PROBLEM-SOLVING PERFORMANCE AND MATHEMATICS ACHIEVEMENT: THE MEDIATING ROLE OF EYE TRACKING MEASUREMENTS

Research article

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Abstract

The purpose of the present study was to propose a model for mathematics achievement considering the mediating role of eye tracking measurements in the relationship between problem solving performance and mathematics achievement. In this sequential explanatory mixed method research design, a geometry test was conducted to 381 7th grade students. Their problem-solving process was recorded using eye tracking technology. Also, their mathematics achievement scores were acquired from their schools. Afterwards, semi-structured interviews were conducted to 15 students. Based on the results, it was observed that there was a positive relationship among problem-solving performance and mathematics achievement while eye tracking measurements were negatively correlated to problem solving performance and mathematics achievement. Qualitative findings also confirmed these results. Moreover, the hypothesized model could approximately express 22% of the variance on mathematics achievement.

Keywords: eye tracking, mathematics achievement, problem solving, students

1. Introduction

Problem solving is composed of actions performed through exploration of a path beginning at a problem statement to reach the anticipated goal (Dhillon, 1998; Jackson, 1975). In this respect, problem solving taking place in mathematics lessons as in many disciplines have effects and reflections on individuals' real life. The more equipped with problem solving the students become through the lessons, the more successful they are to face with and remove complex problems in their real life outside the school (Irwanto, Saputro, Rohaetti, & Prodjosantoso, 2018).

Problem solving is critical in mathematics education taking place at the core of the teaching process (NCTM, 2000). Moreover, previous researches show that students' problem-solving performance is in connection with their academic achievement in the lessons (Aka, Guven & Aydogdu, 2010; Cheng, She & Huang, 2018; Gok, 2014; Hu, Xiaohui & Shieh, 2017; Saputro, Irwanto, Atun & Wilujeng, 2019). This connection can be emphasized by the quote "You do not know anything until you have practiced" by R. P. Feynman because problem solving provides the opportunity of testing the knowledge that the students acquire. Also, in problem solving, the students make searching to create new strategies and paths to reach a solution using

previously learned knowledge rather than applying the learned rules simply (Woolfolk, 1993). In this respect, it can be stated that problem solving performance can be an indicator to estimate the students' achievement. Furthermore, it is stated that the performances that the students represent through problem solving can provide information about their conceptual understanding of the content and achievement in the lessons (Gok, 2014). Moreover, Serway and Beichner (2000) strongly insist on the usage of problem solving to test the students' knowledge in order to make them acquire the necessary knowledge about the concept and represent academic achievement.

In the process of examination of individuals' problem-solving processes, the solution paths and the results are examined simultaneously in order to extract their problem solving performance, and their views about the process may provide more information about the context. In this process, the individuals may be blocked in some parts of the problem or the solution. Sometimes, they could not explicitly explain this case and the reasons for their difficulty that can prevent to perform effective feedback and hint and to represent their coping behaviors explicitly. The researchers suggest benefiting from eye tracking measures in the cases including these opaque problem-solving processes by collecting data about the individuals' visual attention using temporal and spatial resolutions while they engage in solving problems (Rayner, 1998). In this respect, the problem-solving performance can be examined in detail in a different perspective with the help of eye tracking proposing attention-related evidences for the problem solving processes (Johnson & Mayer, 2012). Moreover, the mathematical achievement that the students represent their mathematical conceptualization can be referred to their cognitive processes. In this respect, eye tracking can be beneficial since by enhancing the exploration of cognitive structures and processes because the related literature have showed that how individuals behave can be explained by considering what they think (Hyöna, 2010; Jang et al., 2014; Jarodzka, Scheiter, Gerjets, & Van Gog, 2010; Mayer, 2010). To line with this view, eye tracking methods can suggest detail and holistic view to represent the mental operations, the cognitive processes and the actions occurring in individuals' mind focusing on their visual attention (Andrá et al, 2015; Johnson & Mayer, 2012; Obersteiner & Tumpek, 2016).
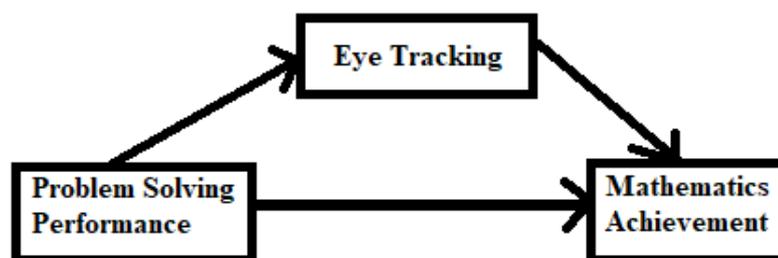
1.1. Problem Solving Performance, Mathematics Achievement and Eye Tracking

