International Journal of Progressive Education, Volume 16 Number 2, 2020

Analysis of the Relationships between Mathematics Achievement, Reflective Thinking of Problem Solving and Metacognitive Awareness

Çetin Toraman [i]
Çanakkale Onsekiz Mart University

Şenol Orakcı [ii]
Aksaray University

Osman Aktan [iii]
Ministry of Education

Abstract

In this study, it was examined that in what level reflective thinking towards problem solving skills and metacognitive awareness explained maths course achievements of the students and the relationships among these variables were studied. Relational survey model was applied. The study included 412 seventh grade students from two different secondary schools within each of the three central districts located in Ankara. In this study, "Reflective Thinking Skill Scale to Problem Solving (RTSSPS)" and "The Metacognitive Awareness Inventory for Children (MAI-C)" and the scores of "Maths Course Achievement" were used. "Pearson Momentler Correlation Coefficient" and "Multiple Linear Regression Analysis" were used in the correlational and regression analysis. It was determined that there was a strong positive significant correlation between students' maths achievement, reflective thinking towards problem solving and metacognitive awareness. It was also determined that there was a strong positive significant correlation between reflective thinking towards problem solving and metacognitive awareness.

Keywords: Achievement, reflective thinking, problem solving, metacognition

DOI: 10.29329/ijpe.2020.241.6

---

[i] **Çetin Toraman,** Assist. Prof.Dr., Faculty of Medicine, Çanakale Onsekiz Mart University, ORCID: 0000-0001-5319-0731

[ii] **Şenol Orakcı,** Assist. Prof., Education Faculty, Aksaray Unıversıty, ORCID: 0000-0003-1534-1310

[iii] **Osman Aktan,** Assist. Prof.Dr., Duzce Rehberlik ve Araştırma Merkezi, Ministry of Education, ORCID: 0000-0001-6583-3765

Correspondence: karakteregitimi@gmail.com

INTRODUCTION

Today, it has become evident that the student is not considered to be as a passive receiver of knowledge, but as an active participant. The perception of the education in which information is actively structured in the mind through basic skills in learning environments has gained more importance. In this sense, it is aimed to equip students with high level cognitive skills among which are critical thinking, analytical thinking, reflective thinking, inquiry, research, adaptation to new situations, decision making, implementing, and problem solving (Orakci, Durnali, & Aktan, 2019). Maths is accepted as one of the courses aiming to enable students to gain these skills and abilities.

Maths is a discipline that develops estimating, calculating, counting, drawing and measuring via human mind to solve problems in daily life throughout environmental influences (Altun, 2013). It is a universal language created as a result of using relations and symbols between sizes, numbers and shapes (Baykul, 2009) and that develops analytical thinking (Minisker, 2006). The main purpose of teaching maths is to gain a problem solving approach with basic knowledge and skills towards maths through mathematical concepts and systems required in a person's daily life and to gain a system of thinking that solves events through problem solving approach (Altun, 2014). Maths makes it possible to understand the relations in our close environment and to make logical inferences by analyzing the available information (Baykul, 2009). It is a subject in which students can experience problems and failures in terms of teaching maths. The abstract concepts in Maths can be cited as one of the reasons for this situation (Baykul, 2009; Ernest, 2010).

Achievement can be expressed as an individual's achievement of a desired goal. When it comes to education, it is linked to the fact that the student exhibits their behaviour with regard to the objectives of the curriculum (Demirtas & Gunes, 2002). Besides, it is an indication of the degree to which the objectives of the course or curriculum are realized (Ozguven, 2002). The variables affecting the students' maths achievement should be determined and the necessary precautions should be taken to contribute to the students' achievement (Dursun & Dede, 2004). Different studies revealed that academic achievement was in relation with cognitive processes such as learning style and intelligence, and affective factors including personality traits, motivation, self-efficacy, and devoting enough time for course responsibilities, as well as parental attitude, family income, teaching leadership specification of the teachers, teachers' competence, their attitudes towards students and other environmental factors (Howie & Pieterson, 2001; Wang, 2004).

According to maths educators, the improvement of problem-solving skills of students is the primary goal of maths education (Karatas & Guven, 2003; Kilic, 2013; Lester, 2013; Szabo, 2017). A problem is mainly identified as a conflict situation where an individual has more than one possibility of reaching a goal, disturbing the individual mentally or physically from being obstructed (Cubukcu, 2011; Karasar, 2009). In other words, problem is a situation that confronts the individual with the need of decision on the choice of strategy that can be used to solve the problem. In a problematic situation, there should be a difference between the current situation and the ideal situation (Kneeland, 2001). Problem solving is the process of choosing and implementing the tools and behaviours that are effective and useful to overcome the difficulties that an individual faces to fulfil his or her purpose (Bingham, 2004; Duman, 2013). Problem solving is a skill that an individual acquires by learning and should develop constantly (D'Zurilla & Chang, 1995; D'zurilla & Nezu, 2006; Krulik & Rudnick, 1989). Problem solving is high-level goal-oriented mental skill that requires a conscious and rational effort.

Problem solving skills include the skills necessary for an individual to solve the problems when faced with a difficult problem (Yetkin & Bascan, 2008). Problem solving is an important part of an individual's mental behaviour. One's self-perception is important in the development of problem-solving skills. In addition, the individual sometimes solves the problem by trial and error, sometimes solves with the scientific method (Miller & Nunn, 2001). It is a complex process and it is recommended by experts that this process be performed gradually because the gradual division of the problem-solving process facilitates both teaching and learning (Senemoglu, 2013). It is clear that

problem solving is divided into various steps for the improvement of cognitive and metacognitive skills related to problem solving by many researchers (Dewey, 1910; D'Zurilla & Goldfried, 1971; Garofalo & Lester, 1985; Krulik & Rudnick, 1989; Morgan & Williams, 2007; OECD, 2004; Polya, 1997; Stevens, 1998; Schoenfeld, 1985).

Problem solving, which is a vital skill in all of the training programmes, has become the main goal of maths education with George Polya's work (1887-1985) (Okur, Tatar, & Isleyen, 2011). Polya (1997) identified four stages of problem-solving process. These steps are; "understanding the problem, preparing the plan for the solution, applying the prepared plan and evaluating the solution". It is essential for learners to have problem-solving skills from early ages and to use this skill at school and in their everyday life (Sahin, 2015). Enabling learners to acquire problem solving skills increases the achievement of the course with regard to academic achievement and, from a social perspective, helps an individual and group live in harmony (Altun, 2013, Kahramanoglu & Deniz, 2017, Senemoglu, 2013). Problem solving is not only a scientific method but it also allows students to learn and use different upper mental skills including critical, creative, analytical and reflective thinking in the solution phase of the problem (Posamentier & Krulick, 2009).

Within the process of problem solving, student's thinking about problem and combining their own knowledge and experience for the solution are related to reflective thinking (Buzdar & Akhtar, 2013). Reflective thinking is a cognitive feature that is learned and developed consciously. Therefore, it is important to have this type of thinking in school environment (Wilson & Jan, 1993). Reflective thinking, which is an inquiry approach that cares about constructivism in education, enables the individual to reconstruct his/her experiences (Mahnaz, 1997). John Dewey, one of the most famous educators in the USA, proposed the concept of reflective thinking to improve students' problem-solving skills and to encourage a spirit of research in students in the direction of supporting student-centered education. According to Dewey (1933), reflective thinking is an effective, coherent and careful thinking of an information structure that supports any thought or knowledge and the achievement of its intended results. When students encounter any problems, they believe that systematic, careful and disciplined thinking around a scientific roof will help them to solve problem. The aim of reflective thinking is to understand a situation or a problem and to find a better solution (Kizilkaya & Askar, 2009). In this context, problem solving is one of the most important skills that an individual should have and reflective thinking is thought to contribute to the problem-solving process. Actions carried out in reflective thinking process are questioning, reasoning and evaluation (Kizilkaya & Askar, 2009; Hong & Choi, 2011). RTTPS encourages students to take initiation to solve, to keep up their interests, and to create an understanding of environmental control in them (Epstein, 2003). The development of reflective thinking skills in students has an influence on their success in maths and problem solving (Meissner, 2006).

