PROSPECTIVE PRIMARY SCHOOL TEACHERS' ACTIVITIES WHEN DEALING WITH MATHEMATICS MODELLING TASKS

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Abstract

The current teaching of mathematics is guided by recommendations that suggest the implementation of various activities in order to raise the understanding of mathematical knowledge. This diversity is related to the characteristics of the tasks proposed in the learning contexts. Among all tasks, the modelling ones call for the application of activities through different representations. So, it is important that teacher training courses promote experiences involving prospective teachers with this type of task. Based on this assumption, we intend to identify the activities that prospective primary school teachers perform in solving modelling tasks, the difficulties experienced in these tasks and the value of the models they determine. From the analysis of the resolutions of two tasks, we find that the prospective teachers translate the information of the data available in tables through graphs and analytical expressions. Some discuss models that determine which best fits the data. In the activities carried out, difficulties arise in determining the proportionality constant that best translates the problem situation, discussing the reasonableness of the values generated by the model, and sketching the graph of the model that best fits the experimental data. As for the usefulness of the model they determine, few prospective teachers are predicting outcomes.

Keywords: Teaching of mathematics, Modelling tasks, Teacher training, Primary education

Abstrak

Pengajaran matematika saat ini dipandu oleh suatu rekomendasi yang menyarankan implementasi berbagai kegiatan untuk meningkatkan kemampuan pemahaman matematis. Keberagaman ini terkait dengan karakteristik tugas yang diusulkan dalam konteks pembelajaran. Di antara semua tugas, soal pemodelan membutuhkan aktivitas pembelajaran melalui representasi yang berbeda. Jadi, penting bahwa pelatihan guru mendorong pengalaman yang melibatkan para calon guru dengan jenis tugas ini. Berdasarkan asumsi ini, kami bermaksud mengidentifikasi kegiatan yang dilakukan calon guru sekolah dasar dalam menyelesaikan tugas pemodelan, kesulitan yang dialami dalam tugas ini, dan model yang mereka tentukan. Dari analisis terhadap dua tugas yang diberikan, kami menemukan bahwa calon guru menerjemahkan informasi dari data yang tersedia dalam bentuk tabel melalui grafik dan ekspresi analitis. Beberapa mendiskusikan model yang menentukan mana yang paling cocok dengan data. Dalam kegiatan yang dilakukan, kesulitan muncul dalam menentukan konstanta proporsionalitas yang paling baik menerjemahkan situasi masalah, membahas kewajaran nilai yang dihasilkan oleh model, dan membuat sketsa grafik model yang paling sesuai dengan data eksperimen. Adapun kegunaan model yang mereka tentukan, hanya sedikit diantara para calon guru yang memprediksi hasil dari permodelan yang diberikan.

Kata kunci: Pengajaran matematika, Soal pemodelan, Pelatihan guru, Pendidikan dasar

Mathematics is one of the school subjects that occupies a larger space in compulsory education curricula of most countries. This is often justified for the important role that mathematics plays in citizens' social and professional environments, as well for the connections it has with other science and also for the contribution it gives to understanding everyday phenomena (Maass et al., 2019; Risdiyanti & Prahmana, 2020). Such vindication highlights the role of mathematics as an application tool. A consequence of this is that mathematics teaching and learning should focus not only on conveying concepts to students but also on the engaging them on using those concepts to give meaning and to make sense of in real, daily

life contexts (Ginting et al., 2018; Österman & Bråting, 2019). This is one of the reasons why the educational value of mathematical tasks has been increasingly recognized (Simon & Tzur, 2004; Stein & Smith, 1998; Viseu & Oliveira, 2012; Viseu, 2015; Riyanto et al., 2019). There is a variety of mathematical tasks, ranging from exercises to investigation tasks, that Ponte (2005) distinguishes according to the degree of challenge (high/low), the degree of structure (open/closed), the duration (short/medium/long) and the context (reality/semi-reality/pure mathematics) that they offer.

Modeling tasks are open tasks that offer a high challenge, and may take a long time to perform mainly because they require a planned integration of mathematics concepts and facts and phenomena that take place in real contexts (Chong et al., 2019; Dede, Hidiroglu, & Güzel, 2017; Riyanto et al., 2019) which the modeler needs to be deeply understand from a mathematical point of view before being able to answer to the question they pose (Kaiser & Sriraman, 2006).

Kaiser and Maaß (2007) describe modelling as a process in which a given situation is solved through the application of Mathematics, what Barbosa (2006) calls the use of ideas and/or mathematical methods to understand and solve problem situations arising from areas of knowledge other than mathematics. For Silva and Barbosa (2011), mathematical modeling aims to develop an understanding of how mathematics is used in social practices, through the critical analysis of the dominant culture through mathematics. A learning environment that encourages questioning and investigating situations originating in other areas of reality involves students in activities of schematizing, developing arithmetic operations, generating equations, drawing drawings, graphing, and, mainly, producing speeches (Silva & Barbosa, 2011).

