


# The professional development of primary school mathematics teachers through a design-based research methodology

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This article sets out a professional development programme for primary school mathematics teachers. Clark and Hollingsworth's model of teacher change provided the theoretical framework necessary to understand teacher change. A design study allowed for increased programme flexibility and participator involvement. Five volunteer primary school teachers teaching at South African state schools were involved in the programme for a period of one year and their pedagogy, use of mathematical content and context developed during the programme. Twenty lessons were observed over the year-long period. An observation rubric that specifically focused on mathematical pedagogy, use of context and mathematical content scale guided the researcher to gauge global changing teacher practices. Teacher growth was evident through their professional experimentation and changes in their personal domain. The design features emanating from the study are that teachers be given opportunities to experience reform tasks (e.g. model-eliciting tasks) in the role of learners themselves and teachers should be encouraged to use contextual problems to initiate concept development. More mathematical detail in lesson planning is also necessary. Furthermore, teachers need appropriately designed resource materials to teach in new ways. It is recommended that professional development includes teachers engaging collaboratively in solving rich tasks. This study adds to the growing body of knowledge regarding teacher development programmes that focus on how teachers change their own classroom practices.

**Keywords:** Mathematics teacher professional development; primary school mathematics teachers; in-service training; design-based research; model-eliciting tasks.

## Introduction

The need to develop mathematics teacher practices is known worldwide (Borko, 2004; Jaworski, 1998; Koellner, Seago, & Jacobs, 2018). Professional development (PD) environments in which teachers will 'take up' (Adler, 2005) new paradigms and practices are necessary to make a difference in classrooms. Often, teachers take part in PD workshops but are not able to integrate new ideas into their current practices (Clark & Hollingsworth, 2002; Richardson & Placier, 2001). Cai et al. (2017, p. 234) explain that research findings should be of the correct 'grain size' for teachers to be able to use them in their own lesson planning. Although Brown (1992) reminds us that interventions should be able to transfer to everyday classrooms; interventions are only meaningful if they result in changes in mathematics classrooms (Wilson, 1998).

Professional development programmes that have been successful include those that focus on: representations, explanation and communication (Hill & Ball, 2009; Kilpatrick, Swafford, & Findell, 2001), engagement in a variety of activities (Porritt & Earley, 2010), learner centeredness (Polly et al., 2015) and a strong connection to classroom contexts and practices (Chigonga & Mutodi, 2019). From a mathematical models and modelling perspective, researchers propose that teachers should have the same experiences that they want their learners to learn from (Schorr & Lesh, 2003). Materials and tasks also affect learning in PD programmes (Ferrini-Mundy, Burril, & Schmidt, 2007; Hill, 2004) while PD that focuses on reflective thinking (Artzt, Armour-Thomas, Curcio, & Gurl, 2015) shows positive results. Elements such as learner centeredness, engaging in a variety of activities and teacher reflection have always resonated with me as the researcher as important aspects of successful mathematics classrooms. Merchie, Tuytens, Devos and Vanderline (2018), in their evaluation of the effectiveness of PD initiatives, set out that extended engagement, collective learning and active participation are hallmarks of effective PD interventions. This study wanted to ascertain if these PD features could be translated into a successful PD programme for a local context, and furthermore if teachers changed their classroom practices.

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The teachers involved in this study were Grade 5 and Grade 6 teachers in South African primary schools. All teachers had at least three years of teaching experience and were teaching in well-resourced suburban schools. All schools and classes were mixed in terms of cultural background, ethnicity, gender and ability. The language of teaching and learning in all schools was English. However, for about one-third of the learners in the classes, English was not their home language. Only one teacher specifically trained to be a mathematics teacher. The challenges and economic effects of poor-quality mathematics education in South Africa are well documented (Spaull, 2013) and development of mathematics teachers is considered a national priority. However, how to effectively change teacher actions and decisions at a classroom level is less apparent.

Schoenfeld (2011) set out that teachers make decisions based on three interlinking aspects: their resources (which includes various forms of knowledge), their orientations (which includes their beliefs) and their teaching goals. This PD programme set out to resource teachers by providing both knowledge, classroom materials and activities to support teacher professional experimentation and decision-making. The aim of the development programme was to facilitate teacher decision-making towards increased use of contextual problems at the outset of teaching a section of work, more robust mathematical discussions and increased learner activity during mathematics lessons. This study concurs with the ideas of Chirinda and Barmby (2017) that design-based PD interventions should allow teachers to construct knowledge through being actively engaged in the PD. This PD ran in three cycles, and included teachers being actively involved and not simply listening to presentations or lectures.

In a review essay that critically examined mathematics teacher change, Goos and Geiger (2010, p. 505) found that a common thread in the literature on teacher change was the role of 'productive tensions in creating opportunities for mathematics teacher change'. The study reported on here proposed that model-eliciting activities may provide productive tension since these types of open problems were new to the teachers involved. Engaging collaboratively with this productive tension may catalyse professional experimentation and possible changes in teachers' classroom practices. Model-eliciting tasks were used as a 'springboard' for the PD sessions. Modelling problems are open tasks where learners are presented with a real-world problem where the instruction explicitly requests that a model for thinking about the problem is generated (see Figure 2 as an example). Model eliciting tasks have been found to be beneficial both to learner mathematical competencies (Maaß, 2006) and for teacher professional development (Doerr & Lesh, 2003). Borromeo Ferri and Blum (2013) lamented that very few studies involving primary school teachers were evident in the literature. In South Africa specifically, limited research exists on mathematical modelling in studies involving both learners or teachers, although the results are very positive (see Biccard, 2013; Durandt, 2018).