Some researchers have associated problem solving process with reflective thinking skills. According to Ferri (2003), reflective thinking skills are related to examining the problem in various dimensions rather than finding out what the result is when a problem is encountered. These thinking skills include evaluating the assumptions put forward in the problem-solving process (Mezirow, 1991). Reflective thinking skills allow the evaluation of the solutions for the problem and choosing the best solution for it (Bingham, 2004). According to Tripp (2003), reflective thinking starts with defining the problem, then things to be done in the context of the problem are listed. Reflective thinking takes place at this stage. Following this stage, a plan is made for the application, and it is put into practice. The things that have been done by that time are evaluated by explanations and this process continues with a rotating cycle. It can be said that reflective thinking skills towards problem solving are a combination of reflections made during problem solving stages. In other words, RTTPS are a high-level thinking style which is carried out in the process of problem solving from the understanding of the problem to the resolution, in the stages of focusing, planning, finding possible solutions, decision making and evaluation (Saygılı & Atahan, 2014)

The concept of metacognition can be generally identified as awareness of an individual's own cognitive processes (Dunlosky & Metcalf, 2009; Nelson, 1999), observing and controlling them

(Flavell, 1987) and their arrangement (Kuhn & Dean, 2004). The knowledge about cognition including the achievement of certain goals of individuals, and arrangement of one's cognitions are two main elements of metacognition (Flavell, 1976; Garofalo & Lester, 1985; Lester, Garofalo & Kroll,1989; Veenman, 2006). In short, metacognition can be expressed as mental processes involving cognitive awareness (Martinez, 2006). It has been among the prominent concepts in recent years. This is due to the assumption that metacognition is effective in gaining competencies such as learning to learn and self-learning, which are beginning to dominate education with regard to the paradigm of our era (Akpunar, 2011). Metacognition is employed to regulate and monitor cognitive processes (Chauhan & Singh, 2014), such as communication, convincing, memory, language learning, reading and comprehension, interest, social awareness, problem solving, self-control, and individual learning (Flavell, 2004). It was stated that the individual, task and strategy variables, and the interactions of these variables play an important role in the formation of metacognitive knowledge, (Flavell, 1979; Veenman, 2005). Flavell (1979) explained these variables as "The category of individual encompasses everything about the beliefs about you and the nature of other people in the context of cognitive processes. The category of task shows the individual's knowledge on nature of the situation and the necessity of a particular task. The category of strategy deals with the information on what kinds of cognitive attempts will be likely to be effective in achieving which of sub-goals and targets they can achieve. "Metacognitive knowledge is related to the fact that individuals are aware of what they know, whereas metacognitive skills, including the actions of observation and regulation are related to their ability to know when and how to use this knowledge is related to (Depaepe, Corte and Verschaffel, 2010; Özcan, 2015; Schoenfeld, 1987).

Problem solving skills include a metacognitive process based on the awareness of one's own mental skills, his/her ability to monitor and control the problem-solving process (Demirel, 2012; Mayer, 2001; Schoenfeld, 1992). Metacognition is a key concept in problem solving skills (Jacobse & Harskamp, 2012; Lee, Chang, & Lee, 2001; Schoenfeld, 1985; Scott & Berman, 2013). Derry and Hawkes (1993) discussed two important metacognitive skills in problem solving. These were "self-monitoring" and "planning". "Self-monitoring" means the individual's ability to create self-control in the process of problem solving. "Planning" involves the individual's ability to break the problem into sub-targets that can be separately solved. Diverse studies have determined that there existed a meaningful relationship between metacognitive skills and problem solving; the teaching of metacognition skills improved students' problem solving ability and as a result, they organized their mental processes more effectively (Bayat & Tarmizi, 2010; Demir, 2016; Kahramanoglu & Deniz, 2017; Kaplan, Duran, & Bas, 2016; Karakelle, 2012; Kramarski, Mevarech, & Arami, 2002). Centered on the examinations of the definitions of metacognition in the related literature, there are six main aspects: "awareness, control, evaluation, planning, monitoring and self- efficacy" while there may be other aspects of metacognition. As for MA, it is linked to an individual's character of consciousness about his/her learning, building and developing way of knowledge. (Anderson & Nashon, 2007).

MA is the ability of the individual to be aware of his or her own cognitive processes and to be able to control the processes by keeping them under control (Sari, 2015). It is the ability of an individual to plan, sort, monitor and use his or her cognitive processes in a better way (Schraw, & Dennison, 1994). In short, the individual is aware of the learning strategies that are appropriate for him or her (Demirsoz, 2010). According to Mason and Santi (1994), MA is in existence at several levels: the awareness of what one knows, why one knows that this is true, awareness of knowledge building procedures, and awareness of changes in one's conceptual structures. These are phases of dialogue argumentation as individuals come into the group environment. Here, it is seen that individual thought has a relationship with group dynamic (as cited Nielsen, Nashon & Anderson, 2009, p.7). MA provides planning, sequencing, and monitoring in learning process for individuals, which directly improves performance (Schraw & Dennison, 1994).

When the studies were examined in the related literature, it can be said that the studies showing the relationship between metacognition and problem solving have started to increase in recent years. In the studies, it was revealed that problem solving skills centred on metacognitive understanding was a method that improved self-regulation skills (Ay & Bulut, 2017). In some studies,

examining the relationship between metacognitive skills and problem solving (Balci, 2007; Bars, 2016; Demir, 2016; Kaplan, Duran & Bas, 2016; Karakelle, 2012; Kramarski, Mevarech & Arami, 2002; Lee, Teo & Bergin, 2009; Swanson, 1992; Teong, 2002), it was found that the level of MA improved the level of RTTPS. It was also revealed in different studies that metacognitive skills increased the success of problem solving (Ozsoy, 2007, Ozsoy & Ataman, 2009; Sengul & Isik, 2014), similarly, metacognitive skills affected maths course achievement positively (Ataalkin, 2012; Deniz, 2017; Kahramanoglu & Deniz, 2017; Kapa, 2001; Pehlivan, 2012). In another study by Tat (2015), it was determined that the most important variables RTTPS were the level of students and teachers. In some studies, investigating the relationship between problem solving and reflective thinking skills and maths achievement (Bas& Kivilcim, 2013; Sen, 2011), it was found out that the students who had RTTPS were more successful. The fact that students have RTTPS enables them to spend more time on recognizing, understanding, and solving problems, thus increasing their course achievement (Kizilkaya, 2009; Serin & Korkmaz, 2018).

When the results of the above-mentioned studies were examined, it was revealed that problem solving skills centred on metacognitive skills improved self-regulation skills and having reflective thinking skills towards metacognitive skills and problem solving were important variables in increasing problem solving skills and achievement of the course. On the other hand, maths is seen as a course that most of the students are afraid of. Why so many students are afraid of maths may be that students cannot escape from their prejudice including failure. Students can overcome their fears when they increase their success in maths (Ekenel, 2005). MA and RTTPS are two important concepts that increase success (Alcı, 2007; Çelik, 2012; Reçber, 2011). In addition, this research can give new ideas to researchers who will conduct research in the field of maths education. Based on the results of the research, it can also help teachers who are the practitioners of the maths education program to provide appropriate learning environments to enable them to develop their own perceptions and MA. It is believed that this research can help to improve the quality of education, and to guide teachers on why it is necessary to increase their MA and RTTPS when organizing teaching processes.