Mathematical modelling of a phenomenon may consist of analyzing and demonstrating the elements and relationships present in the phenomenon, adapting and solving the real situation based on Mathematics, interpreting the results and confronting them with the phenomenon under study and taking the relevant conclusions. For the modelling of a given situation to take place, it is necessary to follow a path defined by delineated and arranged steps in a sequential order. This procedure is described by a cycle that will be repeated as many times as necessary to obtain the model closest to the situation under study (Verschaffel, Greer, & De Corte, 2000). Although the modelling cycle is structured and dissected in different ways by different authors, this cycle follows a guideline: identify the real situation; translate the obvious aspects of the situation into the construction of a mathematical model; investigate the mathematical model; obtain new information on the situation under study by interpreting the data (obtained from the model) with regard to the real situation; evaluate the adaptation and adjustment of the results to the real situation.

The definition of mathematical modeling is not consensual in the literature (Barbosa, 2006; Kaiser & Sriraman, 2006). Based on the assumption that students and professional modelers have different conditions and interests and that the practices conducted by them are different, Barbosa (2006) distinguishes the mathematical modeling done by professional modelers from the modeling activity that is performed in the classroom. At the classroom level, the modelling activity consists of analyzing and

demonstrating the elements and relationships present in a given situation, solving the situation based on Mathematics, interpreting the results and confronting them with the phenomenon under study and taking the relevant conclusions (Barbosa, 2006; Chong et al., 2019). Verschaffel et al. (2000) identify a sequence of six phases as shown in Figure 1.
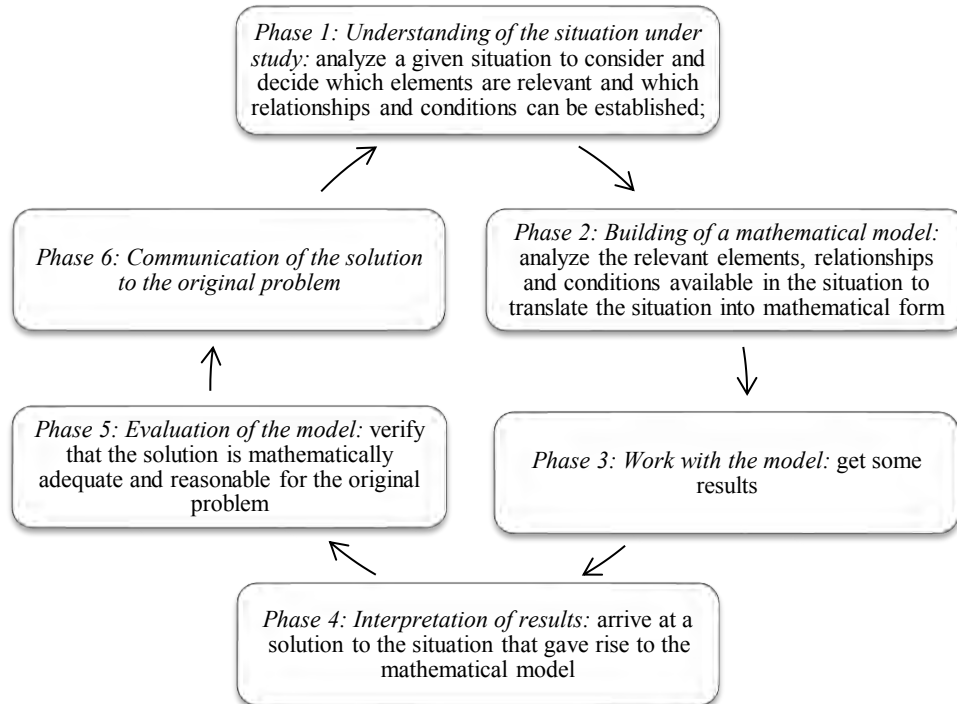
Phase 1: Understanding of the situation under study: analyze a given situation to consider and decide which elements are relevant and which relationships and conditions can be established;

Phase 6: Communication of the solution to the original problem

Phase 2: Building of a mathematical model: analyze the relevant elements, relationships and conditions available in the situation to translate the situation into mathematical form

Phase 5: Evaluation of the model: verify that the solution is mathematically adequate and reasonable for the original problem

Phase 3: Work with the model: get some results

Phase 4: Interpretation of results: arrive at a solution to the situation that gave rise to the mathematical model

Figure 1. Phases of a modeling activity (adapted from Verschaffel et al. (2000))

When we are faced with a given problem, we need to choose a mathematical structure to represent it, thus constructing a mathematical model (Verschaffel et al., 2000). As soon as the problem is represented mathematically, we try to use available mathematical contents to analyze it, in order to get to new conclusions, considering that these conclusions have to be interpreted according to the situation from which everything started. Thus, the model is evaluated, deciding whether or not it is appropriate. If not, we try to redefine the problem, consider new variables and establish new relationships between variables. The cycle is repeated until a satisfactory result is reached (Zeytun, Cetinkaya, & Erbas, 2017). Thus, mathematical modelling is a complex cyclical process which consists of structuring, generating facts from the real world and data, mathematizing, working in mathematics and interpreting or validating (Blum, Galbraith, Henn, & Niss, 2007).

