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Mathematics Teachers Reflection on the Role of Productive Pedagogies in Improving their Classroom Instruction

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Abstract: This paper sought to investigate the mathematics teachers' reflection on the role of Productive Pedagogies' framework in fostering effective classroom instruction. Two research objectives were used to discuss the reflection of the four mathematics teachers. A qualitative case study was adopted as the design of the study. Classroom observations, Research Journal and Reflective Interviews were organised to collect data for the study, while the narratives approach to data analysis was used to analyse the data collected. From the study, the mathematics teachers were able to gradually adjust to the new framework through effective reflective meetings with their colleagues and with the support of the researcher. They (mathematics teachers) reflected that, the framework helps them developed confidence, positive attitude and interest to their teaching practice. It helped fostered effective collaboration and created an atmosphere of trust between the mathematics teachers and their students. The study also suggested that the pedagogy influence the mathematics teachers to provide and received feedback from their colleagues and from their students. They were also of the view that there was a gradual shift from the traditional teacher centred instruction they are more conversant with to a more student-centred pedagogy. The implication of the study suggests the need to adopt the Productive Pedagogies framework into the Nigerian mathematics classroom to improve mathematics teacher's classroom instruction.

Keywords: *Productive pedagogies; improving classroom teaching; teachers' reflection to their teaching practice; student centred learning; teacher centred learning.*

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Introduction

Theories of teaching and learning based on constructivist principles suggest that students as active learners view mathematical learning as active manipulation of meanings (Davis, et al, 1990). This means that mathematics teachers desiring to achieve these constructivist principles in their classroom instruction should be able to provide learning environments that will help challenge their students' critical thinking. This constructivist epistemology which guides students' thinking suggests a shift from the traditional teaching strategies which seek to transmit fixed, well-structured knowledge with a firm external control of content, sequence and pace of learning by the mathematics teacher to a more relaxed, collaborative and participative student-centred learning. In this classroom, students construct their own knowledge, learn more independently and in the process acquire self-reliance (Heinich, et al, 2001). Nesher and Kilpatrick (1990) state that Piaget's contributions are essentially built on the basic idea that knowledge derives from the adaptation of the Learner's environment, is much richer than the knowledge received by mere recitations or procedural presentation of facts, as promoted by the traditional teacher centred epistemology. Apart from students' interaction with the environment within and outside the classroom, social interactions among students also play a major role in students learning, particularly during mathematics classroom practice. This reflects the Vygotskian perspectives on social constructivism. Similarly, and closely related the Vygotskian epistemology of social interaction is the work of Bruner's (1990) who was of the view that "*Culturally adapted way of life depends upon shared meanings and shared modes of discourse for negotiating differences in meaning and interpretation*" (p. 13). The view of Bruner centred on the concept of shared ideas among students and negotiation between learners during mathematics classroom practice.

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The Traditional Mathematics Classroom Teaching in Nigeria

Teaching of mathematics in Nigeria is characterized by the traditional formula- based approach with emphasis on computation with little reference to mathematical reasoning and problem solving. According to the Nigerian Education and Research Development Council (NERDC) this type of teaching is mechanical and teacher-centered; it is outdated, since in this approach, mathematics is taught and learned instrumentally by rote and memorization, without meaningful understanding of concepts taught (NERDC, 2013). Students become frustrated in the face of apparently meaningless symbols that are manipulated. They regard mathematics as a static subject with a set of algorithms to be applied mechanically, as it is learnt through exercises or drill and practice (Igbokwe 2000; NERDC, 2013; Ogunbiyi, 2004).

The review of the performance of students in mathematics from the national examinations conducted in Nigeria suggest that, a large percentage of students are finding it difficult to perform moderately complex tasks that requires higher order thinking skills in mathematical problems (Adedayo, 2001). This is because most mathematics teachers are unskillful in implementation the modern teaching strategies (Bature, et al, 2016; Mustapha, 2001). There is also the unrepentant attitude of most mathematics teachers in Nigeria to the implementation of the traditional approach to classrooms practice (Bature & Jibrin, 2015). In view of these, most mathematics teachers in Most mathematics classrooms in Nigeria find it difficult to help their students make use of higher order thinking skills to reconstruct these complex mathematics tasks (Bature & Jibrin, 2015; Nneji 1998)

It is a common knowledge to the happening in the Nigerian mathematics classroom that mathematics teachers most at times feel satisfied when they can make their students successfully perform routine, computational tasks using formulas or other traditional teacher centred approach to classroom practice (McKnight, et al, 1987). These teachers tend to feel satisfied without really checking whether these students can consistently and successfully solve problems requiring applications higher order thinking skills or are having the ability to solved more complex and difficult tasks that require students' reconstruction of knowledge (Atweh & Goos, 2011; Dossey, et al, 1988; Lowe, 2013).

Researchers globally had lent their voices to the discussions on the traditional, didactic teaching strategies in their countries (Porter, et al, 1988; Westwell, 2013). They were of the view that most mathematics teachers concentrate their attentions on developing the computational teacher given or students receiving curricula, which is the reflection of the traditional mathematics classroom practice that had dominated the Nigeria classrooms (Lowe, 2013; Westwell, 2013). The development of teacher giving computational skills and the students receiving recitations skills occupies almost three-quarters of their time spend during mathematics classroom practice (Bature & Atweh, 2016; 2019). This could be similar, to the argument of Schoenfeld (1988) who identified mechanical procedures and symbolic manipulation as believed by most mathematics teachers to mean mathematical learning as against the view of Campbell and Bolyard (2018) who believed that students need more time to talk during classroom instruction.

Abimbade and Afolabi (2012) complained that some mathematics teachers have consistently stuck to their traditional teacher-centred mathematics teaching strategies in most Nigerian schools. However, Igbokwe (2000) and Ogunbiyi (2004) reflected from their research that this traditional-centred teaching strategy is generally due to the lack of adequate instructional materials and teachers' unskillfulness in the use of effective teaching strategies. For these teachers, teaching mathematics simply implies memorizing rule-bound algorithmic skills and procedures that are devoid of context or meaning (Bature, et al, 2016; NERDC, 2013). NERDC (2013) when further to observed that this type of mathematics classroom practice illustrates the trivialized, superficial learning that lacks deep understanding, deep knowledge as promoted in Productive Pedagogies framework.