Taking into account to these facts, the purpose of this study is to analyze whether RTTPS and MA predict students' maths achievement significantly. In this context, the research questions below will be addressed:

1. Is there a meaningful relationship between RTTPS and students' maths achievement?
2. Is there a meaningful relationship between MA and students' maths achievement?
3. Is there a significant relationship between RTTPS and MA?
4. Are the levels of MA and RTTPS are significant predictors of maths achievement?

METHODOLOGY

In the study, it was examined that in what level RTTPS and MA predicted maths course achievements of the students and the relationships among these variables were studied. In the study, relational survey model was applied. The researchers focused on how phenomena interacted with each other in the natural environment without establishing any experimental designs. It is undisputed that establishment of an experimental design including a causal and controlled investigation will be a more effective way. However, relational research based on the interaction of variables is determined in the natural environment of events and phenomena. Relational survey model researches based on the interaction of variables also contribute to science by providing a perspective to possible prospective research. The selection of the method as descriptive-based relational comparison restricts the interpretation of a causal effect even if there are possible relationships and interactions among variables.

Research Sample

The study consisted of 501 seventh grade students from two different secondary schools within each of the three central districts (Cankaya, Kecioren and Yenimahalle) located in Ankara. Since the data did not show a normal distribution, the outlier labeling technique was used. As a result of these procedures, 412 seventh grade middle school students are the sample of the study. Of these students, 203 (49%) are female and 209 (51%) are male. The "purposeful sampling strategy" was chosen since it is easy to collect information from participants who are easily accessible to the researcher (Senol, 2012).

Research Instrument

In this study, "Reflective Thinking Skill Scale to Problem Solving (RTSSPS)" and "The Metacognitive Awareness Inventory for Children (MAI-C)" were utilized as data collection instruments. The achievement scores of the maths course of the students were obtained from the school administrators.

RTSSPS

The scale developed by Kizilkaya and Askar (2009) aims to determine seventh grade students' RTTPS. The scale is a five-point Likert-type scale with a Cronbach's Alpha value of 0.83. "Questioning" (5 items), "Reasoning" (4 items) and "Evaluating" (5 items) are sub-dimensions of the scale. The highest score to be obtained from the scale is 70 while the lowest is 14. With regard to the confirmatory factor analysis done for the the validity studies of the scale, the fit indices are "X^2/sd=2.69", "GFI=0.92", "AGFI=0.89", "NNFI=0.93", "CFI=0.95", "RMSR=0.08", "RMSEA=0.071". The Cronbach α is 0.73 for the "questioning" sub-dimension of the scale, "0.71" for "reasoning", and "0.69" for "evaluation", and "0.83" for the whole of the scale. Although these reliability values are not excellent, they are acceptable levels of reliability. However, this scale was preferred instead of any other tools for reflective thinking in problem solving. The reasons for this are that it is a measurement tool developed with regard to the students in target audience in Turkey and was developed with regard to Turkish culture. The fact that there were not too many items in the measurement tool led the researchers think that middle school students would answer questions without losing their attention. This situation was expected to increase the reliability of the research. Reflective thinking is embodied as being aware of what people are doing and thinking about what they are doing. The items of this scale also question students' thoughts and evaluations on how they solve the problem after they solved it.

MAI-C

The Turkish version of the scale was developed by Sperling, Howard, Miller and Murphy and adapted by Karakelle and Sarac (2007). It consisted of A and B forms. The A form with 12 items developed for third, fourth and fifth graders is a 3-point Likert type scale changing from 1 ("never") to 3 ("always") whereas the B form with 18 items developed for sixth, seventh, eighth and ninth graders is a 5-point Likert type scale changing from 1 ("never") to 5 ("always"). In this study, the B form was used as it was conducted with seventh graders. The highest score that can be got from the scale is 90 while the lowest score that can be got from the scale is 18. The Cronbach α of the scale that consists of one dimension is 0.72. The reliability value of this scale is not perfect, but it is within acceptable limits. First of all, the research instrument that is appropriate for the students in the target audience of the research and specific to the Turkish culture was searched. Since this did not happen, it was decided that this scale was appropriate to use. The fact that there were not too many items in the measurement tool led the researchers think that middle school students would answer questions without losing their attention.

Maths Achievement

At the beginning of the research, researchers thought to measure maths achievement by a multiple-choice achievement test. In order to conduct this, it is essential to develop the achievement test, to perform the test application, to calculate the item analysis and the KR-20 reliability value of it and to give the final version of it. It was thought that this process would take time and extend the duration of the research. In addition, it is not possible to measure all the achievements in the 7th grade maths course program with multiple choice items. From time to time, items that require open-ended or progressive response to show the steps of problem solving should also be used. All these reasons led to researchers use achievement measured by teachers in classroom environment as a data. This has advantages and disadvantages. While measuring maths achievement, teachers use multiple choice, true false question types, open-ended item types that require short answer, oral and written exams, and project assignments as performance indicators. In this way, the process is better observed. On the other hand, problems such as objectivity, validity and reliability of teachers' measurements constitute the disadvantage of the measurements made by them.

Research Process

The schools where the research process was implemented were visited a week in advance. The class list and maths achievement scores of the students were requested from the school administration. It was interviewed with the teachers of the classes to be practiced and information was given about what needs to be done on the day of the application. On the day of the application, information about the research was presented before the application of the scales to the students. It was emphasized that participation was centred on the principle of volunteering in the study and it was stated that the students who did not want to be a participant had this right. Each of the scales was marked with a different sign. The students were not asked to write their names in the measurement tools. They were asked to bring the scale to the researcher. The student who brought the scale was told to show his or her name silently in class list, and so it was made sure that the scales belonged to the student.

Data Analysis

In data analysis within the direction of research questions, relation analysis and linear regression analysis were realized. The fact that the data showed a normal distribution enabled "Pearson Moments Multiplication Correlation Coefficient" and "Multiple Linear Regression Analysis" to be used. "Kolmogorov Smirnov Normal Distribution Test" was applied to the scores got from the scales of MAI-C, RTSSPS and from maths course (Buyukozturk, 2013, Kalayci, 2005).

The results of the "Kolmogorov Smirnov Normal Distribution Test" of the data got from 501 students are summed up in table 1.

Table 1. The Results of the Kolmogorov-Smirnov Normal Distribution Test

Scales	N	p
Reflective Thinking Skill Scale towards Problem Solving (RTSSPS)	501	0,044
The Metacognitive Awareness Inventory for Children (MAI-C)	501	0,112
Maths Course Achievement Scores	501	0,039

As seen in Table 1, the scores obtained from MAI-C showed a normal distribution ($p>.05$), but scores from RTSSPS and Maths Course Achievement scores did not show a normal distribution ($p<.05$).

In the data of the scales whose scores did not show a normal distribution, outliers were examined (Kalayci, 2005). After outliers were examined, the "Kolmogorov Smirnov Normal Distribution Test" was again applied to the scores of the remaining 412 participants. The test results are summed up in table 2.

Table 2. The Results of the Kolmogorov-Smirnov Normal Distribution Test After the Examination of Outliers

Scales	N	p
RTSSPS	412	0,200
MAI-C	412	0,200
Maths Course Achievement Scores	412	0,200

As shown in Table 2, after the examination of outliers, the scores of the remaining 412 participants showed a normal distribution. Based on these results, it was decided to use "Pearson Moments Correlation Coefficient" in the correlation analysis and "Multiple Linear Regression Analysis" in regression analysis.

FINDINGS

The relation between RTSSPS, MAI-C and Maths Course Achievement

The relation between levels of students' reflective thinking, MA and maths course achievement were investigated. The results obtained are summed up in table 3.