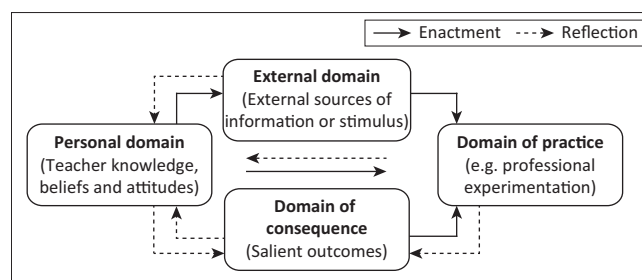
Teacher change should be conceptualised holistically through interventions, resources and change processes within the classrooms. However, teacher development programmes should empower teachers to determine their own growth and change paths (Frid & Sparrow, 2009). Teachers need to be in the driving seat of how change will take place in their classrooms for this change to be meaningful to them. Deficit type narratives (Adler & Sfard, 2017) are not particularly useful in change discourse; rather change should be seen as teachers understanding and appreciating the flexibility of their teaching knowledge and practices (Wilson & Cooney, 2002). In this study, teachers were not expected to reproduce any of the modelling tasks or PD activities in their classrooms; they were requested to use what was useful to them from the PD programme to transfer to their own teaching and classrooms.

In this study, Clark and Hollingsworth's (2002) adaptation of Guskey's (2002) alternative model is employed to understand global changes and growth in teachers' classroom practices. Where Guskey advocates that changes in teacher actions precede changes to their belief systems, Clark and Hollingsworth provide a more authentic model for how teacher professional development can take place through an interconnected model. The model proposes the concurrent activation of four domains: the external domain, the domain of practice, the domain of consequence and the personal domain (see Figure 1).

The research question guiding this design study was: *What changes are evident in primary school mathematics teachers' classroom practices when participating in a design-based professional development programme?*

This leads to the sub-question: *What are 'model-eliciting activities' and how do they exemplify a design-based PD?*

The idea around change and growth in teachers is complex. The word change may denote any change (positive or negative) while growth suggests improvement in teaching practices. In this study the term 'change' was used since I did not necessarily pre-empt a positive change in teachers due to the professional development intervention. In the next section, a model to understand change from teacher professional development is set out.



Source: Adapted from Clark, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education*, 18, 947–967 (p. 951). [https://doi.org/10.1016/S0742-051X\(02\)00053-7](https://doi.org/10.1016/S0742-051X(02)00053-7)

FIGURE 1: Model of teacher change.

## Theoretical model of teacher change

Clark (1998 in Clark & Hollingsworth, 2002) set out that Guskey's model could be more useful if it is seen as a cycle with multiple entry points rather than a linear process. Guskey's model proposes that teachers must first experience a change in their classroom practices before they will generate changed beliefs. Philipp (2007) suggests that supporting teachers to jointly change their beliefs and actions may be more important than worrying about which comes first (beliefs or actions).

Clark and Hollingsworth's (2002) model (see Figure 1) explains that teacher change and growth is a process of enactive and reflective relationships between four inter-related domains. The model provides a more holistic, flexible and integrated model for understanding teacher change through intervention studies. According to the change environment, the *external domain* initiates the change cycle by providing new information or stimulus for teachers. In the case of this study, a professional development programme that would introduce teachers to model-eliciting activities as a springboard for discussing mathematics teaching is the external stimulus. The external domain then promotes a change in the *domain of practice* through enactment of professional experimentation, that is, the teacher may try something different in the classroom. Through professional experimentation, certain salient outcomes are evident (e.g. increased learner participation or enhanced learning); this is the *domain of consequence* (the classroom). These consequences of a teacher's experimentation may result in changes to their *personal domain* (knowledge, beliefs, attitudes or orientations).

Clark and Hollingsworth (2002) conceptualise the interconnectedness between the four domains as comprising two inter-related processes of enactment (solid arrows) and reflection (dashed arrows). Reflecting on the domain of practice may change the personal domain while the enactment of the personal domain will result in changes to the domain of practice. Clark and Hollingsworth acknowledge the complexity of teacher learning and that teacher change can take place through 'multiple growth pathways' (p. 950). What may be deduced from their model is that the external domain (professional development for example) does not have an immediate effect on the domain of consequence. External sources of information or external stimulus catalyse changes in the domain of practice or the personal domain. Through reflecting on these changes in the personal and practice domain, salient outcomes in the domain of consequence (the classroom) may be evident. The professional development programme designed in this study makes provision for both teacher enactment of change (through classroom observation) as well as reflection (on the various PD activities). This article focuses on the enactment of change in teachers' own classrooms.

A guiding principle in this study was to acknowledge that teachers learn in ways that they find most useful (Clark & Hollingsworth, 2002) and that teachers need to be in control

of the decisions they make in a classroom, if teacher change is meant to be significant (Richardson, 1990). What follows in the next section is an explanation of the methodology as well as setting out the procedures in more detail.