Despite the fact that mathematical modelling has been for long taught and learned around the world, research on the effects that teaching and learning modelling has for students is a much more recent topic of research (Schukajlow, Kaiser, & Stillman, 2018). Modelling tasks are cognitively demanding, as they involve the translation between mathematics and reality, and for this it is necessary to have appropriate mathematical ideas and knowledge of reality (Carreira, 2011; Oliveira & Barbosa, 2011). On the other hand, modelling is closely linked to other mathematical skills, such as the design and application of problem solving strategies, interpretation of the questions, and reasoning ability

(Jurkiewicz & Fridemann, 2007). Lee (2000) states that there is no modelling in mathematics classrooms when teachers show their pupils examples, cases or images of real situations to introduce or to explore a mathematical topic. When this happens, pupils get to follow the ideas presented by the teacher and nothing else. Mathematical modelling is more than that. Pupils should work out the way of working, its quality and not just the final result (Lee, 2000). Seto et al. (2012) concluded that a primary school teacher was able to analyze the potential of mathematical modeling tasks and was surprised by the quality of the mathematization processes during a task on the route of a bus powered by a platform, namely: (a) the identification of variables and their relationships; (b) relate mathematical knowledge and school skills to real-world experience; and (c) justification of the mathematical models developed.

Ng et al. (2013) concluded that the adoption of a listening-observing-questioning pattern helped a primary school teacher to understand students' thinking on modeling tasks and the different ways in which the proposed tasks were interpreted and represented. Besides, teacher's use of metacognitive strategies, sensitiveness to the blocks faced by students and awareness of the connections between the real world and mathematics were pointed out as key elements in the process of mathematizing the problem situation. Besides, as Seto et al. (2012) stated, having students justifying their models, helps them to explicitly communicate their thoughts. However, Dede (2016) found that elementary mathematics prospective teachers show difficulties when asked to solve modeling mathematics problems, even when working in small groups. Their difficulties have to do with simplifying, mathematizing, interpreting, validating and selecting overcoming strategies.

Self-efficacy expectations towards modelling tasks and the value attributed to them seem to be lower than for other types of mathematical tasks (Krawitz & Schukajlow, 2018). This may help to understand why some authors (Dawn, 2018) argues that if secondary school mathematics teachers are to use modelling in their mathematics classes, they need to be trained on how to model and use modeling of real world problems. A similar argument is made by Anhalt, Cortez, and Bennet (2018) who concluded that even though prospective mathematics teachers expressed struggle and reward during a training program on modelling, they developed relevant competences associated with modeling and teaching about modeling, These results suggest that if prospective primary school teachers, who have a weaker background on mathematics, are to use modeling in their future classes, initial teacher training courses should provide them opportunities can learn how to do mathematical modelling, as students, and how to integrate it in their pedagogical practices, as teachers.

Despite the importance of modeling in mathematics education, research focusing on the prospective teachers' knowledge and perceptions on mathematical modeling are scarce (Han, 2019). Based on this assumption, this study is intended to identify the activities that prospective primary school teachers perform in solving modelling tasks, the difficulties experienced in these tasks and the utility of the models they determine. The attainment of these objectives requires attention to be paid not only to cognitive but also to metacognitive aspects, which, according to Schukajlow, Kaiser, and Stillman (2018), have not been enough addressed by research in the area.

METHOD

Given the nature of the objectives of this study, a qualitative approach, as defined by McMillan and Schumacher (2014), was adopted in order to achieve a deep understanding of the meanings that prospective teachers attribute to the activities they perform with modelling tasks. This study involved the 25 prospective teachers who formed the class of the 1st year of the Master of Pre-School and Basic Education (Primary School) of a public university in the north of Portugal. The Portuguese education system encompasses 12 years, prior to entry into higher education, as in most countries. The first nine of these years comprise basic education and the last three are secondary education. Basic education consists of three cycles: the first, whose pupils start at the age of 6, lasts four years with just a single teacher; the second lasts two years, and the third lasts three years. During these nine years the mathematics curriculum is same for all students. In the three years of secondary education, where students begin to be routed to a group of higher education courses, the mathematics curriculum varies according to whether courses in sciences, humanities, arts or technology are followed.

These prospective teachers have an undergraduate degree in Basic Education. In this degree, they took mathematics courses such as Elements of Mathematics, Numbers and Sequences, Geometry and Measure Probability and Statistics, Patterns and Problem Solving, Complements of Mathematics, and Didactics of Elementary Mathematics. In the study of the topics that integrate these courses, prospective teachers deepen their knowledge acquired in basic and secondary education. An example of this is the study in the 1st year of the degree on the topic 'Functions as relations', which according to the school curriculum in Portugal begins to be formally studied from the 7th year of schooling. In their studies in basic education, prospective teachers studied in the 7th year the function of direct proportionality and in the 9th year the function of inverse proportionality. In this school level, prospective teachers did not work with mathematical modeling situations, since the curriculum guidelines in force during their school studies did not suggest this activity.