Table 3. The Relationships between Reflective Thinking Skill Scale towards Problem Solving, Metacognitive Awareness and Mathematics Achievement

	Mathematics Achievement	Reflective Thinking towards Problem Solving	Metacognitive Awareness
Mathematics Achievement	1		
Reflective Thinking towards Problem Solving	0.794**	1	
Metacognitive Awareness	0.785**	0.989**	1

**p<.01, n=412

As seen in Table 3,

- It was revealed that there existed a strong positive meaningful relationship between RTTPS and students' maths course achievement (r=0.794, p<.05). It can be said that as the level of RTTPS increased, maths course achievement also increases.
- It was revealed that there existed a strong positive meaningful relationship between MA and students' maths course achievement (r=0.785, p<.05). It can be said that as the level of MA increased, math course achievement also increased.
- It revealed that there existed a strong positive significant relationship between students' RTTPS and MA (r=0.989, p<.05). It can be said that as the level of MA increased, RTTPS also increased.

The Effect of the Levels of MA and RTTPS on Maths Course Achievement

Whether the levels of students' MA and RTTPS were significant predictors or not was examined. The model formulated is as follows:

$$\hat{Y} = b_0 + b_{ref\ think\ prob\ solving} X_{ref\ think\ prob\ solving} + b_{metacog\ awere} x_{metacog\ awere}$$

This analysis was performed by multiple linear regression analysis. Multiple linear regression is applied in cases where the number of explanatory variables is at least two and more predicted variables are single. The results are summarized in Table 4.

Table 4. The Effect of the Levels of Metacognitive Awareness and Reflective Thinking towards Problem Solving on Maths Course Achievement

Model	B	Std. Error	t	p	Collinearity Statistics (VIF)	F	df	p
Constant	26.810	4.137	6.481	0.000				
Reflective Thinking towards Problem Solving	1.053	0.261	4.035	0.000	46.482	349.58	2	0.000
Metacognitive Awareness	-0.034	0.213	-0.158	0.874	46.482			

When Table 4 looked at carefully, it is seen that the constant is significant. It can be interpreted that other variables that were not formulated except explanatory variables (MA and RTTPS) to be formulated were significantly predictive of Maths course achievement. RTTPS is a significant predictive of Maths course achievement ($p<.05$), but MA is not a significant predictive of it ($p>.05$).

Variance Inflation Factor (VIF) shows multicollinearity between predictive variables. In the event that VIF is "1", there are not multicollinearity between predictive variables. There is multicollinearity between predictive variables in the event of $1\leq VIF\leq 10$. In the event of $VIF>10$, there are strong multicollinearity between predictive variables, the model formulated is invalid and a regression model should be formulated with nonparametric or biased estimation methods (Ozdamar, 2013). The ANOVA Model test results show that the model is significant ($F_{(2)}=349.581$, $p<.05$). Although this result shows that the model formulated is significant and appropriate, the validity of the model formulated becomes controversial because of the strong level of autocorrelation among the variables in the VIF statistic. The model formulated was re-established by being divided into two linear regression models instead of multiple linear regression to prevent misleading and bulging correlation values as a result of autocorrelation. The results are summed up in table 5.

Table 5. The Effect of Reflective Thinking towards Problem Solving on Maths Course Achievement

Model	B	Std. Error	t	p	R	R^2	F	df	p
Constant	27.377	2.062	13.278	0.000					
Reflective Thinking of Problem Solving	1.012	0.038	26.473	0.000	0.794	0.631	700.802	1	0.000

When Table 5 looked at carefully, it is seen that the constant is significant ($p<.05$). As a result, it can be interpreted that other variables that are not modelled apart from the level of RTTPS were predictive of Maths course achievement. The level of RTTPS has an effect on maths course achievement and is a significant explanatory factor ($p<.05$). The regression model formulated as a result of ANOVA test is appropriate and significant ($F_{(1)}=700.802$, $p<.05$). The results of the analysis of the effect of the level of MA on the achievement of the maths course are summarized in table 6.

Table 6. The Effect of Metacognitive Awareness on Maths Course Achievement

Model	B	Std. Error	t	p	R	R^2	F	df	p
Constant	42.338	1.546	27.394	0.000					
Metacognitive Awareness	0.816	0.032	25.658	0.000	0.785	0.616	658.347	1	0.000

When Table 6 looked at carefully, it is seen that the constant is significant ($p<.05$). As a result, it can be interpreted that other variables that are not modelled apart from the level of MA were predictive of Maths course achievement. The effect of the level of MA is effective in the achievement of the maths course and a significant explanatory factor ($p<.05$). The regression model formulated as a result of ANOVA test is appropriate and significant ($F_{(1)}=658.347$, $p<.05$).

RESULTS AND DISCUSSION

With regard to the results of the research, it was determined that there existed a strong positive significant correlation between students' maths course achievement and RTTPS. The way to succeed in maths is directly related to good problem solving. In this sense, how problem-solving process works in teaching maths is very important (Olkun & Toluk, 2004). Problem solving skills arise as one of the most important skills in an individual. In this context, reflective thinking skills are thought to contribute to problem solving process (Kızılkaya, 2009). Similarly, it was determined that students who reflected on problem solving environment spent more time on questions and gave up less (Gama, 2004). In this context, it can be stated that reflective thinking can only be observed in the problem-solving process in the best way based on the fact that when it arises at a particular problem is perceived (Shermis, 1992). Being a successful problem solver also brings critical thinking, decision making, MA, reflective thinking, asking questions, analyzing and synthesizing (Hacısalihlioğlu, Mirasyedioğlu & Akpınar, 2003). It can be stated in this study that as the level of RTTPS increased, achievement of math scores also increased. In a study by Bas and Kivilcim (2013) investigating the relationship between high school students' RTTPS and their academic achievement in maths and geometry courses, a strong positive meaningful relationship was found, which supports the findings of this study. Similarly, in a study by Sen (2011), it was revealed that there existed a strong positive significant correlation between RTTPS and students' maths course achievement. Similarly, in a study by Kizilkaya (2009), it was determined that there existed a strong positive significant correlation between students' reflective thinking skills and their maths course achievement. With regard to the findings of that study, it can be stated that students' having reflective thinking skills has a positive influence on their being successful in maths and solving problems (Meissner, 2006). It can be stated that students who had RTTPS, spent more time to recognize, understand and solve problems, combined knowledge and experience by rethinking problems and thus it increased the success of the course (Buzdar & Akhtar, 2013; Kizilkaya, 2009; Serin & Korkmaz, 2018).

With regard to the results of this research, it was found out that the level of RTTPS is an effective and significant explanatory factor on maths course achievement. According to the results of the similar studies, it was determined that the teaching of metacognition skills increased achievement in problem solving hence the students organized their mental processes more effectively (Bayat & Tarmizi, 2010; Demir, 2016; Kahramanoglu & Deniz, 2017; Kaplan, Duran, & Bas, 2016; Karakelle, 2012; Kramarski, Mevarech, & Arami, 2002; Schoenfeld, 1985). RTTPS includes concepts such as inferencing, generalizations, reasoning between events and relations, hypothesis creation, hypothesis evaluation, recalling, problem solving, using necessary information to understand the problem, analyzing information and discussion (Erginel, 2006). Students benefit from reflective thinking skills in developing new learning strategies by questioning activities in the learning process (Kahyaoglu & Elcicek, 2016; Uygun & Cetin, 2014). Based on the results of the research, the structuring of teaching activities for students with activities that improve their reflective thinking skills can make contributions to their course achievement and the development of mental processes.

In addition, it was revealed that there existed a strong positive significant correlation between students' maths course achievement and MA. It can be stated that as the level of RTTPS increased, maths course achievement also increased. Some similar studies examining correlation between MA and students' maths course achievement (Ataalkin, 2012; Deniz, 2017; Kahramanoglu & Deniz, 2017; Kapa, 2001; Pehlivan, 2012) supported the findings of this study. MA is the ability of students to control their individual learning, and to have sufficient skills for problem solving strategies and problem-solving stages (Akyolcu, 2013). Based on the research results, it is thought that students who take more responsibility in their own learning, control their learning, have metacognitive skills and use them effectively in problem solving processes will be more successful in the course (Ozturk, 2017).