## The study: methodology, participants and procedures

### Methodology

The methodology for the study is an intervention study, which can be seen as a type of design study, which is a common type of design study (Cobb, Jackson, & Dunlap, 2015). Design-based research (DBR) typically takes place in three phases (Bakker, 2004). A planning phase where the local instruction theory is formulated (Gravemeijer & Cobb, 2006). The second phase is the actual teaching experiment where designing, redesigning and ongoing analyses of the instructional activities take place (Gravemeijer & Cobb, 2006). The third phase, the retrospective analysis, sets out to contribute to a local instruction theory. Although DBR is not common in professional development of mathematics teachers (Cobb, Confrey, DiSessa, Lehrer & Schauble, 2003; Sztajn, Wilson, Edgington, Myers, & Dick, 2013), its flexibility is of value when dealing with teachers who already are in service and have a bank of experiences and beliefs regarding mathematics teaching. Cobb and Steffe's describe DBR as (in Cobb et al., 2003, p. 9; addition by author) 'a sequence of teaching sessions with a small number of learners [teachers] aiming to create a small-scale version of a learning ecology so that it can be studied in depth and detail'. The aim of the PD programme was to present teachers with model-eliciting tasks and to use teachers' first experiences with these tasks in the PD programme (both as participants in model-eliciting group work and by observing learners solving the same tasks) to springboard discussions on more learner-centered and more problem-centred approaches.

### Participants and procedures

Five mathematics teachers volunteered to be part of the PD programme. The teachers indicated that they were interested in learning about new ways of teaching mathematics.

These teachers were teaching Grade 5 and Grade 6 learners (10 to 12 years old). They were teaching at three schools that formed a convenience sample. The schools were suburban government primary schools that were well resourced. The classes were of mixed ability, cultural background and gender with around 33 to 35 learners in each class. The study ran for a period of one year, although actual contact with the teachers was for a period of nine months. The relevant stakeholders and gatekeepers gave their permission for the study to take place.

The PD ran in three cycles (see Figure 3) with classroom observation visits taking place before the sessions of researcher-teacher intervention commenced. During Session 1 of each cycle, teachers would take positions as learners

themselves where they solved a mathematical modelling problem as a group. An example of a modelling task (Figure 2) given is the one used for the third task in the study.

During Session 2 of each cycle, the teachers would observe small groups of learners solve the same problem. The teachers were asked to become critical reflectors during this session. During Session 3 of each cycle, I considered the discussions in Session 1 and Session 2 and provided support and scaffolding that the teachers indicated they required. PD through DBR requires both engaging teachers in one setting (the PD setting) while trying to reorganise their practices in another setting (the classroom) (Cobb, Zhao, & Dean, 2009). Session 3 of each cycle was adapted and designed as the programme unfolded and as I gained a better understanding of the dynamics of the PD.

Observing and recording holistic changes in a mathematics teacher's lessons is not a simple task. For this purpose, the rubric created by Fosnot, Dolk, Zolkower, Hersch and Seignoret (2006) was used during classroom observations since classroom observations can be used to gauge general teacher quality (Merchie et al., 2018). It includes a pedagogy scale, use of context scale and mathematical content scale. The design concept for this study is congruent with that of Fosnot et al. (2006). Their framework is set within the theoretical work of Realistic Mathematics Education where 'mathematics is a constructive, cognitive activity of making meaning in a social world'. In order for teachers to teach mathematics in this way,

we engaged in-service teachers in experiences that involved action, reflection, and conversation within the context of learning/teaching. We took the perspective that teachers need to construct new gestalts, new visions of mathematics teaching

and learning. To do this they need to be learners in an environment where mathematics is taught as mathematizing, where learning is seen as constructing. (p. 7)

For validity purposes, an existing observation framework was used to understand the observed lessons and to document the teacher's development through the programme. The rubric is too extensive to repeat here, so a summary is presented in Figure 4. The coding protocol for this study is included in brackets.

These formulations allowed me to consider teacher development across three interrelated domains that could capture some elements of teacher change in a mathematics classroom. The rubric is detailed enough to ensure consistency in its interpretation and use. The rubric focuses on teacher actions within the classroom and the repercussions for learning through these actions.

This PD followed a similar approach to that of Cobb et al. (2009) in that I did not work directly with teachers in their classrooms, but by presenting collaborative professional development sessions the mobility of the innovation and how it can be supported was the primary focus. There was also no expectation for the teachers to implement model-eliciting tasks in their classrooms, but to make changes that they felt would improve the teaching and learning in their own classrooms. The study reported on here proposed that model-eliciting activities (as the external stimulus) would catalyse professional experimentation and possible growth in teachers' personal domains. Teachers were asked to teach lessons that would meet the requirements of the curriculum and content that they needed to teach for that day. The research was not to interfere with their normal teaching schedule.

#### Hire or fire?

Mr Ndlovu runs a fast food business. Last year he had nine people who sold food on Saturday mornings at the market. This year, he can only afford to pay two people. These are the records he kept from last year's sales for each person. Who should he hire this year ... and why?

