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## Determining the Geometry Problem Posing Performances of Eighth Grade Students in Different Problem Posing Situations\*

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### Abstract

In this study, it was aimed to determine the geometry problem posing performance of eighth grade students in different problem posing situations. For this purpose, the convergent parallel mixed model accepted as one of the mixed method designs was preferred. The participants consisted of 151 eighth grade students from the same school. The "Geometry Problem Posing Test" was used as a data collection tool which consists of a total of six open-ended problems including free, semi-structured, and structured problem posing situations developed by the researchers. An analytical rubric including seven criteria was used for the analysis of the student posed problems. In the research, a significant difference was found among the problem posing situations. In order to examine this difference in depth, the rubric criteria were analysed descriptively with a qualitative approach. After the analysis, it was concluded that the success in structured problem posing situations was lower than the success in other problem posing situations. Therefore, it can be said that the structured problem posing situations can be more challenging for students in geometry problem posing.

**Key words:** Geometry problem posing, Middle school students, Problem posing situations

### Introduction

Rich and useful discussions can take place when students interpret a problem in different ways since they can find different answers. Therefore, not only solving a problem but also posing a problem, classifying the problems and finding different ways to solve the problems are important activities (Walter, 1980). Olkun and Toluk-Uçar (2014) stated that problem solving includes pre-stages such as noticing the problem, determining the limits and characteristics of the problem, recognizing the problem, and posing the problem besides solving the problems determined by others. In this context, it can be said that problem posing has importance in mathematics teaching as well as problem solving.

Stoyanova (1997) defined problem posing as the process in which students form their personal interpretations from concrete situations and formulate them as meaningfully structured mathematical problems based on their mathematical experience. At the same time, problem posing is one of the high level active learning tasks that are important for students' development, and it is a term that suggests a link between higher order inquiry skills and problem based learning (Nardone & Lee, 2011). In mathematics education, the problem posing approach is not only seen as a means of understanding mathematical thinking of students but also as a tool for understanding mathematics (Cai & Middleton, 2015). Kilpatrick (1987) mentioned that the experience of discovering and creating one's own math problems should be a part of every student's education because students take on a new and more active role in their learning when they are encouraged to pose their own problems (Brown & Walter, 2005).

Cai (2003) stated that problem posing is a key element of mathematical discovery and determined that problem posing focuses on the study of examining students' thoughts from different perspectives. In this respect, Tichá

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and Hošpesová (2009) stated that problem posing efforts contribute to a deeper understanding of mathematical concepts. It is stated that problem posing in mathematics courses can be applied as a teaching strategy or as a purpose of mathematics education (Kilpatrick, 1987). Kılıç (2013) stated that problem posing activities as well as problem solving activities should be included in courses. Similarly, Leung and Silver (1997) proposed to make a wide range of problem posing activities in the classroom.

Suggesting the implementation of problem posing activities in classrooms can be based on the idea that there is a strong relationship between problem solving and problem posing (Cai, 1998; Cai & Hwang, 2002; Silver & Cai, 1996). Moreover, it is stated that problem posing can encourage students for creating original ideas (Brown & Walter, 2005). Kar (2014) stated that problem posing is related to conceptual understanding, creativity, problem solving and reasoning skills, and conducting courses with problem-posing activities will contribute to the development of these skills.

Researches on problem posing have revealed that problem posing activities produce positive results in problem solving skills of students (Cai, 1998; Cai & Hwang, 2002; Lavy & Bershadsky, 2003), problem posing skills (English, 1997; Lavy & Bershadsky, 2003), mathematical thinking (Cai, 2003; Silver, 1997), and their tendency towards mathematics (Dickerson, 1999; Kilpatrick, 1987; Silver & Cai, 1996; Turhan & Güven, 2014). Apart from these, Brown and Walter (2005) stated that the problem posing is a critical component in dealing with math anxiety because it is less scary than responding to the problem.

It was stated that problem posing based education significantly improves students' problem solving skills (Abu-Elwan, 2002; Cankoy & Darbaz, 2010; Cifarelli & Cai, 2006; Turhan & Güven, 2014). It was determined that there are positive differences in attitudes and behaviours of students towards mathematics (Turhan & Güven, 2014). Furthermore, it is stated that problem posing activities support the development of advanced mental skills of students such as analysis, synthesis, and inductive thinking (Cai, 2003; Silver, 1997) and increase motivation (English, 1997). As can be seen, it can be said that the implementation of problem posing activities in the classroom contributes to the cognitive and affective development of students.

It is seen that there has been an increase in the number of studies on problem posing when the studies are examined in the literature in recent years. However, it is noticed in general that the conducted studies have focused on the numbers and operations (Bonotto, 2013; Bunar, 2011; Cai, 1998; Stoyanova, 2005), fractions (Atalay & Güveli, 2017; Bunar, 2011; Kar & Işık, 2015; Toluk-Uçar, 2009; Turhan & Güven, 2014), sets (Bunar, 2011; Şengül & Katrancı, 2012), ratio-proportion (Bayazit & Kırnep-Dönmez, 2017; Çelik & Yetkin-Özdemir, 2011), probability (Silber & Cai, 2017; Yıldız & Baltacı, 2015), and algebraic expressions (Akkan, Çakıroğlu, & Güven, 2009; Ünlü & Aktaş, 2017). In the literature, there is a limited number of studies in the field of geometry learning (Abu-Elwan, 2011; Chua & Wong, 2012; Kanbur, 2017; Lavy & Shriki, 2010; Singer, Voica, & Pelczer, 2017; Şengül-Akdemir & Türnüklü, 2017; Türnüklü, Ergin, & Aydoğdu, 2017). One of the frequently encountered situations related to geometry problem posing in the literature is problem posing in dynamic geometry environment. It has been seen that there are studies about this (Abu-Elwan, 2011; Christou, Mousoulides, Pittalis, & Pitta-Pantazi, 2005; Fukuda & Kakihana, 2009; Kanbur, 2017; Lavy & Shriki, 2010; Leikin, 2015), but there is a need for further studies related to problem posing without the environment of dynamic geometry because there is a gap in this field in the literature. It is thought that this study will contribute to the gap in the related field.

In the present study, a research was conducted towards the free, semi-structured, and structured problem posing situations suggested by Stoyanova and Ellerton (1996). In the literature, there has been a limited number of studies investigating different problem posing situations (Çarkçı, 2016; Kılıç, 2013; Kırnep-Dönmez, 2014; Ngah, Ismail, Tasir, & Said, 2016; Özgen, Aydın, Geçici, & Bayram, 2017; Özgen, Aydın, Geçici, & Bayram, 2019). Moreover, there has been no consensus on which problem posing situation is more challenging for students (Kılıç, 2013; Ngah et al., 2016; Özgen et al., 2017). In the current study, it was aimed to determine the geometry problem posing performance of eighth grade students in different problem posing situations. Furthermore, the frequently mentioned statement “different problem posing situations” in the study describes the classification posed by Stoyanova and Ellerton (1996).

