LEARNING MATHEMATICAL MODELLING WITH AUGMENTED REALITY MOBILE MATH TRAILS PROGRAM: HOW CAN IT WORK?

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Abstract

The aim of this study is to investigate how an augmented reality mobile math trails program can provide opportunities for students to engage in meaningful mathematical modelling activities. An explorative research design was conducted involving two mathematics teachers and 30 eight grades in Semarang, Indonesia. An Augmented Reality Mobile Math Trails App was created, and several math trail tasks were designed, then students run the activity. Data were gathered by means of participatory observation, interviews, questionnaires, tests, and worksheets. Data analysis began with the organisation, annotation, description of the data and statistic tests. The findings indicate that an educational program was successfully designed, which offered students a meaningful mathematical experience. A mobile app was also developed to support this program. The mobile app with augmented reality features is helpful for students as a tool that bridges the gap between real-world situations and mathematical concepts in problem-solving following the mathematical modelling cycle. The program thus contributes to a higher ability in mathematical modelling. The study identified a link between instrumented techniques in programs and mathematical modelling, as built during the instrumentation process. Further studies are essential for project development and implementation in other cities with different situations and aspects of study.

Keywords: Math trails, Augmented reality, Mathematical modelling, Mobile learning

Abstrak

Tujuan dari penelitian ini adalah untuk menyelidiki bagaimana program *Augmented Reality Mobile Math Trails* dapat memberikan kesempatan bagi siswa untuk terlibat dalam kegiatan pemodelan matematika yang bermakna. Sebuah penelitian dengan desain penelitian eksploratif dilakukan dengan melibatkan dua guru matematika dan 30 siswa SMP di Kota Semarang, Indonesia. Sebuah aplikasi *Augmented Reality Mobile Math Trails* telah diciptakan, beberapa tugas *math trails* telah didesain, kemudian siswa menjalankan kegiatan. Data dikumpulkan melalui observasi partisipatif, wawancara, pemberian angket, tes, dan lembar kerja. Analisis data dimulai dengan pengorganisasian, anotasi, deskripsi data, dan uji statistik. Temuan menunjukkan bahwa program pendidikan ini berhasil dirancang untuk menawarkan pengalaman matematika yang bermakna kepada siswa. Aplikasi perangkat seluler juga dikembangkan untuk mendukung program ini. Aplikasi ini dilengkapi dengan fitur *augmented reality* yang bermanfaat bagi siswa sebagai alat yang menjembatani kesenjangan antara situasi dunia nyata dan konsep matematika dalam pemecahan masalah mengikuti siklus pemodelan matematika. Dengan demikian, program ini berkontribusi positif pada kemampuan siswa dalam pemodelan matematika. Studi ini mengidentifikasi hubungan antara teknik instrumen dalam program dan pemodelan matematika, yang dibangun selama proses instrumentasi. Studi lebih lanjut sangat penting untuk pengembangan dan implementasi proyek di kota-kota lain dengan berbagai situasi dan aspek studi.

Kata Kunci: *Math trails*, *Augmented reality*, Pemodelan matematika, *Mobile learning*

Integrating applied mathematics into mathematics learning leads to the ability to identify questions, variables, relationships, or assumptions that are relevant in real-world situations, and transfer them into mathematics. One can then further interpret and validate solutions for given situations and

analyse or compare models by investigating assumptions. This ability is explained by Niss, Blum, Galbraith (2007) as mathematical modelling competence. Modelling is a process, which is the process of translating a problem situation into the mathematical term and vice versa (Drijvers et al., 2019). Modelling competencies can be described in detail via several sub-competencies developed through the modelling cycle, namely: competence to understand real world problems and build real models; competence to create mathematical models from real world models; competence to solve mathematical problems in mathematical models; competence to interpret mathematical results into real world models or real situations; competence to test solutions and, if necessary, to carry out other modelling processes (Kaiser, 2007).

