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Find the Measure of Angle *x*: Teacher's Technology Investigation

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Abstract

This paper discusses strategies that two high school geometry teachers employed to solve a challenging geometry problem with the aid of dynamic geometry software (The Geometer's Sketchpad (GSP). The teachers were part of a group of teachers who participated in a summer professional institute funded by the National Science Foundation. The paper demonstrates the use of technology to construct, explore, and prove, incorporating the measurement tool in GSP. Although the two teachers showed similar approaches, they displayed interesting proof strategies.

Introduction

In recent years, there have been examples of exploration of open geometry problems in Dynamic Geometry Environments (DGEs). Research indicates "that a DGE impacts students' approach to investigating open problems in Euclidean Geometry, contributing particularly to students' reasoning during the conjecturing phase of open problem activities" (Baccaglini-Frank, Antonini, Leung, & Mariotti, 2017, p. 103). Dynamic geometry tools enable students as well as teachers to construct, explore, conjecture, and possibly prove a conjecture. The software tool can create a drawing, measure, and drag the figure to explore that may lead to conjecture and proof (Hollebrands, 2007; Mariotti, 2000). The National Council of Teachers of Mathematics (2000) points out that when the opportunity is offered to use dynamic geometry technology, it "extends their ability to formulate and explore conjectures" (p. 310). Research demonstrates that both high school students and teachers do struggle with writing proofs due to their inability to justify and reason through a given geometry situation (Kotelawala, 2016). The goal of this paper is to discuss two high school teachers that used dynamic geometry software to explore conjecture and prove an open geometry problem. The teachers were part of a group of teachers who participated in a summer professional institute funded by the National Science Foundation.

The problem in Figure 1 (find the measure of angle x) was presented to in-service high school eometry teachers who participated in NSF funded summer instate professional development project. The teachers reported in this paper; Jane and Luke (all pseudonyms) were selected from the group to participate in this study because they demonstrated unique approaches to solving the problems. Jane has taught high school geometry for six years, whereas Luke has been teaching for four years. The two teachers were presented with the following problem:

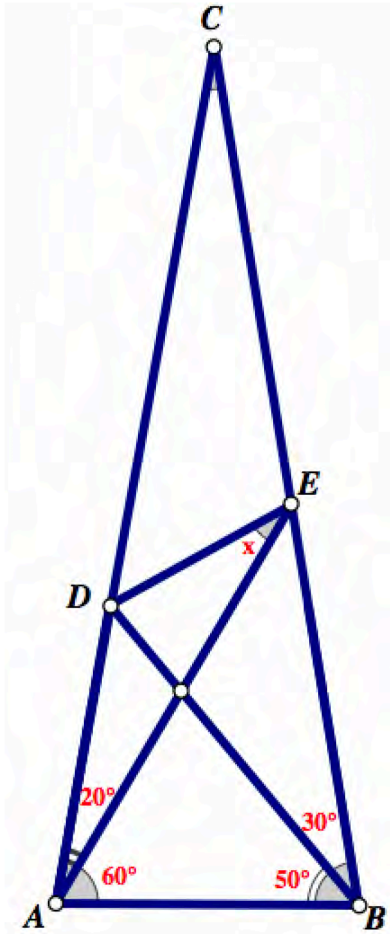


Figure 1

Jane

Jane was first introduced with Figure 1 (not drawn to scale) on a piece of paper and was asked to find the measure of ∠x. Jane took time to figure out the best way to construct the figure that when dragged around, maintained its geometric configuration to focus on the parts of interest to the problem. Initially, Jane's construction was not done correctly more so with base angle measure because when dragged, the geometric configuration of the figure was not maintained. In summary, after some trials, Jane finally constructed a figure that passed the drag test, as stated in her construction steps outlined below:

Step 1: Constructed an arbitrary line segment AB.

Step 2: Double clicked point A and then selected segment AB and point B then under transformation on GSP, selected rotate ∠80˚to construct a new segment and a point.

Step 3: Using point A and the new point, constructed a ray from A through the new point.