Problem solving is an important indicator of achievement in the lessons performs through cognitive processes and goal-directed actions necessitating mental representations and decision making (Metallidou, 2009; Simon & Simon, 1978). Polya (1945) describes problem solving process as linear and hierarchical. In this process, the individuals are expected to keep previously acquired information in working memory, take back schema from long-term memory and glide their visual attention on important parts (Kester, Kirschner, & van Merrienboer, 2005). To line with this view, it can be stated that cognitive and mental operations are necessitated in solving problems. At that point, eye tracking can provide beneficial information about this process (Andrá et al, 2015; Jacob & Karn, 2003; Obersteiner & Tumpek, 2016; Rayner, 1998). Measurements acquired through eye tracking can provide information about the actions and individuals' thinking in solving problems. For example, fixation as a kind of eye tracking measurement can inform researchers about the problem solvers' cognitive attention and difficulty and the increase in fixation measurements can be interpreted as having difficulty in solving the problem at particular stages (Andra et al., 2015; Epelboim & Suppes, 2001; Jacob & Karn, 2003; Liversedge & Findlay, 2000). In this respect, eye tracking can provide information in direct connection with problem solving performance.

In solving problems by going through the stages, the students may have difficulty. The previous researches explain that the students' difficulty in solving problems may reason from

insufficient previous knowledge such as not knowing the hierarchical relationship among quadrilaterals, perceptual issues such as not recognizing the geometrical shapes appropriately and inability to make reasoning about the content and process accurately and sufficiently (Bransford, Sherwood, Vye, & Rieser, 1986; Cooper & Sweller, 1987; Lin & Lin, 2014). Based on this given explanation, two implications can be made. Firstly, the problem-solving performance and process is directly related to their understanding of the content and also mathematics achievement. Secondly, problem solving process includes the actions affected by perceptual actions and visual attention. To line with this view, the students can focus on important parts of the problem and perform a cognitive process including decision making and reasoning by visual attention. Hence, it can be stated that problem solving process giving information about the students' achievement can be explored through eye tracking. By this view, there exist research exploring the individuals' actions through the problem-solving stages (Knoblich, Ohlsson & Raney, 2001; Thomas & Lleras, 2007). Previous research show that eye tracking provides information about the individuals' relational schemata and thinking, reasoning and difficulty in understanding and engaging in achievement tests (Muldner & Burleston, 2015; Schindler et al., 2016; Tsai, Hou, Lai, Liu & Yang, 2012; Thomas & Lleras, 2007).

Based on the given explanations and the research showing that there is direct relationship between problem solving performance and achievement regarding visual attention and cognitive operations, it can be stated that eye tracking can take a role in the relationship between problem solving performance and mathematics achievement. Moreover, there exist previous research in the related literature providing information about the relationship between problem solving and mathematics achievement. The current study differentiates from these researches by focusing on the mediating role of eye tracking on the direct relationship between problem solving performance and mathematics achievement. In other words, the purpose of the current study is to explore whether eye tracking has a mediating role on problem solving performance and mathematics achievement by providing the theoretical model of these explained variables represented in Figure 1.

Figure 1. The theoretical model for the relationship between problem solving performance, mathematics achievement

In order to examine the theoretical model explained in Figure 1, the mediating role of the construct of eye tracking is focused on. In the literature, eye tracking has been explored by various measurements proposed by eye tracking technology and used for the implementation of visual attention and cognitive actions based on eye-mind assumption (Just & Carpenter, 1980). For example, fixation duration which refers to the period of time that the eyes focus on zone and provides information about visual attention and difficulty in cognitive processes (Andra et al., 2015; Epelboim & Suppes, 2001; Jacob & Karn, 2003; Lin & Lin, 2014). This issue is emphasized by the statement "what a person looks at is assumed to indicate the thought 'on top of the stack' of cognitive processes" (Jang et al., 2014, p. 318). In this respect, longer

fixation duration can be interpreted as the existence of complex context, difficulty in conceptualizing and deeper reasoning. Another measurement is glance duration provides information about visual distraction and perception load. The increase on glance duration while engaging in a problem or task can be interpreted as the existence of visual distraction (Bao, Kiss, & Wittmann, 2002; De Waard, 1996; Noy, Lemoine, Klachan, & Burns, 2004). Based on these explanations, it is necessitated to determine the variables included in the construct of the eye tracking. Hence, in the current study, this construct was analyzed by considering the variables of fixation time, dwell time, glance duration and diversion duration by confirmatory factor analysis.