	Hours worked last Summer				Money collected last Summer		
	Dec-12				Dec-12		
	Busy	Steady	Slow		Busy	Steady	Slow
Maria	12.5	15	9	Maria	690	780	452
Karabo	5.5	22	15.5	Karabo	474	874	406
Tony	12	17	14.5	Tony	1047	667	284
Jose	19.5	30.5	34	Jose	1263	1188	765
Chad	19.5	26	0	Chad	1264	1172	0
Lethabo	13	4.5	12	Lethabo	1115	278	574
Robin	26.5	43.5	27	Robin	2253	1702	610
Tony	7.5	16	25	Tony	550	903	928
Siya	0	3	4.5	Siya	0	125	64

Source: Biccard, P. (2013). *The didacticisation practices in primary school mathematics teachers through modelling*. Unpublished doctoral dissertation, Stellenbosch University, Stellenbosch, South Africa (p. 328). Retrieved from <http://hdl.handle.net/10019.1/85598>

Note: The columns *Busy*, *Steady* and *Slow* refer to times when park attendance was high (busy), medium (steady), and low (slow).

FIGURE 2: An example of a modelling task.

## Ethical considerations

The research was granted ethical clearance by the overseeing university (Reference no. DESC\_Biccard2012). Permission to conduct the research was also given by the overseeing provincial department of education. Principals of the schools also granted permission for the study to take place. Participants took part voluntarily in the study and were assured confidentiality. Participants signed informed consent documents and were allowed to withdraw from the research at any point.

## Findings and discussion

The findings of some of the other areas of this study have been partially disseminated (see Biccard, 2013, 2018; Biccard & Wessels, 2015, 2017) which is consistent with design research in that 'within a larger study, several sub-studies often take place' (McKenney & Reeves, 2012, p. 15). The findings presented here set out the global development in the classroom practices of the five volunteer teachers.

Twenty lessons were observed through the programme and the development of the teachers gauged using the rubric in each

<b>Observation</b>	<b>Researcher observes teachers' lessons</b>
<b>Baseline lesson observation</b> <b>Design principles: teacher as learner, moving contextual problems forward.</b>	
Cycle 1 session 1	Modelling task for teachers to solve. Discussion, reflection. The task used is the airplane task and is described in Lesh and Doerr (2003, p. 7).
Cycle 1 session 2	Teachers observe a small group of learners solve the same problem. Discussion and reflection.
Cycle 1 session 3	Guidelines for classroom practice. Discussion around: moving contextual problems to the beginning and not the end of a section of work. Advantages of working with problems. Matching contextual problems with number sentences. Which work better for learning concepts?
<b>Observation cycle 2</b> <b>Design principles: lessons planning, using a variety of representations and examples.</b>	
Cycle 2 session 1	Modelling task for teachers to solve. Discussion and reflection on the task. The task is the tangram toys task adapted from Brousseau (1997, p. 177).
Cycle 2 session 2	Teachers observe a small group of learners solve the same problem. Discussion and reflection.
Cycle 2 session 3	Guidelines for classroom practice. Discussion on lesson preparation and the mathematical goal of a lesson. Discussing hypothetical learning trajectories in teacher lesson planning.
<b>Observation cycle 3</b> <b>Design principles: using existing resources and finding new resources.</b>	
Cycle 3 session 1	Modelling task for teachers to solve. Discussion, reflection. The task is the Hire or Fire task (see Figure 2). Researcher facilitates a discussion on why proportional reasoning is difficult. Difference between additive and multiplicative reasoning and how to facilitate a shift from additive to multiplicative reasoning. Presentation of a variety of classroom tasks that can facilitate this (Van de Walle, Karp, & Bay-Williams et al. 2010, pp. 352–355).
Cycle 3 session 2	Teachers observe a small group of learners solve the same problem. Discussion and reflection.
Cycle 3 session 3	Guidelines for classroom practice. Resourcing teachers for a changing paradigm.
<b>Final observation</b>	Researcher observes teachers' lessons.

Source: Adapted from Biccard, P. (2013). *The didacticisation practices in primary school mathematics teachers through modelling*. Unpublished doctoral dissertation, Stellenbosch University, Stellenbosch, South Africa (p. 145). Retrieved from <http://hdl.handle.net/10019.1/85598>

**FIGURE 3:** Design-based research cycles.

<p><b>Pedagogy scale</b></p> <ul style="list-style-type: none"> <li>• Teaching by telling, which involved teacher explanations, teacher acknowledging of correct answers and learners practising teacher-shown procedures (P1).</li> <li>• Signs of change. This meant that the teacher allowed time before learners answered questions, allowed learners to work in pairs or groups. This was more for sharing of ideas than scaffolding of vertical mathematization (P2).</li> <li>• Facilitating learner construction. Encouragement of learner dialogue that focuses on building reasoning is evident. Facilitates puzzlement around big ideas in mathematics (P3).</li> </ul>
<p><b>Use of context</b></p> <ul style="list-style-type: none"> <li>• Working entirely in the domain of bare numbers or stereotypical word problems that do not serve to advance mathematical ideas (C1).</li> <li>• Word problems are used as a starting point for construction, in contrast to the application of previously learned knowledge as depicted in level one. However, this serves as the purpose of motivation or to elicit children's thinking; no attention is paid to the process whereby mathematical ideas and/or strategies may emerge (C2).</li> <li>• Use of realistic contexts and truly problematic situations as a didactic to facilitate mathematical development. Contexts are purposely designed so as to develop mathematical big ideas, models, and strategies (C3).</li> </ul>
<p><b>Mathematical content</b></p> <ul style="list-style-type: none"> <li>• The teacher's questioning is focused on skills, procedures or tool use (M1).</li> <li>• The teacher questions around concepts and not necessarily the connections between ideas and concepts (M2).</li> <li>• Mathematics is understood as mathematizing and the potential for the development of it is noticed and acted upon throughout the lesson. The discussion is around 'big ideas' and the teacher works towards connecting learner ideas to this 'big idea' (M3).</li> </ul>