Of the 25 prospective teachers who participated in this study, 20 of them carried out modelling tasks while taking their degree in Basic Education. The data was collected through the resolution of two modelling tasks, called 'Birthday candle flame task' and 'Pressure task', performed in a group of three or four elements, and with the use of a spreadsheet. Each group worked in each task, in the classroom context, for an hour and a half and, in the final, submitted its solutions on handouts. The information coming from this method is presented by Gn, where n denotes the number of the group ($n \in \{1,2,3,4,5,6,7,8\}$). The modelling tasks required mathematical knowledge acquired up to the 9th grade of schooling, an option resulting from the fact that some prospective teachers had only attended mathematics courses until this level of schooling. For this reason, the expected models that prospective teachers should develop involved direct proportionality (Birthday candle flame task) and inverse proportionality (Pressure task). In each task there were two questions asking the opinion of the prospective teachers about the difficulties found during the task and the usefulness of the model created by them.

Following the recommendations of McMillan and Schumacher (2014) on how to organize and systematize the information collected, data analysis is based on two dimensions. The first dimension, description of the activity of the prospective teachers with modelling tasks, results from the analysis of this

activity through the modelling phases proposed by Verschaffel et al. (2000). Here we emphasize two of them: (i) Building of the mathematical model; and (ii) Evaluation of the mathematical model. In the building of a mathematical model, it is intended that prospective teachers establish a functional relationship between the data obtained, while in the evaluation of the model it was intended that they do so by adding the least squares or viewing the adjustment of the model graph determined to the points that translate the obtained data. In relation to the second dimension, the difficulties they felt and the usefulness of the model they determined, we analyzed the answers that the prospective teachers gave to questions that focused on these two aspects.

RESULTS AND DISCUSSION

Building of the mathematical model from birthday candle flame task

In the first problem situation, seen in Figure 2, from which the values recorded in an experiment in the classroom, the prospective teachers had to complete a table with the height values of a candle burned in a given time interval.

You usually celebrate someone's birthday with a cake and candles. Maria noticed that the height of a birthday candle decreases with time. Her curiosity led her to look for a relationship between the length of time the candle was lit and the length of the candle that burned at each interval.

To respond to her curiosity, Maria performed the following experiment: she recorded the candle measurement on a table before lighting it. Then she lit the candle and, after burning it for 20 seconds, put it out. She measured the candle again and recorded the height of the candle. She repeated the procedure five more times.

Time (in seconds) of burning (t)	0	20	40	60	80	100	120
Height of the candle (in cm)	8,8	8,6	8,3	8	7,4	7,1	6,9
Height of candle burned (in cm) in each interval of time (hcb)							

What is the relation between **t** and **hcb**?

Figure 2. Birthday candle flame task

In the interpretation of what was requested, four groups (G1, G3, G4 and G5) considered the height of the candle burned in each time interval studied and four other groups (G2, G6, G7 and G8) focused their attention on the height of the candle burned considering its initial height, as exemplified by the responses of groups G1 and G2 (see Figure 3).

Figure 3. Tables completed by G1 and G2, respectively

Regardless of the interpretation of the proposed task question, prospective teachers analyzed the behavior of variable values to identify regularities, which translates the comprehension of the studied situation, as referred by Verschaffel et al. (2000). Through the meaning of this behavior, the prospective teachers identified as a possible model that represents the situation a function of direct proportionality, as exemplified by the following statements:

> Function of direct proportionality, because as time values increase the height values of the candle burned also increase (G1, G2, G5, G6, G7, G8).
>
> We used a graph to understand the relationship between the data, since the values did not increase or decrease in the same proportion. By looking at the graph, we are apparently faced with a situation of direct proportionality (G3).
>
> Despite some irregularities, we consider that we are facing a relation of direct proportionality (G4).

After studying direct proportion, the study of function of direct proportionality, i.e., functions defined by $y = kx$, with positive k, is carried out on the 7th grade of Basic Education. Furthermore, only two groups highlighted that the values of the variables are not proportional, nor do the values of the dependent variable have the same variation. Most prospective teachers do not safeguard these conditions, idealizing beforehand a linear function that best represents the experience data. For example, group G4, despite highlighting the existence of irregularities between the data, which tends to refute the same variation between the values of the variables, identifies as a possible model of the situation a function of direct proportionality, translating the determination of the constant of proportionality by the expression "$k = T \times A$" as if the situation were represented by an inverse proportionality function. In determining the proportionality constant, other groups also made errors in considering that "$k = \frac{0}{0} = \emptyset$" (G1) or that "$k = \frac{0}{0} = 0$" (G2, G3, G5, G6, G7, G8).

The determination of the constant of the possible proportionality relation had the purpose of defining an expression that best represented the collected data, which leads, as Verschaffel et al. (2000) states, to the construction of a mathematical model. In this activity, the prospective teachers were expected to determine: the ratio between the respective values of the variables; the average value of these reasons as representative of the proportionality constant; a linear model as a function of this average value; the square of the difference between the experimental values and the values generated by the model; and the sum of the squares of the differences between the experimental values and the values generated by the model. The determination of this sum aims to evaluate the reasonableness of the model that best fits the experimental data, which exemplifies phase 5 of the modeling activity suggested by Verschaffel et al. (2000).