Mathematical modelling provides an opportunity for students to learn mathematics in a variety of ways (Zbiek & Conner, 2006). In many places around the world, there are special locations where mathematics can be experienced in everyday situations (Prasetyo et al., 2019; Cahyono & Ludwig, 2018; Cahyono & Ludwig, 2019; Ludwig & Jesberg, 2015). Real problems in daily life can be used to design problems for mathematics lessons (Putri & Zulkardi, 2020; Oktiningrum et al., 2016; Prahmana & Suwasti, 2014) with several criteria that must be met. These problems can be solved by following a mathematical modelling cycle with a simple category. Problems with the type of mathematical modelling can be designed in the surrounding environment. Some mathematics tasks related to the locations can be designed by teachers. Tasks located in an area can then be linked, so that routes connecting several task sites can be formed. Students can then explore mathematics in the environment by tracing a planned path consisting of a series of stops, called as math trails (Shoaf et al., 2004). Mathematics trails around their school is one of the tasks that were asked to undertake involved by students (Lee & Johnston-Wilder, 2013).

By taking advantage of the recent developments in mobile technology, math trail tasks can be localised with GPS coordinates, embedded into digital maps through web portals and stored in databases. Trail walkers can then access tasks and carry out mathematical trace activities with the help of mobile applications that support GPS. The combination of the math trail concept and the use of mobile technology has resulted in a new software called the mobile math trail developed through the MathCityMap-Project (Cahyono, 2018; Cahyono & Ludwig, 2019). The Project, which began in 2012, combines the idea of math trail with the use of digital technology (Ludwig & Jesberg, 2015; Cahyono & Ludwig, 2018). The project facilitates student learning of mathematical modelling directly in the surrounding environment by utilising the mobile phone application, which is supported by GPS features. The project continues to be expanded and developed according to needs and technological developments.

By paying attention to current technological trends in learning mathematics (Saadati et al., 2014; Muhtadi et al., 2018) and the characteristics of technology in the Industrial Revolution era 4.0 (Christensen & Eyring, 2011), namely: The Internet of Things (IoT), Mobility, AI, VR/AR, and Automation, the apps used in the mobile math trail need to be developed in the direction of utilising

the features of Augmented Reality (AR). AR is a real-time display directly or indirectly from a physical real object by adding objects from the virtual world to expand information on an existing real object. AR is a technology that allows computer-generated virtual imagery information to be overlaid onto a live direct or indirect real-world environment in real time (Azuma, 1997). The use of AR can be used to feel and see shapes (Hegedus & Moreno-Armella, 2011).

The advantages of AR in education indicate that there is significant potential to integrate AR in teaching and learning, especially for the subjects that require students to visualise (Saidin et al., 2015). By blending virtual and real contents, AR can help to build virtual space that is helpful in the mathematics learning, especially in providing 3D models from real problem (Banu, 2012; Liu et al., 2019). AR-based applications can combine real objects and virtual objects that exist and display this on the smartphone layer. This means that AR provides the opportunity to interact with and indulge in real objects and then manipulate them into images that can then be manipulated again into symbols. AR technologies have positive potential advantages that can be adapted in education (Saidin et al., 2015). In mathematics education, some studies report that AR has positive effect on students' learning performance and attitude in mathematics, especially in solid geometry class (Liu et al., 2019) and for the more complex geometric concepts (Thamrongrat & Law, 2019). This potential can be exploited for improving students' engagement and quality of mathematics learning.

With the notion of engagement and mathematical modelling, the question of this study is: How can an Augmented Reality Mobile Math Trail Program support mathematical modelling ability? The research question raises the following two sub-questions: (a) How the program can contribute to the improvement of mathematical modelling abilities? (b) What is the relationship between the instrumented techniques in the program and mathematical modelling, as built during the instrumentation process?