## Method

In this study, a mixed research approach that intertwined with quantitative and qualitative research approach was adopted as the research method. Yıldırım and Şimşek (2016) stated that the data collected from different methods confirmed each other and that the credibility of the results was strengthened as one of the important

features of mixed research. In the research, as stated by Creswell (2014), convergent parallel mixed model was preferred as one of the mixed method designs. In this design, qualitative and quantitative data are collected at almost the same time period. However, the data are analysed separately, and the findings are compared with each other. The quantitative data of the study consisted of the points obtained by the students in the problem posing test. Qualitative data were obtained by in depth analysis according to some criteria. These criteria were explained in detail in the data analysis.

### Participants

The study was held at a state school located in the southeast region of Turkey. The study group consisted of 151 students studying in the eighth grade from the same school. A total of 151 students participating in the study were selected with purposeful sampling method.

### Data Collection Instrument

The "Geometry Problem Posing Test" was prepared for triangles and parity-similarity sub-learning areas. These issues related to the geometry learning area of the eighth grade curriculum in Turkey were selected. In the eighth grade, the total number of acquisitions in geometry and measurement learning area and the number of items in the test are shown in Table 1.

Table 1. The total number of acquisitions in the geometry and measurement learning area in the eighth grade and the number of items in the test.

Sub-Learning Areas	Total Acquisitions Number	Frequency of Acquisitions	Number of Items in the Test
Triangles	5	29.4	4
Parity-Similarity	2	11.8	2
Transformation Geometry	4	23.5	-
Geometric Objects	6	35.3	-
Total	17	100	6

Table 1 shows that in the eighth grade mathematics curriculum, there are acquisitions in four sub-learning areas in geometry and measurement learning area. In this study, which aims to reveal the students' geometry problem posing skills, it has been seen that asking problem posing situation about each learning area will reveal too many problem posing situations. Therefore, all problem posing situations are constructed for triangles and parity-similarity sub-learning areas. Another reason why the test is to measure triangles and parity-similarity sub-learning areas is that the triangles sub-learning area occupies a large place in the eighth grade curriculum. In addition, triangles are an important topic in geometry learning area (Türnüklü et al., 2017). In the eighth grade curriculum, it was found sufficient to 6 problem posing situations in order to measure 7 acquisitions in triangles and parity-similarity sub-learning areas. There is no specific relationship between problem posing situations and acquisitions, and is designed suitable to student levels.

The test consisting of six open-ended problem posing activities contains questions about Pythagorean relation and parity-similarity concepts for free problem posing situations. For the semi-structured problem posing situations, the problems involving the angle-edge relationship and the triangle inequality context were asked. For the structured problem posing situations, there were problems including the basic similarity theorem and the auxiliary elements of the triangle. Two academicians specialized in their field were consulted in order to ensure the validity and reliability of the test. A pilot study was also conducted. Language, level, content, and scope of the questions were provided in accordance with expert opinions and pilot study. The "inter-rater adjustment" method was applied in order to ensure the reliability of the test.

### Analysis of Data

Problems posed by students were scored with the rubric developed by Özgen et al. (2017). There are seven criteria in the rubric. The criteria scored according to 4 levels (see Appendix-1). The rubric also covers many criteria from the literature. For example;

- Mathematical expression (Gonzales, 1994; Stoyanova, 2005),
- Grammar and expression (Arıkan & Ünal, 2013; Cankoy & Özder, 2017; Gonzales, 1994),
- Suitability to acquisitions (Gonzales, 1994; Şengül-Akdemir & Türnüklü, 2017),

- Quality and quantity (Chang, Wu, Weng, & Sung, 2012; English, 1998; Kaba & Şengül, 2016; Kılıç, 2013; Silver & Cai, 2005),
- Solvability (Cankoy & Özder, 2017; Çelik & Yetkin-Özdemir, 2011; Silver & Cai, 1996),
- Originality (Chang et al., 2012; Gonzales, 1994; Silver & Cai, 2005),
- Solving the problem posed by the student (English, 1998; Şengül-Akdemir & Türnüklü, 2017).

The answers of the students were scored independently by two people, one was a mathematics teacher and the other one was the researcher of this study. Then the reliability percentage was calculated according to the formula “[ $(\text{Agreement})/(\text{Agreement}+(\text{Disagreement}))\times 100$ ]” suggested by Miles and Huberman (1994). According to this formula, inter-rater compliance was found to be 78%. The proximity of the compliance percentage indicated that performed scoring was consistent. A common decision was reached by discussing when there was an inconsistent score. In this way, the inconsistency was eliminated in the scoring.

The obtained data were analysed descriptively in order to determine the geometry problem posing skills of students. The findings are presented in the frequency and percentage tables. The problems posed by the students were graded according to seven criteria and the levels were presented descriptively that emerged according to each criterion. Moreover, in order to support statistical data and to increase the internal validity of the research, direct quotations were made from the students' answers. Each quote was coded as “S- (Student ID) - (Code of problem posing situation)” in order to indicate which activity belonged to which student and which problem posing situation. “1” was used for free problem posing activities, “2” for semi-structured problem posing activities, and “3” for structured problem posing activities. For example, code S146-3 referred to the quotation from the responses to the structured problem posing activity of the student number 146. The normality of the quantitative data was checked by Kolmogorov-Smirnov test. For normal distribution, coefficients of kurtosis-skewness, Histogram and Q-Q plots were checked and it was seen that the data showed a normal distribution. Therefore, a single-factor ANOVA test was used for the related samples in order to determine whether the students' skills showed a significant difference in different problem posing situations.

## Results and Discussion

The descriptive statistics information of the student scores obtained from the Geometry Problem Posing Test is presented in Table 2.

Table 2. Student scores in terms of problem posing situations

Problem Posing Situation	n	Min.	Max.	$\bar{X}$	SS
Free	151	.00	41.0	21.79	11.78
Semi-structured	151	.00	41.0	19.06	13.57
Structured	151	.00	39.0	16.18	12.26
Total	151	7.00	120.0	57.03	27.75

According to the obtained data in problem posing test, the arithmetical average of the points of the students who participated in the research was calculated as 57.03. A student's total score from the Geometry Problem Posing Test was at least 7.00 and at most 120.0. It was seen that some of the students received “0” point because of incorrect answers to the questions in the geometry problem posing test or the posed problems which did not meet the requirements in the criteria. When each problem posing situation was considered one by one, it was seen that the arithmetic average of the scores obtained from free problem posing activities was more than the semi-structured problem posing and structured problem posing activities.