In determining the ratio between the respective values of the variables, all groups showed the same difficulty when the values of the variables simultaneously assume value zero, except for group G4

that did not feel this difficulty because it determined the product instead of the quotient. This difficulty had an influence on the calculation of the average value of the determined ratios, since most of the groups considered zero. These difficulties were also referred in the study developed with preservice teachers by Crespo and Nicol (2006).

From the average value of the ratios between the respective values of the variables comes the building of the linear model (4th column), the square of the difference between the experimental values and the values of the model and the sum of these squares (5th Column), as exemplified by the resolution performed by groups G1 and G6 (Figure 4).

Figure 4. Determination of an expression that translates the relationship between the height of the candle burned and the time during which it burned by groups G1 and G6

The implementation of such activities did not occur equally in all groups:

1. Groups G4 and G5, although they considered it to be a situation that could be represented by a direct proportionality function, in the building of the model they performed calculation procedures as if it were a function of inverse proportionality.

2. Groups G2, G3 and G7 did not determine the sum of the squares of the difference between the experimental values and the values generated by the model.

Evaluation of the mathematical model from birthday candle flame task

After building the mathematical model, prospective teachers could discuss their reasonableness through the order of magnitude of the sum of the squares of the difference between the experimental values and the values generated by the model or the analysis in the graphical representation of the adjustment of the graphic sketch of the model established to the scatterplot resulting from the points representing the values of the variables.

In order to minimize the sum of the squares of the difference between the experimental values and the values generated by the model, some groups (G1, G5, G6 and G8) assigned other values to the 'proportionality constant' depending on the average value they determined of the ratios between the values of the variables. This procedure did not translate into a better model that would fit the experimental data to that already determined, as illustrated by the process performed by group G1 (Figure 5).

(x)	(y)	$y = k(medio) \times x$	$(v.\, \text{frp} - v.\, \text{op.})^2$
0	0	$y = 0,002 \times 0 = 0$	$(0, -0)^2 = 0$
20	0,2	$y = 0,002 \times 20 = 0,004$	$(0,2 - 0,004)^2 = 0,038$
40	0,3	$y = 0,002 \times 40 = 0,008$	$(0,3 - 0,008)^2 = 0,085$
60	0,3	$y = 0,002 \times 60 = 0,12$	$(0,3 - 0,12)^2 = 0,032$
80	0,6	$y = 0,002 \times 80 = 0,16$	$(0,6 - 0,16)^2 = 0,194$
100	0,3	$y = 0,002 \times 100 = 0,20$	$(0,3 - 0,20)^2 = 0,01$
120	0,2	$y = 0,002 \times 120 = 0,24$	$(0,2 - 0,24)^2 = 0,04$
	Fmédio 0,002		$\sum = 0,399$

Figure 5. Looking for the model that best fits the data (G1)

This finding led these groups to consider that the best model that fits the data is the one obtained from the average value of the ratios between the values of the variables. In addition to discussing the reasonableness of the model through numerical procedures, prospective teachers could do so from the graphic sketch of the problem situation under study. Groups G3, G4 and G7 did not sketch any graph of the model they defined. Of the remaining groups, G2 and G5 sketched a line graph (Figure 6).
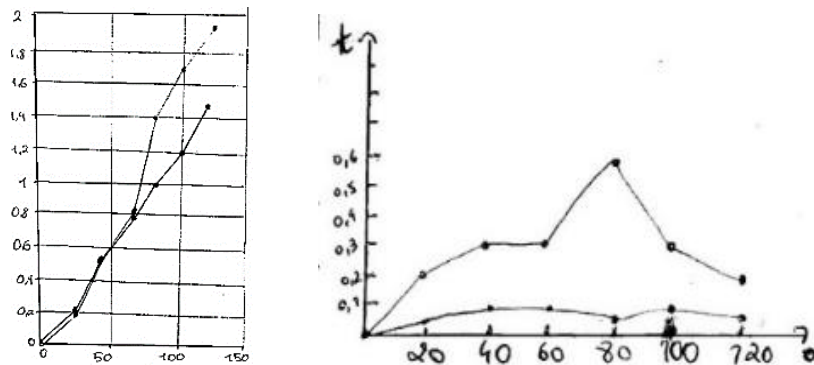

Figure 6. Representation of the situation under study through a line graph (G2 and G5)

These two groups outlined a line graph for both the experimental values and the model-generated values they defined, instead of sketching the model graph in order to find out their reasonableness in representing the values of the experiment. It is also observed that group G5 indicates the labels of the Cartesian axes, which is no longer the case in the G2's graph. The care of presenting such labels is revealed in the graphic sketches created by groups G6, G8 and G1 (Figure 7).
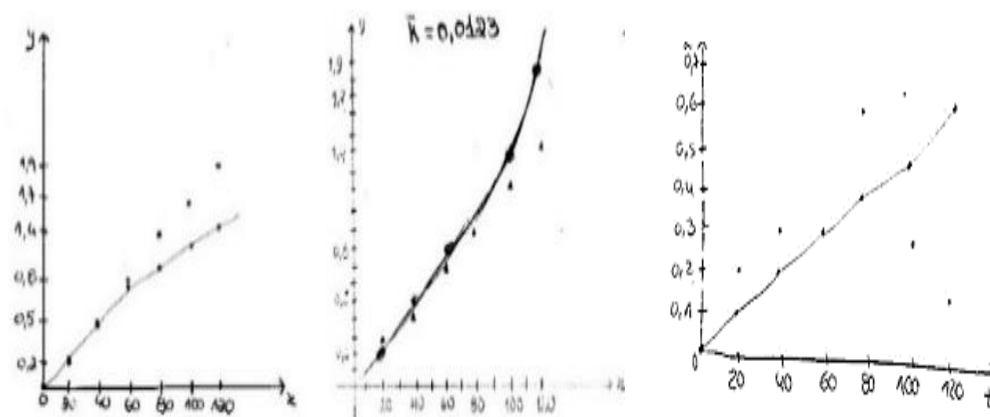

