Remedial and Special Education
2020, Vol. 41(4) 194–206

DOI: 10.1177/0741932519887235
rase.sagepub.com

Applying the Universal Design for Learning Framework to Mathematics Instruction for Learners With Extensive Support Needs

Jenny R. Root, PhD, BCBA[1], Sarah K. Cox, PhD[1], Alicia Saunders, PhD[2], and Deidre Gilley, MS[3]

Abstract

The purpose of this study was to evaluate the effect of a mathematics intervention that utilized the Universal Design for Learning (UDL) framework on mathematical problem solving skills for three middle school students with extensive support needs (ESN). Participants were taught to solve percent of change word problems related to personal finance (calculating the final price after leaving a tip or purchasing a discounted item). Visual analysis of the multiple probe across participants design indicated a functional relation between the mathematics intervention and an increase in mathematical problem solving skills. Results are discussed in terms of acquisition and generalization of mathematical problem solving skills. Implications for application of the UDL framework to mathematics instruction for learners with ESN are discussed.

Keywords

autism, general curriculum access, intellectual disability, mathematics, universal design for learning

Mathematical competence is not only crucial for academic success but contributes to independence in daily life (Browder et al., 2018). Positive school experiences in mathematics influence postsecondary and vocational opportunities (Wang, 2013). All students benefit from increased mathematical competence. Yet some students, such as those with Extensive Support Needs (ESN), face challenges when it comes to accessing mathematics instruction and "showing what they know." These may include difficulties with executive functioning, metacognition, working memory, and limited literacy or numeracy skills (Browder et al., 2018; Spooner, Saunders, Root, & Brosh, 2017). Our use of the term ESN is referring to students who participate in their state's alternate assessment which is aligned to alternate achievement standards (AA-AAS), who may be eligible for special education services under the categories of intellectual disability, multiple disabilities, and autism (Quirk, Ryndak, & Taub, 2017). The past decade of research has shown these students can make progress in higher level mathematical concepts when instruction explicitly supports these needs (Spooner, Root, Saunders, & Browder, 2019).

Universal design for learning (UDL) is a framework for instructional design that identifies barriers in the learning environment and curriculum that prevent meaningful access and increases opportunities for learning by providing multiple options to reach and measure goals (Meyer, Rose, & Gordon, 2014). The UDL framework consists of nine guidelines and 31 checkpoints that can be applied flexibly to provide multiple means of engagement, representation, and action/expression (CAST, 2018). The inclusion of UDL in federal legislation (e.g., Every Student Succeeds Act of 2015) reflects the magnitude of importance that has been placed on UDL for providing access to the general curriculum. In fact, a panel of leaders at the 2015 National Goals conference identified UDL as critical to improving the education of learners with intellectual and developmental disabilities (Thoma, Cain, & Walther-Thomas, 2015). As such, panelists urged researchers to provide empirical evidence of its effectiveness.

One example of using the UDL framework to design instruction that incorporates research-based practices can be seen in the work of Browder, Mims, Spooner,

[1]Florida State University, Tallahassee, USA
[2]University of North Carolina at Charlotte, USA
[3]Powell High School, TN, USA

Corresponding Author:
Jenny R. Root, Assistant Professor in Special Education, School of Teacher Education, Florida State University, 1114 W. Call St., Tallahassee, FL 32306, USA.
Email: jrroot@fsu.edu

Ahlgrim-Delzell, and Lee (2008). Researchers used the UDL framework to evaluate barriers and solutions for the engagement of elementary students with ESN within a shared story intervention. During baseline, students were provided with accessible adapted books and engaged by their teacher in shared story reading. A 16-step task analysis (TA) from prior research directly measured literacy behaviors (e.g., select book, use switch to complete repeated storyline, make prediction, answer comprehension question). Analysis of baseline data showed that simply providing accessible materials, such as a read aloud, and opportunities to respond did not increase independence in students' literacy behaviors. Prior to intervention, the researchers and teacher worked together to individualize the TA using the UDL framework to improve student responding. For example, team members asked the following: (a) "Is there a better way to present this opportunity?" (i.e., representation), (b) "Is there an alternative way the student could respond?" (expression), and (c) "What prompt could be used to get the student to make the response?" (engagement). By considering each step of the TA in terms of representation, expression, and engagement, team members were able to identify barriers and corresponding research-based practices to support student learning. For example, the team modified the augmentative and alternative communication (AAC) devices of two students and the placement of response items for a third to address barriers to expression. Browder et al.'s intervention data show that once participants were provided with individualized systematic instruction on how to engage in each step of a shared story, they were able to increase independent literacy behaviors.

In the area of mathematics instruction for learners with ESN, Modified Schema-Based Instruction (MSBI) uses the principles of UDL to identify barriers and create opportunities to support the specific needs of learners. Figure 1 demonstrates the alignment between MSBI and the UDL guidelines and checkpoints (CAST, 2018). Spooner et al. (2017) explain that MSBI was developed with the intention of adding supports to traditional schema-based instruction (Powell, 2011) to overcome barriers students with ESN face in accessing problem solving instruction. The principles of UDL are intended to be used in this exact fashion. MSBI supports students in (a) accessing the problem, (b) conceptually comprehending the problem and mathematical content, (c) procedurally solving the problem, and (d) generalizing problem solving skills in multiple ways (Spooner et al., 2017).

MSBI emphasizes the use of thematic word problems to provide opportunities to apply targeted mathematical calculations to real-world situations, reflecting a contextualized approach to mathematics instruction (Root, Cox, Hammons, Saunders, & Gilley, 2018). This contextualization optimizes the relevance of mathematics skills (Checkpoint 7.2) and may influence transfer and generalization (Checkpoint 3.4). For example, Root, Saunders, Spooner, and Brosh (2017) used MSBI to teach three middle school students with moderate intellectual disability to solve personal finance word problems involving finding the final price when leaving a tip or using a coupon. The targeted mathematical skill was presented in a personally relevant context. Participants were taught to solve and discriminate between operations of addition and subtraction (Checkpoint 3.2) and generalize to different types of calculators (Checkpoint 5.2).

