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CONCEPTUAL UNDERSTANDING, PROCEDURAL KNOWLEDGE AND PROBLEM-SOLVING SKILLS IN MATHEMATICS: HIGH SCHOOL GRADUATES WORK ANALYSIS AND STANDPOINTS

Masooma Ali Al-Mutawah[1+]
Ruby Thomas[2]
Abdulla Eid[3]
Enaz Yousef Mahmoud[4]
Moosa Jaafar Fateel[5]

[1,2,4,5]*Bahrain Teachers College, University of Bahrain, Bahrain.*
[1]*Email: malmutawah@uob.edu.bh Tel: +97317437152*
[3]*College of Science, University of Bahrain, Bahrain.*

(+ Corresponding author)

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ABSTRACT

This study measures the mathematical abilities high school graduates' in Bahrain. Mathematical abilities encompass conceptual understanding, procedural knowledge and problem-solving skills in the five content domains which are Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. While procedural understanding focusses on performing facts and algorithms, conceptual understanding reflects a student's ability to reason and comprehend mathematical concepts, operations and relations which will be helpful in solving non-routine problems. A test consisting of questions from the five content domains was administered to students where they demonstrated conceptual understanding and procedural knowledge which enabled them to solve problems in various real-life situations. Structured interviews were also conducted to test their mathematical abilities and suggest ways to improve proficiency in mathematics and eliminate misconceptions. The results show that conceptual understanding and problem-solving skills are positively correlated. This research also endeavors to correlate students' performance in this test with their high school GPA.

Contribution/Originality: This study explores the relationship between mathematical abilities: conceptual understanding, procedural knowledge and problem-solving skills in high school graduates in the five mathematics content domains: number and operations, algebra, geometry, measurement, and data analysis and probability.

1. INTRODUCTION

Even after graduating from high school, it is apparent that students do not possess an appropriate level of conceptual understanding in the five content domains. This adversely impacts on their problem solving capabilities. Problem solving is one of the major processes defined in the National Council of Teachers of Mathematics (NCTM) Standards for School Mathematics (National Council of Teachers of Mathematics, 2000). Problem solving involves students applying four processes: reasoning, communication, connections, and representation. Problem solving can also provide opportunities for students to apply content knowledge in all five mathematic domains. Problem solving provides a window into children's mathematical thinking and is consequently a major vehicle for assessment. Learning with understanding is essential to enable students to solve emergent problems throughout their lives.

The National Research Council (2001) set forth a list of five strands in its document *Adding It Up: Helping Children Learn Mathematics*, which include conceptual understanding. Conceptual understanding helps students to avoid errors of magnitude in particular. *Procedural fluency* refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skills in performing them flexibly, accurately, and efficiently.

Mathematical competence relies completely on children developing and connecting their knowledge of concepts and procedures. Many children perform poorly in problem solving because of lack of conceptual understanding. For decades, the major emphasis in school mathematics was on procedural knowledge, now referred to as procedural fluency. Rote learning was the model, with little attention paid to the understanding of mathematical concepts. In recent years, major efforts have been made to focus on what is necessary for students to learn mathematics, and what it means for a student to be mathematically proficient.

A good starting point to appreciate conceptual understanding is to review *The Learning Principle* (NCTM (2000). It is one of the six principles put forward, and states:

> Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. Mathematics assessment tools often focus solely on this procedural side of understanding mathematics instead of the equally important conceptual aspect of learning mathematics.

1.1. Misconceptions

When students systematically use incorrect rules or the correct rule in an inappropriate domain, there are likely to be misconceptions. Knowledge of mathematical concepts has been enriched by a combination of experimental survey research and observational studies. These have challenged prevailing theories about how children think and learn in various mathematical domains (Wood, 1998). A misconception can be the result of lack of understanding, or in some cases a misapplication of a rule or mathematical generalization (Spooner, 2002).

The ideas about how students develop 'misconceptions' are emphasized by most of the empirical studies on learning mathematics in recent decades. Piaget's repeated demonstration in the late 1970s that children think about the world in very different ways than adults resulted in educational research, and a renewed interest in what students were saying and doing on a variety of subject matters (Smith, 1993).

1.2. Mathematical Abilities

According to National Centre for Education Statistics (NCES) the following are considered to be mathematical abilities.