Figure 7. Graphical representation of the model that best fits the data (G6, G8 and G1)

From the graphs represented by these groups, the one closer to what was expected is that created by group G1. One can see that the prospective teachers of groups G6 and G8 joined the points generated by the model without meeting the condition of the linearity that characterizes the graph of a function of direct proportionality. From the graphs created by these three groups it is verified that groups G6 and G1 delimited the graph to the experimental points, which reveals the difficulty of realizing that the model that best fits the experimental data allows for inferences in relation to the values of one of the variables from the knowledge of values of the other variable.

Building of the mathematical model from pressure task

In the second problem situation, seen in Figure 8, from which the values recorded in an experiment in the classroom, the groups, upon the behavior of the values of the variables, identified an inverse proportionality function as a possible model. As group G3 states: "The analysis of the data shows that there is a regularity between the variables, as the volume decreases, the pressure values increase. Thus, we are faced with an inverse proportionality relation", phase 1 of the modelling activity proposed by Verschaffel et al. (2000).

When compressing a given amount of gas contained in an enclosed vessel, the volume (**V**) of the gas and the pressure (**P**) exerted by it vary. An experiment was carried out in which the values presented in the table, which translate the Volume and the Pressure of a gas contained in a syringe whose piston was being pushed, were obtained:

V	P
20	0.996
18	1.08
16	1.2
14	1.32
12	1.49
10	1.68
8	1.89

What is the relation between **V** and **P**?

Figure 8. Pressure task

The identification of a family of functions capable of modeling the situation under study made it possible for the groups to perform the expected activities in their establishment, as illustrated by what G5 group did (Figure 9).

Figure 9. Looking for the model that best fits the data (G1)

Such activities explain the determination of the product of the values of the variables, the average of these products as a representative of the proportionality constant, the values generated by the model and the sum of the squares of the differences between the experimental values and the values generated by the model.

Evaluation of the mathematical model from pressure task

In the accomplishment of such activities there were no striking difficulties, which was not the case with the graphic representation:

1. One group did not draw the graph (G7);
2. A group sketched a line (G1), which may be due to the scale used in the subdivision of the Cartesian axes (Figure 10).
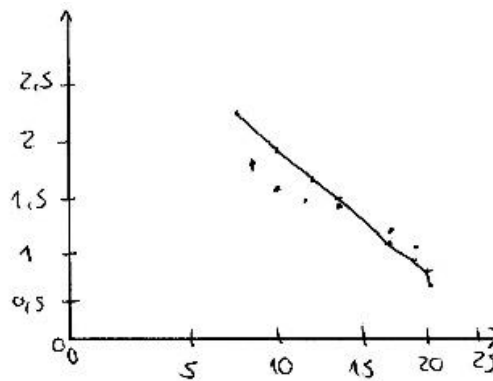

Figure 10. Representation of the best model that fits the experimental data according to G1

3. The other groups outlined a curve, but with some peculiarities. Groups G2, G4 and G5 sketched a curve delimited by the points that translate the values of the experiment and do not indicate the labels of the Cartesian axes. In addition to these details, group G5 sketched a curve to represent the experimental points and another to represent the points generated by the model (Figure 11).
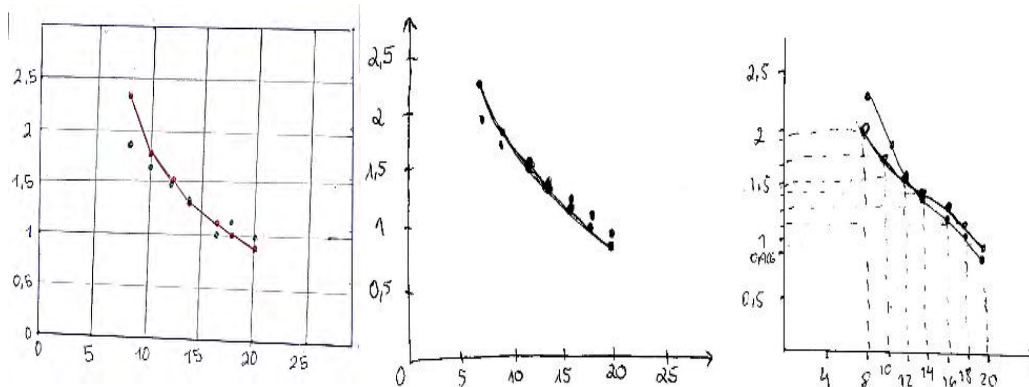