1.2. Theoretical foundations and theoretical framework of research

In information processing theory, the knowledge is acquired through the processes including paying attention to, perceiving, storing in mind, remembering. The individuals receiveexternal stimuli with their hands, eyes or ears and records these stimulants but a few of them switch into short-term memory with the help of selective perception. The new information obtained in this way is compared with the information acquired previously and stored in long-term memory and then stored by being organized (Simon & Simon, 1978). Previous research show encouragement and description of the processes and actions about decision making, learning and problem solving by information processing theory (Peterson, Sampson, Reardon, &Lenz, 1996; Sampson, Peterson, Lenz &Reardon, 1992). Given these explanations show that visual attention can have important role in learning, problem solving and performing the tasks (McGivney & DeFranco, 1995; Simon & Simon, 1978). In this respect, eye tracking can provide a contribution to information processing theory by examining cognitive processes focusing on visual attention and behaviors of the eyes. Hence, the current study has been organized with the purpose that eye tracking related to visual attention can provide useful information about detailing and explaining the students' problem solving and mathematics achievement.

2. Method

In order to examine the connections of the variables of problem-solving performance, eye tracking measurements and mathematics achievement and provide a mediational model explaining the mediating role of eye tracking measurements in the relationship between problem solving performance and mathematics achievement in detail, the present study was carried out based on sequential explanatory mixed method research design. In this research design, the quantitative data dominates on the qualitative data and qualitative data is used in order to comprehend and implement the quantitative data more clearly in detail (Creswell, 2013). In this respect, the quantitative data was used in order to establish the relationships among problem solving performance, eye tracking measurements and mathematics achievement and provide a mediational model. Afterward, the qualitative data were used in order to detail and look at the relationships among variables with respect to holistic perspective.

2.1. Participants

In the quantitative part of the study, the multiple-choice and matching geometry test was conducted to 381 seventh grade students who were selected by the typical sampling strategy. In this respect, 5 schools from low, middle and high socioeconomic level districts of İstanbul were selected. These students participated in the process of solving problems on the test. Afterwards, in the qualitative part of the study, 15 students different from these 381 students were selected based on cluster sampling strategy. With respect to this sampling strategy, three students were identified from each school. These students' problem-solving processes were not

recorded via eye tracking technology. Hence, quantitative data belonged to these students were not collected. They were conducted to semi-structured interviews.

2.2. Data Collection

In the data collection process, in order to collect quantitative data about the students' problem solving performance, a test was prepared based on the common geometrical subjects that the students had misconception and difficulty by a group of researchers having Ph.D.degree in mathematics education and teaching technology. The questions were prepared about the angle and angle measure (Devichi ve Munier, 2013; Doyuran, 2014), the concept and definition of polygon (Akuysal, 2007), triangle (Clements et al., 1999; Tsamir, Tirosh & Levenson, 2008; Tsamir, Tirosh, Levenson, Barkai & Tabach, 2014), triangle and altitude of triangles (Alatorre & Saiz, 2010; Uygun & Akyuz, 2019; Hershkowitz, 1990), quadrilaterals and their hierarchical relationship (Monaghan, 2000; Pickreign, 2007; Erez & Yerushalmy, 2006; Fujita & Jones, 2008), and prism and opening form of geometric solids (Gökkurt, Şahin, Soylu & Doğan, 2015; Tsamir et al., 2014). Mathematics middle school curriculum, geometry textbooks and studies about geometrical misconceptions in the literature were examined before the construction of the test. Eight multiple-choice questions and one matching question in the test (see Appendix for sample questions) were represented to the students using web 2.0 tools on a computer screen. The maximum score that the students could acquire for each question was 1 and the minimum score was 0. They answered the questions on the test one by one. They identified the correct answers by distinguishing them from irrelevant items. After completing the selection of relevant items of the question, they moved to the next question. The students' scores for each question was determined and analyzed separately.