- *Conceptual Understanding*

  Students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; identify and apply principles; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts. Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

- *Procedural Knowledge*

  Students demonstrate procedural knowledge in mathematics when they select and apply appropriate procedures correctly; verify or justify the correctness of a procedure using concrete models or symbolic methods; or extend or modify procedures to deal with factors inherent in problem settings. Procedural knowledge encompasses the abilities to read and produce graphs and tables, execute geometric constructions, and perform non-computational skills such as rounding and ordering. Procedural knowledge is often reflected in a student's ability to connect an algorithmic process with a given problem situation, to employ that algorithm correctly, and to communicate the results of the algorithm in the context of the problem setting.

- *Problem Solving*

  Students demonstrate problem solving in mathematics when they recognize and formulate problems; determine the consistency of data; use strategies, data, and models; generate, extend, and modify procedures; use reasoning in new settings; and judge the reasonableness and correctness of solutions. Problem-solving situations require students to connect all of their mathematical knowledge of concepts, procedures, reasoning, and communication skills to solve problems.

This research follows strands are intertwined and include notions suggested by NCTM in its Learning Principle. To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification; and
- Productive disposition: a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own aptitudes.

The National Assessment of Educational Progress's definition for mathematical abilities is conceptual understanding, procedural knowledge, and problem solving. Therefore, in this research, each participant will be tested to determine their mathematical abilities according to the following:

i. Each concept question will be identified by five domains:
   - D1: Numbers & Operations (comparing, multiplying, dividing, finding percentage).
   - D2: Algebra (simplifying expressions, writing equations, solving equations, substitution).
   - D3: Geometry (perimeter, surface area, volume, circle).
   - D4: Measurement (capacity, mass, distance, time).
   - D5: Data Analysis & Probability (reading graphs, mean, sample space, finding probability).

ii. Each Domain will have four different math concepts, each divided into three different ability questions:
   - K: Procedure Knowledge.
   - C: conceptual understanding.
   - P: problem solving scale.

iii. The test will have a total of 60 questions, 20 questions for each different type of ability. Questions will be numbered as the following:
   - D1-1K, D1-1C, D1-1P and so on.

iv. Each question will be graded using a four-point Likert scale.
   - 0: if they did not write anything.
   - 1: for wrong answer.
   - 2: for part of the process or understanding.
   - 3: for correct answer.

v. Data will be represented in graphs to:
   - Classify students' test score based on the mathematical ability level.
   - Classify students' test score based on the accuracy level (getting 0-3).
   - Classify students' test score based on the mathematical domain.

Finally, the research will attempt to answer the following questions:

1. What percentage of mathematical abilities' do recent high school graduates show in different mathematics domains?

2. What are students' misconceptions when solving conceptually orientated tasks involving different mathematical domains?
3. Is there a correlation between the conceptual understanding and problem solving in mathematics?
4. Are there any differences between the mathematical abilities test score and high school students' GPA, and specializations?

2. LITERATURE REVIEW

For decades, the major emphasis in school mathematics was on procedural knowledge. Rote learning was the norm, with little attention paid to understanding mathematical concepts. Rote learning is not the answer in mathematics, especially when students do not understand the fundamentals. In recent years, major efforts have been made to focus on what is necessary for students to learn mathematics, and what it means for a student to be mathematically proficient (Balka *et al.*, 2014). The debate over conceptual understanding versus procedural knowledge has caught the eye of many teachers in school systems around the world. Conceptual understanding is the comprehension of not only what to do, but also why it is done. Procedural knowledge, also known as imperative knowledge, is exercised in the performance of a task. In both cases, students understand how to complete an assignment, but the way they think about it differs. One thing that many teachers agree on is that students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (Cummings, 2015).

The National Assessment of Educational Progress (NAEP) identifies mathematical abilities as: procedure knowledge, conceptual understanding and problem solving. While the National Council of Teachers of Mathematics (NCTM) identifies these three types of understanding as three of the main strands to be mathematically proficient (NCTM, 2000). NCTM defines problem solving as a strand that involves students applying four other processes: reasoning, communication, connections, and representation which provide opportunities for them to apply content knowledge in all mathematics domains. While conceptual understanding helps students avoid many errors in solving problems, procedural knowledge helps them to use the knowledge of procedures. The knowledge of when and how is the appropriate developments of skills in performing them accurately and efficiently.

Learning with understanding is essential to empower students to solve those problems they will inevitably face in the future, but even after graduating from high school, it is apparent that some do not possess conceptual understanding or problem solving skills in the five content domains: number and operations, algebra, geometry, measurement, and data analysis and probability.

According to Hasnida and Zakaria (1991) the primary goals in mathematics teaching have shifted towards procedural and conceptual understanding. The importance of gaining these is aligned with the objective of mathematics education. Using a survey method, they carried out a study in secondary schools and the data were analyzed descriptively to determine students' procedural and conceptual understanding of mathematics. Pearson's correlation was used to determine the relationship between procedural and conceptual understanding. The findings revealed that the students' level of procedural understanding is high whereas the level of conceptual understanding is low. They suggested that a reformation in teaching is needed to boost conceptual understanding among students to minimize the use of procedures and memorization.