Although there is growing evidence that MSBI is an effective practice for improving mathematical problem solving skills for students with ESN (Spooner et al., 2017), the targeted mathematical skills have primarily been limited to addition and subtraction in one-step problems. For example, although Root et al. (2017) found MSBI to be effective in teaching personally relevant and grade aligned mathematics, many real-world scenarios require calculating a tip or discount as a percent of the original cost (e.g., 15% tip or 20% discount). This task involves completing multiple operations (multiplication and addition or subtraction). This increase in complexity requires mastery of more difficult mathematical skills and puts additional demands on students' executive functioning and self-regulation, thus requiring a greater level of effort and persistence. The UDL guidelines explicitly emphasize providing options for executive functions and self-regulation to provide multiple means of action and engagement. Therefore, they can be utilized to support students in these areas.

The purpose of this study was to draw on UDL guidelines to remove barriers and support learners in learning a complex mathematical skill (percent of change) to extend the work of Root et al. (2017). Following Browder et al.'s (2008) example, we systematically identified barriers and examined each guideline and checkpoint in the UDL framework to identify corresponding practices to support student learning (see Figure 1). We sought to answer the following research questions:

Research Question 1: Is there a functional relation between a universally designed mathematics intervention and an increase in mathematical word problem solving skills by middle school students with ESN?

Research Question 2: Do effects of a universally designed mathematics intervention on problem solving skills generalize to novel problems unrelated to money for middle school students with ESN?

Method

Participants

Institutional Review Board approval was received from the university and school district prior to recruitment. Students

Multiple Means of Engagement	Multiple Means of Representation	Multiple Means of Action
Options for Recruiting Interest	Options for Perception	Options for Physical Action
Student selection of daily theme *(7.1 Optimize individual choice & autonomy)*	Color coding of graphic organizer, text to speech with human voice *(1.1 Offer ways of customizing the display of information)*	Support on request, option to use stylus or finger, & "speak or show" response option *(4.1 Vary methods for response & navigation)*
Real-world themes & video anchors from student's community *(7.2 Optimize relevance, value, & authenticity)*	Text to speech with human voice *(1.2 Offer alternatives for auditory information)*	
Token economy indicated number of problems to solve for the day *(7.3 Minimize threats & distractions)*		
Options for Sustaining Effort & Persistence	Options for Language & Symbols	Options for Expression & Communication
Students state goal at beginning of each lesson *(8.1 Heighten salience of goals & objectives)*	Explicit instruction in vocabulary & symbols *(2.1 Clarify vocabulary & symbols)*	Calculator provided *(5.2 Use multiple tools for construction & composition)*
Behavior specific praise *(8.4 Increase mastery-oriented feedback)*	Considerate text & text to speech with human voice *(2.3 Support decoding of text, mathematical notation, & symbols)*	System of least prompts used to gradually release support with increasing independence *(5.3 Build fluencies with graduated levels of support for practice & performance)*
	Represented information from word problem onto schema *(2.2 Clarify syntax & structure)*	
Options for Self-Regulation	Options for Comprehension	Options for Executive Functions
Daily goal setting, self-monitoring, & self-graphing of progress in Excel *(9.3 Develop self-assessment & reflection)*	Video anchors to supply background knowledge with daily discussion of student experiences *(3.1 Activate or supply background knowledge)*	Goal setting instruction *(6.1 Guide appropriate goal setting)*
	Explicitly taught discrimination between problem types based on task analysis & rule *(3.2 Highlight patterns, critical features, big ideas, & relationships)*	Task analysis & instructor think alouds *(6.2 Support planning & strategy development, 6.3 Facilitate managing information & resources)*
	Explicit opportunities to generalize learning *(3.4 Maximize transfer & generalization)*	Daily goal setting, self-monitoring, & self-graphing of progress in excel *(6.4 Enhance capacity for monitoring progress)*

Figure 1. Alignment of intervention to UDL framework based on CAST (2018) Universal Design for Learning guidelines version 2.2.
Note. UDL Checkpoints are noted in italics. UDL = universal design for learning.

were eligible to participate in the study based on the following criteria: (a) identified as having ESN by receiving special education services under the eligibility areas of intellectual disability or autism and participating in the state AA-AAS, and (b) performed adequately on the researcher-created screening measure. After consent and assent were obtained, researchers administered the mathematics subtest of the Woodcock Johnson Tests of Achievement, 3rd edition (WJ-III; Woodcock, McGrew, & Mather, 2001), the Everyday Mathematics and Attitude toward Math subtests of the third edition of the Test of Mathematical Achievement (TOMA-3; Brown, Cronin, & Bryant, 2012), and a researcher-created screening tool.

The screening tool allowed researchers to ensure participants had sufficient mathematical skills to access the instruction but had not already mastered the targeted skill. Results of the screening were also used to tailor instruction to individual needs of participants. The tool assessed participants' ability to (a) receptively and expressively identify prices (e.g., US$4.25, US$3.00, US$19.89), (b) identify and draw shapes, (c) transfer numbers to an iPhone calculator, (d) solve double-digit addition and subtraction problems with and without decimals using an iPhone calculator, (e) solve multiplication problems with and without decimals using an iPhone calculator, (f) identify and describe the purpose of receipts and coupons, and (g) solve percent of change word problems using a coupon (e.g., 20% off a haircut) or leaving a tip (e.g., 15% tip to a hairstylist). Participants met eligibility criteria if they were able to complete Items a through c with at least 75% accuracy and Item g with no more than 25% accuracy.

Three middle school students participated. Leona (a self-selected pseudonym) was a 13-year-old Black female student in the eighth grade receiving special education services under the categories of intellectual disability and other health impairment. She participated in her state's alternate assessment. Standardized assessment information regarding Leona's adaptive or cognitive ability was not available from the school. Leona received additional services in language therapy. Based on the WJ-III, she had an overall mathematical standard score of 50 (1st percentile), with strengths in applied problems (2nd percentile) and weakness in math fluency (<1st percentile) and calculation (<1st percentile). Leona's knowledge of everyday mathematics was ranked as "very poor" using the TOMA-3 (<1st percentile). Leona's observed barriers were in self-regulation and self-monitoring.

Faith (a self-selected pseudonym) was a 12-year-old Black female in sixth grade receiving special education services under the categories of intellectual disability and language impairment. She participated in her state's alternate assessment. On the Wechsler Intelligence Scale for Children (WISC-V), faith had an overall full-scale intelligence quotient (IQ) of 54. Based on the WJ-III, she had an overall mathematical standard score of 64 (1st percentile), with strengths in applied problems (12th percentile) and weakness in calculation (<1st percentile). Faith's knowledge of everyday mathematics was ranked as "very poor" using the TOMA-3 (1st percentile). Observed barriers for Faith were in her ability to monitor her own progress and a lack of autonomy or independence.