METHOD

To address the 'how can' research question with regard to exploring the potential of the AR-MobileMathTrails program for the engagement in mathematics education, and the learning opportunities for mathematical modelling through outdoor activities in particular, this study used an explorative research design approach. A mathematics educational program and a mobile math trail app were designed; corresponding mathematics teaching and learning activities through a sequence of pilots were set up. This study was conducted in the city of Semarang, Indonesia, involving 30 eighth grade students and two mathematics teachers. Research was conducted through one introductory phase and three following phases (prototypical design, a small-scale field experiment, and a large-scale experiment). Researchers observed the students' activities during the program by accompanying the students outside as participating researchers; notes were made, and students' portfolios were collected. A debriefing session was conducted afterward, and individual mathematics pre- and post-mathematical modelling tests were also given before and after several activities. Student involvement

was measured using the self-reported Situational Motivation Scale (SIMS) developed by Guay et al. (2000). Data analysis began with the organisation, annotation, and description of the data for qualitative data analysis. For the quantitative analyses, data was collected from students in the empirical field studies.

RESULTS AND DISCUSSION

The AR-MobileMathTrails program, as it resulted from the design process, is a math trails program supported by the use of an android mobile phone app with an AR functionality and an on-screen map. This educational Program has been designed by following the concept of MathCityMap (Cahyono, 2018; Cahyono & Ludwig, 2019) with the addition of AR features. The program can be run by six groups, with five students per group. Each group uses a mobile app to find the location of the objects being targeted. The app displays a map showing the locations of the objects (Figure 1). The first group starts the activity by searching for the first object, the second group looks for the second object, and so on. Students then move on to other objects until they find six objects overall. After arriving at the different locations, students scanned the marker that had been provided; a three-dimensional image appears which is a geometry object that resembles the real object being targeted. At the same time, a problem text is also given on the screen (Figure 1).
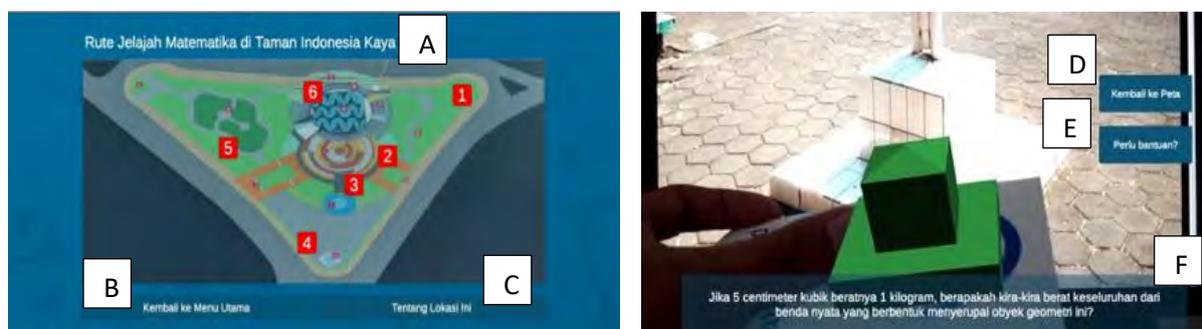
Translations: (A) Math trail route in Indonesia Kaya Park; (B) Back to Main Menu; (C) About this Location; (D) Back to Map; (E) Need Help? (F) If 5 cubic centimetres is 1 kilogram in weight, what is the overall weight of the real object that looks like this geometry object?

Figure 1. Math Trails route map and the app displays 3D objects that resemble the target object