It was determined that there existed a strong positive significant correlation between students' RTTPS and MA. It can be said that as the level of MA increased, RTTPS also increased. Some similar studies (Balci, 2007; Bars, 2016; Demir, 2016; Kaplan, Duran & Bas, 2016; Karakelle, 2012; Kramarski, Mevarech, & Arami, 2002; Lee, Teo, & Bergin, 2009; Swanson, 1992) examining

correlation between RTTPS and MA supported the findings of this study. The development of reflective thinking skills and MA in students is effective in both maths and problem solving (Mayer, 2001; Meissner, 2006; Schoenfeld, 1992). MA enables students to embody maths course composed of abstract concepts and learn it better (Ozturk, 2017). The study results show that RTTPS and MA can be effective variables in increasing maths achievement. In the light of these results, it can be said that students who take responsibility in their own learning processes and have the skills to control learning processes can increase course achievement.

In this study, it was determined that both the level of MA and the level of RTTPS were significant explanatory variables on maths course achievement. With regard to the research results, it may be stated that students who have higher MA and RTTPS can be more successful in maths courses. The regression models that were formulated also showed that other variables that were not modelled except explanatory variables (RTTPS and MA) to be modelled have also an effect on maths course achievement.

With regard to the results of the study, it is advised to include activities that will improve MA and reflective thinking skills in teaching and textbooks for students. In addition, it is suggested to diversify teaching methods for the maths lesson in a way that will provide these skills, and to support teachers' MA and RTTPS with in-service training activities.

REFERENCES

Akpunar, B. (2011). Biliş ve üstbiliş (metabiliş) kavramlarının zihin felsefesi açısından analizi (The analysis of the concepts of cognition and metacognition in terms of the philosophy of mind). *Turkish Studies, 6*(4), 353-365.

Akyolcu, R. (2013). *Resim-iş eğitimi anabilim dalı öğrencilerinin üstbilişsel farkındalıkları ile okul başarıları arasındaki ilişkinin incelenmesi (An investigating of metacognitive awareness and academic achievement among the department of art teaching students)*. (Master's Thesis). Gazi University Institute of Educational Sciences, Ankara.

Alcı, B. (2007). *Yıldız Technical University students' perceived mathematical achievements, problem-solving skills, self-efficacy perceptions, metacognitive self-regulation strategies and descriptive and predictive relations pattern between OSS numerical scores* (Doctoral Dissertation). Yıldız Teknik University, Institute of Educational Sciences, İstanbul.

Altun, M. (2013). *Eğitim fakülteleri ve sınıf öğretmenleri için matematik öğretimi (Mathematics teaching for faculties of education and teachers)*. Bursa: Aktüel Alfa.

Altun, M. (2014). *Ortaokullarda (5, 6, 7 ve 8. sınıflarda) matematik öğretimi (Teaching mathematics in secondary schools [5th, 6th, 7th and 8th grades])*. Bursa: Alfa Akademi Yayıncılık.

Anderson, D., & Nashon, S. (2007). Predators of knowledge construction: Interpreting students' metacognition in an amusement park physics program. *Science Education, 91*, 298-320. https://doi.org/10.1002/sce.20176

Ataalkin, N. A. (2012). *Üst bilişsel öğretim stratejilerine dayalı öğretimin öğrencilerin üst bilişsel farkındalık ve becerisine, akademik başarı ile tutumuna etkisi (The effects of teaching based on metacognitive teaching strategies in science and technology education on students' metacognitive awareness and skills, academic achievement and attitudes)*. (Master's Thesis). Akdeniz University, Institute of Social Sciences, Antalya.

Ay, Z. S., & Bulut, S. (2017). Üst bilişsel sorgulamaya dayalı problem çözme yaklaşımının öz-düzenleme becerilerine etkisinin araştırılması (Investigating the effects of problem-solving approach based on metacognitive questioning on self-regulation skills). *İlköğretim Online, 16*(2), 547-565. htpps://doi.org//10.17051/ilkonline.2017.304716

Balci, G. (2007). *İlköğretim 5. sınıf öğrencilerinin sözel matematik problemlerini çözme düzeylerine göre bilişsel farkındalık becerilerinin incelenmesi (The examination of metacognitive skills of fifth grade students in terms of their levels of solving verbal math problems)*. (Master's Thesis). Cukurova University, Institute of Social Sciences, Adana.

Bars, M. (2016). *Öğretmen adaylarının üstbilişsel farkındalıkları, öğretmenlik mesleğine yönelik öz yeterlikleri ve problem çözme becerilerine ilişkin algılarının incelenmesi (A study of the perceptions of prospective teachers regarding their metacognitive awareness, self-efficacy for the teaching profession and problem-solving skills)*. (Doctoral Dissertation). Dicle University,Institute of Educational Sciences, Diyarbakır.

Bas, G., & Kivilcim, Z. S. (2013). Lise öğrencilerinin problem çözmeye yönelik yansıtıcı düşünme becerileri ile matematik ve geometri derslerindeki akademik başarıları arasındaki ilişki (The correlation between reflective thinking skills towards problem solving and academic success in mathematics and geometry courses of high school students). *Ahi Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi (KEFAD) 14*(3), 1-17.

Bayat, S., & Tarmizi, R. A. (2010). Assessing cognitive and metacognitive strategies during algebra problem solving among university students. *Procedia Social and Behavioral Sciences, 8*, 403-410. https://doi.org/10.1016/j.sbspro.2010.12.056.

Baykul, Y. (2009). *İlköğretimde matematik öğretimi (6-8. sınıflar) (Teaching mathematics in primary school [grades 6-8])*. Ankara: Pegem Akademi Yayıncılık.

Bingham, A. (2004). *Çocuklarda problem çözme yeteneklerinin geliştirilmesi (Developing problem solving skills in children) (A. F Oğuzkan, Çev. [Trans.])*. İstanbul: Millî Eğitim Basımevi.

Buzdar, M., & Akhtar, A. (2013). Development of reflective thinking through distance teacher education programs at AIOU Pakistan. *International Review of Research in Open and Distance Learning, 14*(3), 43-58. https://doi.org/10.19173/irrodl.v14i3.1350

Buyukozturk, S. (2013). *Sosyal bilimler için veri analizi el kitabı (Manual of data analysis for social sciences)*. Ankara: Pegem Akademi.

Chauhan, A., & Singh, N. (2014). Metacognition: A conceptual framework. *International Journal of Education and Psychological Research, 3*, 21-22.

Cubukcu, Z. (2011). Düşünme becerileri (Thinking skills). Filiz, S.B. (Ed.), *Öğrenme öğretme kuram ve yaklaşımları içinde* (Chapter in learning theories and approaches) (pp. 279-334). Ankara: Pegem Akademi Yayınları.

Celik, E. (2012). *Matematik problemi çözme başarısı ile üstbilişsel özdüzenleme, matematik özyeterlik ve özdeğerlendirme kararlarının doğruluğu arasındaki ilişkinin incelenmesi (Investigating the relationships among mathematical problem-solving achievement and metacognitive self-regulation, mathematics self-efficacy, accuracy of self-evaluations)* (Doctoral Dissertation). Marmara University, Institute of Educational Sciences, İstanbul.

Demir, O. (2016). Ortaokul öğrencilerinde problem çözme ve bilişsel farkındalık beceri düzeylerinin incelenmesi (An investigation of secondary school students' levels of problem solving and metacognitive skills). *Kastamonu Eğitim Dergisi, 24*(2), 789-802.

Demirel, O. (2012). *Öğrenme sanatı (The art of learning)*. Ankara: Pegem A Yayıncılık.