Uma (a self-selected pseudonym) was a 15-year-old White female in the eighth grade receiving special education services under the categories of autism and intellectual disability. She participated in her state's alternate assessment. Standardized assessment information regarding Uma's cognitive or adaptive functioning was not available. Other related services Uma received, included language therapy, occupational therapy, and physical therapy. Her fine motor skills were weaker than the other two participants, and she expressed some frustration with the instructional materials (i.e., stylus). Based on the WJ-III, she had an overall mathematical standard score of 26 (<1st percentile), with strengths in applied problems (0.5 percentile) and weakness in calculation (<1st percentile) and math fluency (<1st percentile). Uma's knowledge of everyday mathematics was ranked as "very poor" using the TOMA-3 (<1st percentile). Uma's observed barriers were primarily in fine motor skills and self-monitoring.

Settings and Interventionists

Sessions for each participant occurred one-on-one 3 days per week and lasted approximately 25 min. Sessions were conducted in a private room at a public school in the southeastern United States. Participants received daily whole and small-group mathematics instruction from their special education teacher, who used Unique Learning Systems curriculum, as adopted by the school district. During the time of the study, instruction focused on addition and subtraction of double-digit numbers and did not include word problem solving instruction. Two members of the research team (second and fourth authors) were interventionists. Both were licensed teachers enrolled in graduate programs in special education. The first author used role-play and modeling to train both interventionists to 100% fidelity using an 11-item checklist.

Targeted Mathematics Skills

The targeted word problems for this study were all percent of change problems, which align with middle grades mathematics standards. Two mathematics content standards from the Common Core State Standards (National Governor's Association Center for Best Practices & Council of Chief State School Officers, 2010) were the focus: (a) "Solve problems involving finding the whole, given a part and the percent" (6.RP.A.3.C), and (b) "Use proportional relationships to solve multistep ratio and percent problems" (7.RP.A.3).

Independent Variable

A universally designed mathematics treatment package was used to teach the percent of change word problems. We used the UDL framework to identify components of MSBI that could be further individualized to address identified barriers of these participants to this complex problem solving task (Cook & Rao, 2018). Figure 1 demonstrates how components of the intervention align with the UDL framework.

Materials. Materials included the following: (a) researcher-created video anchors; (b) electronic grid of community themes (Checkpoint 7.1, 7.2); (c) electronic worksheets displaying TA (Checkpoint 6.2, 9.3), a word problem (Checkpoint 2.3), and the graphic organizer (Checkpoint 1.1, 2.2); (d) iPad with stylus (Checkpoint 4.1, 5.2); (e) calculator app on iPhone (Checkpoint 5.2); and (f) self-graphing template on excel workbook (Checkpoint 6.1, 6.4, 9.3).

Themes and video anchors. Participants selected one of 15 community-based themes from a 3 × 5 grid display on the *GoWorksheet App* in each sesssion and were required to select a new theme each day until all themes had been selected (Checkpoint 7.1). The research team created 20 to 30 s video anchors for each theme to provide real-world examples of using the targeted percent of change skills, such as using a coupon at the grocery store or tipping at the nail salon (Checkpoint 7.2, 3.1). Participants watched the videos on an iPhone.

Worksheets and word problems. Electronic worksheets on the *GoWorksheet App* were displayed on an iPad during all sessions. Worksheets contained a six-step TA, percent of change word problem aligning to the selected community theme, and a graphic organizer (see Figure 2). All text on the worksheet contained a text-to-speech option with a human voice (Checkpoint 1.2). Adhering to recommendations from Spooner et al. (2017), word problems followed a specific pattern to reduce cognitive load (Checkpoint 2.3). All problems were written in four lines. The first line introduced the characters and theme. The second line provided the total cost of the item or service. The third line indicated the percent and type of change (i.e., tip or sale/discount). The final line asked, "What will the total cost be?" Dollar amounts and percent of change amounts were whole numbers appropriate for the context of the problem.

Generalization worksheets followed the same structure, but the participants were not able to select a theme, and the word problems contained percent of change problems that were not tip or sale (Checkpoint 3.4). For example, "The doctor measured Arnold's weight at his physical. Last year he weighed 90 pounds. This year he weighs 5% more. How much does Arnold weigh now?"

Dependent Variables and Measurement

Data were collected during probes and instructional sessions. The primary difference between probes and instructional sessions was the absence of prompting and feedback on probes. Participants solved two problems from each type (percent increase or decrease) during probe sessions for a total of four problems solved each session. Probes were administered during baseline, at the end of each intervention phase, and during maintenance as described in the procedures. Students solved two word problems of the targeted problem type in each instructional session. Data were taken to determine participants' progress toward mastery of the targeted skills for each intervention phase (see procedures). Participants solved two problems of each type during generalization probes for a total of four problems. Generalization probes were administered in baseline and at the end of each intervention phase.

The primary dependent variable was mathematical problem solving skills, measured by the total number of points a participant received by independently performing the six steps of the TA. Participants could earn a total of 11 points for each problem, as three of the steps contained multiple behaviors which were measured separately (see Table 1). Participants could earn a total of 22 points for each problem type across the two problems in each session. Measuring steps of the TA allowed for analysis of progress in each step of the chained task provided evidence of skill growth and helped diagnose any errors in the problem solving process. This is especially important for students with ESN for whom mastery of the entire skill (i.e., solving the problem) will likely not be immediate upon entering intervention.

The secondary dependent variable was generalization of problem solving skills, measured by the total number of points a participant received by independently performing the six steps of the TA when given a word problem depicting percent of change in a novel context (i.e., not tip or sale). Participants could earn the same 11 points for each generalization problem, resulting in a total of 22 possible points for each problem type (percent increase or decrease).

Experimental Design

A multiple probe across participants design (Ledford & Gast, 2018) was used to investigate the effectiveness of a universally designed mathematics intervention on the mathematical problem solving skills of the three participants. The implementation of the design adhered to the criteria established by the What Works Clearinghouse (WWC; Kratochwill et al., 2013). There were three experimental conditions (baseline, intervention, and generalization) for determining existence of a functional relation between the intervention and dependent variables. The intervention condition consisted of three phases, including percent increase, percent decrease, and discrimination. A three-session probe was conducted between each intervention phase to measure maintenance of treatment effects and, in the first probe, generalization of treatment effects to the untaught problem type (percent decrease). A one-session generalization probe was also conducted once in baseline and after participants met mastery in each intervention phase to assess generalization to percent of change problems in a novel context (Ledford & Gast, 2018). The third and final series of probes following discrimination training also served as a maintenance measure.