Step 4: Double clicked point B and selected segment AB and point A then under transformation on GSP, selected rotate ∠-80˚ to construct a new segment and a point.

Step 5: Selected point B and the new point to construct a ray to meet the other ray at point C.

Step 6: To construct segment AE, double clicked point A and then ray that start at point A. Then under transformation menu of GSP, selected rotate ∠-20˚to construct a new ray to meet the ray BC at point E.

Step 7: Then double click point B and select ray BE and rotate through ∠30˚to meet ray AC at point D. Finally, she constructed segment AC and BC and then hide ray AC and BC to create Figure 1.

To investigate the problem, Jane started first by measuring all angles of interest to conjecture how she could find ∠x using the measuring tool in gsp. First, she measured ∠x (m∠DEA = 30˚). She conjectured that the m∠DEA = 30˚. It was now a challenge for Jane to show that m∠DEA was actually 30˚ (Note that m∠DEA means measure ∠DEA).

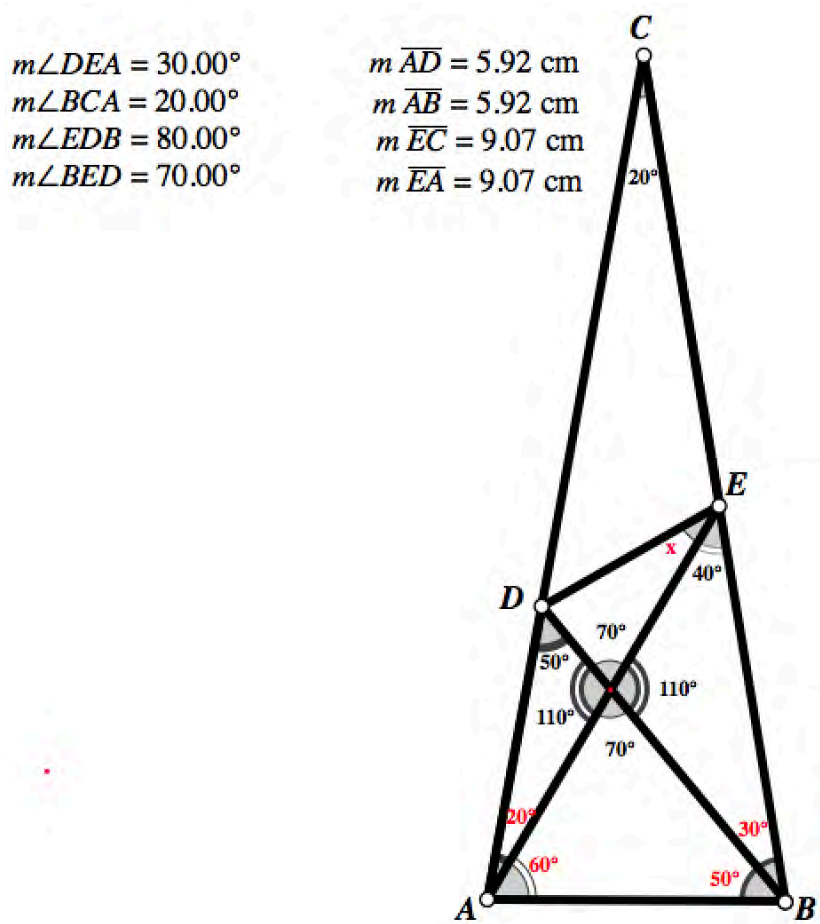


Figure 2

To investigate the conjecture, Jane found all angles represented (in black) in Figure 2 to find other missing angles. Asked to explain the strategy, Jane noted

> I first summed up the base angle for example m∠CAB = m∠CBA = 80 and therefore angle m∠ACB = 180˚ - (m∠CAB + m∠CBA) = 180˚ - (80˚ + 80˚) = 20˚.