The traditional mathematics classroom is that classroom environment that is traditional. The teacher is believed to be in possession or in control of the growth and development of knowledge and understandings. In this type of classrooms, students are asked to individually solve exercises in a teacher-dominated environment (Lyman & Foyle, 1990; Slavin, 1990). Teachers are seen to dominate classroom discussion and maintain structures that heavily rely on teacher-student recitation (Good & Brophy, 1987). Researched into the Nigerian mathematics classrooms suggests that these traditional patterns of mathematics classroom practice had remained largely unchanged and unchallenged. These traditional teachers centred pedagogical patterns reflect the authoritarian, didactic approach to classroom teaching where the mathematics teacher monopolized classroom activities (Adamu, 1992).

This traditional classroom practice patterns, can be likened to a transaction in a bank. Instead of assisting students to develop knowledge with other students during classroom instructions, the teacher deposits information, which students patiently receive, memorize, and repeat exactly the same back to the teachers when asked (Bature & Atweh, 2019; Freiere, 1996). This is the 'banking' concept of education, in which the scope of action allows students to receive, file, and store deposits of facts delivered to them by the teacher (Bature et al, 2015). Similarly, one could continue by asserting that these traditional teaching strategies still being adopted by mathematics teachers in Nigeria are based on the objectivist epistemology which has been questioned by many researchers globally and locally (Jerinic, 2014; Afolabi, 2001; Okwo, 2000; Usman, 2001).

One of the major challenges in these traditional teaching strategies is that the teacher's perspective becomes the major determining factor in the resulting classroom environment (Bature & Atweh, 2016; Ezekute, 2000). For example, what teachers believe about classroom discourse influences the types of discussion that occur within their classrooms, so students have little or no influence in the direction of the discussion (Cohen, 1990; Thompson, 1994; Wilson & Lloyd, 2000). The effect of this traditional classroom setting could be viewed as described by NERDC (2013) that the natural curiosity of students who are eager to understand their environment are often diminished by instructions that discourages inquiry or students' self-discovery learning due to lack of expertise by most mathematics teachers who use the traditional approach during classroom instruction. Obviously, this pedagogical pattern is not equipping the students mathematically to live effectively in our contemporary age of science and technology as stipulated in the National Policy on Education (FME, 1998; NERDC, 2013).

In view of these, Mansaray and Amosun (2002) suggested that it is necessary for mathematics teachers to focus their classroom practice towards developing students' critical thinking capabilities to succeed in resolving the challenges facing their classroom instruction. Similarly, Abimbade and Afolabi (2012) suggested that to develop students' critical thinking, deep knowledge and deep understanding during mathematics classroom practice mathematics teachers should be able to develop teaching strategies that elicit thought provoking responses from their students. To develop such critical thinking capabilities of students, Obanya (1999) suggested that there is need for emphasizing teaching strategies that encouraged critical analysis of issues, and a democratic classroom atmosphere that encourages exchange of ideas and opinions among students, which is in line with the Intellectual Quality dimension of Productive Pedagogies.

This suggests the need for the replacement of these traditional classroom practice that supports drill and memorization of mathematical procedures with classroom teaching that is student-centred and that supports students' engagement with conceptual issues of inquiry, collaboration and problem solving. It also helps in developing students' mathematical insight and understanding, students' learning, participation and engagement during mathematics classroom practice (Baroody, 2004; Clements, et al, 2004; NCTM, 2000). This implies that there is the need to shift from the traditional teacher-centred classroom practice to a more student-centred classroom instruction as suggested by Wilson and Lloyd (2000) who were of the view that this classroom pattern can be altered if mathematics teachers and their students are willing to shift their roles and beliefs. For them, shifting the expectations of traditional mathematics classroom practice to a more engaged system where everyone is involved makes mathematics classroom instruction real and engaging as promoted by the Productive Pedagogies framework.

Similarly, mathematics teachers who want to shift from the traditional mathematics classroom to more student-centred classrooms approaches should learn to adopt strategies that will encourage interactions, collaboration, students' participation and engagement between the teachers and his students and between students. Wilson and Lloyd (2000) were of the view that this concept of shift from the teachers' control of the mathematics classroom teaching to a more relaxed classroom environment where learning will be done collaboratively in a more non-authoritative way does not necessarily imply that the mathematics teachers' authority is completely discounted. Rather, it makes mathematics teachers emphasize student involvement by decreasing their traditional role of givers of knowledge and playing a more relaxed role of guiding students' knowledge and understanding. Suggesting that mathematics classroom where students' ideas are solicited and valued as important contributions to developing knowledge and understanding of concepts and problems helped such classrooms become collaborative, learning environment are evolves.

Productive Pedagogies Framework

Productive Pedagogies is a framework developed by the School Reform Longitudinal Study (SRLS) research team of the University of Queensland to enhance quality classroom teaching (Lingard, et al, 2001). It is made up of twenty elements that have been broken down into four main dimensions (See attached appendix). This include Intellectual Quality, Supportive Classroom Environment, Connectedness, and Recognition of Differences. Intellectual Quality is an important dimension in achieving quality classroom teaching. Previous studies indicate that high Intellectual Quality classrooms help improved students' performance during classroom instruction (Boaler, 1997; Hayes, et al, 2006). The Supportive Classroom Environment dimension is needed to create and enable learning environments involving support and engagement in order to foster high Intellectual Quality and Connectedness.

Connectedness attempts to make mathematics more relevant to students' life by connecting students' life experiences either at home or in the society with their lives in school (curriculum and content) or other school subjects. This attempt is with the view to making mathematics more 'relevant' and provide students with more meaningful life experiences. Finally, on Recognition of Difference, Hayes et al. (2006) was of the view that teachers should give more emphasis and need to recognize the diversities that exist among students with different cultural backgrounds and beliefs during classroom instruction to provide an equitable classroom environment for all students.

Research studies in the literature have demonstrated the role of Productive Pedagogies in mathematics classroom instruction. Similarly, there are professional development programs being developed in many areas today, focusing on Productive Pedagogies to train teachers on how quality classroom teaching could be achieved (Gore, et al, 2001, 2002).