All participants entered baseline simultaneously. A response guided approach was used to make decisions about introduction of participants to intervention (Ledford & Gast, 2018). The first participant (Leona) entered intervention after she had a stable pattern of responses over a minimum of five data points. She was selected to go first

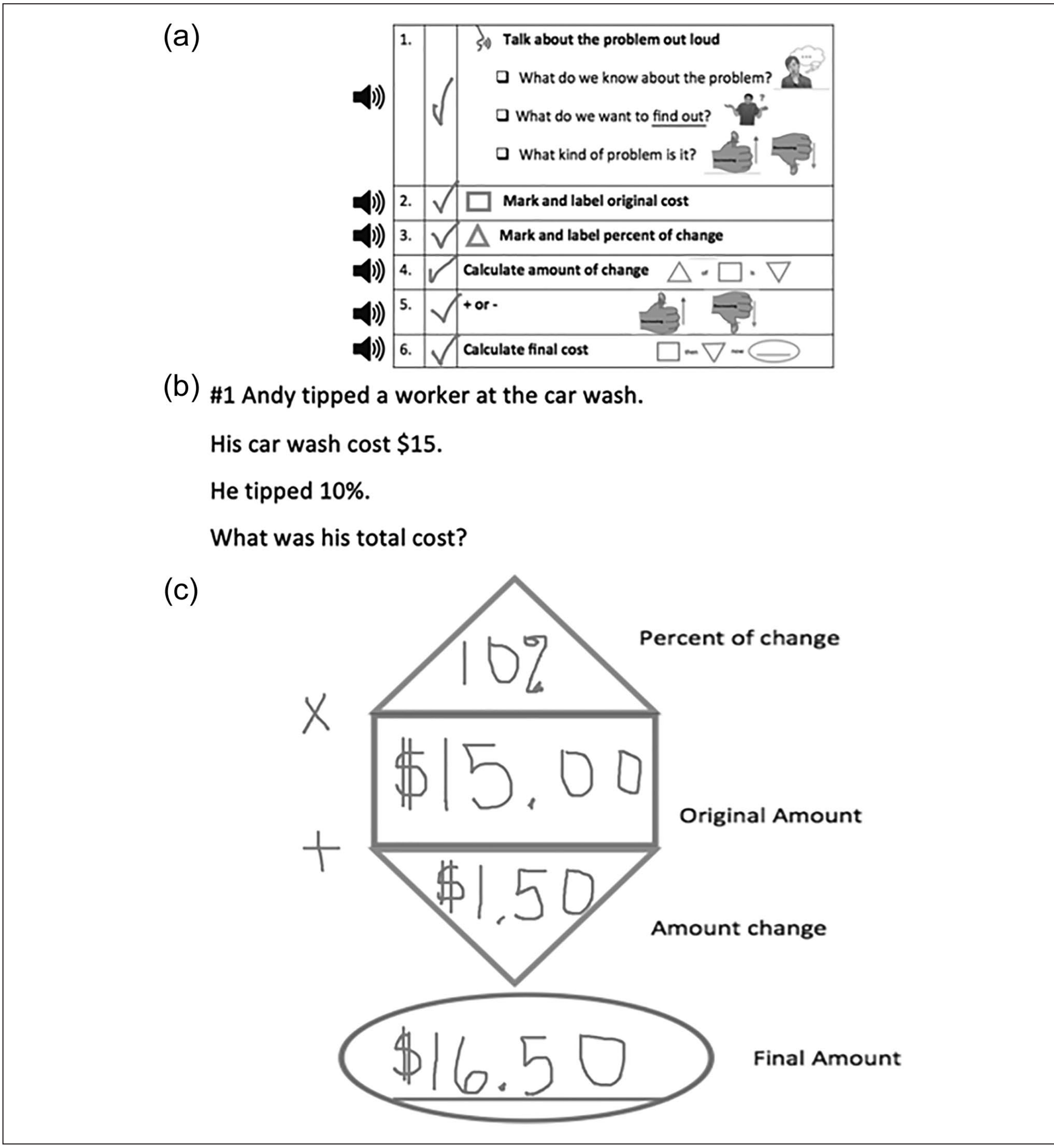

Figure 2. Example of completed student worksheet displaying (a) task analysis (checklist), (b) percent decrease (sale) word problem, and (c) graphic organizer (schema).

because she was displaying agitation in baseline over "not being taught." After the first participant (Leona) showed a clear accelerating trend or improved level of a minimum of three data points during percent increase intervention, the second participant (Faith) entered intervention. This systematic introduction to intervention continued for the third participant (Uma). Participants continued through the phases at their own pace of learning, moving from intervention to the three-session probes as previously described after meeting mastery criteria of 80% of steps completed independently correct (19/22), which had to include Steps 2 to 6, for two sessions. Thus, participants had to solve the

Table 1. Expected Student Responses for Each Step of the Task Analysis.

Step	Expected student response
1. Talk about the problem out loud. 1a. What do we know about the problem? 1b. What do we want to find out? 1c. What kind of problem is this?	1a. States what we know about the problem (original cost and amount of tip/sale) 1b. States the question 1c. States the type of problem with reason (tip = increase, sale = decrease)
2. Mark and label original cost.	2. Writes original cost on graphic organizer and includes the label (i.e., $, lbs, oz)
3. Mark and label percent of change.	3. Writes the percent of change on the graphic organizer and includes the % symbol
4. Calculate amount of change.	4a. Multiplies percent of change by original amount 4b. Writes the amount of change onto the graphic organizer, including the label (i.e., $, lbs, oz)
5. "+" or "–"	5a. Says or shows correct rule/think aloud for problem type 5b. Writes or says correct operation ("+" for increase or "–" for decrease)
6. Calculate final cost.	6a. Correctly adds or subtracts 6b. Writes correct final cost on graphic organizer, including label (i.e., $, lbs, oz)

problems correctly (Step 6) prior to moving to the next phase.

Procedures

General procedures. At the start of every session, participants selected a theme and watched a video involving a percent of change problem in a community setting (Checkpoint 7.1, 7.2, 3.1). As shown in Figure 2, participants had access to the iPad displaying the worksheet with a TA at the top and color-coded graphic organizer at the bottom (Checkpoint 1.1) along with a stylus and iPhone calculator for all sessions. Participants were taught to use the app (e.g., navigate to the next question, text-to-speech), operate the stylus, and clear the calculator before baseline began (Checkpoint 4.1, 5.2).