Jane used the angle sum property of triangles to find all other angles (in black), as shown in figure 2. Next, Jane measured all segments (Figure 2) and noted that segment AD = AB and justified that by noting, "note that ∠ADB = ∠ABD = 50˚, therefore, making triangle DAB isosceles triangle." She also stated that segment EC = EA which she justified by pointing to the fact that ∠DAE =∠ECD = 20˚, making triangle AEC an isosceles triangle therefore segment EC = EA. One issue that Jane had misconception was that measuring angles or segments couldn't be regarded as formal proof in geometry. It can only help to make conjectures.

At this point, Jane did not know how to move forward and was somewhat discouraged.

> I have found all possible angles and sides but do not seem to see anything to help me solve the problem.

But without giving up, Jane explored the idea of constructing circles. It was not clear why Jane decided to construct the five circles, but when asked, she noted

> I'm playing around with GSP to see if something comes out.

She constructed five circles with circle (c_1) center at point A radius AD, circle (c_2) center at point B and radius BA, circle (c_3) center at point C and radius CE, circle (c_4) center at point D and radius DA, circle (c_5) center at point E and radius EB. Among all the five circles she constructed, the one with center A and radius AD resulted in interesting conjectures, as shown in Figure 3. She dragged the figure around to make sure it was constructed correctly (drag test).

In the construction, Jane noticed that (c_1) that goes through point B intersects segment BC at point F. From her observation point D appeared to be on the circle but she was not certain that it actually was on the circle. Jane created point F and constructed triangle ADF. She wanted to examine the characteristics of triangle ADF to determine if the triangle offered any insight into the proof process. At this point, the researcher asked Jane a question: What do you notice from the circle that you have constructed?

Jane responded

> I see that segment AD ≅AF ≅AB ≅FE, and that the circle intersects triangle ABC at point B, F and point D. That meant that AF ≅AB that makes a triangle ABF isosceles triangle.

Jane's findings that triangle ABF is isosceles was important in that she was able to find that since ∠ABF ≅ ∠AFB, then ∠AFB = (50˚+30˚) = 80˚. Using a triangle property, Jane justified that since ∠AFB ≅ ABF = 80˚, then ∠FAB = (180˚-(80˚+80˚)) = 20˚. At this point, Jane did not know the way forward but went back to GSP measurement for some observation as she noted:

Since AD ≅AF ≅FD, then the conjecture is that triangle ADF is an equilateral triangle.