Demirsöz, E. S. (2010). *Yaratıcı dramanın öğretmen adaylarının demokratik tutumları, bilişüstü farkındalıkları ve duygusal zekâ yeterliliklerine etkisi (The effects of the creative drama on the democratic attitudes, metacognitive awareness and emotional intelligence abilities of the teacher trainees)*. (Doctoral Dissertation), Dokuz Eylül University, Institute of Educational Sciences, İzmir.

Demirtas, H., & Gunes, H. (2002). *Eğitim yönetimi ve denetimi sözlüğü (Glossary of educational administration and supervision)*. Ankara: Anı Yayıncılık

Deniz, T. (2017). *Ortaokul öğrencilerinin üstbiliş becerileri, matematik özyeterlikleri ve matematik başarısı arasındaki ilişkinin incelenmesi (An investigation of the relationship between middle school students' metacognitive skills, mathematics self-efficacy and mathematics achievement)*. (Master's Thesis), Gaziantep University,Institute of Educational Sciences, Gaziantep.

Depaepe, F., Corte, E. D., & Verschaffel, L. (2010). Teachers' metacognitive and heuristic approaches to word problem solving: Analysis and impact on students' beliefs and performance. *ZDM Mathematics Education*, *42*, 205-218. https://doi.org/10.1007/s11858-009-0221-5.

Derry, S. J., & Hawkes, L. W. (1993). Local cognitive model of problem-solving behavior: An application of fuzzy theory. *Computers as Cognitive Tools*. Lajoie, Susanne P. and Derry, Sharon J. (eds.) Lawrence Erlbaum Associates.

Dewey, J. (1910). *How we think, the problem of training thought*. Retrieved from http://rci.rutgers.edu/~tripmcc/phil/dewey-hwt-pt1-selections.pdf at 05.07.2018.

Dewey, J. (1933). *How we think: A restatement of the relation of reflective thinking to the educative process*. Boston: D. C. Heath Publication.

Duman, B. (2013). Problem çözme ve problem çözme terminolojisi (Problem solving and problem-solving terminology). Ocak, G. (Ed.), *Öğretim İlke ve yöntemleri içinde (Chapter in teaching principles and methods)* (pp. 415-423). Ankara: Pegem Akademi Yayınları.

D'Zurilla, T. J., & Chang, E. C. (1995). The relations between social problem solving and coping. *Cognitive Therapy and Research, 19*, 547-562. https://doi.org/10.1007/BF02230513.

D'Zurilla, T. J., & Goldfried, M. R. (1971). Problem solving and behavior modification. *Journal of Abnormal Psychology, 78*, 107-126. https://doi.org/10.1037/h0031360

D'Zurilla, T. J., & Nezu, A. M. (2006). *Problem-solving therapy: A positive approach to clinical intervention*. New York: Springer Publishing Company.

Dunlosky, J., & Metcalf, J. (2009). *Metacognition*. Thousand Oaks, CA: Sage.

Dursun, S., & Dede, Y. (2004). Öğrencilerin matematikte başarısını etkileyen faktörler: Matematik öğretmenlerinin görüşleri bakımından (The factors affecting students' success in mathematics: Mathematics teachers' perspectives). *Gazi Eğitim Fakültesi Dergisi, 24*(2), 217-230.

Ekenel, E. (2005). *Matematik dersi başarısı ile bilişötesi öğrenme stratejileri ve sınav kaygısının ilişkisi (The relation between metacognitive learning strategies and examination anxiety and success in mathematic lessons)* (Master's Thesis). Anadolu University, Institute of Social Sciences, Eşkişehir

Epstein, A. S. (2003). *How planning and reflection develop young children's thinking skills* [Electronic version]. Beyond the Journal. Young Children on the Web, September 1–8. Retrieved fromhttps://www.brandeis.edu/lemberg/employees/pdf/planningandreflection.pdf. at 05.07.2018

Erginel, S. Ş. (2006). Developing reflective teachers: A study on perception and improvement of reflection in pre-service teacher education (Yansıtıcı düşünen öğretmen yetiştirme: Hizmet öncesi öğretmen eğitiminde yansıtıcı düşünmenin algısı ve geliştirilmesi üzerine bir çalışma). (Doctoral Dissertation). Orta Doğu Teknik University, Institute of Social Sciences, Ankara.

Ernest, P. (2010). Mathematics and metaphor. *Complicity; An International Journal of Complexity and Education, 7*(1), 98-104. https://doi.org/10.29173/cmplct8844.

Ferri, R. B. (2003, February). *Mathematical thinking styles-An empirical study.* Proceedings of the Third Conference of the European Society for Research in Mathematics Education, Bellaire, Italy.

Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. Resnick (Ed.), *The nature of intelligence* (pp. 231-236). Hillsdale, NJ: Erlbaum.

Flavell, J. H. (1979). Metacognition and cognitive monitoring a new area of cognitive—developmental inquiry. *American Psychologist*, *34*(10), 906-911. https://doi.org/10.1037/0003-066X.34.10.906

Flavell, J. H. (1987). Speculation about the nature and development of metacognition. In F. Weinert & R. Kluwe (Eds.), *Metacognition, motivation, and understanding*, pp. 21-29. Hillsdale, NJ: Lawrence Erlbaum.

Flavell, J. H. (2004). Theory of mind development: retrospect and prospect. *Merrill, 50*(93), 274-291. https://doi.org/10.1353/mpq.2004.0018.

Gama, C. (2004). *Integrating metacognition instruction in interactive learning environments*. (Doctoral Dissertation). University of Sussex the Graduate School of Education, Sussex.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16*, 163-176. https://doi.org/10.2307/748391.

Hacısalihlioğlu, H. H., Mirasyedioğlu, Ş., & Akpınar, A. (2003). *Matematik öğretimi (Teaching mathematics)*. Ankara: Asil Yayın Dağıtım.

Hong, Y. C., & Choi, I. (2011). Three dimensions of reflective thinking in solving design problems: a conceptual model. *Educational Technology Research and Development, 59*(5), 687-710. https://doi.org/10.1007/s11423-011-9202-9.

Howie, S. J., & Pietersen, J. J. (2001). Mathematics literacy of final year students: South African realities. *Studies in Educational Evaluation, 27*, 7-25. https://doi.org/10.1016/S0191-491X(01)00011-6.

Jacobse, A. E., & Harskamp, E. G. (2012). Towards efficient measurement of metacognition in mathematical problem solving. *Metacognition and Learning, 7*, 133-149. doi: 10.1007/s11409-012-9088-x.

Kahramanoglu, R., & Deniz, T. (2017). An investigation of the relationship between middle school students' metacognitive skills, mathematics self-efficacy and mathematics achievement. *Inonu University Journal of the Faculty of Education, 18*(3), 189-200. https://doi.org/10.17679/inuefd.334285.

Kahyaoglu, M., & Elcicek, M., (2016). Eğitsel bilgisayar oyunlar ile desteklenen fenbilimleri öğretiminin öğrencilerin motivasyon ve yansıtıcı düşünmebecerileri üzerine etkisi (The effects of educational computer supported science lesson on motivation and reflective thinking skills). *Electronic Turkish Studies, 11*(14), 349-360. https://doi.org/10.7827/TurkishStudies.9563

Kalayci, S. (2005). *SPSS uygulamalı çok değişkenli istatistik teknikleri (Multivariate statistical techniques with SPSS)*. Ankara: Asil Yayın Dağıtım.