During the process of answering, eye tracking data was recorded by SMI Experiment 2 to represent visual stimuli. The scores of eye tracking measurements were collected for each question for each student separately. Hence, 3081 scores for problem solving performance and eye tracking measurements could be acquired. The process of students' problem solutions through eye tracking technology was illustrated in Figure 2. The data collected through eye tracking technology were analyzed and reported using BeGaze program. By this vehicle and program, the measurements of fixation duration, fixation dispersion, saccade duration and blink duration were collected. Then, in order to acquire the quantitative data belonged to each question provided by each student, an eye tracking measurement was calculated by getting the mean value using the measurements of fixation duration, fixation dispersion, saccade duration and blink duration. Afterward, in order to collect data about students' mathematics achievement, their mathematics grades that they acquired in the previous semester for their mathematics lessons were taken from their schools.

Figure 2. An illustration for data collection process by eye tracking

2.3. Data Analysis

In the quantitative data analysis, descriptive statistics, Pearson's correlation method, an approach based on Ordinary Least Squares Regression, and Bootstrapping were used. Mahalanobis distance values were calculated and no extreme outliers were found. To check normal univariate distribution, the values of kurtosis and skewness were calculated. The results showed acceptable ranges in the region of -1.5 to +1.5 (as shown in Table 1), it was implemented that the students' scores for the variables tended to show the normal distribution (Tabachnick & Fidell, 2013).

In the present study, a mediation model (Preacher & Hayes, 2008, p. 880), was performed with the help of mediating analysis. Hayes (2009) explains "if zero is not between the lower and upper bound, then the analyst can claim that the indirect effect is not zero with $ci\%$ confidence" (p. 412). In order to find stronger mediators and specific indirect of the variables, a contrast test was used. The Bootstrapping analyses were performed based on "Mediation Model 4" using PROCESS Macro 3 through IBM SPSS 24.0 (Hayes, 2012/2013) with the statistical significance p-value of .05.

In the qualitative part of the present study, 15 students were conducted to semi-structured interviews. The questions such as "Why and how did they select or eliminate the particular options?, How did they spend their time in solving problems?" were asked to the students. Each interview lasted approximately 30 minutes and were recorded by audio recordings. After completing the interviews, the audio recordings were transcribed verbatim and related quotations were given in the results.

3. Results

3.1. Descriptive Statistics and Correlations

In order to examine the relationship among the variables of problem-solving performance, mathematics achievement and eye tracking measurement, Pearson's correlation coefficients were calculated and represented in Table 1. The values of mean, standard deviation, skewness and kurtosis belonged to these variables, and correlation values for the relationships among these variables were illustrated in Table 1.

Table 1. *Descriptive statistics and bivariate correlations among variables*

Variables	Mean	Sd.	Skewness	Kurtosis	1	2	3
1. Problem Solving Performance	.46	.36	.22	.44	--		
2. Eye Tracking	977.28	313.61	1.21	1.41	-.35**	--	
3. Mathematics Achievement	67.60	22.47	-1.28	-1.19	.63**	-.45*	--

N=3081, *p*<.05, **p*<.01.

In Table 1, it is observed that there exist statistically significant relationships among the variables based on Bivariate correlations (Table 1). In other words, Table 1 illustrates that eye tracking is negatively correlated to mathematics achievement and problem solving performance while problem solving performance and mathematics achievement are positively correlated.

3.2. Mediation Model Analyses

When the mediating effect of eye tracking measurements was tested in the relationship between problem solving performance and mathematics achievement, the mediational model could be proposed. The findings of this mediational model explaining the mediating role of eye tracking measurements in the relationship between problem solving performance and mathematics achievement are presented in Figure 3.
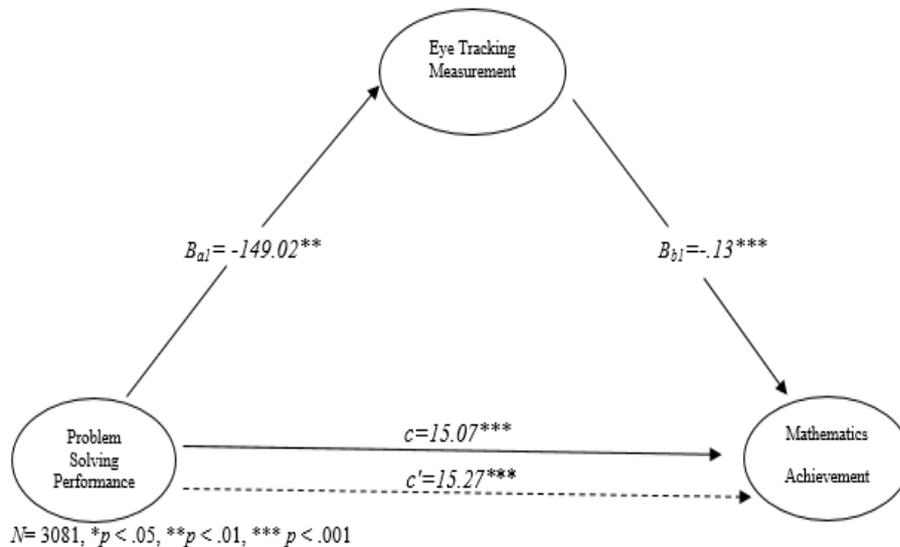