What the Queensland School Reform Longitudinal Study (QSRLS, 1999) report has termed Productive Pedagogies therefore is crucial to improving classroom instruction and engagement for all students during mathematics classroom practice (Lingard et al., 2001). However, the QSRLS (1999) pointed out two facts necessary to make the research on Productive Pedagogies open to other social settings for further research and investigation. First, they asserted that, while each of these dimensions are readily defended in an ideal setting (Australian setting), there is no research basis for believing that school systems everywhere have been consistently overly successful in providing high levels of any of these dimensions to a large proportion of student populations in all settings. Secondly, the four dimensions of Productive Pedagogies in an ideal world or learning environment may be necessary and sufficient for all students to perform well in school.

However, there is no substantial evidence or research basis for believing that all the dimensions are equally required for success in all socio-cultural societies (Atweh, et al, 2011). Atweh et al (2011) supported the assertion of QSRLS (1999) by stating that different segments of classroom activities may reflect some of these dimensions more than others. Secondly, these dimensions are not mutually exclusive; that is, one activity can demonstrate high Intellectual Quality while others may not. Thirdly, the demonstration of the various dimensions in a classroom may not necessarily be reflected in the selection of a specific activity; and lastly, this framework does not attempt to provide reliable quantitative measure of the implementation of various dimensions. One may conclude here by saying that Productive Pedagogies may work in one environment and fail to work in other environments depending on the prevailing circumstances surrounding the environment (Atweh, 2007; 2014). Furthermore, this distinction implies an open challenge for researchers to test the workability of the experiment in their environments because both researchers QSRLS (1999) and Atweh (2007) maintain that there are possibilities of having different results in other environments.

This paper therefore was set out to investigate the perception of mathematics teachers on the introduction of Productive Pedagogies as to tool for reforming mathematics classroom practice in Nigerian schools. More specifically the following research objectives guided the study.

1. To Investigate the mathematics teachers' Initial perceptions on Productive Pedagogies Framework as a tool for reforming their mathematics classroom practice
2. To Investigate the mathematics teachers' reflection on the benefits of Productive Pedagogies Framework

Reflection on Classroom Practice

Reflection on classroom practice could take place at any time. The reflection in this context had to do with what the mathematics teachers felt after using the Productive Pedagogies framework to achieved quality classroom instruction. It suffices to suggest that the most important reflection in mathematics classroom practice is the one that occurs immediately after classroom instruction. This is in line with the view of Cooper (1999) who believed that effective reflection occurs when teachers takes their mind away from the hustle and bustle of classroom interactions. This is because effective reflection involves self-evaluation through critical analysis of teaching, decisions on the outcomes of a classroom activity or a teaching strategy to determine how effective the activity was, and how beneficial they were to the teacher and the students in achieving the objectives of the lesson or activity (Cooper, 1999).

Dewey (1933) noted that reflection is an essential component of professional development and can place a novice teacher on the path to becoming an expert teacher. There is a connection between reflection and constructivism (Schunk, 2012). This is because it is through reflection that mathematics teachers can mentally construct new knowledge about themselves, their teaching, their students and can continually improve and modify their concept for the betterment of the entire system (Snowman, et al, 2012). Similarly, Osterman and Kottkamp (2004) were of the view that reflective practices incorporate key elements of constructivism, experiential learning, and situated cognitive learning experiences. Effective learning must involve active processes through which knowledge construction and experience occur as the result of collaboration of ideas between students and their teachers.

Methodology

The Design

This study employed a qualitative case study approach to seek data for this research. This was adopted for this study because the qualitative case study epistemology provides tools for researchers to study complex phenomena within a given context. It also allows researchers to explore subjects of their study through complex interventions, relationships, communities of practice or programs (Yin, 2006), thereby promoting the deconstruction and the reconstruction of the phenomena under study. When the approach is applied correctly, it becomes a valuable approach to conducting qualitative research with the view of developing interventions. This is one of the key reasons for adopting the qualitative case study in the research. The desire to improve classroom instruction was the focus of the researcher, hence adopting the case study. Stake (2005) and Yin (2006) view qualitative case study in different perspective, however, they both agreed that a case study epistemology seeks to ensure that the concept under study is well explored.

The Participants

This research was aimed at studying classroom instruction of four mathematics teachers. The researcher observed each of the mathematics teachers for a period of ten weeks for the first phase and five weeks for the second phase spread over two years. In the first phase the researcher worked with four mathematics teachers (Audu, Abba, Aminu and Amina) for a period of 10 weeks equivalent to a teaching term in Nigerian secondary schools. In the second phase, the researcher adopted a follow-up approach to explore the progress of two (Aminu and Abba) out of the four mathematics teachers. The two teachers were selected because as at the time of the follow up they were the only available and willing teachers to participate in the research. The other two teachers were busy with other official engagements. The two available teachers took one class each to explore their further understanding of the Productive Pedagogies' framework. The two mathematics teachers voluntarily choose one senior secondary one class each to implement the Productive Pedagogies framework in their classroom instruction. Aminu selected a senior secondary one science class while Abba selected one Senior secondary one class Arts. Both teachers were teaching in the same school and the content of their mathematics was the same. This suggest that what Abba was teaching in the arts class Aminu was teaching the same in the science classes.

Workshops/ Intervention

Since Productive Pedagogies was a new concept of classroom practice in Nigeria, there was the need for the mathematics teachers to be acquainted with the framework and developed understanding on how to implement the framework during their classroom teaching. Therefore, before the research started, the researcher had to provide an intensive one-week workshop to familiarize the mathematics teachers on the concepts and dimensions of Productive Pedagogies (See Appendix) and how they can use the Productive Pedagogies framework to achieved quality classroom instruction.

Data Collection Instrument

The instrument used for data collection in this study is reflective interviews. These reflective interviews were videotaped or recorded. There were two-hours of reflective interviews after every three weeks of the mathematics teacher's classroom practice. The researcher and the mathematics teachers will meet and review the classroom practice of each of the four mathematics teachers one after the other. During these periods, the researcher and the mathematics teachers as a team discussed the weaknesses and strengths of each teacher before launching into the next three weeks. There was a total of four reflective interviews throughout the one and half years of the research. As stated above, since this study was part of a major study, the analysis concentrated on the reflective interviews with the teachers. This is because the researcher in this article concentrated on the views of the teachers on the benefits, they obtained in implementing the Productive Pedagogies framework during their classroom instructions. However, the researcher also kept a research Journal to note or record data collected outside the reflective meeting sessions. These could be problems encountered by mathematics teachers on issues they needed clarifications or observations made by the researcher before, during or after any reflective interview.