Baseline and ongoing probes. After viewing the video anchor, the researcher asked participants to "show me how to solve this problem." Participants were praised for effort but no specific feedback was given. Technical assistance with navigating the iPad app was provided as needed (Checkpoint 4.1).

Preunit. Following baseline but prior to intervention, each participant completed a one-session preunit (15 min) that was designed to target skills in isolation using explicit instruction (Model-Lead-Test procedure) to reduce cognitive load during intervention. The preunit covered the following skills: (a) reading and writing dollar amounts and percentages and (b) understanding place value representation of dollar amounts in the calculator, such as 4.5 is the same as US$4.50 (Checkpoint 2.1).

Intervention. Intervention began with 2 days of modeling, (a) real-world examples of leaving a tip or using a coupon with play money (Checkpoint 7.2), (b) mathematical terminology instruction using constant time delay (percent, increase, decrease, multiplication symbol, dollar symbol, addition sign, subtraction sign; Checkpoint 2.1), and (c) completing percent of change word problems with the TA and graphic organizer (Checkpoint 6.2, 6.3). No data were collected during the 2 days of modeling, as there was no opportunity for the participant to respond independently.

Beginning on the third session, the interventionist gave participants an opportunity to solve the problems independently using (a) specific positive feedback for independent correct responses to each step of the TA, (b) a system of least prompts if the student did not respond within 5 s (Checkpoint 5.3), or (c) error correction in the form of a model-retest for an incorrect response. A three-level prompting hierarchy included the following: (a) a verbal prompt directing the participant to use the checklist (TA) to listen to the step (e.g., "let's listen to the checklist one more time"), (b) a specific verbal prompt providing the participant with more information on what to do to complete the step (e.g., "find the original cost of ___ in the problem and label it on your graphic organizer"), and finally (c) a model of how to complete the step (e.g., "in this problem, we know the original cost of ___ is $____. Mark the cost of ___ with a square, and label $___ on your graphic organizer"). The error correction procedure was to model the correct response and retest the participant. Specific feedback was thinned as participants progressed through the phases (Browder et al., 2018; Checkpoint 8.4).

Each intervention session began with participants stating the type of problem they were solving and reviewing the goal they set at the end of the previous session (Checkpoint 8.1). Each intervention session concluded with self-graphing progress and goal setting using an Excel spreadsheet on the iPad (Checkpoint 6.1, 6.4, 8.4, 9.3). At the end of the session, the interventionist facilitated a review of progress using the following prompts: (a) "What type of problems did you solve?"; (b) "What was your goal today?"; (c) "You were able to get ___ steps correct by yourself. Let's graph your progress"; and (d) "What would you like your goal to be tomorrow?" The interventionist helped the participant set appropriately ambitious goals.

Following recommendations provided in prior MSBI studies to reduce cognitive load (i.e., Spooner et al., 2017) and in alignment with principles of explicit instruction (Engelmann & Carnine, 1991), percent increase problems were taught first to mastery, followed by a series of three probes as described in experimental design (Checkpoint 3.2). Percent decrease problems were taught next, following the same procedures. During the third intervention phase, participants were taught to discriminate between percent increase and percent decrease problems using a T-chart and multiple exemplar training (Checkpoint 3.2). The interventionists used think-alouds to model identifying the problem type and sorting problems into the two columns (percent increase or percent decrease), and then gave students an opportunity to independently practice sorting the problems into the two problem types. During discrimination intervention sessions, participants solved four word problems (2% increase and 2% decrease) with the visual aid of the T-chart. Once the student met mastery criteria of two sessions with a score of 19 or higher, a final three-session probe and generalization probe was administered.

Generalization probes. Generalization probes included 4% of change word problems (two increase and two decrease) that depicted novel situations that were unrelated to purchasing (Checkpoint 3.4). Participants were provided with electronic worksheets in the same format as probe and intervention, the stylus, and iPhone calculator. No prompting or specific feedback occurred.

Interobserver Agreement (IOA) and Procedural Fidelity (PF)

IOA and PF were collected across all conditions and phases to assess reliability and fidelity of implementation. Two undergraduate research assistants were trained to use the checklist to code the PF using videos of sessions. The checklist contained 11 items per problem, and measured whether the interventionist provided all necessary materials and followed the scripted procedures for the specific phase (e.g., provision of feedback or prompting). Research assistants used the same data sheet as interventionists to code the dependent variable. IOA and PF were calculated both in vivo and through video recordings. Agreement was calculated using a point-by-point comparison (Ledford & Gast, 2018), dividing the number of agreements by the number of agreements plus disagreements.

IOA was collected for all dependent variables. IOA was calculated for an average of 46% of baseline/probe sessions (range 44%–53%) across participants, with 98% agreement (range 97%–100%). IOA was calculated for an average of 38% of intervention sessions (range 33%–45%) across participants with 93% agreement (range 82%–100%). IOA was calculated for an average of 61% of generalization sessions (50%–75%) with 95% agreement (range 82%–100%). PF was measured during the same sessions for which IOA was calculated. Overall PF was 99% (86%–100%) across all participants and conditions (100% baseline, 98% intervention). Any disagreements were discussed among the research team to come to consensus.

Social Validity

Two direct measures of social validity were administered to assess the impact of participation in the intervention in the participants' attitude about mathematics and the perceived importance and usefulness of the intervention. To assess participant attitudes about mathematics, the participants completed the TOMA-3 Attitude Toward Math subtest pre- and postintervention. To assess perceptions of the intervention, participants were asked eight open-ended questions at the conclusion of the study by the second author.

Results

Mathematical Problem Solving Skills

During baseline, each participant had a stable pattern of responding (see Figure 3). None of the participants were able to correctly solve any problems. Upon entering intervention, each participant showed an immediate increase in level with an increasing trend and no overlapping data with baseline performance. Mastery criteria for moving to subsequent phases mandated that each participant solved both problems for two sessions (Steps 2–6). Data from the first probe for Leona and Faith show maintenance of treatment effects for percent increase problems and some generalization to the percent decrease problems. Leona solved all three problems correctly during the first probe, and Faith solved two problems correctly. Uma demonstrated a decreasing trend during the first probe sessions for percent increase and did not solve any problems correctly. She did demonstrate similar generalization to percent decrease problems as the other two participants.

