “twice the product of x and y subtracted from the quotient of a and b ” is difficult for them. One student said during the FGD “*I think it is the most difficult topic in algebra for me. I find it confusing and I can’t understand what the statement mean*”. Another student responded that “*this was the most confusing part. I was an idiot because I didn’t know the difference between ,representing and finding*”. We had a lesson back in high school that we have to find someone’s age in our algebra class but the methods of doing that are not anymore fresh in my mind” These two statements from the education students reflected that comprehension is very important in the analysis of word problems. Likewise, they need to understand what condition the given statement conveys. According to Tuohimaa, et. al. (2008) mathematical word problems was strongly related to performance in reading comprehension. They contend that technical reading skills increased the mathematical word problem solving skills of the students. However, even after controlling for the level of technical reading involved, performance in mathematical word problems was still related to reading comprehension, signifying that both of these skills require overall reasoning abilities. Thus, it is conclusive that comprehension is related to students word problem solving skills.

Table 2. Mean Percentage Errors for Each Competencies

Competencies	Mean Percentage Error
Classification of algebraic expressions according to the number of terms	37.72
Classification of algebraic expressions according to the degree	74.18
Addition and subtraction of algebraic expressions	52.14
Translations	84.82
Multiplication of algebraic expressions	61.26
Division of algebraic expressions	37.77

Note. The mean percentage error is interpreted as follows: 0.00 – 20.00 (Very Low); 20.01 – 40.00 (Low); 40.01 – 60.00 (Average); 60.01 – 80.00 (High); and 80.01 – 100.00 (Very High)

The competencies of algebraic expression presented in Table 3 does not call much attention to the researcher because result shows that preservice teachers find it easy as indicated by the mean percentage error of 37.72.

This requires only low level thinking skills. However, in the expression $(2x + y) - (7x - 2y)$, majority of the preservice teachers are confused of counting the number of terms, some of them counted that as four terms because they count it without considering parenthesis and they named it as polynomial. This means that parenthesis confuses them in identifying algebraic expressions according to the number of terms. The presence of parentheses in algebraic expressions conveys different meanings that confuse students. In this regard, students should take into account the roles and functions of parentheses in an algebra. Although the mean percentage error obtained in this competency is low, results presented in Table 3 likewise reflect that errors are still committed by the preservice teachers. Their responses during the focus group discussion revealed that some of their errors are due to their carelessness and on recalling what has been discussed previously. They were not able to recall what has been discussed by their mathematics teachers during high school. In this regard, they suggested during the FGD that a review on previous topics should be taken into consideration by the mathematics teacher before the start of the new lesson especially if the previous topic is a prerequisite to the new one. One participant quoted that *"I feel this topic quite easy, however, I need to have a review on this, that's why I need my mathematics teachers not to proceed to the new topic instead a review of the previous topic should be undertaken"*. This response is a reflection that mathematics teachers should take time reviewing the previous lesson before a new is introduced and discussed.

Table 3. Mean Percentage Errors in Classifying Algebraic Expressions According to the Number of Terms

Test Items	Number of Incorrect Responses	Percentage	Mean Percentage Error
$2x + 3y$	25	32.1	37.72
$25a^2b^3c^4$	30	38.5	
$(2x + y) - (7x - 2y)$	43	55.1	
$x + y + 5$	24	30.8	
$x^3 + 5x^2 - 3x + 5$	25	32.1	

Note. The mean percentage error is interpreted as follows: 0.00 – 20.00 (Very Low); 20.01 – 40.00 (Low); 40.01 – 60.00 (Average); 60.01 – 80.00 (High); and 80.01 – 100.00 (Very High)