Source: Adapted from Fosnot, D.T., Dolk, M., Zolkower, B., Hersch, S., & Seignoret, H. (2006). *Mathematics in the city: Measuring teacher change in facilitating mathematization*. Paper used at the coaching institute for numeracy and literacy leaders.

**FIGURE 4:** Classroom observation rubric.

case by the same researcher. A brief summary of the lessons is presented (see Table 1) as well as the overall rubric score (P1, P2, P3 for the three ratings on the pedagogy scale; C1, C2, C3 for the three ratings on the use of context scale and M1, M2, M3 for the three ratings on the mathematical content scale). These results reflect the enactment of the PD on the teachers' domain of practice through professional experimentation.

Teaching by telling was observed as the dominant pedagogy during the baseline lessons. The learners were mostly seated and silent during these lessons. Only one

lesson involved pupils working in groups to calculate averages followed by a whole class discussion. However, this was to present answers and not to facilitate learner constructions. This lesson was identified as showing 'signs of change' (P2).

During the weeks that followed the baseline observations, teachers were involved in Cycle 1 of the PD programme (see Table 1). The design principle of teacher-as-learner and moving problems to the beginning of a lesson were the focus of this cycle. During session 2, the teachers worked as a group

to solve the Airplane modelling problem. Their initial responses revolved around how the problem was unsuitable to Grade 6 learners because no methods were given nor suggested and that the question was too open. However, when they observed how groups of Grade 6 learners approached and solved the problem, they were surprised that learners (who were learners at one of the schools) were able to organise the problem and model an answer. During the final session for this cycle, I provided the teachers with printed cards that had either a contextual (word) problem or a decontextualised problem. Teachers had to 'match' the contextual problem with its more traditional partner. We then discussed when and how giving learners a contextual problem at the outset (before showing methods) could assist with conceptual development. An example of the type of matching problem is provided in Figure 5.

Teachers discussed how two different types of questions on the same concepts could either alienate students or be more inclusive:

- Teacher E: These [*referring to number-only problems*] are all memory, they must remember what to divide and when.
- Researcher: And if I don't remember?
- Teacher A: Then it's over.
- Teacher C: But that [*problem-centred questions*] they can still figure out – it's words.
- Teacher A: They don't have to remember that you have to do that, and then that...

**TABLE 1:** Summary of baseline lessons.

Baseline lessons	Rubric rating
<b>Teacher A</b>	
Word problems on percentage increase and decrease	P1
The teacher explained a 'method' during a whole class discussion by discussing calculating a 'tip' for a waiter. Thereafter learners worked individually on similar problems.	C1 M1
<b>Teacher B</b>	
This was a revision lesson. Learners were generating mind maps on the concepts of Area and Perimeter. The teacher reminded learners of the various formulae for calculations. Learners worked individually on their mind maps.	P1 C1 M1
<b>Teacher C</b>	
The teacher presented methods to convert fractions to decimal numbers and percentages through using area representations. The lesson was concluded with an individual worksheet.	P1 C1 M1
<b>Teacher D</b>	
The teacher presented learners with 'sum search' sheets where they had to find addition and subtraction problems. Learners worked in groups but worked individually to find the 'sums'. Groups had to calculate the average number of sums they found.	P2 C2 M2
<b>Teacher E</b>	
The teacher asked learners to write out tables multiplying by 10, 100 or 1000. The teacher explained that this was holding learners up in their multiplication methods, so she spent the lesson trying to get them to identify patterns such as: $3 \times 10 = 30$ $3 \times 100 = 300$ , etc.	P1 C1 M1

C, context scale; M, mathematical content scale; P, pedagogy scale.

Around March, each year the Grade 6s and 7s go on camp. There are 105 learners going on camp this year. At the camp, there are canoes to rent. There are 2, 3, 4 or 5 seater canoes. Mrs Smith prefers to order the same size canoe, how many canoes of what size must she order?	Is 105 divisible by 2, 3, 4, or 5?
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**FIGURE 5:** Example of card matching activity.

- Teacher E: You see, I am still telling them...I have to go to where they work it out for themselves...that is where I am struggling. (Biccard, 2013, p. 191)

What is evident here is that teacher change must be approached holistically to include teachers' personal domain since this is a strong enabler of changing a teachers' domain of practice to develop improved classroom outcomes.

I visited the teachers at the beginning of the next school term; Table 2 reflects a summary of the lessons observed.

During the follow-up lessons in Cycle 2, two lessons demonstrated 'signs of change' where teachers allowed ideas from learners to guide the discussion while the question and answer sessions were more in-depth and focused on connecting the mathematical ideas.