Immediately after receiving instruction on percent decrease problems, all three participants showed a change in level and increase in trend, with no overlapping data with baseline or probe performance. Data from the second probe for each participant show maintenance of treatment effects for percent decrease problems, but a decrease in percent increase problems, indicating they could not discriminate

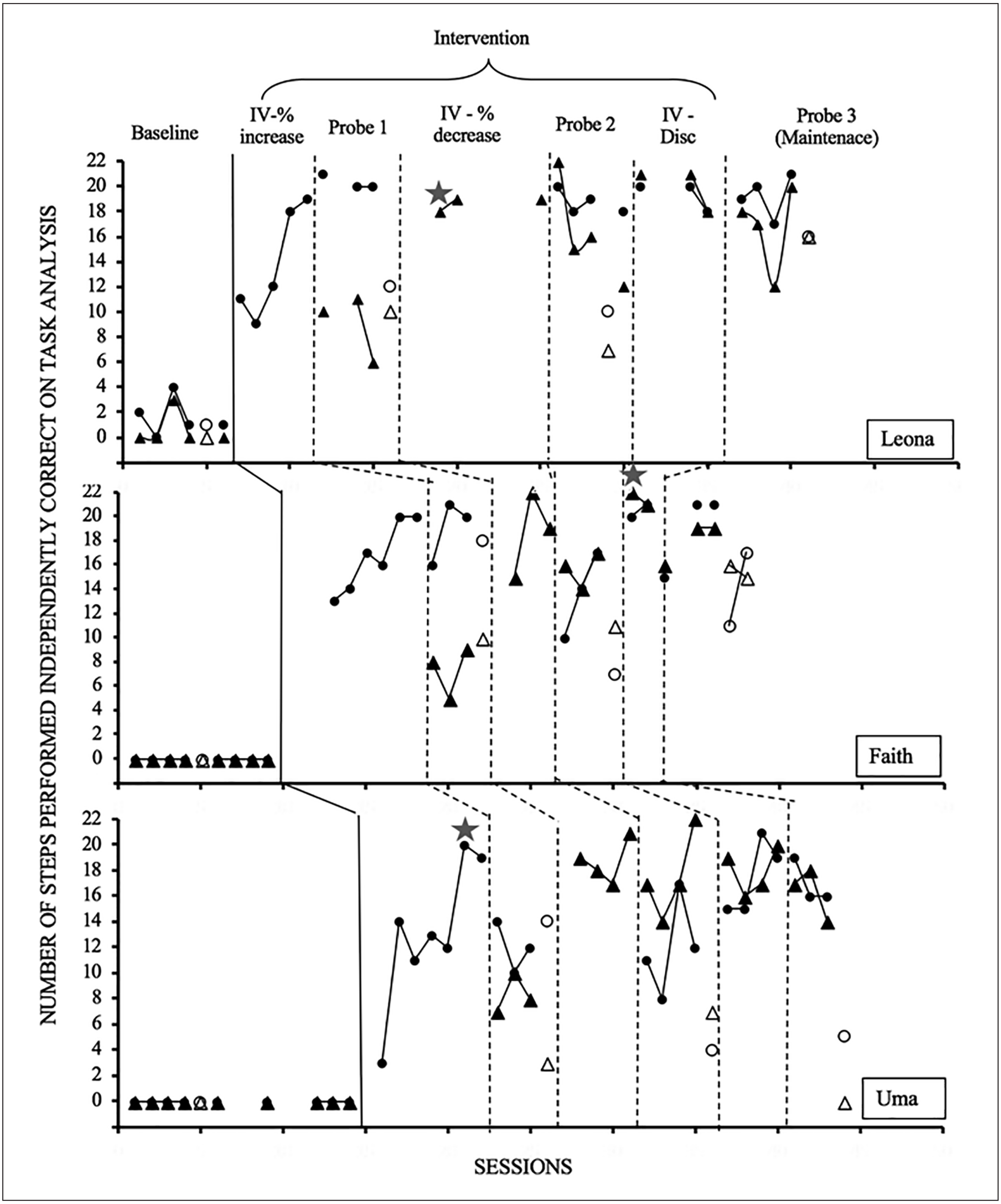

Figure 3. Number of points each participant received for independent performance on the task analysis steps across percent increase (closed circle) and percent decrease (closed triangle) problems, as well as performance on generalization probes (open circles and triangles).

Note. Star indicates provision of personalized token economy. Not pictured on the graph are the two training sessions at the beginning of each intervention phase, as students were not given the opportunity to make an independent response.

between the two. Immediately upon receiving discrimination training, all participants increased independence in solving both problem types and were able to meet mastery criteria to move to the third probe. Data from the final probe for each participant show maintenance of treatment effects and discrimination of problem types. Inclusive of all phases, Leona engaged in 31 sessions (25 exclusive of baseline), Faith engaged in 34 sessions (24 exclusive of baseline), and Uma engaged in 36 sessions (26 exclusive of baseline). Visual analysis of the graph shows a functional relation between the universally designed mathematics intervention and mathematical problem solving skills.

Generalization of Problem Solving Skills

Baseline generalization performance for each participant was commensurate with the rest of baseline in that no participant solved any generalization problems correctly. Leona was the only participant to solve any problem solving steps correctly during baseline generalization sessions (percent increase = one step), whereas all other participants completed zero steps independently correct. Each participant showed some degree of generalization to novel problems following the introduction of intervention, but not at the same level as her intervention performance. During the maintenance probe, Leona's number of independent steps increased to 16 points for both problem types. On the final generalization probe in maintenance, Faith completed 17 and 15 problem solving steps independently for percent increase and percent decrease problems, respectively. Uma had the most difficulty with generalization and showed a decrease in her performance on her final generalization probe.

Social Validity

Two of the three participants' attitude toward mathematics improved after participating in the study, based on the TOMA-3 Attitude Toward Math subtest. Leona reported the highest gains in her attitude toward math, self-evaluating in the 50th percentile before the study and moving into the 75th percentile after the study. Before the study, Uma rated her attitude toward math as "very poor" (<1st percentile). After the study, Uma improved her ratings to below average (9th percentile). Faith's self-reported attitude toward math scored in the average range both before (63rd percentile) and after (50th percentile) the study.