Figure 11. Representation of the best model that fits the experimental data according to G2, G4 and G5

As for groups G3 and G8, they tend to consider values other than the experimental ones in their sketch, although only G8 group considers the labels of the Cartesian axes (Figure 12).
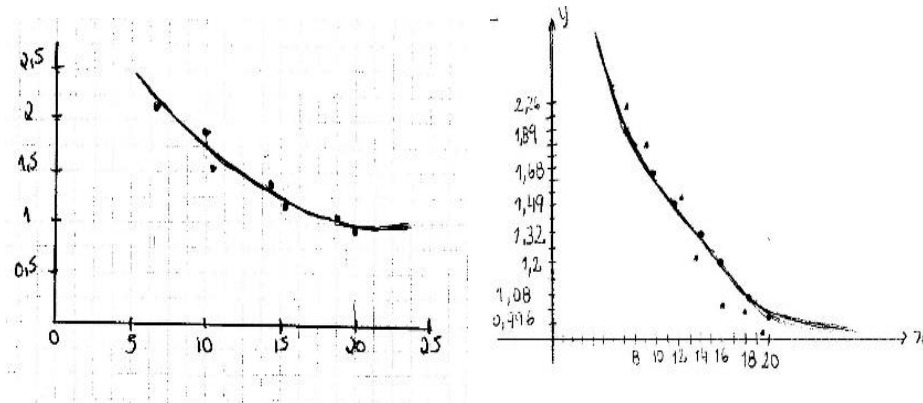
Figure 12. Representation of the best model that fits the experimental data according to G3 and G8

Finally, the graphic sketch created by group G6 presents the labels of the Cartesian axes, delimits the graph at the point of the biggest abscissa, but does not denote attention to the behavior of the function in the neighborhood of zero by values to its right. This group stands out from the others for the letters they attribute to the variables (Figure 13). Despite all these difficulties and constraints in the graphical representation of the data, all groups identified the idea of inverse proportionality underlying the problem.
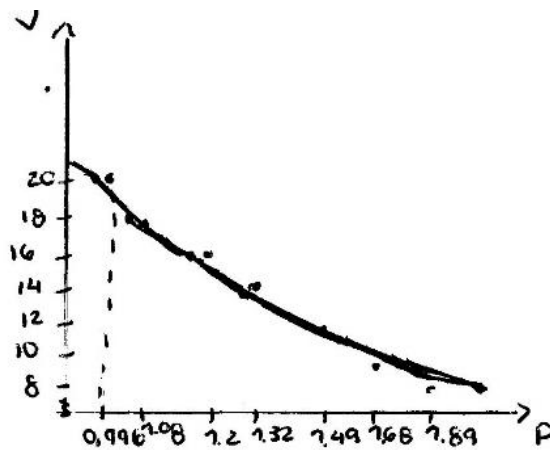

Figure 13. Representation of the best model that conforms to the experimental data according to G6

Difficulties experienced by the prospective teachers

In the resolution of the two proposed modelling tasks, the prospective teachers showed difficulties in determining the value of the proportionality constant (k), in the creation of graphs and, only in the 'Task under Pressure', to recall concepts (Table 1).

Table 1. Difficulties experienced by the prospective teachers in solving tasks

	Candle Task	**Pressure Task**
Construction of graphs	G1, G2, G3, G7, G8	G1, G2, G3, G8
Determining the most appropriate k value	G1, G4, G6	G6, G7
Construction of the table	G5, G7	G4
Remembering concepts	–	G4, G5

The greatest difficulty laid in the graphical representation of both the dispersion of the points that translate the experimental data and the model that best fits these data. One of the reasons for this difficulty is the scale to be used in the subdivision of the Cartesian axes, as well as in distinguishing the line graph from the graph of a function. The difficulties that prospective teachers had to sketch graphs are similar to the ones identified by the study that Viseu, Martins and Rocha (2019) carried out recently.

Another difficulty was to determine the constant of proportionality, which is certainly related to the habit of working with precise values, in which there is no need to discuss which constant best fits the data experiments. This difficulty had repercussions in the construction of the table, which combined the experimental values, the values generated by the model determined by each of the groups and the search to minimize the sum of the squares between the experimental values and the model values.

The difficulty in remembering concepts, translated in a model that was expected to be from the family of the functions of inverse proportionality, indicates that earlier learning tends to be more resilient than that acquired *posteriori*. In fact, the functions of direct proportionality are studied from the 7th level of schooling while the functions of inverse proportionality are studied in the 9th level of schooling. On the other hand, the graphical representation of the first type of functions, straight lines, is more often worked during the basic education than the graphical representation of the second type of functions, branch of hyperbole.

Usefulness of the model determined by the prospective teachers

After determining the best model that fits the experimental data of each of the proposed modelling tasks, the prospective teachers identified the usefulness of these models in the organization of the data, the calculation of the value of one of the variables knowing the value of the other and the forecast of results (Table 2).

Table 2. Usefulness of models for the prospective teachers.