Table 4 reflects that the mean percentage error in classifying algebraic expression according to the degree is high as indicated by the mean percentage value of 74.18. Examining each item on the number of incorrect responses, results show that items $xyz + 1$, $2xy^4 - 5x^2y^2z^2 = 2$, and $a + b + 3^2$ obtained the highest percentage errors of 93.7, 93.7, and 91.1, respectively. However, items $2a + 3b + 4c$ and $7x^8 + 2x^6 - 5x^2 + 3 = 3$ obtained the lowest percentage error of 46.8 and 45.6, respectively. Although this is just counting the exponents of the variables, however, students find it difficult. They find this area very confusing may be because this topic requires skill to memorize which students in mathematics find not significant. According to Baki and Kartal (2004), algebraic comprehension depends not on knowledge of the students on the formulas and understanding the calculations right, but instead understanding of the concepts and operations, and development of mathematical thinking. Therefore, concepts and relations should indicate importance instead of procedural means of solution, and learning should be realized through conceptual learning that involves the knowledge of operations and concepts in a balanced manner. Responses of the preservice teachers during the FGD show that majority of them thought that the highest exponent alone is the basis in determining the degree of an algebraic expression. However, there are rules to follow which they were not able to get the correct answer. One student responded that *"I made a guess because I don't exactly know the procedures in finding the degree. Others say the rules are confusing"*.

Table 4. Mean Percentage Errors in Classifying Algebraic Expressions According to the Degree

Test Items	Number of Incorrect Responses	Percentage	Mean Percentage Error
$xyz + 1$	74	93.7	74.18
$a + b + 3^2$	72	91.1	
$2a + 3b + 4c$	37	46.8	
$7x^8 + 2x^6 - 5x^2 + 3 = 3$	36	45.6	
$2xy^4 - 5x^2y^2z^2 = 2$	74	93.7	

Note. The mean percentage error is interpreted as follows: 0.00 – 20.00 (Very Low); 20.01 – 40.00 (Low); 40.01 – 60.00 (Average); 60.01 – 80.00 (High); and 80.01 – 100.00 (Very High)

It can be deduced from Table 5 that the mean percentage error of the preservice teachers in addition and subtraction of algebraic expression is only average as indicated by the overall mean percentage error of 52.14. However, examining item *"add $2a + 3b - c$, $a - c$, $5a - b + 7c$ "*, result reveals that preservice teachers have

experienced some difficulties in subtracting algebraic expressions with regards to how it is written, using the vertical positioning of variables made them difficult to simplify. Likewise, majority of them forgot to change the sign of the subtrahend before they proceed to the addition rule. They directly proceed to the process without considering the operation. Furthermore, it can be gleaned from the results that students have difficulty in understanding the statement „subtracted from and subtracted to” what they do is they did not consider the statement but directly proceed to the operation of addition and subtraction. Analysis of their answers on the test revealed that they have difficulties also with the signs. One student responded in the FGD saying that “*I am wrong in this item because I was confused with the sign*”. Majority of the preservice teachers were incorrect because of the signs. They said that “*a recall of this is necessary*”.

Table 5. Mean Percentage Errors in Addition and Subtraction of Algebraic Expressions

Test Items	Number of Incorrect Responses	Percent-age	Mean Percentage Error
$(2a + 3b) + (5a + 6b)$	20	25.3	
$5x + 3y - 2x - y$	52	65.8	
Add $2a + 3b - c$, $a - b$ and $5a - b + 7c$	31	39.2	
Subtract $4a + 5b - c$ from $a - 2b + 3c$	48	60.8	52.14
Take $4x^2 + 2x^2y^2 + 7y^2$ from $5x^2 + 2x^2y - 10y^2$	55	69.6	

Note. The mean percentage error is interpreted as follows: 0.00 – 20.00 (Very Low); 20.01 – 40.00 (Low); 40.01 – 60.00 (Average); 60.01 – 80.00 (High); and 80.01 – 100.00 (Very High)