Students look for the object in question and then solve the problem by collecting data (counting, measuring, etc.) directly on site. For example, in a task about stairs, students take measurements, then using the gradient concept, they determine the slope of the stairs. Students write answers on a worksheet (Figure 2).
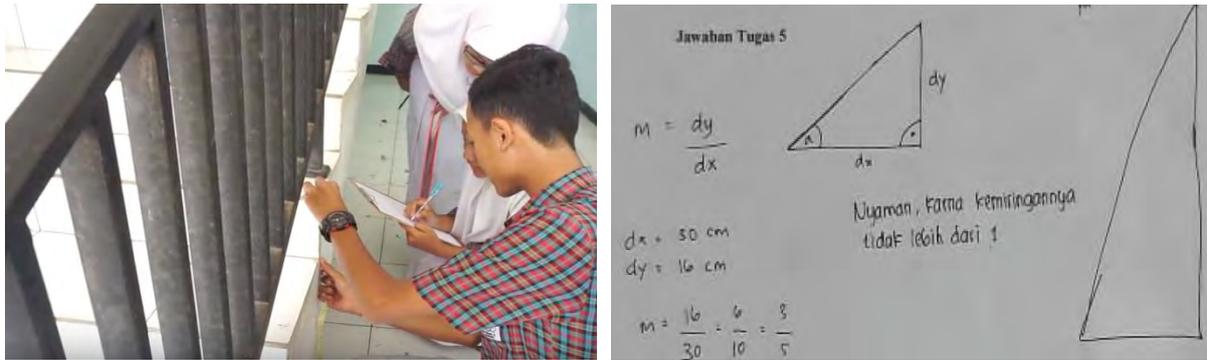
Translations: Task 5 Answer
m = dy/dx
dx = 30 cm
dy = 16 cm
m = 16/30 = 6/10 = 3/5
Comfortable because the slope is no more than 1.

Figure 2. Example of students' activity and students' work

Based on the results of the calculation, students interpreted that with a slope of 3/5, the stairs are still comfortable to use. Through interviews, they argued: *"Stairs as being uncomfortable if the slope is greater than 1 or more than 45° which means that the stairs are too upright, and this makes it more tired if walking up and this is not safe."* Students make illustrations through a picture beside their work. After solving the problem at the particular location, the group continues to the next location following the route directed on the application.

Some groups experience obstacles in a location. They have no idea to solve the problems encountered. However, the app has been equipped with features to anticipate this situation. If they are unable to solve the given problem, students can scan the barcode again to get help in the form of information displayed through three-dimensional images and text (Figure 3). For example, the geometric object of the flagpole base is manipulated into three sections: cuboid, cube, and pyramid, so that students are more easily connected to the mathematical concepts they have learned. Another example is the geometry object of the cupboard, whereby the surface will be painted and manipulated into nets of cuboid; students may then more easily understand that the problems asked can be solved by looking for the area of the cuboid.
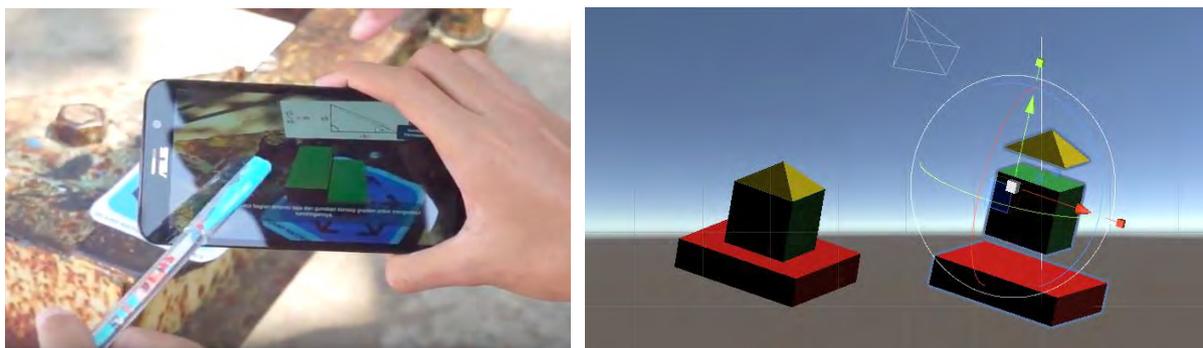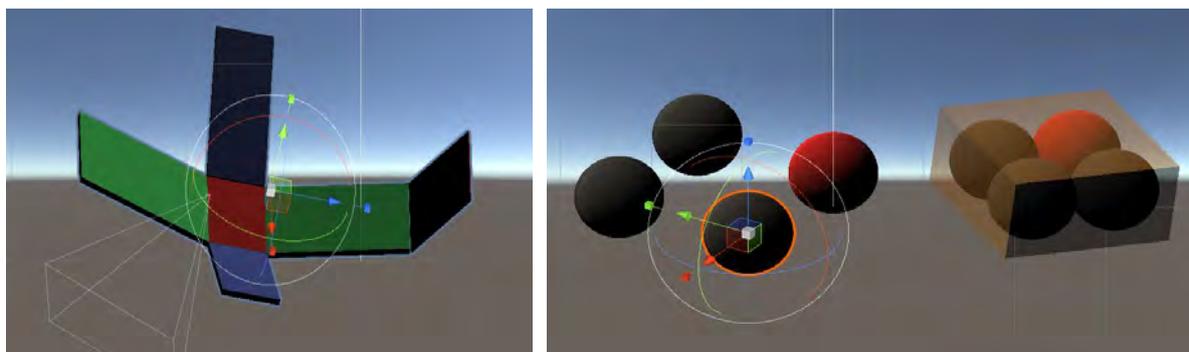
Figure 3. Examples of hints displayed by app