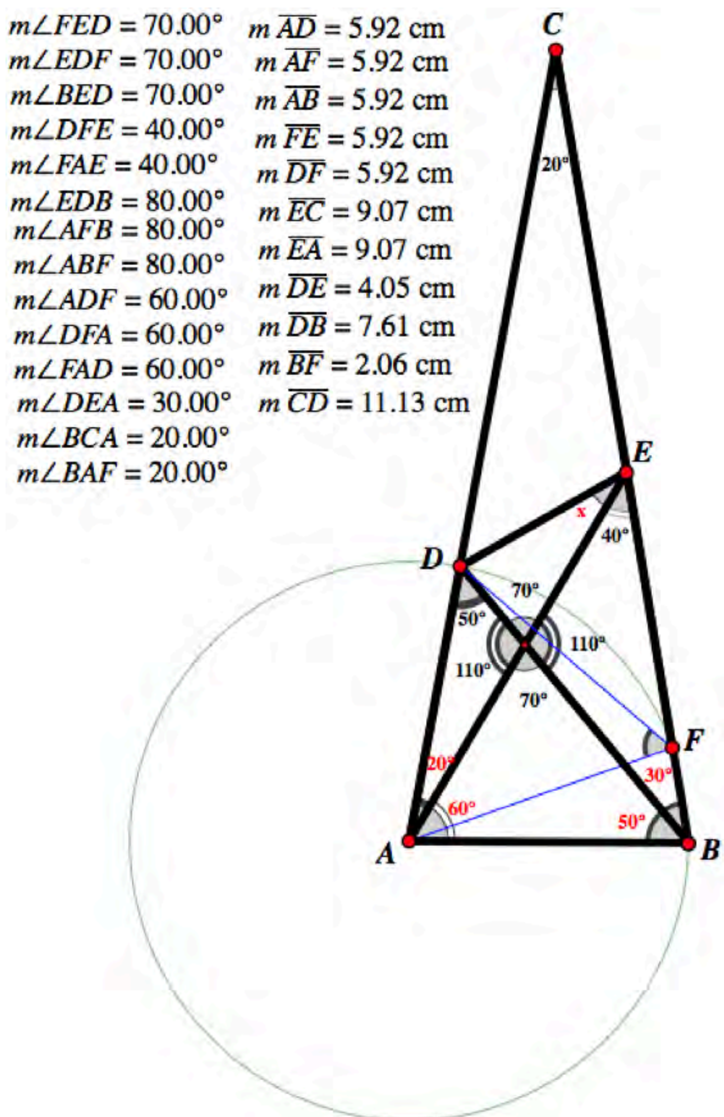


Figure 3

The researcher asked: How sure are you that it is an equilateral triangle? Jane could not answer this question immediately. She paused and said:

Since ∠DAE = 20˚ and ∠EAB = 60˚ and since ∠FAB = 20˚, then ∠DAF = 60˚. Now we know from construction that segment AF ≅ AD and ∠DAF = 60˚. If two legs have the same length, then the base angles have the same angle measure. That means that ∠AFD ≅ ∠ADF but since ∠DAF = 60˚, then

$\angle AFD \cong \angle ADF = \frac{1}{2}(180° - 60°) = 60°$, therefore, making triangle AFD equilateral.

Introducing a circle in the figure helped Jane visualize what needed and relationships that existed. After this point, Jane moved on to triangle BDE that contains the angle of interest ∠AED = x˚. Based on measurement provided by GSP, Jane noticed that there was not a relationship between the lengths of the legs of triangle BDE and therefore could not draw any conjecture. She then moved on to triangle FDF and noticed that the measure of segment FE ≅ FD. Now the challenge for Jane was to prove why FE ≅ FD as illustrated in the GSP measurement in Figure 3 above.

To prove that FE ≅ FD was a challenge for Jane because according to her, it was not something obvious. To investigate this, I did ask her: What is your challenge at this stage? Jane responded:

> "To prove that FE ≅ FD has been a challenge at this time. But after thinking over and over, I decided to revisit the GSP file (Figure 3). I selected the following measurements and placed them side by side to analyze them for any clues as shown below.

$m\,\overline{AD} = 5.92$ cm
$m\,\overline{AF} = 5.92$ cm $m\,\overline{FE} = 5.92$ cm
$m\,\overline{AB} = 5.92$ cm $m\,\overline{DF} = 5.92$ cm

Comparing the GSP measurements <u>invoked the transitive property</u> in Jane's mind. As Jane noted:

> "It was obvious from my initial investigation that length AF ≅ FE, on the other hand as noted earlier, AF ≅ FD, therefore using the transitive property, FD ≅ FE."

Based on that, Jane noted that since FD ≅ FE, then triangle FDE is isosceles which means that FDE ≅∠FED.

In the interview with Jane, I noticed that now she had a clear idea of what she was going to do. She restated that triangle FDE is isosceles and therefore ∠FDE ≅∠FED. She continued to state that:

> Since BFE lie on a straight line, then ∠BFA + ∠AFD + ∠DFE = 180˚. But we do know that ∠BFA = 80˚and ∠AFD = 60˚ which means ∠DFE = 180˚ - (80˚+60˚) = 40˚. Let ∠FDE = y, and we know that triangle FDE is isosceles, then we can state that: x +40˚ = y ..(i) 40˚+40˚+x+y = 180, which means x + y = 100 (ii), solving the two equations, x + x + 40 = 100, which means 2x = 60, x = 30˚

In the end, Jane made very clear that without GSP, it could have been hard or even impossible to solve the problem:

> The dynamic nature of the GSP enabled me to construct, measure, and conjecture. That, in turn, forced me to ask why it appeared to be true, which prompted me to prove the conjecture. GSP enables the user to make constructions using different

colors and allows measurements of all parts of the construction. These are some of the attributes of GSP that make exploring geometry problems exciting.