During the interview with the second author, all three participants stated they learned something new during the project. Two of the three participants felt that the TA made solving the problems easier, and all three participants agreed that the graphic organizer was very useful when solving the percent of change problems. Two of the three participants felt that the most difficult aspect of solving the problems was "talking about the problems out loud" (Step 1 of the TA). When asked for an example of when they might use this new information in the community, all three participants gave examples from the video anchors (e.g., using a coupon at a store). During the intervention sessions, participants utilized the additional supports of the text-to-speech and calculator.

Discussion

The purpose of this study was to evaluate the effects of a universally designed mathematics intervention on mathematical problem solving skills for three middle school students with ESN. We used UDL guidelines to increase flexibility and access for learners with ESN (see Figure 1). Results showed a functional relation between the universally designed mathematics intervention and participants' problem solving skills. This study extends previous work of Browder et al. (2008) on using UDL to provide learners with ESN access to the general curriculum as well as research on MSBI by targeting mathematics content of greater complexity, specifically two-step problems that require multiple operations (multiplication, addition, and subtraction). The UDL framework was used to identify potential challenges and solutions to increase student participation and success. We will discuss the findings as they relate to specific UDL guidelines and implications for practitioners.

Multiple Means of Engagement

This study demonstrates how the UDL guidelines can be used to reduce potential barriers to mathematical problem solving skills prior to instruction. Instructional decisions were made after considering the individual student's needs and the existing effective instructional practice (i.e., MSBI) to provide access to the general curriculum for individuals with intellectual disability. A key tenant of MSBI is the use of thematic story problems that contextualize the mathematical content within a setting or activity relevant to the participants current or future environments (Spooner et al., 2017). In this study, participants were given a menu of themes to choose from at the start of each session to further increase their engagement in the story problems. In addition, the video anchors aligned to each theme were created in the participants' community. These features of the treatment package provided explicit opportunities to generalize learning (Checkpoint 3.4), activate or supply background knowledge (Checkpoint 3.1), and options for recruiting interest (Checkpoint 7.1, 7.2). Each participant referenced knowledge she gained from the video anchors during the social validity interviews. Contextualizing mathematics instruction within real-world applications of the targeted skills may optimize the relevance, value, and authenticity of the learning objective (Checkpoint 7.2) and heighten the salience of goals and objectives (Checkpoint 8.1).

Multiple Means of Representation

A distinguishing feature of MSBI is the emphasis on supporting student conceptual comprehension of "what is happening" in the story problem, as opposed to instructional strategies that promote searching for "key words" as clues to selecting operations (Spooner et al., 2017). All schema-based strategies (traditional and modified) teach students to use schematic diagrams for this purpose (Powell, 2011). Students with ESN may experience barriers to comprehension due to fine motor and planning difficulties that would make physically drawing their own diagrams difficult. Providing graphic organizers and considering incorporation of color-coding may support their perception (Checkpoint 1.1). Furthermore, explicitly teaching students how to represent the information from the problem onto the schema (Checkpoint 2.2) supports comprehension of critical mathematical relationships. In this study, symbols were displayed in the participants' TA (see Figure 2) that matched the graphic organizer. This was intended to serve as discriminative stimuli for completing the graphic organizer (Checkpoint 2.2). Practitioners can use the graphic organizer and student TA from this study (see Figure 2) as a model for using the UDL framework to design instructional materials that simultaneously meet students' representational needs and support independence in problem solving.

A second key component of supporting students' representational needs we wish to highlight is a systematic instructional sequence that supports student discrimination of problem type. A particularly important component of the instructional sequence for learners with ESN is discrimination training, whereby one concept (problem type) is taught to mastery, then a second problem type is taught to mastery, followed by explicit instruction in discriminating between the two (Browder et al., 2018; Root et al., 2017; Spooner et al., 2017). This instructional sequence proactively supports comprehension by highlighting patterns and relationships (Checkpoint 3.2). Learners with ESN likely do not have the fact recall to make procedural solving as efficient as it would be for students with high-incidence disabilities (Spooner et al., 2017). Thus, students with ESN are acquiring both conceptual knowledge and procedural knowledge during instruction. To reduce cognitive load, MSBI divides instruction into phases where students learn to either solve a problem type to mastery or discriminate between problem types. Similar to the findings in Browder et al. and Root et al., participants in the current study followed a predictable pattern of behavior in that their performance in percent increase problems decreased following instruction on percent decrease problems (see Figure 3), thereby justifying the need for a separate discrimination between problem type phase. Practitioners should collaborate with general education teachers to ensure that students are provided with a logical sequence of instruction that is both mathematically sound and meets students' needs for understanding relationships and critical features of the targeted problem type.

Multiple Means of Action and Expression

To support the executive functioning needs of students with ESN, students learn how to self-monitor using the TA (Checkpoint 6.3). Instructors use think-alouds to model strategy employment (Checkpoint 6.2). In the current study, we added two additional components to further support participants in becoming strategic and goal-directed, namely daily goal setting and self-graphing of progress, both of which are critical self-determination skills, particularly for secondary students with ESN (Wehmeyer, Field, Doren, Jones, & Mason, 2004). Participant baseline performance pointed to the need for these additions. Despite being shown how to use the read-aloud features to listen to each step of the TA (Guidelines 2 and 4), none of the participants engaged in self-monitoring or attended to the TA in baseline. However, once they began intervention, all participants immediately began self-monitoring and demonstrated excitement over setting and meeting their goals each day, as evidenced by their requests to set goals and clapping for themselves when they reached their goal. To enhance their capacity for monitoring progress (Checkpoint 6.4), token economies were aligned to preferences and interests of two participants (music notes for Leona, superheroes for Uma). Practitioners and future researchers should analyze students' data when they are provided with supports for goal-setting and related skills to determine if or when students with ESN need additional reinforcement.

Limitations and Future Research

There are several limitations to the study that should be addressed by future research. Although all phases of intervention (percent increase, percent decrease, and discrimination) are considered one experimental condition in visual analysis, there was some growth on the percent decrease problems before they were taught. This can be attributed to the steps of the TA that require the same behavior across both problem types (Steps 1–3). However, participants were not able to identify the rule or calculate the final cost, demonstrating a ceiling on their abilities prior to instruction. Only after explicit discrimination training, participants were able to demonstrate accuracy for solving both problem types.