The results of the experiment showed that the activity ran smoothly, the mobile application worked well, and the rules and objectives of the program proved understandable. The findings suggest that students easily engaged in the program during all pilot studies. Students report that the rules and objectives of activities were easy to understand. Through the Situational Motivational Scale (SIMS), students were asked about their motivation to be involved in the learning activities.

The findings show that the average SIMS score for four sub-scales varies, starting from 1.00 to 7.00. Standard deviation indicates adequate variability in all sub-scales. This suggests that the motivation of the students to engage in these activities varies. Students were enthusiastic about participating in the activities and gave a great deal of useful feedback. The Kruskal-Wallis independent-sample test reveals a significant difference between four SIMS sub-scales (IM, IR, ER and AM) with Chi-Square = 73,067 and p = 0.000.

Based on the test results of Kruskal-Wallis, compared to other sub-scale scores, the amotivation subscale has the lowest SIMS score average (Mean Rank AM = 30.05). This low score suggests that students enjoy the activity and find meaning in it, which is reflected in the intrinsic motivational score (the Mean Rank of IM = 97.95) and the identified regulation score (Mean Rank IR = 74.07). Students also report motivated or reacted to external requests, which is an extrinsic regulation indicator (Mean Rank ER = 39.93) indicating that students tend to be neutral on this sub-scale.

In addition, to determine the relative level of student motivation, the four scores on each subscale were then used to calculate a single motivational value called Self-Determination Index (SDI) for each student. The SDI score of 30 students shows that all students have a positive SDI score (ranging from MIN = 1.00 to MAX = 14.00 with an average SDI ± standard deviation of SDI = 7.71 ± 3.51). The results further show that in general, the motivation of students to engage in this activity is more determined by themselves. The positive value in this case therefore indicates that the internalised motivational forms, namely intrinsic motivation (IM) and identified regulations (IR), are more dominant. Students consider the activities they follow to be interesting or enjoyable (IM indicators) and meaningful or valuable (IR indicators). The interview result show that students engage in activities for their own enjoyment or satisfaction, and their involvement is voluntary. Based on the

interview, students reported that the use of mobile devices with AR feature for outdoor mathematics learning activities has become an attraction and they learned how to apply mathematics in the real world by the help of mobile app. These findings suggest, therefore, that the program has been successful in offering activities that can motivate students intrinsically to engage in mathematical learning. The students were pleased to do mathematical trails around their school, as had been stated by Lee & Johnston-Wilder (2013).

However, motivation is just a stimulus for students to increase their involvement in mathematics. As a serious mathematics education program, the main focus of this activity is to support students in learning mathematics and build their own mathematical knowledge. The program has been designed to offer students the opportunity to do so by practicing the application of mathematics to solve real problems following mathematical modelling cycles. The mathematical modelling competence in the study was measured in five phases: competence to understand real-world problems and build reality models (phase I), competencies for creating mathematical models of real-world models (phase II), Competence to solve mathematical problems in mathematical models (phase III), competence to interpret mathematical outcomes into real world models or real situations (stage IV), and competence to test solutions and, if necessary, to perform other modelling process (phase V).