Luke

Luke's construction was not that different from Jane's as shown in Figure 4(a). Luke investigated the problem by finding a point that he thought was equidistant from the points A, D and E. Luke tried several scenarios in looking for that point on BE, calling it point F. Luke explained what he thought he could show that points A, D, E lie on the circle with a center somewhere on CB. He now could use the theorem that states the measure of a central angle of a circle is twice the measure of any inscribed angle subtended by the same arc on the circumference, which means that angle AFD is twice angle AED = x.

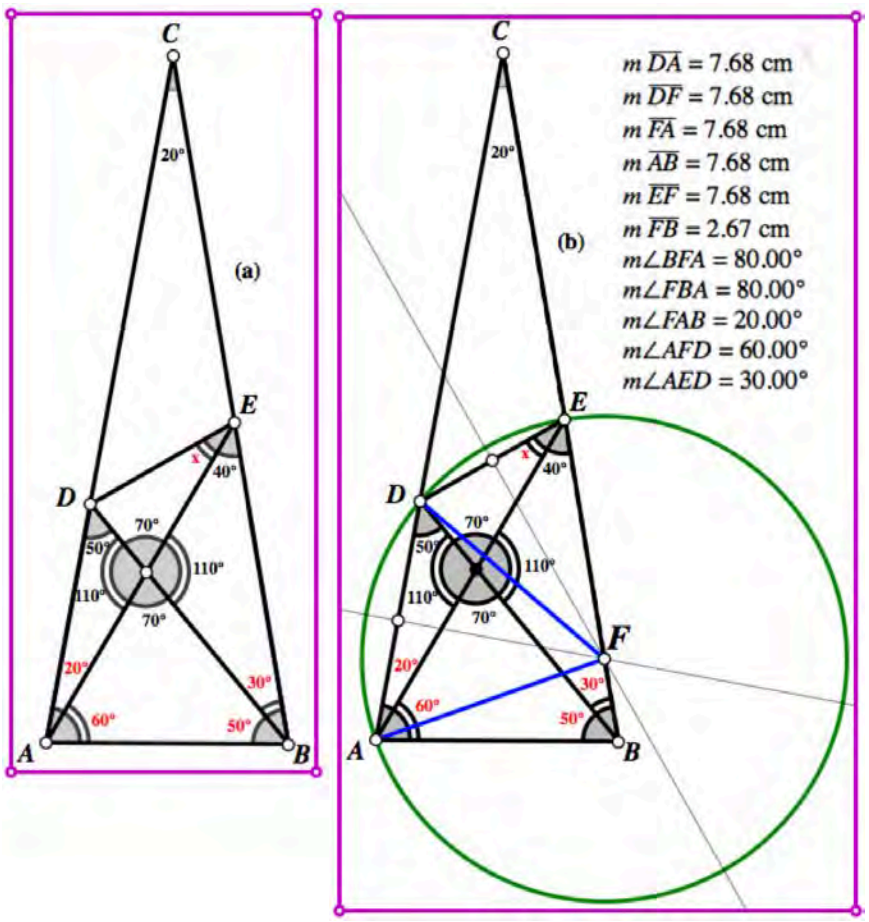


Figure 4

To investigate the problem, Luke thought of a way of finding that point F such that F is the center of the circle. He did so by first constructing a perpendicular bisector of segment AD and DE to locate the circle that goes through point A, D and E. By coincidence; both bisectors seemed to meet on segment BC at a point F (Figure 4 (b)).

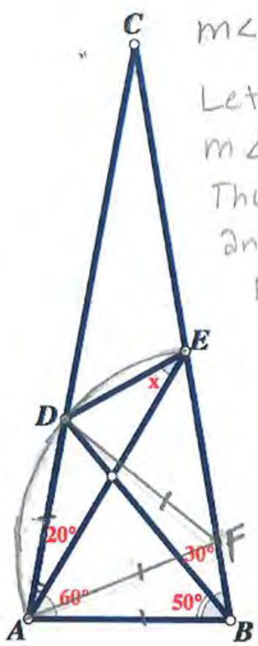
Using point F, Luke used the measurement tool to investigate what might be going on. This is what Luke said:

> As shown in figure 4(b), it appeared that AD ≅AF ≅AB ≅FE ≅FD and angle ∠BFA ≅ ∠ABF= 80˚. Which means ∠FAB = 20˚
>
> .