In order to test whether students' skills in different problem posing situations showed a significant difference, a single-factor ANOVA test was used for the related samples. Analysis results are given in Table 3.

Table 3. Single-factor ANOVA results for related samples of students' scores in geometry problem posing test

Source of Variance	Sum of Square	Sd	Mean of Square	F	p	Meaningful Difference
Interpersonal	38508.94	150	256.72			
Measurement	2381.67	2	1190.83	11.008	.000*	1-3, 2-3
Error	32452.32	300	108.17			
Total	73342.93	457				

1-Free problem posing, 2-Semi-structured problem posing, 3-Structured problem posing

A statistically significant difference was found between students' scores in free, semi-structured, and structured problem posing activities [ $F(2,300)=11.008, p<.05$ ]. The mean score ( $\bar{X}=16.18$ ) in structured problem posing activities was lower than the mean score ( $\bar{X}=21.79$ ) in free problem posing activities and the average score ( $\bar{X}=19.06$ ) in the semi-structured problem posing activities. There were no statistically significant differences between the other problem posing situations. According to this finding, it can be said that the points obtained from free problem posing situations and semi-structured problem posing situations were almost similar, but the obtained points differed in structured problem posing situations.

In order to find an answer to the other sub-problem of the study, the obtained data were separately analysed in terms of seven criteria of rubric. The frequency and percentages of the students' skills in geometry problem posing in terms of criteria are shown in Table 4.

Table 4. Geometry problem posing performances of students in terms of criteria

Criterion	Problem Posing Situation	Level 1		Level 2		Level 3		Level 4		Total	
		f	%	f	%	f	%	f	%	f	%
Using the Language of Mathematics	Free	77	25.5	75	24.8	91	30.1	59	19.6	302	100
	Semi-Structured	127	42.1	22	7.3	58	19.2	95	31.4	302	100
	Structured	131	43.4	59	19.5	67	22.2	45	14.9	302	100
	Total	335	37	156	17.2	216	23.8	199	22	906	100
Grammar and Expression Suitability	Free	103	34.1	67	22.2	47	15.6	85	28.1	302	100
	Semi-Structured	128	42.4	59	19.6	30	9.9	85	28.1	302	100
	Structured	148	49.1	44	14.5	47	15.6	63	20.8	302	100
	Total	379	41.8	170	18.8	124	13.7	233	25.7	906	100
Suitability to Acquisitions	Free	81	26.8	54	17.9	5	1.7	162	53.6	302	100
	Semi-Structured	130	43	15	5	3	1	154	51	302	100
	Structured	134	44.4	37	12.3	11	3.6	120	39.7	302	100
	Total	345	38.1	106	11.7	19	2.1	436	48.1	906	100
Quality and Quantity of Data	Free	84	27.8	12	4	40	13.2	166	55	302	100
	Semi-Structured	130	43.1	6	2	10	3.3	156	51.6	302	100
	Structured	139	46	9	3	32	10.6	122	40.4	302	100
	Total	353	39	27	3	82	9	444	49	906	100
Solvability	Free	81	26.8	44	14.6	11	3.6	166	55	302	100
	Semi-Structured	131	43.3	15	5	2	0.7	154	51	302	100
	Structured	134	44.3	35	11.6	5	1.7	128	42.4	302	100
	Total	346	38.2	94	10.4	18	2	448	49.4	906	100
Originality	Free	119	39.4	117	38.7	41	13.6	25	8.3	302	100
	Semi-Structured	142	47.1	104	34.4	43	14.2	13	4.3	302	100
	Structured	164	54.3	92	30.5	34	11.3	12	3.9	302	100
	Total	425	46.9	313	34.5	118	13.1	50	5.5	906	100
Solving the Problem Posed by the Student	Free	133	44.1	18	5.9	6	2	145	48	302	100
	Semi-Structured	160	53	21	7	4	1.3	117	38.7	302	100
	Structured	186	61.6	19	6.3	6	2	91	30.1	302	100
	Total	479	52.8	58	6.4	16	1.8	353	39	906	100

It is seen that almost 54% of the students who posed problems are at Level 1 and Level 2 according to the skill of using the mathematical language. Using the language of mathematics has great importance when expressing relationships in shapes presented in geometry problems. It is seen that half of the students who posed problems have failed in terms of this criterion. The most problematic problem posing situation is structured problem posing in students' use of mathematical language. It is seen that half of the answers given by students in free and semi-structured problem posing situations are in the Levels 3 and 4. This shows that the participants were more successful in terms of "using the language of mathematics" criterion in free and semi-structured problem posing activities compared to structured problem posing situations.

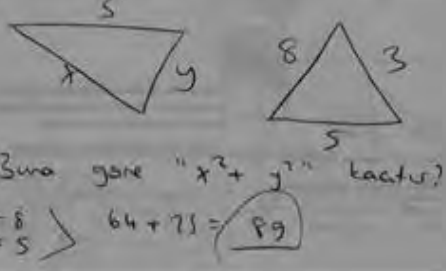
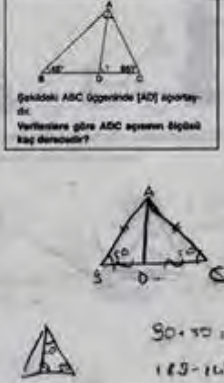
<p>3) "Eşitlikli benzer üçgenlerin eşit ve eşy ölçüleri eş olan köşgenlere eş köşgenler denir." "Açıların ölçüleri eşit karşılıklı benzer üçgenlerin oranları olan köşgenlere benzer köşgenler denir." Yukarıdaki eşit ve benzerlik kurallarından yararlanarak verilenleri, şekle karşılıklı eşit veya benzerlik ile ilgili soruların doğru çözümlerini yazınız. (Serbest Problem Kurma Durumu)</p> 	<p>3) </p> <p>Yanda verilen problemi(leri) gibi öğrenim yardımcı elemanlarından (Açortay-Benzerlik-Yükseklik) oluşan bir problem kurunuz ve çözünüz. (Yapılandırılmış Problem Kurma Durumu)</p>
What is the result of " $x^2+y^2$ " according to this?	Accordingly, what is the result of $\widehat{DAC}$ ?