Generalization and transfer of knowledge and skills are critical for learners to become competent mathematical problem solvers. The strategies employed in the current study to support this were not adequate, as demonstrated by participants' data in Figure 3. With regard to the generalization measure, researchers intended for the universally designed intervention to program common stimuli (Stokes & Baer, 1977) and therefore maximize generalization and transfer (Checkpoint 3.4). However, it appears this was not sufficient and may have more closely resembled "train and hope" (Stokes & Baer, 1977). This is likely because the content in the problems was not equivalent to what was trained in intervention, especially when the units of measure were different (e.g., ounces rather than dollars). In addition, during Step 1, the participants sometimes stated the problem was about people but continued to write the dollar symbol, thus lowering their overall score. It is likely participants needed explicit training to generalize to novel problems that were not related to money but still used the same percent of change procedures. Given that a common learning characteristic of individuals with ESN is difficulty with generalization, future researchers can consider how to "train loosely" (Stokes & Baer, 1977) by varying the format of word problems (e.g., location of key information, number, and structure of sentences) to maximize transfer and generalization (Checkpoint 3.4).

An additional limitation lies in the first step of the TA. Talking about the problem out loud was laborious and an aversive task for all three participants. This was a shared sentiment all participants told interventionists throughout the study, and two participants needed added positive behavioral supports in the form of a token economy to consistently engage in this step. Future research should consider ways to provide options for expression and communication (Guideline 5) that will build students' capacity to engage in the mathematical communication tasks required in problem solving and emphasized by standards of mathematical practice (National Governor's Association Center for Best Practices & Council of Chief State School Officers, 2010).

The purpose of this study was to extend the work of Root et al. (2017) and teach a more complex mathematical skill (percent of change) than had previously been attempted in empirical literature, drawing on the UDL guidelines to remove barriers and support learners. The remaining question is, how to apply such strategies in more natural school and community environments so more learners, perhaps even those without ESN, may benefit from the explicit strategies used in this intervention to enhance their conceptual and procedural understanding in mathematical problem solving.

Acknowledgments

The contents of this manuscript were developed in part under a grant from the U.S. Department of Education, H325D140074. However, those contents do not necessarily represent the policy of the U.S. Department of Education, and you should not assume endorsement by the Federal Government. Authors would like to thank Grace Holley and Thomas Ayzenshtat for their assistance in conducting this research.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

References

Browder, D. M., Mims, P. J., Spooner, F., Ahlgrim-Delzell, L., & Lee, A. (2008). Teaching elementary students with multiple disabilities to participate in shared stories. *Research and Practice for Persons With Severe Disabilities*, *33*, 3–12.

Browder, D. M., Spooner, F., Lo, Y. Y., Saunders, A. F., Root, J. R., Ley Davis, L., & Brosh, C. R. (2018). Teaching students with moderate intellectual disability to solve word problems. *The Journal of Special Education*, *51*, 222–235. doi:10.1177/0022466917721236

Brown, V., Cronin, M., & Bryant, D. (2012). *The Test of Mathematical Abilities* (3rd ed.). Austin, TX: Pro-Ed.

CAST (2018). Universal Design for Learning Guidelines version 2.2. Retrieved from http://udlguidelines.cast.org

Cook, S. C., & Rao, K. (2018). Systematically applying UDL to effective practices for students with learning disabilities. *Learning Disability Quarterly*, *41*, 179–191. doi:10.1177/0731948717749936

Engelmann, S., & Carnine, D. (1991). *Theory of instruction: Principles and applications*. Eugene, OR: NIFDI Press.

Every Student Succeeds Act, 114 U.S.C. § 1177 (2015).

Kratochwill, T. R., Hitchcock, J. H., Horner, R. H., Levin, J. R., Odom, S. L., Rindskopf, D. M., & Shadish, W. R. (2013). Single-case intervention research design standards. *Remedial and Special Education*, *34*, 26–38. doi:10.1177/0741932512452794

Ledford, J. R., & Gast, D. L. (Eds.). (2018). *Single case research methodology: Applications in special education and behavioral sciences*. New York, NY: Routledge.

Meyer, A., Rose, D., & Gordon, D. (2014). *Universal design for learning: Theory and practice*. Wakefield, MA: CAST.

National Governor's Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/Math/

Powell, S. R. (2011). Solving word problems using schemas: A review of the literature. *Learning Disabilities Research & Practice*, *26*, 94–108.

Quirk, C., Ryndak, D. L., & Taub, D. (2017). Research and evidence-based practices to promote membership and learning in general education for students with extensive support needs. *Inclusion*, *5*, 94–109. doi:10.1352/2326-6988–5.2.94

Root, J. R., Cox, S. K., Hammons, N., Saunders, A. F., & Gilley, D. (2018). Contextualizing mathematics: Teaching problem solving to secondary students with intellectual and developmental disabilities. *Intellectual and Developmental Disabilities*, *56*, 442–457. doi:10.1352/1934-9556-56.6.442

Root, J. R., Saunders, A., Spooner, F., & Brosh, C. (2017). Teaching personal finance mathematical problem solving to individuals with moderate intellectual disability. *Career Development and Transition for Exceptional Individuals*, *40*, 5–14.

Spooner, F., Root, J., Saunders, A., & Browder, D. (2019). An updated evidence-based practice review on teaching mathematics to students with moderate and severe developmental disabilities. *Remedial and Special Education*, *40*, 150–165. doi:10.1177/0741932517751055

Spooner, F., Saunders, A., Root, J., & Brosh, C. (2017). Promoting access to common core mathematics for students with severe disabilities through mathematical problem solving. *Research and Practice for Persons With Severe Disabilities*, *42*, 171–186. doi:10.1177/1540796917697119

Stokes, T. F., & Baer, D. M. (1977). An implicit technology of generalization. *Journal of Applied Behavior Analysis*, *10*, 349–367.

Thoma, C. A., Cain, I., & Walther-Thomas, C. (2015). National goals for the education of children and youth with intellectual and developmental disabilities: Honoring the past while moving forward. *Inclusion*, *3*, 219–226.

Wang, X. (2013). Why students choose STEM majors: Motivation, high school learning, and postsecondary context of support. *American Educational Research Journal*, *50*, 1081–1121. doi:10/3102/002831213488622

Wehmeyer, M. L., Field, S., Doren, B., Jones, B., & Mason, C. (2004). Self-determination and student involvement in standards-based reform. *Exceptional Children*, *70*, 413–425.

Woodcock, R. W., McGrew, K. S., & Mather, N. (2001). *Woodcock-Johnson III Tests of Cognitive Abilities*. Rolling Meadows, IL: Riverside.