Jazuli *et al.* (2017) stated that most students find it difficult to understand and apply the concept of mathematics in a real-world context. They argue that the difficulty is due to conventional learning strategies which are unable to improve the students' abilities. They performed an experimental study intended to discover whether the implementation of a contextual learning strategy would improve conceptual understanding and problem-solving skills. These two issues have been analyzed by using a pre-test and post-test, and comparing the results using a control group with conventional learning. The results showed that the contextual learning strategy significantly affects conceptual understanding and the ability to solve mathematical problems.

Rittle-Johnson and Alibali (1999) suggested that procedural knowledge may influence gains in conceptual knowledge by helping children to identify and eliminate misconceptions. Conceptual knowledge may influence improvements in procedural knowledge by improving problem representation and facilitating the adaptation of known procedures in problem solving.

Children must learn both basic concepts and correct procedures to solve problems. Mathematical competencies depend on their ability to connect the knowledge of fundamental mathematical concepts and procedure to real life situations. Observations show that students who possess procedural knowledge alone were unable to solve real life problems due to a lack of conceptual understanding. They were unable to connect the concepts to the problem-solving situations.

2.1. Conceptual Understanding and Procedural Knowledge

Concepts are the building blocks of knowledge (Charlesworth, 2012). Conceptual understanding and procedural knowledge are essential to the development of problem-solving skills (Geary, 2004). These skills contribute towards the effective processing of information when solving problems. The five strands of mathematical proficiency (Kilpatrick *et al.*, 2001) represent the interdependence of the five components of a learner's proficiency in mathematical problem-solving. Use of problem-solving to teach mathematics not only develops knowledge and skills but also helps students make sense of it. They will be able to see how new concepts connect to their existing knowledge which help them to be mathematically proficient (Mayer, 2008).

Conceptual knowledge is in general abstract knowledge that addresses the essence of mathematical principles and relationships between them, while procedural knowledge consists of symbols, conditions, and processes that can be applied to complete a given mathematical task (Hiebert and Lefevre, 1986). Procedural knowledge is meaningful only if it is connected to a theoretical fact. Faulkenberry (2003) suggests that conceptual knowledge is rich with relationships and refers to the basic mathematical constructs and relationships between the ideas that illustrate mathematical procedures, and give them meaning. On the other hand, procedural knowledge addresses the mastery of mathematical skills, and acquaintance with the procedures to determine the mathematical components, algorithms, and definitions. Many researchers suggest that both conceptual knowledge and procedural knowledge are important components in understanding mathematics (Desimone *et al.*, 2005; Hiebert *et al.*, 2005).

For instance, in understanding of area measurement, procedural fluency, and reflections on the accuracy of solutions for measuring areas represents higher-order thinking skills (Lehrer, 2003). While investigating children's conceptions of area measurement and their strategies for solving area measurement problems, Huang and Witz (2012) found that children who had a good understanding of the concept of area and the area formula (by using the property of multiplication) exhibited competency in identifying geometric shapes, using formulas for determining areas, and self-correcting mistakes. Those children who had a good understanding of multiplication underlying the area formula, but misunderstood the concept of area, showed some ability to use area formulas. Conversely, the children who were unable to interpret the property of multiplication underlying the area formula irrespective of their conceptions of area exhibited the common weaknesses in identifying geometric shapes and differentiating between area and perimeter. The general concept of area refers to the amount of a 2-D region within a boundary, while area measurement concerns measuring the quantity of a surface enclosed within a 2-D region (Lehrer, 2003). This incorporates the prior concept of area and measurement skills. The strategic knowledge of area measurement contains a conceptual understanding of basic facts and the knowledge of efficient strategies in solving problems with justified reasoning. Though there are various ways to solve area measurement problems, appropriate use of formulas based on conceptual understanding can be considered an efficient strategy (Lehrer *et al.*, 2003).

Moreover, it is noted by Siegler and Alibali (2005) that when comparing fractions with physical models, students could easily see the largest fraction. When the physical model was not being used, some students still had to draw the model to compare the size of fractions. In order for fraction and decimal number sense to be acquired, there are three foundational concepts agreed upon by researchers (Barnett-Clarke *et al.*, 2010). These concepts along with conceptual understanding and procedural knowledge, cognitive theories, and instructional theories will create a suggested series of tasks to develop fraction and decimal number understanding, and gain understanding for long term application (Watanabe, 2006; Walle, 2007).