Figure 1. S82-1 coded activity

Figure 2. S93-3 coded activity

The free problem posing activity posed by the S82 coded student related to parity and similarity issue is shown in Figure 1. Here, the student has drawn two triangles based on the idea of drawing identical triangles to each other. However, the corner pointing the triangles has not been named. It has not been pointed out that the concurrence of the triangle or which sides are identical, and the lengths are not indicated in any unit. Therefore, it was scored as Level 2 according to the criterion of "using mathematical language" since there is lack of the mathematical concepts that should be mentioned in this problem.

The structured problem posing activity of the S93 coded student related to the auxiliary elements of the triangle is presented in Figure 2. The mathematical concepts were correctly stated on the figure in the student posed problem, and corner points of the triangle was indicated as well as describing the triangle as isosceles and stating [AD] as the median. However, it was seen that the student did not explain them as a text. Therefore, the posed problem was found to be incomplete in terms of "using mathematical language" criterion and it was considered as Level 3.

The criterion of grammar and the suitability of an expression is related to being appropriate with the rules of the language, not including incoherency or spelling mistakes. Approximately 42% of the students who posed problems are at Level 1 according to this criterion. This finding shows that almost half of the posed problems were scored as "0" according to this criterion. 25.7% of the posed problems were able to get a full score according to this criterion. It is seen that the students showed similar performances in different problems. In the structured problem posing situations, the percentage of responses in Level 1 (49%) is higher than the other problem posing situations. Here, it can be interpreted that students had more difficulty in the structured problem posing situations.

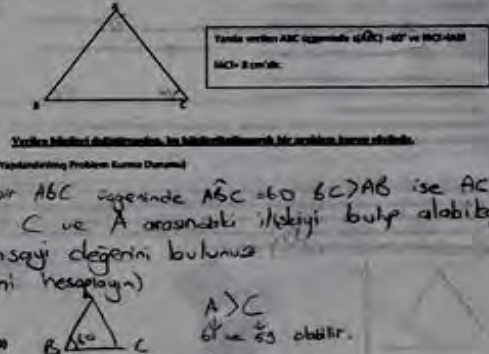
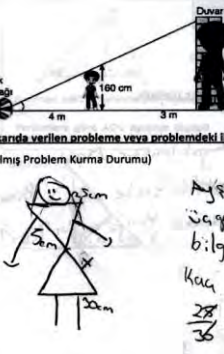
<p>3) </p> <p>Yukarıda verilen problemi(leri) gibi öğrenim yardımcı elemanlarından (Benzerlik-Benzerlik) oluşan bir problem kurunuz ve çözünüz. (Yapılandırılmış Problem Kurma Durumu)</p>	<p>3) </p> <p>Yukarıda verilen probleme veya problemdeki ilişkiye benzer başka bir problem kurunuz ve çözünüz. (Yapılandırılmış Problem Kurma Durumu)</p>
In ABC triangle, $\widehat{ABC}=60^\circ$ , $BC>AB$ , and $AC=8$ , find the whole number value through recovering the relation between C and A.	The dress of Ayşe has consisted of two similar triangles. How many centimeters is the place indicated by "x" according to the information provided in the next picture?

Figure 3. S16-2 coded activity

Figure 4. S73-3 coded activity

The problem in the semi-structured problem posing activity posed by S16 coded student is presented in Figure 3. It is not understood what the student wants to state by writing “the relationship between C and A” in the posed problem. What was meant here should be expressed more clearly in terms of both mathematical and language skills. Therefore, it is thought that there is an incoherency in this problem. Thus, the student posed problem was deemed to evaluate as Level 2 in terms of “grammar and expression suitability”.

The structured problem posing activity posed by the S73 coded student about parity and similarity issue is shown in Figure 4. Instead of posing a similar problem to the fiction in the given problem, the student posed a problem with a different fiction. In the student posed problem, it was stated that there were similar triangles and the one length of a side was asked by giving some side lengths of the triangles. However, when the problem was expressed, it was stated as “place indicated by x”. Instead, it was thought that a more understandable expression should be used. For this reason, the student posed problem was evaluated as Level 3 in terms of “level of knowledge of the grammar and expression” criterion.

Nearly half of the posed problems by the students (48.1%) were evaluated as Level 4 in terms of the suitability of the problems to the acquisitions. It is understood that the student problems are partially enough in terms of “suitability to acquisitions” criteria. 11.7% of the problems evaluated at the Level 2 were not able to be evaluated as Level 4 due to being convenient with the acquisitions but they were considered as deficient expression or data in the problems. The problems at the Level 3 were posed as expected but they were related to another acquisition. When the data obtained from different problem posing situations are examined, it is determined that most of the posed problems in structured problem posing situations are not suitable for acquisitions. In the free problem posing situations, 60% of the posed problems were in Levels 3 and 4. Here, it can be said that students are more successful in free problem posing situations than other problem posing situations.

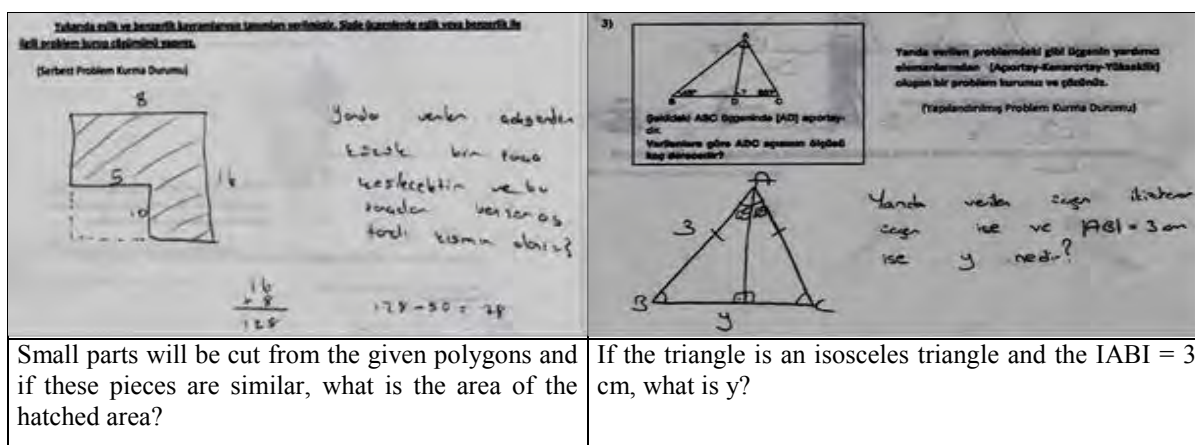


Figure 5. S134-1 coded activity

Figure 6. S34-3 coded activity

The free problem posing activity of the S134 coded student about parity and similarity in triangles is presented in Figure 5. It was seen that the student misunderstood and posed a problem with the area of the rectangle while suppose to pose a problem about congruent and similar triangles in the answer. Although the posed problem was a correct geometry problem, the response of the student was considered as Level 3 since it was related to another acquisition according to the “suitability to acquisitions” criterion.