Kapa, E. (2001). A metacognitive support during the process of problem solving in a computerized environment. *Educational Studies in Mathematics, 47*, 317- 336. https://doi.org/10.1023/A:1015124013119

Kaplan, A., Duran, M., & Bas, G. (2016). Ortaokul öğrencilerinin matematiksel üstbiliş farkındalıkları ile problem çözme beceri algıları arasındaki ilişkinin yapısal eşitlik modeliyle incelenmesi (Examination with the structural equation modeling of the relationship between mathematical metacognition awareness with skill perception of problem-solving of secondary school students). *İnönü Üniversitesi Eğitim Fakültesi Dergisi, 17*(1), 0-0. https://doi.org/10.17679/iuefd.17119785

Karakelle, S., & Sarac, S. (2007). Çocuklar için üst bilişsel farkındalık ölçeği (ÜBFÖ-Ç) A ve B formları: Geçerlik ve güvenirlik çalışması (Validity and factor structure of Turkish versions of the metacognitive awareness inventory for children [Jr. MAI]-A and B forms). *Türk Psikoloji Yazıları, 10*(20), 87-103.

Karakelle, S. (2012). Üst bilişsel farkındalık, zekâ, problem çözme algısı ve düşünme ihtiyacı arasındaki bağlantılar (Interrelations between metacognitive awareness, perceived problem-solving, intelligence and need for cognition). *Eğitim ve Bilim, 37*(164), 237-250.

Karasar, N. (2009). *Bilimsel araştırma yöntemi (Scientific research method)*. Ankara: Nobel Yayın Dağıtım

Karatas, I. & Guven, B. (2003). Problem çözme davranışlarının değerlendirilmesinde kullanılan yöntemler: Klinik mülakatın potansiyeli (Methods used to evaluate problem solving behaviours: Potential of clinical interview.). *İlköğretimOnline, 2*(2), 2-9.

Kilic, C. (2013). Turkish primary school teachers' opinions about problem posing applications: students, the mathematics curriculum and mathematics textbooks. *Australian Journal of Teacher Education, 38*(5),143-155. https://doi.org/10.14221/ajte.2013v38n5.10

Kizilkaya, G. (2009). *Yansıtıcı düşünme etkinlikleri ile desteklenmiş web tabanlı öğrenme ortamlarının problem çözme üzerine etkisi (The effect of web-based learning environments supported with reflective thinking activities to problem solving)*. (Doctoral Dissertation). Hacettepe University, Institute of Social Sciences, Ankara.

Kizilkaya, G., & Askar, P. (2009). The development of a reflective thinking skill scale towards problem solving. *Education and Science, 34*(154), 82-92.

Kneeland, S. (2001). *Problem çözme (Problem solving)*. Ankara: Gazi Kitabevi.

Kramarski B., Mevarech Z.R., & Arami M. (2002) The effects of metacognitive training on solving mathematical authentic tasks. *Educational Studies in Mathematics 49*, 225-250. https://doi.org/10.1023/A:1016282811724

Krulik, S., & Rudnick, J. A. (1989). *Problem solving: A handbook for senior high school teachers*.The USA: Allyn and Bacon.

Kuhn, D., & Dean, D. (2004). A bridge between cognitive psychology and educational practice. *Theory into Practice, 43*(4), 268-273. https://doi.org/10.1207/s15430421tip4304_4

Lee, N. H., Chang, A., & Lee, P. Y. (2001). The role of metacognition in the learning of mathematics among low achieving students. *Teaching and Learning, 22*(2), 18-30.

Lee C. B., Teo, T., & Bergin D. (2009). Children's use of metacognition in solving everyday problems: An initial study from an Asian context. *The Australian Educational Researcher, 36*(3), 89-102. https://doi.org/10.1007/BF03216907

Lester, F. K. J., Garofalo, J., & Kroll, D. L. (1989). *The role of metacognition in mathematical problem solving: A study of two grade seven classes*. Final report. Retrieved from ERIC databases (ED314255).

Lester, F. K. (2013). Thoughts about research on mathematical problem-solving instruction. *The Mathematics Enthusiast, 10*(1 & 2), 245-278.

Mahnaz, M. (1997). Content and nature of reflective teaching: A case of an experiment middle school science teacher. *Clearing House*, *70*(3), 143-151.

Martinez, M. E. (2006). What is metacognition? *Phi Delta Kappan, 1*, 696-699. https://doi.org/10.1177/003172170608700916

Mason, L., & Santi, M. (1994). *Argumentation structure and metacognition in constructing shared knowledge at school*. Paper Presented at The Annual Meeting of the American Education Research Association (New Orleans, L.A, April 4-8).

Mayer, R. E. (2001). *Changing conceptions of learning: A century of progress in the scientific study of learning*. Edit. by Corno, L. Education across the century: The centennial volume One hundredth yearbook of the National Society for the study of Education, Chicago, National Society for the study of Education, 34-75.

Meissner, H. (2006). Creativity and mathematics education. *Elementary Education Online, 5*(1), 65-72.

Mezirow, J. (1991). *Transformative dimensions of adult learning*. San Francisco: Jossey-Bas

Miller, M., & Nunn, G. D. (2001). Using group discussion to improve social problem solving and learning. *Education, 121*(3), 470-475.

Minisker, M. (2006). Matematiğin doğası, yapısı ve işlevi (The nature, structure and function of mathematics). H. Gür (Ed.), *Matematik öğretimi içinde (Chapter in teaching mathematics)* pp.11-17. İstanbul: Lisans.

Morgan, J., & Williams, B. C. (2007). Overview of problem solving. In S. W. Beyerlein, C. Holmes, & D. K. Apple (Eds.), *Faculty guidebook: A comprehensive tool for improving faculty performance* pp. 181-184. Lisle, IL: Pacifi c Crest.

Nelson, T. O. (1999). Cognition versus metacognition, In: P. J. Sternberg (Ed.), *The nature of cognition*, pp. 625-641. Cambridge, MA: MIT Press.

Nielsen, W. S., Nashon, S., & Anderson, D. (2009). Metacognitive engagement during field-trip experiences: A case study of students in an amusement park physics program. *Journal of Research in Science Teaching, 46*(3), 265-288. https://doi.org/10.1002/tea.20266

OECD. (2004). *Problem solving for tomorrow's world: First measures of cross-curricular competencies from PISA 2003*. OECD Publishing. Retrieved from http://www.oecd.org/education/school/programmeforinternationalstudentassessmentpisa/34009000.pdf at 05.07.2018.

Okur, M., Tatar, E., & Isleyen, T. (2006). İlköğretim düzeyinde problem cözme yaklaşımları (Problem solving approaches at primary level). *Journal of Qafqaz University, 18*, 166-170.

Olkun, S., & Toluk, Z. (2004). *İlköğretimde etkinlik temelli matematik öğretimi*. Ankara:Anı Yayıncılık, Ertem Matbaacılık.

Orakci, S., Durnali, M., & Aktan, O. (2019). Fostering Critical Thinking Using Instructional Strategies in English Classes. In Robinson, S. P., & Knight, V. C. (Eds.)*, Handbook of Research on Critical Thinking and Teacher Education Pedagogy*. Hershey, PA: IGI Global.

Ozcan, Z. Ç. (2015). The relationship between mathematical problemsolving skills and self-regulated learning through homework behaviours, motivation, and metacognition. *International Journal of Mathematical Education in Science and Technology*, *46*, 1-13. https://doi.org/10.1080/0020739X.2015.1080313

Özdamar, K. (2013). *Paket programlar ile istatistiksel veri analizi (Statistical data analysis with packet programs)*. Eskişehir: Nisan Kitabevi, 1. Cilt.

Ozguven, I. E. (2002). *Bireyi tanıma teknikleri (Recognition techniques of person)*. Ankara: Nobel Yayın Dağıtım

Ozsoy, G. (2007). *İlköğretim beşinci sınıfta üstbiliş stratejileri öğretiminin problem çözme başarısına etkisi (The effect of metacognitive instruction on problem solving achievement of fifth grade primary school students)*. (Doctoral Dissertation), Gazi University, Institute of Educational Sciences, Ankara.