Two lessons facilitated learners' own constructions, which presents some evidence of professional experimentation in the domain of practice. These lessons saw learners working in groups trying to construct meaning and not working individually and silently as in the first series of lessons. In the one lesson, groups had to use blocks to explain to the rest of the class what  $\frac{2}{3}$  of 18 'looked like' while the second lesson involved learners working in pairs to calculate equivalent fractions of area models. The inverse operations lesson was still traditional in nature. Two lessons were still dominated by teachers specifying methods while three lessons had learners think about methods. The time allocated for student interaction (Teacher A and Teacher C) in the lessons allowed for students to think reflectively about their working with the manipulatives.

**TABLE 2:** Summary of Cycle 2 lessons.

Cycle 2 lesson	Rubric rating
<b>Teacher A</b>	
Creating equivalent fractions.	P2
Learners worked in pairs through a self-discovery worksheet. They had to pack out blocks to determine the equivalent form of various fractions.	C3 M2
<b>Teacher B</b>	
The teacher explained 'inverse' operations using a flow diagram drawn on the board. An extensive teacher-led whole class discussion took place before individual seatwork.	P1 C1 M2
<b>Teacher C</b>	
The teacher asked learners to work in groups. She provided each group with small wooden blocks and a big sheet of paper. She asked learners to calculate $\frac{2}{3}$ of 18 and to show their working (by packing blocks on the projector) to the rest of the class using the blocks. Learners could use the paper for calculations to verify their answers if necessary.	P3 C2 M2
<b>Teacher D</b>	
The teacher led a whole class discussion on presenting equivalent fractions in area models. She held an extensive question and answer sessions and wanted to show connections between learner understandings.	P2 C1 M1
<b>Teacher E</b>	
The teacher led a whole class discussion on different number patterns that she presented on the board. She wanted learners to give the next number in each pattern and explain the pattern. She used both additive and multiplicative patterns. Learners were encouraged to show various methods of working out the next number. The teacher focused on learners discussing how $+ 2$ ; $+ 4$ ; $+ 8$ is the same as $\times 2$ in the pattern: 2; 4; 8 ... The lesson concluded with learners working individually on textbook problems	P3 C1 M2

C, context scale; M, mathematical content scale; P, pedagogy scale.

The first session of PD that followed these lessons had teachers solve the second modelling task in groups. Some teachers struggled with calculating the ratio for the task (e.g. how to increase a 4 cm length to 7 cm) and once again felt that learners would need more guidance. Teachers also worked through a variety of other proportional reasoning problems in a variety of contexts. The shift from additive reasoning to multiplicative reasoning was the main discussion of this session. The summary of proportional research (Van De Walle et al., 2010, p. 350) was also presented to them. During the second session, the teachers observed groups of learners solving the same task. The learners were able to use a variety of techniques with varying degrees of success. Teachers observed both additive and multiplicative reasoning. The teachers did, however, note that the task allowed learner thinking to be more visible.

During the lesson observation, I also noted that teachers' lesson planning was not very detailed. The lesson plans were mostly populated with dates and topics but no further elaboration of concept development. I, therefore, presented a session on thinking about the mathematical goal of the lesson and hypothetical learning trajectories as an important aspect of lesson thinking and lesson planning. I wanted teachers to see that the mathematical goal of the lesson not as the lesson title, for example adding fractions, but rather as *what mathematics* learners would need to learn, for example writing fractions in equivalent forms.

I visited the teachers again and the lessons are summarised in Table 3.

**TABLE 3:** Summary of Cycle 3 lessons.

Cycle 3 lesson	Rubric rating
<b>Teacher A</b>	
Multiplication and division word problems. Learners worked in pairs. The teacher explained that learners should co-construct their understanding of the problem. Both learners had to agree to the solution method and be able to explain why it was correct. A whole class discussion concluded the lesson.	P2 C2 M2
<b>Teacher B</b>	
Learners sat in groups and had to observe and point out the features of a given 3D shape. They were then asked to create the same 3D shape using toothpicks and jelly sweets. The teacher moved around looking at the constructions and asking questions without explicitly telling learners what to do. The learners worked on traditional textbook work on 3D shapes afterwards.	P3 C3 M2
<b>Teacher C</b>	
The teacher asked learners to work in groups. She provided each group with a large sheet of paper. Groups had to make as many factor trees as possible and present their solutions to the rest of the class.	P2 C2 M3
<b>Teacher D</b>	
The teacher revised the names and types of 3D shapes the learners had learnt the year before. She discussed their features. She then took learners on a walk around the school grounds and asked them to identify as many of the 3D shapes as they could. She stopped at strategic places on the school grounds. The lesson concluded with a whole class discussion on the various shapes they identified.	P2 C2 M2
<b>Teacher E</b>	
The teacher provided each learner with a page where some pizzas and some chocolate bars were printed. She then asked them to share the pizzas (3 pizzas shared between 4 children) or 4 chocolates shared between 12 children. Learners worked individually but were in discussion with their peers. She then led a whole class discussion on the different ways that learners used to share out the pizzas or chocolates.	P3 C3 M3

C, context scale; M, mathematical content scale; P, pedagogy scale.