The structured problem posing activity posed by the S34 coded student about the auxiliary elements of the triangle is shown in Figure 6. Here, the student has drawn an isosceles triangle and has posed a problem aiming to find the other side with the help of a height lowered from the peak through giving one of the isosceles. All the data in the posed problems were given on the figure thought to be solved with the help of these data. Moreover, giving the peak angle as 90° indicates that the triangle was a triangular triangle. It was considered as an appropriate problem for the eighth grade acquisitions and is considered as Level 4.

It is seen that approximately half of the posed problems (49%) are at the Level 4 according to the data quantity and quality criteria. Considering the problems of the students at other Levels, it is understood that most of the students were not able to respond to the activities or there was lack of data in their posed problems. It is understood that the answers evaluated at Level 1 in free problem posing situations are less (27.8%) than the other problem posing situations. It is seen that the given answers to structured problem posing activities are at the 4th Level and those who get full score (40.4%) from the evaluation are less than other problem posing



situations. It is seen that students achieve better results in terms of free and semi-structured problem posing situations in terms of “quality and quantity of data” criterion and they have more difficulty in structured problem posing situations.

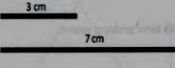
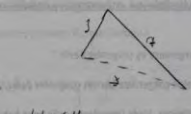
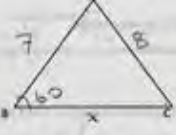
<p>2)</p>  <p>Elinde 3 cm ve 7 cm uzunluğunda iki tahta çubuk bulunan Onur bir tane daha tahta çubuk kullanarak üçgen bir çerçeve yapmak istiyor.</p> <p><b>Yukarıda verilen bilgileri kullanarak bir problem kurunuz ve çözünüz.</b> (Yarı-Yapılandırılmış Problem Kurma Durumu)</p>  <p>Bu üçgen çeşitlenir bir üçgen olabilir mi? Bu üçgen çeşitlenir bir üçgen olabilir mi? Bu üçgen çeşitlenir bir üçgen olabilir mi?</p> <p><math>4 &lt; x &lt; 11</math> 5, 6, 7, 8, 9, 10 5</p>	<p>2)</p>  <p>Verilen veriler ABC üçgeninde <math>\angle A = 60^\circ</math> ve <math> BC  =  AB </math> <math> AC  = 8</math> cm dir. <math> AB  = 7</math> cm dir.</p> <p><b>Verilen bilgileri değiştirilmeden, bu bilgiler kullanılarak bir problem kurunuz ve çözünüz.</b> (Yarı-Yapılandırılmış Problem Kurma Durumu)</p> <p>buna göre <math> BC </math> nin alabileceği değerlerin toplamı kaçtır?</p> <p><math>8 &lt; BC &lt; 15</math> olmalı <math>9 + 10 + 11 + 12 + 13 + 14 = 69</math></p>
<p>Since this triangle is a scalene triangle, how many different wooden bars can come to this?</p>	<p>Accordingly, what is the sum of the values that <math> BC </math> can take?</p>

Figure 7. S23-2 coded activity

Figure 8. S87-2 coded activity

The response of the S23 coded student to the semi-structured problem posing activity related to triangle inequality is presented in Figure 7. The student who posed problems by the given information stated that the triangle to be posed would be a scalene triangle. In this way, the data was diversified in the posed problem. As the data in the posed problem were enough and appropriate, this problem was scored at Level 4 according to “quality and quantity of data” criterion.

The semi-structured problem posing activity of the S87 coded student on the subject of angle-side relationship in the triangles is shown in Figure 8. In addition to the given information here, the student added a side length and asked for the values that the other side could take. Then, the problem was solved through thinking  $[BC]$  will be the longest side. The posed problem was found to be appropriate according to “quality and quantity of data” criterion. Therefore, the student posed problem was seen at Level 4.

Approximately 40% of the posed problems were evaluated as Level 1 according to the solubility criterion. It can be understood that 40% of the student posed problems are the problems that are not possible to be solved. The problems considered as Level 2 (10%) cannot be solved because the data are not enough or appropriate or there is a lack of expression. 2% of the posed problems cannot be solved due to the incoherency or spelling mistakes. It is seen that almost half (49.4%) of the student posed problems are solvable problems. It is seen that the best results in terms of solvability of the problem are obtained in free problem posing situations and worst results are obtained in structured problem posing situations.




<p>3)</p>  <p>Şekilde, Kağan'ın gölgesinin duvardaki görüntüsü ve ışık kaynağı ile Kağan ve duvar arasındaki uzaklık verilmiştir.</p> <p>Buna göre, Kağan'ın gölgesinin boyu kaç cm'dir? 3.20</p> <p><b>Yukarıda verilen probleme veya problemdeki ilişkiye benzer başka bir problem kurunuz ve çözünüz.</b> (Yapılandırılmış Problem Kurma Durumu)</p> <p>Şekilde, Mert'in gölgesinin duvardaki görüntüsü ve ışık kaynağı ile olan uzaklık ve Mert'in gölgesinin boyu verildiğine göre Mert'in gölgesi ile olan uzaklığı kaç m'dir?</p>  <p><math>\frac{160}{4} \cdot \frac{120}{x} = \frac{480}{3} = 160</math> <math>3 = x</math></p>	<p>Şekilde, Kağan'ın gölgesinin duvardaki görüntüsü ve ışık kaynağı ile Kağan ve duvar arasındaki uzaklık verilmiştir.</p> <p>Buna göre, Kağan'ın gölgesinin boyu kaç cm'dir?</p> <p><b>Yukarıda verilen probleme veya problemdeki ilişkiye benzer başka bir problem kurunuz ve çözünüz.</b> (Yapılandırılmış Problem Kurma Durumu)</p> <p>İki kutu bir topun çapı 30 cm'dir. Topun gölgesi duvarda oldu. Gölge kutu top arası 3 cm, top ile gölge arası 2 cm ise gölgenin uzunluğunu bulalım.</p>  <p><math>\frac{3}{5} = \frac{30}{x}</math> <math>x = 50</math></p>
<p>In the figure, the appearance of Mert's shadow on the wall and the distance from the light source, and the length of Mert's shadow are given, how many meters is Mert's distance from his shadow?</p>	<p>The diameter of a light-held ball is 30 cm. As the shadow of the ball is on the wall, let's find the length of the shadow if between the light and the ball is 3 cm and the distance between the ball and the shadow is 2 cm.</p>

Figure 9. S139-3 coded activity

Figure 10. S7-3 coded activity

The problem posed by the S139 coded student is shown in Figure 9. The text in the student posed problem about basic similarity theorem is presented in a comprehensible way. Although the data appeared to be appropriate, the light from the light source was going to create a larger shadow than the person in behind. It was thought that the student who did not consider this did not create a real life related problem. Processing only by numbers did not mean that there was a solvable problem. From this point of view, the student posed problem was evaluated as Level 2 in terms of “solvability” criterion.