Students ' works show that students have different ways of solving a given mathematical problem. This is in line with what Zbiek & Conner (2006) stated that mathematical modelling provides a place where students can learn mathematics in various ways. An example of students' work in solving a problem by following a mathematical modelling cycle is shown in Figure 4. The problem requires students to estimate the number of fish that can be preserved in a pond on the school yard; the pond is a rectangular prism. However, in this example, the group had difficulty determining the mathematical concepts used. They were then able to use hints to find out the illustration of the geometry object of the problem so that they can determine the volume.
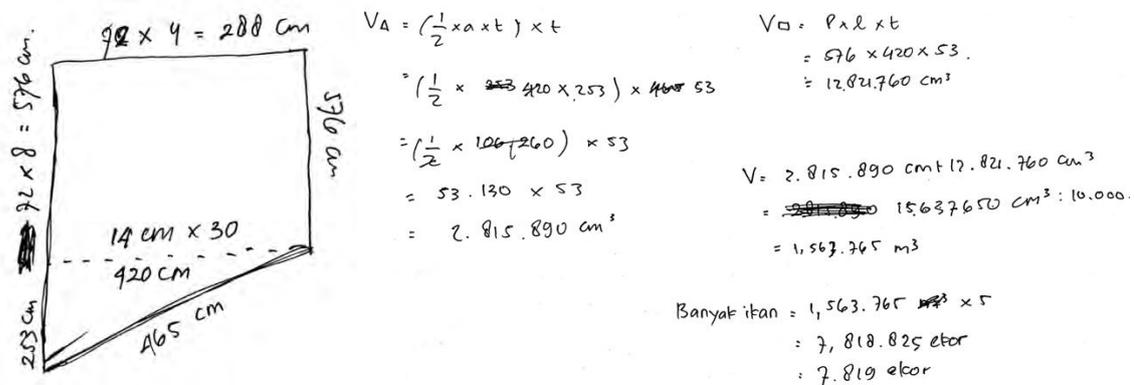

Figure 4. An example of students works in solving the problem by following a mathematical modelling cycle

The hint showed a three-dimensional illustration of a rectangular prism that is partitioned into two parts of a triangular cuboid and a triangle prism. The group is more familiar with the volume

formula of both of these geometry objects; thus, they were able to solve the problem. The application therefore plays a valuable role in helping students to model real situations into mathematical models; students are assisted in the process of using mathematical concepts for solving mathematical problems by following mathematical modelling cycles.

In general, students were able to solve the problems given by following a mathematical modelling cycle. The percentages of student success in achieving each sub-competency in the mathematical modelling process are: 93.06% for phase I, 92.36% for phase II, 90.28% for phase III, 87.50% for phase IV, and 81.94% for phase V. Before and after intervention through this learning program, students worked on mathematical modelling tests. Both tests were used to measure students' ability to solve problems by following a mathematical modelling cycle. Furthermore, to determine the effect of interventions given through the program on the competency of modelling in mathematics learning, statistical analysis of the student scores before and after intervention was conducted. One-sample Kolmogorov-Smirnov test indicates that the students score is a normal distribution ($p = 0{,}200 > 0.01$); t-test was thus used to analyse the data.

The t-test results indicate that there is a significant difference ($p. < 0.001$) between the students' mathematical modelling ability before and after the intervention, with effect size $d = 2.61$. Average of students' mathematical modelling skills after intervention (75.50) was higher compared to the mathematical modelling skills prior to intervention (49.25). Students' mathematical modelling skills were also influenced by the motivation of students to engage in activities designed through this study. The results of a regression test indicated that there is a significant positive influence ($p. < 0.001$) of the student's motivation (x) to the student's mathematical modelling ability (y). The relationship is indicated by the formula $y = 58{,}881 + 2.156x$. Student motivation contributed by 75.5%. This shows that the program implemented has successfully influenced students' mathematical modelling skills. These skills include the ability to translate the problem situation into mathematical term and vice versa (Drijvers et al., 2019)