According to Hasnida and Zakaria (1991) the goal in mathematics teaching has shifted towards an emphasis on both procedural and conceptual understanding. Their research revealed that the students' level of procedural understanding is high whereas the level of conceptual understanding is low and hence they suggested that a reformation in teaching is needed to boost conceptual understanding among students to minimize the use of algorithms and memorization.

Conceptual understanding can be measured in various ways, mainly involving providing definitions, explanations and reasons. Conceptual knowledge in a domain usually requires knowledge of many concepts. Procedural fluency can be measured by checking the accuracy or the procedure of solving problems. When interested in how flexible procedural knowledge is, researchers assess students' knowledge of multiple procedures and their ability to choose among them to solve problems efficiently (Star and Rittle-Johnson, 2008; Verschaffel *et al.*, 2009). This flexibility of procedural knowledge will be a result of conceptual knowledge. The positive correlations between the two types of knowledge have been found in almost all domains. For example, in Number and Operations (Canobi and Bethune, 2008; Jordan *et al.*, 2009) fractions and decimals (Reimer and Moyer, 2005; Hallett *et al.*, 2010) estimation (Dowker, 1998; Star and Rittle-Johnson, 2009) and equation solving (Durkin *et al.*, 2011).

2.2. Problem-Solving

Problem Solving is one of the major processes defined in the National Council of Teachers of Mathematics (NCTM) Standards for School Mathematics (NCTM, 2000). Problem solving involves students in applying four other processes: reasoning, communication, connection and representation. Jonassen (2003) defines problem solving as an individual thought process because the previously learned law can be applied in solving problems in other situations. It is also deemed to be a new type of learning and is the result of the application of knowledge and procedures of the problems (McGregor, 2007).

Problem solving can also provide opportunities for students to apply content knowledge in the areas of number and operations, algebra, geometry, measurement, and data analysis and probability. Problem solving provides a window into children's mathematical thinking and thus is a major technique of assessment.

According to Jawhara (1995) problem solving activities can open opportunities for students to learn freely. In their own ways, students will be encouraged to investigate, seek for the truth, develop ideas, and explore the problem. Kilpatrick *et al.* (2001); Pugalee (2004;2005); Suh and Moyer-Packenham (2007) concluded that increase in the levels of cognitive demand, mathematical intricacy and abstraction balances students' procedural fluency in problem solving, and is based on their ability to use intellectual knowledge and skills in interpreting the problem.

Many mathematics skills are involved in problem-solving. However, large numbers of students have not acquired the basic skills they need in mathematics. As a result, many students were reported to face difficulties in mathematics particularly in mathematics problem solving (Tay, 2005). The ability to use cognitive abilities in learning is crucial for a meaningful learning to take place (Stendall, 2009). There are two major steps in problem-solving: transforming the problem into mathematical statements or equations, and calculation of the required statements. Difficulties faced among students were more noticeable during the first procedural step in problem-solving compared to the other. Polya (1981) stated that problem-solving is a process starting from the instant a student is faced with the problem until the end of the process when the problem is solved. Garderen (2006) stated deficiency in visual-spatial skills might cause difficulty in differentiating, relating and organizing information meaningfully.

3. RESEARCH AIMS

The research aims to:

- Analyze students' work according to five mathematical domains and three types of mathematical abilities;
- Find out the root cause of students' misconceptions and errors.
- Suggest ways to improve the conceptual understanding and problem-solving skills to reach to mathematical proficiency.

4. RESEARCH METHOD

350 students participated in the study; 50 males and 300 females.

- The quantitative approach uses a test comprised of questions from five domain in mathematics (numbers and operations; algebra; geometry; and measurements; data analysis and probability). The test comprised 60 questions; 20 to test students' conceptual understanding, 20 to test procedure knowledge and 20 for problem-solving skills.
- In the qualitative approach interviews were conducted to collect data about students' understanding. Interviews were semi-structured. The authors interviewed participants who were lacking in any one of the mathematical abilities and most recurring misconceptions.

After students took the test, it was corrected, and the results were recorded and analyzed to find out the different type of abilities they possessed. Interviews were conducted using a sample of the student group. Depending on the students' response and type of errors committed in the test items, they were interviewed to identify the root of the misconception. One or two ways of teaching those concepts were then suggested to avoid misconceptions in future.