The structured problem posing activity posed by the S7 coded student about parity and similarity issue is shown in Figure 10. The student posed a problem like the problem given here, the created problem questions the same relationship by replacing the person in the given problem and replacing it with another object. This posed problem was clearly stated and seen as a solvable problem. Therefore, the student posed problem was evaluated as Level 4 in terms of “solvability” criterion.

The obtained data indicate that only 5.5% of the posed problems are original problems. When the obtained percentages from Level 1 and 2 considered, it is understood that approximately 35% of the posed problems are away from the originality. In other words, it shows that the posed problems are the problems that often encountered in the textbooks or evaluated in the type of exercise. Although there are no significant differences in terms of originality criterion among the different problem posing situations, it is seen that students have more difficulty in structured problem posing activities as in other criterions.

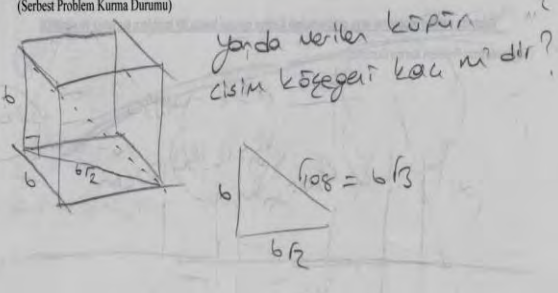
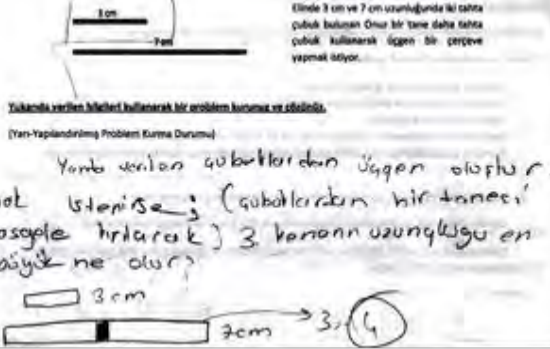
<p>Sizde Pisagor hipotezini içeren bir problem kurunuz ve çözünüz.</p> <p>(Serbest Problem Kurma Durumu)</p> 	<p>Elinde 3 cm ve 7 cm uzunluğunda iki tane çubuk bulunan Ömer bir tane daha tane çubuk kullanarak eşgen bir üçgen yapmayı istiyor.</p> <p>Yukarıdaki verilen bilgilerden bir problem kurunuz ve çözünüz.</p> <p>(Yarı-Yapılandırılmış Problem Kurma Durumu)</p> <p>Yanda verilen çubuklardan üçgen oluşturmak istenirse; (çubuklardan bir tanesini rasgele kırılarak) 3. kenarın uzunluğu en büyük ne olur?</p> 
<p>How many meters are the body diagonal of the cube given?</p>	<p>If you want to create a triangle from the bars (one of them is randomly broken), what is the length of the 3rd edge at most?</p>

Figure 11. S145-1 coded activity

Figure 12. S47-2 coded activity

The free problem posing activity posed by the S145 coded participant about the Pythagorean relation is presented in Figure 11. The student posed problem was expressed in clear way and it was seen that the problem is solvable. While posing the problem, the student, who also stated the solution of the posed problem, relating the problem in geometric objects which is another learning field were indicated the originality of the problem. This answer of the student was evaluated as Level 4.

The semi-structured problem posing activity posed by the S47 coded student about triangle inequality is presented in Figure 12. It is seen that the student guided the problem with a different perspective without changing the given information when posing the problem. When the long bar was broken to create the third side of the triangle, the problem is not an ordinary question, although there is no other option for the third side length. For this reason, the student posed problem was considered as partially original and it was scored as Level 3.

52.8% of the posed problems indicates that the students left the solution empty or made it completely wrong. 39% of the posed problems were solved by the student in a full correct manner. 48% of free problem posing situations, 38.7% of semi-structured problem posing situations, and 30.1% of structured problem posing situations are fully achieved in terms of different problem posing situations. Here, it can be said that students are more successful in solving the problems posed in free problem posing situations than other problem posing situations. The lowest success in solving the posed problems was provided in structured problem posing situations.

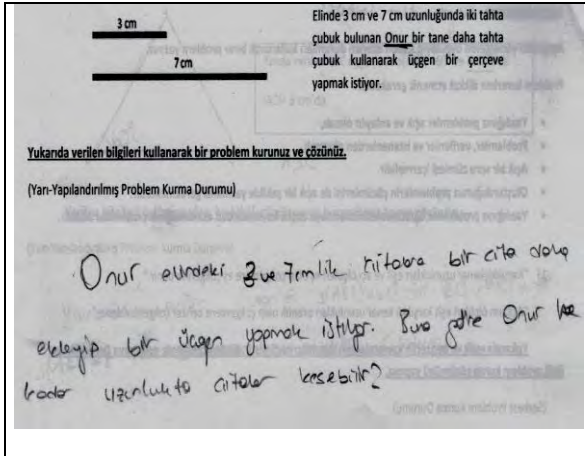
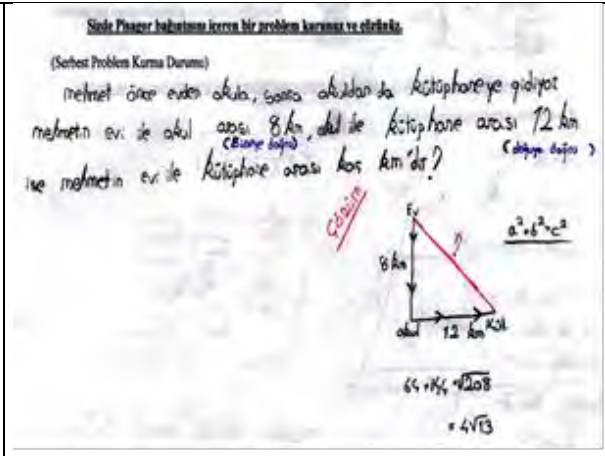
	
<p>Onur wants to add a lath to the 3 and 7 cm laths in his hand and make a triangle. Accordingly, how long can Onur cut the laths?</p>	<p>Mehmet goes from home to school first, and then from school to library. The distance between Mehmet's school and home is 8 km (towards the south), and the distance between Mehmet's school and library is 12 km (east), then what is the distance between Mehmet's house and the library?</p>

Figure 13. S143-2 coded activity

Figure 14. S36-1 coded activity

The semi-structured problem posing activity of the Ö143 coded student about the triangle inequality is shown in Figure 13. Here, the student has posed a solvable problem, but leaves the solution blank. For this reason, the student posed problem was evaluated as Level 2 in terms of being solved by the student.