Implementation of the Study

This study adopted the Productive Pedagogies framework in approach to teaching mathematics as described in the Appendix. Productive Pedagogies are twenty elements divided into four dimensions that could be used to determine quality classroom instruction (Atweh and Goos, 2011). This paper was only interested in the benefits achieved in implementing the framework in Nigerian mathematics classroom and not the identifications of how the elements and the dimensions were implemented or achieved. Therefore, only the responses of the teachers on what the benefited implementing the framework in the classrooms were analyzed and used in this article.

Validation of the Instruments

Several techniques were adopted by the researchers to validate the instruments used in collecting data for this research. The first step of ensuring the validity of the data collected was choosing a well-trained and skilled moderator or facilitator to look at the quality of the data collected. This was done by a senior academic in the faculty of education. The moderator checked personal biases and expectations of the research. While a good moderator is key, another strategy adopted to validate the data is the choice of the sample group. The participants were truly members of the segment from which they are recruited. The research also employed ethical recruiting to collect data from mathematics education specialist who are truly representative of their segment, which the researchers believe will lead to achieving a valid result. The researcher also employed triangulation strategy to validate the data by adopting multiple perspectives of using several moderators, and different locations with the view of getting the results from different angles. There were also the respondent validation strategies, deep saturation into the research with the view of promoting validity and finally, there was the seeking of an alternative explanation of ideas from the participants.

Ethical Issues

Appropriate ethical issues that involve taking permission from the school and the parents of the students used for this study were followed. For example, all the mathematics teachers used in the study voluntarily consented to participate in the teaching and engaged in Reflective interview/meetings, casual discussions and some intermittent interviews were also used as means of providing support to the teachers. Before then, the researcher has obtained relevant ethical permission from the school to carry out the research. Relevant consents were also obtained from the mathematics teachers' head of department.

Data Analysis

Data analyses in this paper concentrated on the reflective interviews on views and the observations of the mathematics teachers and the researcher on their reflection on the use of Productive Pedagogies as a tool to reform their classroom practice. The narrative approach was adopted to analyses the data collected through reflective practice of the mathematics teachers. This agrees with the views of Clandinin and Connelly (2000) who were of the view that narrative could be used in analyzing research data that the researcher might have recorded, or a story through a journal, diary or that the researcher might have observed individuals and records fieldnotes during data collection. During narratives the researcher seeks to identify themes, categories, and patterns in order to determine relationship between the themes identified and the response of the subjects to the research questions. After which the research will begin to make to data collected through a process of rigorous and constant comparisons of related themes (Braun & Clarke, 2006; Clarke, 2006; Creswell; 2007).

While conducting qualitative research, researchers collect large chunks of data, either through observations or as in this case, through reflective interviews. After the data were collected, the researcher sat down and watched the videos, transcribed all that was recorded into a written document. After this process the researchers read through the transcripts and begin to group the ideas into themes, codes and patterns, with the view of making meaning of the data. Research studies suggest that, there are two approaches to thematic analysis (Caulfield, 2019). The researcher could decide to use inductive thematic analysis which allows the data to determine the themes to be used or a deductive analysis where the researcher comes to the data with some preconceived themes (Carter & Fuller, 2016; Caulfield, 2019; Evers, 2016). In this paper, the researcher adopted both the inductive and the deductive themes to approach the data analysis. In data analysis themes majorly, key words used to group the data into reasonable patterns or codes. Therefore, the data collected were coded and all the codes were sorted into different themes by grouping the mathematics teachers' comments, views and perceptions into related patterns. After which, the researcher began to make meanings of themes through the process called analysis.

Results

This paper therefore was set out to investigate the perception of mathematics teachers on the introduction of Productive Pedagogies as a tool for reforming mathematics classroom practice in Nigerian schools. In this section the research will analyse the data according to the research questions postulated above under the following themes. (1) the mathematics Teachers Initial perception of Productive Pedagogies framework and (2) the mathematics teachers' reflection on the benefit adopting Productive Pedagogies framework in their classroom practice. In the analysis two key words would be used Phase 1 and Phase 2 to reflect the phases the data used was collected. The researcher will use P1 for Phase 1 and P2 for phase 2 as keys.

Mathematics teachers initial Perception of Productive Pedagogies.

Objectives 1: To Investigates the mathematics teachers' Initial perceptions on Productive Pedagogies Framework as a tool for reforming their mathematics classroom practice

When this research started the mathematics, teachers greeted the idea with mixed feelings. Some were of the view that this is just another research project, others felt that there is nothing new or good that will come out of it, while others were indecisive. For example, one of the initial comments raised by the mathematics teachers was that:

..., in education, if you are dealing with human beings in the area of teaching-learning you have to be careful. Now, as you know there are so many methods of teaching and learning people are coming out with today..., which have not really helped the system..., Is this Productive Pedagogies framework a savior to the teaching learning problems we have with our students in Nigeria today? (Abba; Reflective interview: P1)

The comment of Abba above suggest that the mathematics teachers had initial skepticisms on the feasibility of Productive Pedagogies framework in the Nigerian mathematics classroom. They wondered, if there are the possibilities of its effectiveness in reforming their classroom practice. For example, coming from the traditional classroom background, the mathematics teachers had initial concerns about their role in implementing the Productive Pedagogies framework as a tool for reforming their classroom practice.

Allowing students to direct the learning as proposed in the Productive Pedagogies' framework (Student Direction) what then is the role of the teacher? Will the class not be rowdy, and the students take over the management of the classroom from the teacher? Because..., (Smile) as a teacher I should have control of my class..., these our students..., Hmn.... (everybody laughs....) (Aminu: Reflective interview: P1)

The view of Jackson above suggests that the mathematics teachers were not comfortable with the concept of student direction as being part of the elements of Productive Pedagogies. They argued that releasing classroom control (Student Direction) to their students could prove counter-productive to effective classroom teaching. Abba supported Aminu and argued by saying;

Sir, if we hope to achieve our objectives of completing what we have plan for the students, allowing students to control their learning as proposed in the Productive Pedagogies framework is not visible.... Some students are generally slow..., some will take advantage of the framework to discuss other things which will make the classroom instruction distractive... so I have my fears... (Abba: Reflective Interview: P1)

Closely related to the argument of Abba above, Aminu came out so bluntly by asserted that mathematics teachers are to explicitly determine what activities students should do and how such activities should be done if teachers hope to meet their learning objectives as suggested by Abba above.