The third cycle lesson observation showed a move away from teachers' teaching by telling toward a greater focus on asking learners to work collaboratively. Lessons now included questions that stimulated learner thinking rather than seeking specific answers. The external domain appeared to have activated a change cycle in the other three domains (domain of practice, domain of consequence and the personal domain).

Three lessons during Cycle 3 showed 'signs of change' while two allowed for learners' own constructions to guide the lesson. In the three that showed signs of change, learners worked in groups and discussed their ideas regarding multiplication and division word problems, factor trees and 3D shapes that they identified around the school grounds. In the two lessons coded as P3, one lesson saw learners cutting paper pizzas and chocolates to share between numbers of learners and in the other lesson, learners had to construct their own 3D prism before moving onto formal terminology, diagrams and concepts. Learners were spending more time working with each other and verifying their work with each other. Teacher B's and Teacher E's lessons included questions that stimulated student thinking rather asking for specific answers. Lessons in this cycle also reflected many more mathematics moments due to the increase in student activity. The professional experimentation results in a change to the domain of consequence (outcomes) and changes to the personal domain. It supports Guskey's (1986, 2002) principle that teachers have to experience learner success in their classrooms to change their beliefs about teaching. Although teachers were asking learners to present their ideas, no explicit connections between the ideas was made by the teachers.

The final sessions of PD once again started with the teachers solving a modelling task (see Figure 2). Teachers found this problem challenging and were starting to talk about the various ways in which the learners would approach it. They expressed concern that learners would not manage to work with so much data. Although the groups of learners did produce models for the problem, they were not able to work with the busy, steady and slow aspects of the data but rather aggregated the data.

Teachers indicated throughout the PD that they needed assistance in finding contextual problems that covered the curriculum topics at the correct level. This is consistent with Borromeo Ferri and Blum's (2013) quantitative findings on the barriers teachers cited to implementing model-eliciting tasks. We discussed how teachers could use the resources that they do have (e.g. textbooks) to facilitate mathematical learning where learners not only follow set methods but are allowed to first tackle problems and think about them. We discussed using textbook problems at different stages of the lesson. Teachers looked at different examples of typical textbook problems and discussed if they should always be used to conclude the lesson. They shared examples of how, at times, it may be beneficial to use the textbook problems at the beginning of the lesson or as pair work during the main part

of the lesson and to use a whole class discussion afterwards. During this final session, I provided URLs to open online resources that teachers could refer to as well as book titles (e.g. *Mathematics in Context* or the *Shell Centre for Mathematics*) that teachers could use.

At the end of this session, I asked teachers to reflect on their own teaching. Their responses show that they are starting to consider their roles as facilitators more critically. They stated the following when asked which aspects of their teaching they still wanted to improve. These responses indicate changes in their personal domain.

To have the patience not to give groups who are struggling the answer, but to guide them patience with their methods/ideas – not to tell them how to work out the answer. Talk less and listen more to facilitate more and control less. (Biccard, 2013, p. 257)

I visited the teachers at the end of the term and the summary of lessons is presented in Table 4. These observations took place either just before or just after the schools' mid-year exams. Teachers were revising or catching up work, before the winter vacation.

In the final lessons, two showed signs of change while three focused on learners' own constructions. The signs-of-change lessons included pair work on word problems with a follow-up whole class discussion while the second lesson involved groups of learners working collaboratively on ordering decimal numbers. For the three learner construction lessons, one involved a model-eliciting problem and two involved

learners' own constructions on profit models and learners constructing their own nets for 3D shapes. Teacher A and Teacher D appeared to have folded back to more traditional teaching for this lesson.

## Summary

The baseline lessons and Cycle 2 lessons were predominantly calculation-based lessons. During Cycle 3 lesson observations, three teachers used some other contexts as starting points (e.g. word problems) while two teachers presented truly problematic situations where no known procedures could simply be applied to the problems (sharing pizzas and constructing 3D objects). In the final lesson observations (almost a year after the start of the programme) three teachers presented truly problematic situations but only two of these teachers guided the classroom discussion to connect learner understandings. It was during this cycle that one of the teachers presented the model-eliciting problem based on finding the winner of an athletics event. The teacher noticed that this problem (presented before moving onto typical textbook problems) assisted her learners in understanding concepts related to ordering decimal numbers. This teacher tried a mini-experiment in her classes (professional experimentation). She had one of her classes work on the textbook problem first and then complete the model-eliciting problem. She found that this class struggled with the textbook problem while those that did the model-eliciting problem first did not.

Teachers' initial lessons were typical mathematics lessons in terms of focusing on bare numbers, procedures or skills. During Cycle 2 observations, some of the teachers started exploring mathematical moments in their classes by allowing learners to enter the mathematical discussion through self-directed activities and group work. By Cycle 3 observations, two teachers had their classes explore mathematical ideas through their own constructions (e.g. pizza sharing lesson). By exposing mathematical ideas through contexts, teachers in this study were able to capitalise on the 'mathematics moments' in their classrooms. Of the 20 lessons observed, only 4 reached the third level on the pedagogy scale. Changing teacher practices is a complex endeavour and the time needed for paradigm shifts in mathematics teaching cannot be underestimated (Guskey, 2002; Wilson & Cooney, 2002). Teacher change is also not a linear process of teachers moving from point A to point B to point C but moving around between the various stages of change or development based on their knowledge, context and goals at the time. Teachers take up certain aspects from the PD to try out in their classrooms when they feel professional experimentation is appropriate.