Ozsoy, G., & Ataman, A. (2009). The effect of metacognitive strategy training on problem solving achievement. *International Electronic Journal of Elementary Education, 1*(2), 67-82.

Ozturk, B. (2017). *Ortaokul öğrencilerinin üstbilişsel farkındalık düzeyi ile matematik öz yeterlik algısının matematik başarısına etkisinin incelenmesi (The analysis of the effect of metacognitive awareness and mathematics self-efficacy perceptions on mathematics achievement of middle school students)*. (Master's Thesis). ESOGÜ, Institute of Educational Sciences, Eskişehir.

Pehlivan, F. (2012). *İlköğretim beşinci sınıf matematik dersinde üstbiliş strateji kullanımının öğrencilerin başarı ve tutumlarına etkisi (Fifth grade elementary school student achievement in math class and attitudes influence the use of metacognitive strategies)*. (Master's Thesis), Niğde University, Institute of Educational Sciences, Niğde.

Polya, G. (1997). *Nasıl çözmeli? (How to solve?)* (Feryal Halatçı, Çev. [Trans.]). İstanbul: Sistem Yayıncılık.

Posamentier, A. S., & Krulick, S. (2009). *Problem solving in mathematics grades 3-6: Powerful strategies to deepen understanding*. Thousand Oaks, CA: Corwin.

Recber, Ş. (2011). *An investigation of the relationship among the seventh-grade students' mathematics self-efficacy, mathematics anxiety, attitudes towards mathematics and mathematics achievement regarding gender and school type* (Master's Thesis). Middle East Technical University, Institute of Educational Sciences, İstanbul.

Sari, S. (2015). *İlkokul dördüncü sınıfta fen bilimleri dersinde üst biliş stratejilerine dayalı öğretim uygulamasının öğrenci erişilerine etkisi (The effect of teaching implement based on metacognitive strategies on student's data in science in fourth class of primary school)*. (Master's Thesis). Dumlupınar University, Institute of Educational Sciences, Kütahya.

Saygılı, G., & Atahan, R. (2014). Üstün Zekâlı Çocukların Problem Çözmeye Yönelik Yansıtıcı Düşünme Becerilerinin Çeşitli Değişkenler Bakımından İncelenmesi (Analyzing reflective thinking skills towards problem solving of gifted children in terms of various variables). *SDÜ Fen Edebiyat Fakültesi Sosyal Bilimler Dergisi*, 31, 181-192.

Schraw, G., & Dennison, R. S. (1994). Assessing metacognitive awareness. *Contemporary Educational Psychology, 19*, 460-475. https://doi.org/10.1006/ceps.1994.1033

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189 -215). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334-370). New York: MacMillan.

Scott, B. M., & Berman, A. F. (2013). Examining the domain-specificity of metacognition using academic domains and task-specific individual differences. *Australian Journal of Educational & Developmental Psychology*, *13*, 28-43.

Serin, M. K., & Korkmaz, I. (2018). İşbirliğine dayalı ortamlarda gerçekleştirilen üstbilişsel sorgulama temelli öğretimin ilkokul 4. sınıf öğrencilerinin matematiksel problem çözme becerilerine etkisi (The effect of metacognitive questioning instruction performed in cooperative learning environments on the mathematical problem-solving skills of 4th grade primary school students). *İlköğretim Online, 17*(2), 510-531.https://doi.org/10.17051/ilkonline.2018.418893

Shermis, S. S. (1992). *Critical thinking: helping students learn reflectively*. Bloomington: EDINFO Press.

Szabo, A. (2017). *Mathematical abilities and mathematical memory during problem solving and some aspects of mathematics education for gifted pupils*. (Doctoral Dissertation). Stockholm University, Department of Mathematics and Science Education, Stockholm.

Senemoglu, N. (2013). *Gelişim öğrenme ve öğretim, kuramdan uygulamaya (Development, learning and teaching, from theory to practice)*. Ankara: Yargı.

Stevens, M. (1998). *Sorun çözümleme (Problem analysis)* (A. Çimen Çev. [Trans.]). İstanbul: Timaş Yayınları.

Swanson, H. L. (1992). The relationship between metacognition and problem solving in gifted children. *Roeper Rev, 15*(1), 43-48. https://doi.org/10.1080/02783199209553457

Sahin, H. (2015). *Psikososyal gelişim temelli eğitim programının anasınıfına devam eden çocukların duygusal zekâlarına ve problem çözme becerilerine etkisi (The effect of the psychosocial development-based education program on the emotional intelligence and skills of problem solving of the children in pre-schoolclasses)*. (Doctoral Dissertation). Gazi University, Institute of Educational Sciences, Ankara.

Sen, H. S. (2011). *İlköğretim öğrencilerinin problem çözmeye dayalı yansıtıcı düşünme becerileri (Elementary school students' reflective thinking skills based on problem solving). 4-8 Ekim*. I. Uluslararası Eğitim Programları ve Öğretim Kongresi, Anadolu Üniversitesi Eğitim Fakültesi, Eskişehir.

Sengul, S. & Isik, C. (2014). 8. sınıf öğrencilerinin üst bilişsel becerilerinin "Webb'in Bilgi Derinliği Seviyeleri"ne ait problemleri çözme süreçlerindeki rolü (The role of primary eighth-grade students' metacognitive behaviours exhibition skills on process of solving problems based on webb's dok levels). *International Journal of Social Science, 24*, 93-127, https://doi.org/10.9761/JASSS2157.

Senol, S. (2012). *Araştırma ve örnekleme yöntemleri (Research and sampling methods)*. Ankara: Nobel Akademik Yayıncılık.

Tat, O. (2015). *Ortaokul öğrencilerinin problem çözmeye yönelik yansıtıcı düşünme becerilerini etkileyen faktörlerin hiyerarşik doğrusal modeller ile incelenmesi (Investigation of elementary school students' reflective thinking ability through problem solving by using hierarchical linear models)*. (Master's Thesis). Yüzüncü Yıl University, Institute of Educational Sciences, Van.

Teong, S. K. (2002). The effect of matecognitve training on mathematical word problem solving. *Journal of Computer Assisted Learning, 19*, 46-55. https://doi.org/10.1046/j.0266-4909.2003.00005.x

Tripp, D. (2003). Action inquiry. Action research e-Reports. Available from http://www.scielo.br/pdf/ep/v31n3/en_a09v31n3.pdf.

Uygun, K., & Cetin, T. (2014). Sosyal bilgiler öğretiminde yansıtıcı düşünme uygulamalarının akademik başarı ve tutuma etkisi (The effect of reflective thinking practices on academic achievement and attitudes in social studies education). *Researcher: Social Science Studies (RSSS), 2*(3), 50-72.

Veenman, M. V. J. (2005). The assessment of metacognitive skills: What can be learned from multimethod designs? In C. Artelt & B. Moschner (Eds.), *Lernstrategien und Metakognition: Implikationen fu¨r Forschung und Praxis* (pp. 75-97). Berlin: Waxmann.

Veenman, M. V. J. (2006). The role of intellectual and metacognitive skills in math problem-solving. In A. Desoete & M. V. J. Veenman (Eds.), *Metacognition in mathematics education* (pp. 35-50). New York: Nova Science Publishers.

Wang, D. B. (2004). Family background factors and mathematics success: A comparison of Chinese and US students. *International Journal of Educational Research, 41*, 40-54. https://doi.org/10.1016/j.ijer.2005.04.013.

Wilson J., & Jan W. L. (1993). *Thinking for themselves: Developing strategies for reflective learning*. Australia: Eleanor Curtain Publishing

Yetkin, D., & Bascan, O. (2008). *Son değişikliklerle ilköğretim programı 1-5 (Primary program 1-5 with recent changes)*. Ankara: Anı Yayıncılık.