Table 6 reveals that the mean percentage error obtained by the preservice teachers in the translations of mathematical phrases and sentences is very high as shown by the mean percentage error of 84.82 most especially on items (Ariel’s age is represented by $3x + 4$. Represent his age 5 years ago), (Ariel’s age is represented by $3x + 4$. Represent his age 3 years ago), and (Ariel’s age is represented by $3x + 4$. Represent his age $x - 6$ years ago). It is very clear from the responses of the students that they have difficulties in the translation of mathematical phrases or sentences into mathematical symbols or equations. Their responses during the FGD reflected that they find it difficult to analyze word problems. When ask, what makes word problems difficult for them? Majority have said that the conditions given in the problem makes it difficult for them to analyze word problem solving. As responded by one preservice teacher “*problem solving is difficult because I don’t know how to analyze. Problem solving is really my problem*”. Siniguan (2013) investigated the difficulties experienced by college students in solving mathematics problem. Major results of his study showed that the students’ difficulties are on the inability to translate problem into mathematical form and inability to use correct mathematics. Furthermore, Mansoor (1989) expressed that a large proportion of college students majoring in science are unable to translate even simple sentences into algebraic equations. He also hypothesized that students who lack formal operational reasoning may experience more problems in the translation of algebraic equations.

Table 6. Mean Percentage Errors in Translating Mathematical Sentences in to Mathematical Symbol

Test Items	Number of Incorrect Responses	Percentage	Mean Percentage Error
Ariel’s age is represented by $3x + 4$. Represent his age 5 years ago	76	96.2	
Ariel’s age is represented by $3x + 4$. Represent his age 3 years ago	76	96.2	
Ariel’s age is represented by $3x + 4$. Represent his age $x - 6$ years ago	75	94.9	84.82
A carpenter has a wood that is $2x + 1$ feet long. Represent the length of the wood after $x - 4$ have been cut off	57	72.2	
Marivic had $d^2 + 4d = 1$ pesos. How much did she have after spending $4d - 1$ pesos	51	64.6	

Note. The mean percentage error is interpreted as follows: 0.00 – 20.00 (Very Low); 20.01 – 40.00 (Low); 40.01 – 60.00 (Average); 60.01 – 80.00 (High); and 80.01 – 100.00 (Very High)

Table 7 indicates that the mean percentage error of the preservice teachers in multiplication of algebraic expression is average as reflected by the obtained mean percentage error of 61.26. Responses of the students in this regard reflected that they have difficulties in the square of a binomial. Examination of the responses reveals that they were not able to get the correct answer because almost all of their answer when $(a+b)^2$ is $a^2 + b^2$ instead of $a^2 + 2ab + b^2$. This error may be due to their carelessness. It can be deduced also from their answers that they have difficulties with the exponents. They are confused whether they will add or multiply the exponents. This is supported by the responses of one student "I got only one check because I forgot whether to add or multiply the exponent. I was confused. I need to have a review of the different rules of the exponent". Mathematics teachers should take note of this. It is necessary that a recall should be done before introducing a new topic.

Table 7. Mean Percentage Errors in Multiplication of Algebraic Expressions

Test Items	Number of Incorrect Responses	Percentage	Mean Percentage Error
$x^2 \cdot x^5$	47	59.5	61.26
$3a(4a) - 5$	43	54.4	
$(a+b)(2a+5b)$	47	59.5	
$(a+b)^2$	56	70.9	
Find the area of a square whose side is $2xy$ cm long	49	62.0	

Note. The mean percentage error is interpreted as follows: 0.00 – 20.00 (Very Low); 20.01 – 40.00 (Low); 40.01 – 60.00 (Average); 60.01 – 80.00 (High); and 80.01 – 100.00 (Very High)