$N= 3081, *p < .05, **p < .01, ***p < .001$

Figure 3. The Mediating Role of Eye Tracking Measurements in the Relationship between Problem Solving Performance and Mathematics Achievement

The mediational model proposed in the present study is illustrated in Figure 3. The total effect of problem solving performance on mathematics achievement is statistically significant ($c = 15.07$, SE = 1.08, t = 13.92, p < .001) (step 1). The direct effect of problem-solving performance on eye tracking measurements is statistically significant (B = -149.02, SE= 48.46, t= -3.07, p < .01) (step 2). The direct effect of eye tracking on mathematics achievement is statistically significant (B = -.13, SE= .04, t= -3.29, p < .01) (step 3). When problem solving performance and mediating variable eye tracking measurements have been taken simultaneously (Step 4), the relationship between problem solving performance and mathematics achievement has increased and the significance value has remained at the same level (c'= 15.27, SE= 1.08, t = 14.11, p < .001). This coefficient is also statistically significant. According to this result, eye tracking measurement is partially mediated in the relationship between problem solving performance and mathematics achievement. These results provide the mediational hypothesis and the model is significant (F(2-3079), p < .001) by explaining approximately 22% of the variance on mathematics achievement.

3.3. Problem Solving Performance and Mathematics Achievement: The Mediating Role of Eye Tracking Measurement

The findings belonged to the comparisons of the total, direct and specific indirect effects of problem-solving performance on mathematics achievement through eye tracking measurements are illustrated in Table 2.

Table 2. *The findings on the mediating role of eye tracking measurements in the relationship between problem solving performance and mathematics achievement*

Effects	Point Estimate	SE	*t*	*p*	Bootstrapping 95% BCa Confidence Interval	
					Lower	Upper
Indirect	-.20	.10	-	-	-.4142	-.0401
Total	15.0738	1.0725	13.9244	.0000	12.9512	16.1964
Direct	15.2708	1.0825	14.1072	.0000	13.1483	13.7533

Note: *N*= 3081, *k* = 5000, **p* < .05, ** *p* < .01, ***p < .001, *BCa*: Bias corrected and accelerated 5000 bootstrap samples

The indirect effect was explored with the help of bootstrapping with 5000 bootstrap samples. The estimates were considered within 95% confidence interval and bias corrected and accelerated results are illustrated in Table 2. The results illustrated in Table 2 emphasize that the indirect effect (the difference between the total and direct effects /c-c') of problem solving performance through eye tracking measurements on mathematics achievement is statistically significant (point estimate= -.20 and 95% BCa CI [-.4142, -.0401].

3.4. Qualitative Findings

Based on the findings of the qualitative data analysis, students' explanations provided evidence and strength for the quantitative findings representing the relationship among the variables of mathematics achievement, problem solving performance and eye tracking measurement and proposed mediation model. For example, a student not liking mathematics made an explanation as follows:

"Mathematics is so difficult for me. My mathematics grades are low. In solving mathematical problems, I had difficulty. Even if I can understand the concepts, I cannot solve problems. I usually look at the problems, focus on it and think about it. Still, I cannot solve it."

Another student liking mathematics made explanation as follows:

"I like mathematics and solving problems, especially about geometric solids. When I face with geometry problems, I spend short time in solving problems that I can understand and solve easily. When I have difficulty in solving problems, these problems are usually about the concepts that I cannot comprehend effectively. Generally, this case can decrease my grade when the exam includes problems with these concepts."

Moreover, it was observed that the students having low scores of problem solving performance were likely to have low scores of mathematics achievement. Hence, a student having low scores of mathematics problem solving performance and mathematics achievement had difficulty in answering the question about the altitude of a triangle. This student answered this question incorrectly. She made her choices based on the idea that the altitude had to be perpendicular to the side. She could not identify the altitudes of obtuse angled triangles. Moreover, she spent much time while answering this question. Hence, it can be stated that this student got higher scores of eye tracking measurements.