When the teacher is in the class, he is supposed to be in control of all the activities in his class. He should be in-charge of directing all the affairs of his class, allowing students to take control of the classroom activities might be counterproductive. (Aminu: Reflective Interview: P1)

Such views about strict control of classroom are widely held among Nigerian mathematics teachers. From the researchers' experience in classroom teaching, the ability to keep your students on checked during mathematics classroom instruction makes you a good teacher in the traditional mathematics classroom setting. This philosophy, though good to some extent, however, if not properly managed during classroom instruction, the tendency to create a gap between the teacher and the students is high, and it sometime leads to the overbearing nature of the traditional teacher. For example, there were evidences of the overbearing-authoritarian mathematics classroom environment created by the mathematics teachers in the first week of the research. This was observed by the researcher in most of the classroom visited during the implementation of the Productive Pedagogies by the mathematics teachers.

I observed that the classroom environment of most of the mathematics teachers were not friendly. The students were not giving the opportunity to relate freely with their classmates, the teachers were just working alone..., they were just writing on the board..., even when they gives students problems to solve, they will not allow them to finish or interact with their colleagues..., they will just go to the board and solve the problem, and students were not part of the lesson..., I also observed that the mathematics teacher were too strict, too serious ..., did not allow the students a breathing space...; shouts down on students when student makes little mistake..., etc, (Researchers Journal: P1).

However, despite these reservations and challenges, the researcher and the mathematics teachers kept faith in the project and kept moving step after step. As the research progresses the perceptions of the mathematics teachers gradually changes. Amina asserted that

There was a gradual development of interest in Productive Pedagogies framework as we progressed day after day..., we begin to see that our students learnt more working and taking responsibility of their learning..., ok..., we can also see what we learnt in the university..., the constructivist view of classroom teaching..., (Amina: Reflective Interview: P1).

Hmnnn.... yes..., you are right Jenny..., the students' self-construction of knowledge..., students having responsibility of their learning..., you know you (the Researcher) said we should find out that in literature..., this is constructivism in Action.... (Aminu Reflective Interview: P1)

Similarly, two other mathematics teachers reflected on the benefits of the framework and felt they will continue with Productive Pedagogies framework in their further study.

The Mathematics Teachers' Reflection on the Benefits of Productive Pedagogies Framework

Objectives 2: To Investigate the mathematics teachers' reflection on the benefits of Productive Pedagogies Framework

In research objective one, the researchers discussed the initial fears the mathematics teachers had when they encountered Productive Pedagogies framework for the first time. In this section the researcher discusses the reflection of the mathematics teachers on their teaching and how the framework helps them improved their practice.

Shift from the Teachers Domineering Classroom Instruction

Data collected suggested a shift from the teachers domineering classroom instruction to a more relax classroom environment. From the reflection of the mathematics teachers, the new framework brought about improvement in their teaching pedagogies thereby helped them capture the minds and interests of their students, for example, Jackson said,

I started slowly..., in my normal domineering classroom teaching..., I generally insist my students to sit quietly and listen to me teach..., but, using Productive Pedagogies framework in my class seems to help me capture the minds and interests of my students easily, as they were all engaged..., motivated and challenged (Aminu: Reflective Interview: P1)

Abba was of the view that adopting Productive Pedagogies framework in his classroom helped him teach mathematics at any time of the day as against the general believe that mathematics can best be taught in the morning hours. He said

..., mathematics teachers in Nigeria and from experience generally believe that mathematics is best taught in the morning hours in primary and secondary schools. But I observed that using Productive Pedagogies in my class..., I could teach mathematics at any time of the day, whether morning or afternoon. (Abba: Reflective Interview).

Abba when further to state that Productive Pedagogies does not only foster effective teaching of mathematics at any time of the day but also it helped arousing interest in mathematics and makes classroom practice livelier.

I think the reasons were because we tried to relax our classroom environment, created opportunities for students independent learning..., this arouse the interest and makes learning livelier. (Abba: Reflective Interview: P2)

Productive Pedagogies help in Improving Mathematics Teachers Teaching Pedagogies

The mathematics teachers were of the view adopting dialogue, discussions, and debates with their colleagues before classroom instruction provided further assistance on areas needing improvement in their practice and believed that this process helped increases there teaching pedagogies. According to the mathematics teachers, the discussions were characterised by comparatively lengthy interactions between the researcher and the mathematics teachers or between the mathematics teachers during reflection meetings in a context of collaboration and mutual support.

The way the program was structured emphasizes very challenging learning objectives, we received and provided support to one another, and not only is feedback given throughout the program, but we actively sought for it, not only from the researcher, but also from colleagues. (Abba: Reflective Interview: P2)

Notes from the researcher's journal also suggested that,

Discussions and dialogue allow the mathematics teachers have thoughts on issues they could not have had on their own, yet to recognize these thoughts as developments of their own thinking. These thoughts help the mathematics teachers, developed their own pedagogies, reflect on their practice and build up confidence to change their initial views of their teaching practice (Researcher: Observations: P1)

Amina and Abba at different times asserted that discussions with colleagues and the research helped them in build a framework that guided their teaching practice during the research. Amina was of the view that this create in them and indeed in mathematics teachers a sense of responsibility in their classroom teaching practice.

During reflective interviews we share ideas, discussed our challenges, and received either support or criticisms from our colleagues. through these we received new insights..., learn new ideas..., create new approaches to classroom teaching..., which I think helped me in my teaching practice. (Abba: Reflective Interview P2).