At this point, Luke said that his goal was to prove that F is the center of the circle and therefore the circle contains points A, D and E. To do this, Luke started by constructing BAF = 20˚. Luke knew that 20˚ couldn't be constructed geometrically. In the conversation, Luke noted:

> Given the information, I can easily construct 20˚ by bisecting ∠BAD (80˚) to get 40˚ and bisect the 40˚ to get the 20˚.

Based on that, Luke provided the following proof:



m∠CAB = m∠CBA ⇒ AC = BC

Let F be a print on BC such that m∠FAB = 20 ⇒ m∠DAF = 60.
Thus m∠BFA = 80 = m∠FBA, thus △AFB is an isosceles △ and AF = AB.
Now △ADB is also isosceles thus AB = AD.
Since AB = AD and m∠DAF = 60, thus △ADF is equilateral and DF = AF.
Now △AFE is isosceles because m∠FAE = m∠FEA, thus EF = AF. From this we have that EF = AF = DF, thus F is equidistant from F and is ∴ the circumcenter for △ADE.
∠AED is inscribed in a circle centered at F and is substanded by the same arc as the central angle, ∠AFD where m∠AFD = 60, thus m∠AED = $\frac{1}{2}$ m∠AFD = 30
∴ x = 30°.

The case for Luke was most impressive in that he relied on technology to construct, explore, conjecture, and finally prove. One issue that Luke struggled with his proof was that:

> When I said to construct 20˚, one thought came to mind that in real life, it is not possible to construct 20˚, but when I realized that two of the angles were 20˚ (∠ACB) and 80˚ (∠BAC), then I had two options, either to duplicate ∠ACB to create ∠BAF or bisect ∠BAC to create angle BAF.

This narrative by Luke indicated that he used technology as a tool to construct, investigate, conjecture and finally prove.

Conclusion

Technology, when appropriately used, can play a significant role in both students and teachers when doing proofs. Teachers, as well as students, can construct, explore, and conjecture to get insights on how to proceed with a proof. The tools that GSP provide enable teachers and students to do math investigations more quickly and efficiently (Selaković et al., 2019). They can construct, explore, and conjecture which may eventually lead to proof. Both Jane and Luke were able to make conjectures by using tools that GSP provides. They were able to conjecture based on data obtained from the measurement tools to proceed with their proofs. Training teachers on how to incorporate technology and encouraging them to use it may help teachers and students to investigate challenging mathematical problems that they previously would not even consider trying. It should be noted here that both teachers understood key geometric concepts that were needed to solve the problem. Knowledge of basic geometric concepts and the software is vital when solving or proving geometric problems. As Powellet et al. (2016) states;

> When learners appropriate a DGE as an instrument, they will be able to use it to demonstrate geometrical concepts and solve geometrical problems. This appropriation may result in knowledge of how to use dynamic geometry software as well as knowledge of geometry (p. 75).

In the problem that the two teachers worked on, they needed to be familiar with theorems, such as Angle at the Center Theorem, Sum of angles in a triangle theorem, and vertical angles. As much as GSP can be used to investigate a geometric problem, one should not be mistaken to think that such an investigation constitutes a proof in geometry. For the two participants, Jane tackled the problem by trial and error, whereas Luke was more thoughtful in the process. He thought about the circle theorem in the first place and received minimal help, whereas Jane required more assistance. Incorporating technology in teaching does make the teaching of geometric concepts easier as students are empowered to discover a particular geometric relationship that might not easily be conceived when using traditional methods of teaching. This may be because the dynamic feature in GSP is exciting to students who use it and it is efficient in constructing geometrical figures and think about mathematics in a different light. As the University of Illinois at Urbana-Champaign. (2010, January 5) notes that "If we help teachers try to understand what kind of thinking students will have when using technology, then we can help students to have a deeper understanding of mathematical ideas"

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