Table 8 reveals that the mean percentage error of the preservice teachers in the division of algebraic expression is low as shown by the obtained mean percentage error of 37.77. These results reflected that preservice teachers have no difficulties in the division of algebraic expression involving monomials. However, results indicate that the mean percentage error in item $x + 1\sqrt{x^2 - 7x - 8}$ is very high as shown by the obtained mean percentage error of 86.11. This result is reflective of the fact that preservice teachers encountered difficulties in simplifying complex algebraic expressions especially in division. According to the response of one student in the FGD conducted "item 6 is difficult for me, I remember that it can be solved using long method and synthetic division, but I cannot remember the process of doing it"

Table 8. Mean Percentage Errors in the Division of Algebraic Expressions

Test Items	Number of Incorrect Responses	Percentage	Mean Percentage Error
$\frac{x^7}{x^5}$	10	12.7	37.8
$\frac{x^5}{x^7}$	20	25.3	
$\frac{x^{4a+5}}{x^a}$	26	32.9	
$\frac{15x^2y^5}{5y^3}$	34	43.0	
$x + 1\sqrt{x^2 - 7x - 8}$	68	86.1	

Note. The mean percentage error is interpreted as follows: 0.00 – 20.00 (Very Low); 20.01 – 40.00 (Low); 40.01 – 60.00 (Average); 60.01 – 80.00 (High); and 80.01 – 100.00 (Very High)

Conclusions

In this study, preservice teachers have experienced difficulties in algebraic expressions. Their difficulties were pointed in the classification of algebraic expressions according to the number of terms and translation of

mathematical phrases or sentences into mathematical symbols and equation. Preservice teachers were not able to master these topics during their high school and they already forgot the process of doing it. Likewise, preservice teachers were not able to develop word problem solving skills. They could not even translate simple mathematical phrases into mathematical symbol and equation. More specifically, they experienced difficulties of determining the correct degree of the given algebraic expression due to the presence of the exponents. In other words exponents made more difficult for them to identify the correct degree.

In the analysis of preservice teachers' errors in algebraic expressions, in terms of the classification of algebraic expressions according to the number of terms, students have difficulties in identifying the number of terms when parentheses were used in the given algebraic terms. On the other hand, students were unable to recall rules and functions of exponents in determining the degree of algebraic expressions. In addition and subtraction, errors of the students were commonly observed on the use of the statement subtracted or added from or subtracted or added to. They misconceived the meaning of "to and from" in the statement.

Furthermore, in the translation of mathematical phrases and sentences into mathematical symbols and equations, the findings conclude that preservice teachers have poor analysis in relation to what mathematical phrases and sentences conveys that may lead to the solution of a problem. In other words, common error in this regard was primarily on representing mathematical phrases or sentences into mathematical symbols or equations. In multiplication of algebraic expression, findings reflected that education students' error in multiplying algebraic expressions was on exponents. They are confused whether they will add or multiply the exponents. Carelessness was another reason cited. However, findings in the division of algebraic expressions concludes that majority of their errors were on complex division where polynomials will be divided by a binomial. They find it difficult to organize the process of dividing a polynomial by a binomial.

Recommendations

The findings and conclusion presented propel the researcher to recommend to the mathematics professors and instructors to design programs and activities that will improve performance of the preservice teachers in mathematics more specifically in algebra. It is also recommended that preservice teachers should exert more efforts especially in the practice of different exercises on translation because constant practice leads us to near perfection.