When problems are generated, we all discussed together to find a common approach that could be used to address such problem, this help me approach my classroom practice with a sense of belonging..., it also gives me the sense that there are people that will support, encourage and criticized me at each stage of your work (Audu: Reflective Interview: P1)

The data collected did not only suggest that, dialogues, debates and discussions were only among the mathematics teachers, but the mathematics teachers created the same atmosphere during their classroom instruction. Aminu Asserted

Discussions with the colleagues makes me adopted the same strategy with my students. This makes my classroom instruction interactive..., My students engaged in discussions dialogues and interactions with their peers in their effort to create their own knowledge (Aminu: Reflective Interview: P2).

And from the reflection of the mathematics teachers, creating classroom atmosphere that encouraged debates, collaborations and dialogues among students, helped students build understanding, explore ideas and practice collaborative thinking. For example, Aminu argued that

The Productive Pedagogies framework helps mathematics teachers build understanding, explore ideas, practise thinking through and expression of concepts among students, that will help them achieve quality classroom practice; (Audu: Reflective Interview: P1).

My students also engaged in substantive conversation, both in the class and outside from the report I gathered from them..., (Aminu: Reflective Interview: P1).

Amina also reflected that Productive Pedagogies helped mathematics teachers developed in their students the ability to construct their own ideas, skills, concepts and/or processes to improve their learning

When problems are generated, we all discussed together to find a common approach that could be used to address such problem. It also helps my students developed self-efficacy..., (Amina: Reflective Interview: P1)

From the view of these mathematics teachers, it suggests that the adoption Productive Pedagogies framework work as a tool for achieving quality classroom instruction and learning requires effective discussions and dialogues. If this is adopted and properly implemented it will help mathematics teachers and their students develop new strategies for reasoning, enquiry and negotiation of ideas and also provide opportunities for cooperation among teachers and students during classroom instruction and among teachers in the staff room.

Data collected in the study suggested that the introduction of production pedagogies to the mathematics teachers' classrooms provided opportunities for them to give and received immediate feedback from their students and their colleagues.

Personally, the experiences of discussing with colleagues help my problem-solving skills, provide opportunities for cooperation, and there is an enhanced level of immediate feedback from my colleagues and sometimes from my students (Aminu: Reflective Interview: P1).

From the view of Aminu, dialogues and discussions provided avenues for the mathematics teachers to receive immediate feedback from professional colleagues. It also helped to strengthen and broaden the understanding of the mathematics teachers and provided feedback on their strengths and weaknesses during classroom teaching. For example, he said,

The way the program was structured emphasises very challenging learning objectives, we received and provided support to one another, and not only is feedback given throughout the program, but we actively sought for it, not only from the researcher, but also from colleagues. (Aminu: Reflective Interview)

Productive Pedagogies helps mathematics teachers Improved Students Confidence and Trust

Data collected in the study suggested that the introduction of the Productive Pedagogies framework in the mathematics teacher's classroom practice increases students' confidence and trust toward mathematics and mathematics teachers.

The general phobia that had bedeviled most students towards mathematics in Nigeria classrooms were much reduced, my students approach problem solving in mathematics classroom with confidence..., they were engaged in effective collaboration among themselves. (Amina: Reflective Interview: P1).

This is as result of the new confidence observed in students during the classroom instruction of the mathematics teachers as they employed Productive Pedagogies principles and strategies. For example, Amina reflected that,

Productive Pedagogies removes this general fear that students have of their mathematics teachers..., In the situation where the students sit quietly in the class, no talking, their responsibility is just to listen and obey whatever the teacher said does not really give them the opportunity to relate well with the mathematics teacher. (Amina: Reflective Interview: P1)

Data collected from the student suggested that the mathematics teachers played a passive role of supervision and support, rather than the normal traditional role of teaching and dishing out of instructions or information to their students. Amina commented

It seems to me like..., the students to some extent have some control over the learning activities in their various groups..., the teacher only supervises and offers assistance when he discovered students are in sort of a problem..., apart from some levels of control they have on their learning activities, they seems to assimilate faster, when I used Productive Pedagogies framework in my class. (Amina: Reflective Interview)

That is true Amina..., this culture of the traditional teacher-centred learning that had dominated the Nigerian mathematics classroom in my own view gave way to a more students-centred learning. My students took control of their classroom learning, I was only providing support to those with learning challenges.... (Audu: Reflective Interview: P1)

In my class, sometimes you wonder who is the teacher...? Or who is the students..., the students were very active..., I wonder where they gather these knowledges to be so independent among themselves..., (Abba: Reflective Interview)

Productive Pedagogies help Mathematics teachers Improved Students' Attitude

Data collected in this research and reflected by the mathematics teachers demonstrated promising improvements in students' attitudes to mathematics when Productive Pedagogies framework was used as a tool to improve their classroom practice. One of the mathematics teachers asserted that;

The students were serious and committed with their learning. From what the teachers said, they were always ready to defend the solution to their problems as against the background of fear and timidity that greeted most of their classroom in the past. (Audu: Reflective Interview)

There were attitudinal changes towards mathematics teaching from the side of the students and towards learning. This is because of the changes in mathematics teachers domineering attitude towards the students. (Amina: Reflective Interview)

There were also attitudinal changes towards mathematics as a subject, because a girl said that students now love mathematics in the class, and don't look at mathematics as that difficult again (Amina: Reflective Interview)

The mathematics teachers were of the view that the result of their students being responsible for their learning makes them develop positive attitudes towards problem solving. Similarly, the mathematics teachers reflected that, there were positive improvements in students' interest to mathematics and mathematics teachers. Audu commented,

The students were very interested in the way I use Productive Pedagogies to set my class. They were so free to communicate with me..., the students were interested..., they were always expecting something new from me..., that makes my mathematics classroom activities lively. (Audu: Reflective Interview).

Productive Pedagogies helps in Improving Students Participation, Engagement, Relationships and Interactions During Classrooms Instruction

Data collected suggested that there was improvement in teacher-students and students-student's relationships. This improvement was demonstrated through the interactivity that existed between students;

I observed that students-students relationship greatly improved, as one can really see the students interacting with one another and sharing ideas among themselves. It's like they see learning as being collaborative rather than it being competitive. (Amina: Reflective interview)

Amina reflected on her classroom practice and was of the view that there was effective student participation and engagement during her classroom practice. Amina was also of the view that inclusivity was demonstrated in her class, students were involved, and ready to assist and contribute to the mathematics classroom activities. She reflected,

I observed that my students' participation increases during my classroom practice, compared to the conventional classroom practice I am used to..., my students were free and friendly with in my class..., they contributed to problem solving activities in the class..., there was interactivity between the teacher and students and between students which was not allowed in my conventional mathematics classroom practice. (Amina: Reflective Interview: P1)

..., the introduction of Productive Pedagogies created good relationship that was fostered and developed by the effective interaction that existed between the students. It helped my students to move freely in the class to seek assistance from their colleagues and give the same assistance to one another (Amina: Reflective interview: P1).