The lesson observation rubric focused on three big areas (pedagogy, context and content) where the potential for mathematisation can be gauged in a classroom. The three elements of the rubric are interrelated in terms of classroom practice. 'Teaching by telling' was often associated with bare

**TABLE 4:** Summary of final lessons.

Final lessons	Rubric rating
<b>Teacher A</b>	
Learners worked in groups on lists of decimal numbers that had to be ordered. The groups had to agree on the order and one person from each group had to present their solutions. A whole class discussion with teacher connecting ideas concluded the lesson.	P2 C1 M1
<b>Teacher B</b>	
The teacher presented learners with a complex problem on calculating profit. It involved repackaging a large packet of peanuts into smaller quantities. Learners worked in pairs to solve it. The teacher gave them big blank sheets of paper to work on and not their usual workbooks.	P3 C3 M2
She concluded the lesson with a whole class discussion on the problem where she tried to connect different ideas presented by the learners.	
<b>Teacher C</b>	
The teacher presented learners with a model-eliciting problem. Learners were given the results of athletics events and the list of 'winners'. Some events are won because of the larger number (e.g. long jump) while other events are won because of a smaller number (e.g. a race). Learners worked in groups to determine if the winners' list provided to them was correct.	P3 C3 M3
<b>Teacher D</b>	
Learners worked in pairs on multiplication and division revision problems. The teacher created two sets of worksheets with word problems using similar mathematical concepts but in different contexts. Learners presented their solutions. She led a discussion on how the problems on the two different worksheets were similar in structure and solution path.	P1 C1 M2
<b>Teacher E</b>	
The teacher provided each group of learners with a 3D shape. She asked them to look carefully at the edges, corners, etc. She provided each learner with a large sheet of paper. Learners had to draw a net of the shape. Thereafter she provided learners with a pre-printed net and asked them to compare their net to the pre-printed one. She asked them to look at what was the same and what was different. She then asked them to construct both nets.	P3 C3 M3

C, context scale; M, mathematical content scale; P, pedagogy scale.



numbers emphasising procedural mathematics (with the teacher sticking to a pre-designed script) while a pedagogy of facilitating learner constructions necessitated the use of truly problematic situations and a focus on the underlying structure of the problem. In the latter lessons, teachers were using learner involvement to a greater extent, although not always making explicit connections between learner ideas. Teachers' evolving pedagogy involved using more contexts and more focus on learners' own constructions during the lessons. This is consistent with the findings of Cobb et al. (2009). The teachers also increased the physical and visual material used in the latter lessons. However, this in itself does not signify change if the materials are used superficially. There is a danger that teachers may use added materials because they think they are expected to in terms of their participation in PD.

## Conclusion

Teachers are the most 'critical layer of the school system in terms of efforts to change what happens in schools' (Smith & Southerland, 2007, p. 397). However, the notion of teacher change is a complex concept that makes professional development of mathematics teachers more a *design* process than an *implementation* process. The formulation of design principles in this study (teacher as learner, bringing contextual problems forward, more detailed lesson planning, using a variety of representations and resourcing teachers) may allow others to reflect and select or design their own principles in their own settings.

This study sought to contribute to the micro theory level by proposing activities such as modelling tasks for teacher professional development and did not necessarily seek replication of tasks in the classroom. In an attempt to move away from deficit-type narratives regarding teachers, the DBR approach in this study did not prescribe which practices teachers *should* take up since there are 'many different ways for teachers to create powerful learning environments' (Schoenfeld & the TRU project, 2016) in their classrooms. Rather Richardson's (1990, p. 14) suggestion that meaningful change will take place if teachers have control over what they 'adapt, adopt or ignore' from the PD was used for this study. The modelling activities became springboards for discussion, and may have created avenues for conceptual change in teachers. The two-tiered process (working on a model-eliciting problem themselves followed by observing learners solve the same problem) appeared to assist teachers in making some changes in their own classes. The modelling problems may have supported the change environment since professional experimentation led to some personal domain changes. The processes of enactment and reflection facilitate changes in one domain being translated to a change in another domain (Clark & Hollingsworth, 2002).

This study was limited to five teachers in one region of South Africa, teaching Grade 5 and Grade 6 mathematics that is congruent with one of the known limitations of

DBR – designing to scale (Cobb et al., 2015). Further research is needed in how to implement DBR at scale. The study did not focus on teachers implementing specific model-eliciting problems in their classrooms. Rather, it sought to work with teachers in terms of where they were and how they chose to develop their own pedagogies. Teachers appeared to have knowledge needs (what is modelling, why present problems at the outset, etc.) and orientation needs (how do learners learn mathematics in meaningful ways?). Teacher change in this study was modest and since this is also not a longitudinal study, it is not known if teachers maintained some of the changes evidenced during the PD intervention time period. Since 'it is what teachers think, what teachers believe, and what teachers do at the level of the classroom that ultimately shapes the kind of learning that young people get' (Hargreaves, 1994, p. ix), there is a need for ongoing research in mathematics teacher professional development that focuses on how teachers can develop their own pedagogy, use of context and mathematical content in their own classrooms.

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I declare that I am the sole author of this article.

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