4. Discussion and Conclusion

Based on the quantitative findings of the current study, it was observed that problem solving performance and mathematics achievement were positively correlated. Moreover, the qualitative findings acquired through the analysis of semi-structured interviews supported evidence and strength for this relationship. This finding can be provided by previous research emphasizing the connection of problem-solving performance to academic achievement in their lessons in the literature (Aka, Guven & Aydogdu, 2010; Cheng, She & Huang, 2018; Gok, 2014; Hu, Xiaohui & Shieh, 2017; Saputro, Irwanto, Atun & Wilujeng, 2019). In the literature, there have been many researches exploring the effects of problem solving performance. These research state that problem solving performance is an important factor related to students' achievement and conceptual understanding (Gok, 2014; Serway & Beichner, 2000). Hence, it can be claimed that problem solving performance can serve as an important predictor for mathematics achievement with respect to the findings of the present study.

Based on the findings, it was observed that eye tracking measurements were negatively correlated with these variables. Moreover, qualitative findings acquired through semi-structured interviews encouraged the expressed the connection of eye tracking measurements to the variables of problem-solving performance and mathematics achievement. The finding of the relationship between problem solving performance and eye tracking measurements can be confirmed by previous research (Johnson & Mayer, 2012; Rayner, 1998) providing attention-related evidences for the problem-solving processes with the help of eye tracking measurements. The eye measurements can provide information and serve as an indicator to interpret the difficulty of the students experiencing in solving problems. For example, the more score of eye fixation duration a student gets, the more difficulty the student has in solving problem (Andra et al., 2015; Epelboim & Suppes, 2001; Jacob & Karn, 2003; Liversedge & Findlay, 2000). Hence, it can be claimed that eye tracking measurements can provide information about the students' problem-solving process and difficulty. In addition, it was observed that there was a statistically significant negative relationship between eye tracking measurements and achievement in the context of mathematics. This finding can be confirmed by the results of the previous research (Muldner & Burleston, 2015; Schindler et al., 2016; Tsai, Hou, Lai, Liu & Yang, 2012; Thomas & Lleras, 2007) stating the connection of eye tracking measurements and academic achievement. Hence, it can be stated eye tracking measurements can serve as an important predictor for academic achievement in the context of mathematics.

It was also shown that eye tracking measurements has mediating role in the relationship between problem solving performance and mathematics achievement and this hypothesized mediation model could approximately express 22% of the variance on mathematics achievement. This model can be confirmed by the studies of Andrá and colleagues (2015), Johnson and Mayer (2012), Just and Carpenter (1980), Obersteiner and Tumpek (2016) emphasizing the beneficial role of eye tracking to represent the students' mental operations, cognitive processes and actions occurring in their minds focusing on their visual attention through solving problems and engaging in achievement tests. Hence, this study can provide beneficial contribution to the literature. The present study differentiates from previous research since it reported the direct relationship of eye tracking measurements with the variables of problem solving performance and mathematics achievement, and their mediating role on these variables. It also differentiates by detailing these relationships with the help of qualitative findings.

5. Limitations and Implications

There exist some limitations in the present study. The participants of the present study were composed of 7th grade students. Moreover, the relationships among the variables were examined in the study. Further research can be conducted to the students from different grade levels and the causality considering the variables of the study can be explored by designing experimental and longitudinal research. Moreover, further research can be performed by using different comprehensive and detailed mathematics tests appropriate for all middle level grade students. The context of further research can also be narrowed down considering the learning areas of mathematics such as algebra, data analysis. Moreover, the present study can contribute to the teachers by representing the connection of visual attention and cognitive process to problem solving performance and academic achievement with the help of eye tracking technology. Therefore, they can analyze their students' visual attention in problem solving processes in order to make a prediction about and help their students increase their problem solving performance and academic achievement. Moreover, by emphasizing the relationship between problem solving performance and mathematic achievement, the teachers can help their students increase their problem-solving performance by providing them with opportunities to solve problems. Hence, students can improve their mathematics achievement. Moreover, the findings can make beneficial contribution to the literature about problem solving and academic achievement using eye tracking by proposing the mediation model. Moreover, teacher educators can educate preservice teachers to consider their students' visual attention to predict and increase the students' problem-solving performance and academic achievement.

6. Conflict of interests

The authors declare that there is no conflict of interest.

7. Ethics Committee Approval

The authors confirm that the study does not need ethics committee approval according to the research integrity rules in their country.

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