The free problem posing activity of the S36 coded student about the Pythagorean relation is shown in Figure 14. The student expressed the posed problem through a clear language and posed a problem that meets the criteria that should be in a problem. Complete operation was performed in the solution of the posed problem. Therefore, the solution of the student posed problem was graded as Level 4.

## Conclusion

In this study, the performance of geometry problem posing of eighth grade students was investigated in different problem posing situations. Research findings showed that the products of students' problem posing activities were generally at the Level 1 or 2 according to the evaluation criteria. Similarly, it has been seen that there are many studies concluding that secondary school students' achievements in problem posing activities are not at the desired level (Gökkurt, Örnek, Hayat, & Soylu, 2015; Kar, 2014; Özdişçi & Kaba, 2018; Özgen et al., 2017). In the literature, there are also studies concluding that students are successful in problem posing activities (Cai, 2003; Lin & Leng, 2008; Şengül-Akdemir & Türnüklü, 2017).

Gökkurt et al. (2015) stated that problem posing skills of eighth grade students were not at the desired level. In addition to this, it was stated that most of the students exactly copied the problem by changing the numerical values in the given problem or posed illogical problems that did not have a solution. In a study by Çarkçı (2016), it was stated that the students of fourth grade had difficulty in posing problems in different situations. Kar (2014) pointed out that the success of secondary school students was low in posing valid problems for collection with fractions and that the factors causing such deficiencies should be determined in order to eliminate such deficiencies in students. The similarity between the results of many studies in the literature and this study is that the student achievement is not at the desired level and this emerges as a subject that is needed to be considered. On this issue, Gökkurt et al. (2015) suggested to make problem posing activities to improve student problem posing skills and to give feedback to the students about the mistakes they made in the problem posing process. Moreover, in order to reach the desired level of problem posing skills of the students, first, problem solving should be endeared to the students. As a final step in problem solving, problem posing skills will be contributed by administering problem posing activities.

When the problem posing situations were examined on their own, it was observed that the average of the scores reached in free problem formation activities was more than the average in other problem posing activities. When

the problem-posing situations were examined statistically, a statistically significant difference was found between the points of geometry problem posing in students' different problem posing situations. A significant difference was detected between structured problem posing situation and other problem posing situations. It can be said that students have more difficulty in structured problem posing activities while posing geometry problems. When the literature is examined, it is observed that there are studies supporting this result (Çarkçı, 2016; Kanbur, 2017; Kılıç, 2013) or there are also other views (Bayazit & Kırnep-Dönmez, 2017; Ngah et al., 2016; Özgen et al., 2017). Data collection tools and study subject or sample can be effective in the formation of these differences. Kanbur (2017) and Kılıç (2013), who worked with primary school students and teacher candidates, found concordant results with this study. However, Özgen et al. (2017) found no significant difference between the problem posing situations. In another study, Ngah et al. (2016) stated that secondary school students experienced more difficulties in posing free problems.

In order to examine the reasons of the result of a significant difference among the problem posing situations, the criteria included in the scoring key were examined separately. The aim was to reveal the students' skills through handling the posed geometry problems in terms of seven criteria of the rubric and to determine in which problem posing situation the students had more difficulty. Almost half of the student posed problems were found to be at levels 1 and 2 in the criterion of using the mathematical language. It is stated by various researchers that mathematical language plays an important role in the process of learning and teaching geometry (Cansız-Aktaş & Aktaş, 2012; Sarama & Clements, 2009). It can be said that using the language of mathematics has a great importance while expressing the relations with the shapes presented in geometry problems. In using the mathematical language, the least achieved problem posing situation is the semi-structured problem posing activities. It was found that more than half of the students' answers in free and semi-structured problem posing situations were at Levels 3 and 4.

The criterion of “grammar and expression suitability” is related to the fact that the wanted to be expressed need to be in accordance with the language rules and not be made incoherency or spelling mistakes. Almost 42% of the student posed problems were at Level 1 according to this criterion. Similar results were observed in the literature (Arıkan & Ünal, 2013; Yıldız & Özdemir, 2015). According to this criterion, it was seen that the students performed similar performances in different problem posing situations. However, in structured problem posing situations, the percentage of responses at Level 1 was higher than other problem posing situations.

It is concluded that the student posed problems were at the intermediate level according to the suitability with acquisitions criteria. This result is in parallel with the results of the study conducted by Şengül-Akdemir and Türnüklü (2017). Şengül-Akdemir and Türnüklü (2017) determined that 54.5% of the student posed problems were curriculum dependent problems. In the current study, it was determined that most of the student posed problems in structured problem posing situations were not suitable for the acquisitions. In free problem posing situations, it was found that most of the posed problems were found to be suitable for acquisitions. In terms of suitability for acquisitions, it can be said that students are more successful in free problem posing situations than other problem posing situations. The reason for this difference was thought to be derived from the allowance of activities given in free problem posing situations to pose too many different problems related to the subject.

When the posed problems were examined in terms of “quality and quantity of data”, it was seen that almost half of them were at Level 4. When the problems at other levels considered, it was understood that most of the students could not respond to the activities or they had a lack of data in posed problems. When the student posed problems examined in terms of the quality and quantity of data, it was concluded that they had better results in free and semi-structured problem posing situations and they had more difficulties in structured problem posing situations. Türnüklü et al. (2017) found that the percentage of the posed problems decreased as the mathematical quality increased, and students had difficulty in writing high quality mathematical problems. Gökkurt et al. (2015) stated that eighth grade students wrote problems using the given problems only by changing the numerical value or posed illogical and unsolvable problems. Similarly, Özdişçi and Kaba (2018) found that secondary school students were inadequate in problem posing. They also emphasized that this was due to the inadequate use of problem solving stages.