Aminu reflected that initially, he viewed interactions between students during classroom teaching practice as distraction and disturbance. However, noted that his strict and domineering mathematics classroom became relaxed and students began to interact with one another, his students were free to move from one place to the other to seek and give assistance to one another. In his view this makes teaching and learning more students centred and the notion that a mathematics teacher knows everything is deemphasized.

I must confess..., my classroom used to be like a graveyard as students dare not talk, but to my amazement when I introduced Productive Pedagogies framework in my class, the class naturally became interactive, the students interacted in their groups, before you know, the solution to the problem is gotten and even those who fear mathematics you see them ready to defend their answers. (Aminu: Reflective Interview: P2)

What do you Mean? (Abba: Reflective Interview P2)

When I used the Productive Pedagogies framework to set up my classroom, I observed that my class naturally became interactive..., my students started interacting with one another and the classroom learning becomes a

different thing entirely..., particularly in this second Phase..., I wondered how the change was dramatic (Aminu: Reflective Interview).

What was the difference from the first Phase? (Researcher: Reflective Interview)

My classroom environment was truly relaxed; I think because I created a friendly atmosphere for teacher-students and students-students interaction. My students were moving from one sit to the other to asked questions and seek assistance from one another. (Aminu: Reflective interview: P2)

Findings and Discussions of the Study

This section discusses the findings of the study, under the following themes. First, the initial perceptions of mathematics teachers on Productive Pedagogies as it affects their classroom instruction. Second, the mathematics teachers' reflections on the benefits of Productive Pedagogies in reforming their classrooms instruction.

Initial Perceptions of Mathematics Teachers on Productive Pedagogies

When this research started the mathematics, teachers greeted the idea with mixed feelings. Some were of the view that this is just another research project, others felt that there is nothing new or good that will come out of it. Despite these reservations and challenges, the researcher and the mathematics teachers kept faith to the project and kept moving step after step and their perceptions gradually took a positive turn. For example, the findings of the study suggested that, the views of the mathematics teachers about the framework and its implementation gradually shifted as the research progresses.

Most of the progress observed were on the importance of shifting their believed about their classroom practice, particularly, the traditionally teachers-control and overbearing mathematics classroom they are used to, to a more democratic system where students will have control over their learning. This agree with the findings of Wilson and Lloyd (2000) who suggested that, the pattern of classroom instruction can be altered if mathematics teachers and their students are willing to shift their roles and beliefs. In view of this, the mathematics teachers observed the gradual nature of their classroom-control giving way to a more democratic student-centred mathematics classroom. The implication of these observations suggested that the introduction of Productive Pedagogies in the mathematics teachers' classrooms reformed their classroom instruction to a more student-centred classroom environment as supported by (Atweh, 2011; Bature & Jibrin, 2015, Bature, et al, 2015).

The finding of the study suggested that the mathematics teachers had initial skepticisms on the feasibility of Productive Pedagogies framework in their classroom practice. They wondered if there are possibilities of its effectiveness in reforming practice. For example, coming from the traditional classroom background, the mathematics teachers had initial concerns about their role in implementing the framework. They had fears of losing control of their classroom, or the students taking advantage of the framework to disrupt classroom activities. However, these fears were allayed in literature that the implementation of framework should not be taken as the opportunity for teachers to lose control of their classroom practice (Bature & Bature, 2007; Bature & Jibrin, 2015). The implication of this was that fact that, the culture of the traditional teacher-centred learning in Nigerian mathematics classrooms should give way to a student-centred learning. mathematics teachers should demonstrate willingness to surrender more of their roles to the students and adopt a passive role of supervision and assistance rather than the normal traditional role of teaching and dishing out of instructions or information to their students (Wilson & Lloyd, 2000; Black, 2007).

The Mathematics Teachers' Reflection on the Benefits of Productive Pedagogies Framework

The principle of student-centred approach in Nigerian classrooms is not new to classroom teachers. There have been series of workshops, seminars, conferences and professional development programs that have been organized to update teachers' pedagogical development especially in using the student-centred instructional approaches (Bature, et al, 2016; NERDC, 2013). However, most mathematics teachers and indeed most teachers generally seem not to have attempted explore this during their classroom practice (Emaikwu, 2012). This had resulted in the Nigerian mathematics classroom teachers to persistently and predominantly continue to implement the teacher-centred teaching approach during their classroom practice (Afolabi & Abimbade, 2012). This teacher domineering teaching approach gives no opportunity for students' participation and/or engagement (NERDC, 2013), with sometimes mathematics teachers viewing themselves as having the monopoly of knowledge (meaning; they possess all the knowledge that the students need) (Igbokwe, 2000).

From the reflections of the mathematics teachers, considerable evidence exists in the study that the application of the framework influenced their classroom practice by challenging their personal views about mathematics classroom instruction as practice in Nigeria. From the finding of the study, this epistemology was challenged by the mathematics teachers at the initial implementation of the framework. However, as the study continued the mathematics teachers developed interest and confidence to adopting the student-centred approach to classroom practice and promoted by the dimensions of the Productive Pedagogies (Atweh, 2007; Bature & Jibrin 2016)

The findings of the study show that the mathematics teachers reflected on the opportunities they had to dialogue, debate and discuss with their colleagues to seek further assistance on areas needing improvement in their practice. These discussions were characterized by comparatively lengthy interactions between the researcher and the mathematics teachers or between the mathematics teachers in a context of collaboration and mutual support throughout the research period. The mathematics teachers use the dialogue debates and discussions to build understanding, explore ideas and practice collaborative thinking among themselves and with their students (Johnson & Johnson, 1999). The findings also reveal that the classroom discussions debates and dialogues help the mathematics teachers build a framework to guide their students to develop and construct their own ideas, skills, concepts and/or processes to improve their practices as supported by Wenger (1998).