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References

- Baki, A. (1998). Matematik Öğretiminde İşlemsel ve Kavramsal Bilginin Dengelenmesi. *Atatürk Üniversitesi 40. Kuruluş Yıldönümü Matematik Sempozyumu*, Erzurum.
- Baki, A. V & Kartal, T. (2004). Kavramsal ve İşlemsel Bilgi Bağlamında Lise Öğrencilerinin Cebir Bilgilerinin Karakterizasyonu. *Türk Eğitim Bilimleri Dergisi*, 2(1), 27-46.
- Boyatzis, Richard E. *Transforming Qualitative Information*. Thousand Oaks, CA: Sage Publications, 1998. Print.
- Braun, V. & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology* 3, 77-101.
- Creswell, J. W. (1998). *Qualitative inquiry and research design: Choosing among five traditions*. Thousand Oaks, CA:Sage.
- Dede, Y., Yalın, H. İ. & Argün, Z. (2002). *İlköğretim 8. Sınıf Öğrencilerinin Değişken Kavramının Öğrenimindeki Hataları ve Kavram Yanılgıları*. UFBMEK-5, ODTÜ, Ankara.
- Dede, Y. & Peker, M. (2007). Öğrencilerin Cebire Yönelik Hata ve Yanlış Anlamaları: Matematik Öğretmen Adaylarının Bunları Tahmin Becerileri ve Çözüm Önerileri. *İlköğretim Online*, 6(1), 35-49.
- Fletcher, J.M. (2005). Predicting math outcomes: Reading predictors and comorbidity. *Journal of Learning Disabilities*, 4, 308-312.
- Gay, L. R., Mills, G. E. & Airasian, P. (2006). *Educational research: Competencies for analysis and applications, Eighth edition*. NJ: Pearson Education Inc.

- Gilbert, S. (1989). *Principles of educational and psychological measurement and evaluation, Third edition*. CA : Wardsworth.
- Herscovics, N. and Linchevski, L. (1994) A cognitive gap between arithmetic and algebra, *Educational studies in mathematics*, Vol. 27, pp. 59-78.
- Holopainen, L. (2002). *Development in reading and reading related skills: A follow-up study from preschool to the fourth grade* (Jyväskylä Studies in Education, Psychology, and Social Research 200).Jyväskylä: Jyväskylä University Printing House.
- Kieran, C. (1989) The early learning of algebra: A structural perspective. In S. Wagner and C. Kieran (eds), *Research issues in the learning and teaching of algebra*, Reston: NCTM.
- Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws. (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. (pp. 390-419), NY: Macmillan Publishing Company.
- Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels: building meaning for symbols and their manipulation. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 707- 762). Reston, VA: NCTM.
- Leppänen, U. (2006). *Development of literacy in kindergarten and primary school* (Jyväskylä Studies in Education, Psychology, and Social Research 289). Jyväskylä: Jyväskylä University Printing House.
- Linchevski, L. and Livneh, D. (1999) Structure sense: The relationship between algebraic and numerical contexts. *Educational studies in mathematics*, 40, 173-196.
- McAlpine, M. (2002). *Principles of assessment*. Bluepaper No. 1, CAA centre, University of Luton. Retrieved April 7, 2010 from <http://www.caacentre.ac.uk/dldocs/Bluepaper1.pdf>
- Perso, T. (1992). Using Diagnostic Teaching to Overcome Misconceptions in Algebra. *The Mathematical Association of Western Australia*.
- Philipp, R. A. (1999). The many uses of algebraic variables. In B. Moses (Ed.), *Algebraic thinking, Grades K-12: Readings from NCTM's school-based journals and other publications*. (pp. 157-162), Reston, VA: NCTM.
- Stacey, K. (1997) Students' understanding of algebraic notation: 11-15, *Educational studies in mathematics*, Vol. 33, pp. 1-19.
- Subramaniam, K. (2004) Naming practices that support reasoning about and with expressions. Regular lecture presented at ICME-10, Denmark.
- Sfard, A. (1991) On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational studies in mathematics*, 22, 1-36.
- Tall, D. et al (1999) What is the object of encapsulation of a process? *Journal of mathematical behavior*, 18(2), 223-241.
- Vlassis, J. (2002). About the flexibility of the minus sign in solving equations. In Cockburn, A.D., & Nardi, E. (eds), *Proceedings of the 26th conference of the International Group of the Psychology of Mathematics Education*: Vol. 4. (pp. 321 – 328). Norwich, UK: University of East Anglia.
- Warren, E. (2003). The role of arithmetic structure in the transition from arithmetic to algebra. *Mathematics of Education Research Journal*, 15, 122-137.