It was concluded that almost half of the student posed problems were solvable problems. The solvability of the posed problem and the control of whether including logical errors are the most important factors that should be taken into consideration in the process of problem formation (Kırnep-Dönmez, 2014). According to different problem posing situations, in terms of solvability of the problem, it was seen that the best results were found in free problem posing situations while the worst results were obtained in structured problem posing situations. In the study analysing the middle school students posed problems, Silver and Cai (1996) determined that the large number of the students posed solvable problems and some of the posed complex problems. Yuan and Sriraman

(2011) emphasized that content knowledge of the students had a great impact on their problem posing success. In this study, it was considered that some of the students' missing information about the content of the subject was a big obstacle in the formation of solvable problems.

Another result of this study was that the very small number of the posed problems was original. It was determined that the student posed problems were the problems frequently encountered in the textbooks or evaluated in the type of exercises. Although there were no significant differences in terms of originality criterion among different problem posing situations, it was observed that students had more difficulty in structured problem posing activities as in other criteria. Restriction effect of the structured problem posing situations was the reason of this situation. Bayazit and Kırnap-Dönmez (2017) stated that the problems posed by the mathematics teacher candidates were the problems far from originality and creativity. As it is seen, both student and teacher candidate posed problems are generally composed of problems in a routine manner. In order to develop this skill of the students, it can be said that the activities that will stimulate the cognitive skills of the students should be applied in the courses. Özgen et al. (2019) mentioned that in order to solve this problem, students should face interesting or daily life problems.

In this study, the students left the solution blank or solved them in a completely wrong way in large number of problems posed by them. Only 39% of the posed problems were correctly solved by the students. It was observed that the students who were able pose problems had difficulties in solving posed problems. When it was examined according to different problem posing situations, it was concluded that the students were more successful in solving the problems they posed in free problem situations and they were less successful in structured problem posing situations. It is thought that facing many problems posed by students in their daily life make contributions such as “recognizing and determining the problem”. However, it can be interpreted that problem solving skills are still not developed. In the literature about this situation, it is stated that problem based mathematics teaching can be applied in order to increase the problem solving performance of students (Dickerson, 1999; Turhan & Güven, 2014).

As a result of the present research, it has been concluded that the success in structured problem posing situations is lower than the success in other problem posing situations. Therefore, it can be said that the structured problem posing situations are more challenging for students in posing geometry problems. The restriction of students' creativity skills in structured problem posing situations is thought to be the reason of this result. Kanbur (2017) stated that mathematical logic, data quality, instructions in posed problems, data quantity, and solvability criteria developed through the free problem posing situation from the structured problem posing situations in geometry problems posed by the pre-service teachers in dynamic environment. Kılıç (2013), in the study conducted with teacher candidates, found that teacher candidates experienced difficulties most in the structured problem posing situations and least in free problem posing situations and stated that this could be caused by the structure of problem posing situations. Considering the results of other studies (Ngah et al., 2016; Özgen et al., 2017), there may be many reasons why the results of the present study are compatible or different with other studies in the literature. In particular, the human factor can be shown as well as time and subject difference, different class level or different data collection tools. Because, even if the students are at the same class level, their performance on any subject may change over time and from region to region. Considering that different researchers can use different measurement tools, it can be considered as a normal situation that the results of research differ on similar subjects.

## **Recommendations**

Based on the results of the research, the following recommendations are made:

- It has been seen that the student posed geometry problems are generally used in short sentences or only question marks are used in terms of mathematical language. So, teachers are suggested to express a problem in mathematics lessons clearly and pay attention to the use of symbols. For this purpose, teachers should provide opportunities to support students to use mathematical language.
- Experimental studies can be carried out to examine students' difficulties in the process of geometry problem posing. In this way, students' geometry problem posing skills and detailed information can be obtained about the difficulties encountered in the process of geometry problem posing.

- It has been noteworthy that students do not pose any original problems. In order to develop this skill, students should be introduced to problem posing activities from an early age. They can gain creativity when mental skills are mobilized.
- It is seen that students have difficulty in solving the geometry problems students have posed by themselves. In order to overcome this problem, problem solving and problem posing can be handled together. Similar problems to solved problems can be posed or solutions can be made for the posed problems.
- In this study, the problems that the eighth grade students posed for the triangles and the parity-similarity sub-learning area were examined. Problem solving skills of secondary school students about other sub-learning areas of geometry can be investigated.
- In other studies where there are different problem posing situations, students' opinions can be consulted.
- In this study, the classification stated by Stoyanova and Ellerton (1996) has been used and there are different problem posing strategies presented by other researchers in the literature. Secondary school students' skills to pose geometry problems can be examined in terms of other strategies.

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## Appendix-1. Rubric towards Evaluation of Problem Posing Skills

	0 Point	1 Point	2 Points	3 Points
<b>Ability to use mathematical language (symbol, notation, and so on) correctly</b>	Null	There is an error in the use of the mathematical language (or concepts).	The mathematical language is/ (or concepts are) used correctly but incompletely.	The mathematical language is/ (or concepts are) used precisely and correctly.
<b>Compliance of the text of the question with grammar rules, whether it contains an incoherency or spelling mistake</b>	Empty, no text, or incoherency or misspelling.	There is no mistake in writing, but there is incoherency.	There is no incoherency, but the writing is wrong.	There are no incoherency and spelling mistakes.
<b>The suitability of instructions used while referring to the operations to be done in problem or stating the problem to the acquisitions</b>	Empty or unclear how the problem will be solved.	The operation to be done for the solution of the problem is suitable for the acquisitions but it is incomplete/wrong.	The operation to be done for the solution of the problem is not suitable for the acquisitions but it is complete/error free.	The operation to be done for the solution of the problem is suitable for the acquisitions and it is complete/error free.
<b>In order for the problem to be solved, the amount of data and expressions contained in the problem, the logical/operational suitability, and the significance of the result</b>	Empty, cannot be understood because it is not clear how to solve it, or there is no data available because there is no shape-text transfer.	There are both invalid and missing data or too much data-expression.	The data is incompatible or there is missing/more data-expression.	The data are adequate and appropriate.
<b>Accessibility of the problem to the desired result (Solvability)</b>	Empty or not be solved because data in the figure cannot be mathematically expressed in text form	Cannot be solved because it is not appropriate or sufficient data, or lack of expression	Although the data are appropriate and sufficient, they cannot be solved because of writing errors and incoherency.	Solvable.
<b>The scenario of the problem text, the originality in terms of the operation steps in order to reach a solution</b>	Empty or cannot be detected	The problem is pretty ordinary (Type of always been to).	The problem is partly original (so unique that it can be distinguished from the ordinary/classical question type).	The problem is largely original (a type of question whose originality is kept on the front line when it is produced, but not in textbooks or other sources).
<b>Case of solving student posed problem</b>	Empty	Could not apply the givens and desired to the solution	The problem is understood correctly and solved but there is an operation error.	The problem solved correctly.