This comment suggests the importance of an embodiment of a new relationship that was observed between teachers. It was believed to have been grounded in an ongoing dialogue; negotiations, debate and in a collaborative climate involving participation, integration and interrelatedness between members of the classroom community (Vitto, 2003). This approach helped students share their experiences and knowledge holistically and celebrate its authenticity through collaboration of ideas. It also ensures the members of the community had involvement in learning communities in which all participants have opportunities to engage in productive discourse (Manoucheri & St John, 2006).

Students' attitude to mathematics and mathematics teachers in Nigerian classrooms has generated debates among mathematics educators, researchers, teachers of mathematics and educational planners. Different views on the causes and remedy of these negative attitudes of students to mathematics in Nigerian mathematics classroom had not yielded much or significant improvement (Oguniyi, 2009; Osuafor, 1999). However, the findings of the study suggested a promising improvement in students' attitudes towards positive students' engagement in mathematics problem solving when Productive Pedagogies framework was used as a tool to improving classroom practice. This resulted to the change in attitude to problem solving among students and the development of students' interest towards mathematics engagement and mathematics teachers as against the findings of Bature & Bature (2005,2006) who asserted that there is a general student negative attitude to mathematics and mathematics engagement in problem solving in Nigerian Classrooms. The findings of the study however suggested an observable increase in students' confidence and trust toward mathematics and mathematics teachers. This was believed by the mathematics teachers as responsible for the willingness of the students to take responsibilities of their learning as claimed by Hom & Murphy, (1985) and Schunk, (1984).

Generally, in most mathematics classroom in Nigeria, there is a lack of interest towards mathematics by students. From experience and as a mathematics teacher, it is no secret that students have lack of interest in mathematics and mathematics classroom instruction. Possible causes of these lack of interest in mathematics to a large extent could be blamed to the teachers' poor pedagogical strategies and approaches to classroom practice (Oguniyi, 2009; Osuafor, 1999). The mathematics teachers reflected that adopting the principles of Productive Pedagogies, provided an observable change in their students' interest towards mathematics and mathematics engagements. Some of these observable changes include the improvement in students' attitude to mathematics, increase students' engagement and the willingness in students to accept responsibilities during classroom practice.

The implication of this findings suggested that mathematics teachers' change in pedagogy has positive influence in their students' interest towards mathematics and its teaching. Bajah (1999) was of the view that productive mathematics teachers are those that have the ability to stimulate their students' interest and have clarity of presentation of mathematical ideas. The important characteristics of productive mathematics teachers include good pedagogical approaches, and interpersonal traits such as helpfulness, openness, and friendliness which could be seen as ingredients for effective mathematics practice (Bature & Bature, 2005; 2006).

The findings of the study suggest that a new friendly classroom atmosphere between teachers and students and between students was created. This was as a result of the relaxed classroom climate created by the mathematics teachers. This agrees with the findings of Evertson and Weinstein (2006) suggest that a focus on relationship between teachers and students is an important aspect of classroom culture. The implication to this finding revealed that the concept of shift from the teacher-control of the classroom environment to a more relaxed classroom could foster effective collaborative, student-centred and student-controlled learning in a non-authoritative classroom environment (Bature & Jibrin, 2015). The mathematics teachers who foster such positive relationships with his/her students create classroom environments that are more conducive for learning and helps fostered effective social, developmental, emotional and academic relationship with their students (Atweh, 2007).

Conclusion and Recommendation

This paper looks at the perceptions and the reflection of four mathematics teachers from three secondary schools Northern Nigeria. These teachers taught mathematics using the Productive Pedagogies framework for a period of fifteen weeks spread over three years in two phases. Two research questions were explored to see the reflection of the mathematics teachers on their engagement in teaching mathematics using the framework. The findings of the study

suggested that the mathematics teachers had initial challenges adopting the new pedagogy as a tool to achieving effective classroom practice. It was discovered that this was because of the traditional teacher centred approach that had prevailed in the Nigerian mathematics classrooms.

From the study, the mathematics teachers were able to adjust to the new framework through effective reflective meetings with their colleagues and with the help of the researcher. They reflected that, the framework assisted them developed confidence in their teaching pedagogies, help their students developed positive attitude and interest to mathematics, fostered effective collaboration between them, and between students and created an atmosphere of trust between mathematics teachers and their students. The study suggested that the mathematics teachers were able to provide and received feedback from their colleagues and they were able to give and received feedback from their students. There was also the shift from the traditional teacher centred learning to a more student-centred pedagogy where students had the control of their learning and developed their own strategies to find solution to their problems.

The implication of this study suggests that, the adoption of the framework helped developed the mathematics teachers' positive relationship with their colleagues and with their students. This pedagogy also helped students develop positive attitude to their learning and to their teachers. Another implication of the study suggests that the introduction of framework assisted the students to take responsibility of their learning, build up new relationships, created and defended their knowledge and also had the privileges to discuss and defend their views before the teachers and their colleagues. These were possible because the mathematics teachers created classroom environment that fostered effective collaboration among students as the mathematics teachers reduced their usual teacher domineering attitude towards the teaching mathematics to their barest minimum.

It is therefore recommended that teachers in Nigerian mathematics classrooms should adopt the Productive Pedagogies framework as a tool for their classroom practice. The universities should also identify and adopt the content of Productive Pedagogies in their curriculum to help mathematics teachers understand the content and the practice of the framework before graduating from the university. Finally, further research could be carried out to investigate the concept of Productive Pedagogies using a wider sample.

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Appendix

The Four Dimensions of Productive Pedagogy

<p style="text-align: center;">Intellectual Quality</p> <ul style="list-style-type: none"> • Higher order thinking • Deep knowledge • Deep understanding • Substantive conversation • Knowledge as problematic • Metalanguage 	<p style="text-align: center;">Connectedness</p> <ul style="list-style-type: none"> • Connectedness to the world • Problem-based curriculum • Knowledge integration • Background knowledge
<p style="text-align: center;">Social Support</p> <ul style="list-style-type: none"> • Student control • Student support • Engagement • Self-regulation • Explicit criteria 	<p style="text-align: center;">Recognition of Difference</p> <ul style="list-style-type: none"> • Cultural knowledges • Inclusivity • Narrative • Group identity • Citizenship