Based on the results of the experiments, it appears that the use of AR provides an efficient way to represent a model that needs visualisation and supports the seamless interaction between the real and virtual environments, allowing a tangible interface metaphor to be used for object manipulation. These advantages match the need for mathematics learning, especially in mathematical modelling where students require a bridge between real situations and mathematical concepts. In line with Kaiser's concept (2007) of sub-competence of mathematical modelling, the app trains students to understand real problems by matching 3D objects that appear in the app with real situations around the site.

In line with the Wang et al. (2014)'s research result, visualisation with AR helps students to create mathematical models from real world models. This is part of the mathematical modeling stage (Kaiser, 2007; Kaiser et al., 2006). These two modelling stages are assisted by the first visualisation in each task. Furthermore, the app's hints feature (if needed when students are unable to solve a problem)

will help students to use mathematical concepts that are more familiar to them in a given mathematical situation.

Referring to the scaffolding concept in mathematical modelling (Stender & Kaiser, 2015), the hints feature takes the role here. With the latest technological developments, digital tools can be used as an aid in learning mathematical modelling (Greefrath & Siller, 2017; Siller, 2011; Greefrath & Siller, 2018a; Greefrath et al., 2011; Greefrath & Siller, 2018b). Along with the condition, in this project, hints provided in the form of a 3D virtual display resembles the real object being targeted. With this visualization, students easily connect real situations with mathematical situations that are appropriate (Liu et al., 2019). As a result, students are able to model real problems into more formal mathematics, as explained in Kaiser (2007)'s theory. With this feature, students are guided to take the next step, solving mathematical problems in mathematical models. They then interpret mathematical results into real world models, situations and testing solutions and, if necessary, to carry out other modelling processes. AR has helped to feel and see shapes, as has been stated by Hegedus & Moreno-Armella (2011) about this new technology.

Findings indicate that AR bridges the gap between the real and the virtual in a continuous way. AR can minimise the misconceptions that happen due to the students' inability to visualise geometrical concepts; AR displays objects and concepts in different ways and angles which helps the students to more easily understand geometric concepts. This condition is in accordance with the findings of Saidin et al. (2015) who reported that there is significant potential to integrate AR in teaching and learning, especially for the subjects that require students to visualise. In addition, students were excited and interested to learn using this AR mobile app and gave positive feedback about their experience in this activity. This finding is in accordance with a statement stating that there has been an increasing interest in applying Augmented Reality (AR) to create unique educational settings (Chen et al., 2017; Ajanki et al., 2011). The role of AR as a bridge between real and virtual situations seems to be able to support students in solving MathCityMap tasks with mathematical modelling cycles. These findings can strengthen MathCityMap's development goals in the improvement of mathematical modelling competences (Gurjanow et al., 2019). In this educational program, AR has presented a virtual concept into the real environment in real time, as mentioned by Azuma (1997) about this technology.

Overall, the findings show that the Augmented Reality Mobile Math Trail Program has a positive impact on students' motivation to be involved in the learning activities. This educational program contributes to the improvement of mathematical modelling abilities. The use of AR technology helps students in the process of mathematical modelling, especially at the stage of understanding real world problems and build real models as well as creating mathematical models from real world models. Moreover, field experiments also showed that there was a relationship between the instrumented techniques in the program and mathematical modelling, as built during the instrumentation process.

CONCLUSION

The results suggest that the AR-MobileMathTrails program was successfully designed and offered a meaningful activity for students, whereby students gained mathematical modelling experience. The mobile app with AR features supported students in bridging between the real-world situations and mathematical concepts in problem solving, through following the mathematical modelling cycle. Further research is needed for the development and implementation of this program. Development aimed at features that were added in accordance with the rapid development of technology by adjusting to the competencies expected from math learning. In addition, expansion of the implementation is important to be examined on other topics in mathematics and its implementation in other places with different characteristics.

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