	Candle Task	**Pressure Task**
Organize data	G1, G5, G7	G1, G5, G7
Calculate values of the variables	G2, G3, G4	G2, G3, G4
Predict results	G6, G8	G6, G8

Since the establishment of the mathematical model that best fits the experimental data results from the activity with the values organized in a table, the tendency of some groups to answer that the usefulness of the model is to organize the data indicates that this organization translates the graphic representation of the experimental data and the graph of the determined model. The other two uses indicated by the other groups acquire some similarity in procedural terms, but in conceptual terms they have different meanings, highlighting the possibility of determining values of a variable from the values of the other variable and the discussion between the plausibility of the model values and the experimental values.

After being given the tasks, prospective teachers carried out several activities, namely reading and interpretation of the questions included in each one of the tasks, and building and evaluation of the

mathematical model that best fits the data. The interpretation of the task aimed at allowing prospective teachers to identify the family of functions that could serve as a model for the situation under study. After analyzing the sets of values and perceiving the trends of variation of those values, prospective teachers conjectured a model for the first problem situation which was a function of direct proportionality and a model for the second problem situation which was a function of inverse proportionality. The graphic representation of the data worked as scaffold for prospective teachers' recognition of the curve that best fits the data and for the identification of the models that describe them, because the ratio between the values of the variables is not constant. This result highlights the importance of different representations in the study of functions as expected based on works previously carried out (Arcavi, 2003; Markovits et al., 1998; Viseu et al., 2019). In fact, graphical information tends to complement what is presented numerically or symbolically.

In order to draw the model for each problem situation, depending on the model they conjectured, prospective teachers determined the quotient, or the product, between the values of the variables in search for a constant of proportionality. As the quotients and the products of the values of the variables were not constant, prospective teachers decided to determine the average value and to acknowledged it as being the proportionality constant. After getting this value, they built a mathematical model for each of the problem situations, which allowed them to generate values for the dependent variable. The process followed by the prospective teaches in order to design the models is consistent with what Kaiser and Maaβ (2007) call the application of mathematics in the resolution of everyday problem situations. Prospective teachers have used knowledge acquired during their school career, about concepts related to functions (dependent variable, independent variable, direct proportionality function, inverse proportionality function) and their representations (tabular, analytical and graphical). This highlights students' recognition of the usefulness of mathematical knowledge beyond the school context.

In order to evaluate the model, prospective teachers used two procedures: sum the squares of the difference between the experimental values and the values generated by the model; and graphical representation of the data obtained for each of the problem situations. The sum of the squares of the difference between the experimental and the model generated values indicated that the model could reasonably fit the experimental values. Some prospective teachers sought to minimize the value of this sum, varying the constant considered, in the search of the model that best fits the experimental data. The graphic representation of the model drawn for each problem situation allowed prospective teachers to perceive whether the model fits the experimental values or not. Some of them, after defining several models for the same problem situation, became aware of the effect of the variability of the data in the adjustment of the graph to the experimental values.

As for the difficulties revealed by the prospective teachers, some of them arise from calculation procedures of the ratio between the values of two variables, in particular when both values were zero. According to Crespo and Nicol (2006), this difficulty derives from the fact prospective teachers' knowledge about division by 0 derives from rote learning rather than from meaningful conceptual understanding of that

mathematical operation. Another difficulty revealed by prospective teachers has to do with the distinction between the mathematical expression of a direct proportionality function and that of an inverse proportionality function. This difficulty emerged when discussing the reasonableness of the model for the first problem situation, that prospective teachers designed based on the sum of the squares of the differences between the experimental values and the model generated values. This result suggests that teachers face challenges like children when discriminating among relationships that are and are not direct proportions (Jacobson & Izsák, 2014). However, the difficulty exhibited by most of the prospective teachers laid on the drawing of graphs. Thus, for making the graphical representation of the first situation, some prospective teachers used a line graph. This may be due to the lack of knowledge of the differences between a statistic graphic and a cartesian graph of a function. This procedure, which reveals a lack of critical ability, also emerged when some prospective teachers represented a curve instead of a straight line, in the first problem situation, or a straight line instead of a curve, in the second problem situation.

As for the usefulness of the models designed, only a few prospective teachers were able to mention their usefulness, stating that they are useful for predicting results. This finding seems consistent with the one got by Krawitz and Schukajlow (2018), as these authors also concluded that the value of modeling tasks is hardly perceived by the modelers, which value them lower than the traditional ones.

CONCLUSION

The prospective primary school teachers need to engage into modelling tasks during their undergraduate education so that they become prepared to use mathematical modelling in their future classrooms, as teachers. Such training needs to comprise three dimensions: a cognitive dimension, encompassing two sub dimensions, one related to mathematics concepts and another one related to modeling itself; a metacognitive dimension, aiming at developing students' critical ability of validating mathematical models; and an affective dimension, related to the value of mathematical models and modeling and their usefulness in daily life. Besides, as daily life situations are multidisciplinary and most of them include a science dimension, training on modeling should call for a cooperation between mathematics and science subjects. Research encompassing these dimensions and requirements would be needed to assess the effect of training on students' knowledge and self-efficacy beliefs related to modeling and to collect data required to monitor and improve prospective primary school teacher education on both mathematical modeling and teaching though mathematical modeling.

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