Test results were analyzed using different SPSS tests as the following:

- Classified students' test score based on mathematical ability level.
- Classified students' test score based on accuracy level (getting 0-3).
- Classified students' test score based on the mathematical domain.
- Classified students' high school GPA (Below average, Average, & Above average).
- High school GPA – 3 mathematical proficiency level scores – correlations, Pearson, Spearman, or Scheffe's test.
- High school GPA – high school specialization (science, commercial, and others) – Analysis of Co-Variance (ANCOVA).

5. DATA ANALYSIS

In this paper, we examined the three mathematical abilities (procedural knowledge, conceptual understanding, and problem-solving skills) in 350 recent high school graduates that were enrolled in the foundation pre-college year at the Teachers College within the University of Bahrain. Students were given five separate tests in each domain of mathematics (number and operations, algebra, geometry, measurement, data analysis and probability) and each test covered the three mathematical abilities.

It was found that in general, students performed well in procedural knowledge and poor in conceptual understanding. The main reasons behind this are the fact that the procedural knowledge problems are those type of problems that are familiar to the students from their K-12 education in schools in Bahrain. The teaching approach and the exams in the school system in Bahrain emphasizes mainly how and what to do instead of why and when to do. This is confirmed by Hasnida and Zakaria (1991) where the authors conducted a survey study on the results of high school graduates. They found that the students' level of procedural understanding was high whereas the level of conceptual understanding was low.

The low level of conceptual understanding of the high school graduates leads them to be mathematically incompetent (NCTM, 2000). This eventually leads to many misconceptions and errors in problem-solving. This is also aligned with the results found in Jazuli *et al.* (2017) where the authors concluded that most students find it difficult to understand and to apply the concepts of mathematics to real life problems. The authors refer the reasons to the same findings by this study which is mainly on the conventional teaching strategy that is used, and argued that changing that strategy to one of contextual learning would improve both mathematical conceptual understanding and problem-solving.

While Rittle-Johnson and Alibali (1999) suggested that procedural knowledge may improve the conceptual understanding by helping the students to identify and eliminate then, we have found several cases where students could efficiently use their procedural knowledge and reason to eliminate misconception, but failed to do so. One such example is a question asking why 1/6 is less than 1/3, where students could easily compare the two fractions to reason why 1/6 is less than 1/3. It has been found that many of the participants did not use the proper logic to state a valid reason.

Based on the results we also found that, in the five domains of mathematics, among the 350 students the conceptual understanding and the problem-solving skills are positively and significantly correlated with each other according to the Pearson correlation. This confirms the urgent need to focus on this issue and to find a solution to improve the conceptual understanding of students as such will ultimately improve their problem-solving skills.

We found also that the mathematical background of students plays an important role in their performance in the three levels of mathematical ability. ANOVA and Scheffe's tests were applied to the students' high school specializations and their performance in the five content areas and it was found that students from scientific and commercial tracks who usually have taken up to elementary level of calculus in high school did much better when compared with students from a literature background who usually have taken mathematics at an elementary level of algebra. We noted that the difference is not that significantly high in the procedural knowledge, but it is much more recognizable in conceptual understanding and problem-solving skills.

The same test results were compared with students' high school GPA and their performance in the five tests and it was found that students whose GPA was above 95 percent did better in the three areas compared to other students. Again, it was noted that difference is not recognizable in procedural knowledge but was clear in conceptual understanding and problem-solving. This confirms this study's main contention that knowledge understanding is the norm for the high school educational system, while the conceptual understanding level is high mostly for students with a solid background in mathematics.

Referring to the four research questions on page four, the following table represents the data obtained and analysis thereof.

5.1. What Percentage of Mathematical Abilities' does the Recent High School Graduates Show in Different Mathematics Domains?

Table 1 shows the ability to understand concepts in the three domains of numbers, algebra and geometry are very low compared to other domains and abilities. Overall, the table below shows the descriptive statistics of mathematical abilities' that recent high school graduates achieved in different mathematical domains.

Table-1. Students' mean and standard deviation in the five domains and the three mathematical abilities.

Domain	Knowledge		Concept		Problem solving	
	Mean	Std. D.	Mean	Std. D.	Mean	Std. D.
Number & Operation	2.13	0.91	0.63	0.65	1.82	1.02
Algebra	1.95	0.89	0.74	1.15	1.24	1.24
Geometry	2.04	0.74	0.61	1.03	1.55	1.31
Measurement	2.39	0.75	1.74	1.38	2.07	1.24
Data analysis & Probability	1.78	0.74	1.34	0.97	1.33	1.19

Source: Test scores.

Table-2. Students' mean in the three mathematical abilities.

N.	Domains	Mean (out of 3)
1	Procedural knowledge	2.06
2	Conceptual understanding	1.01
3	Problem-solving skills	1.60
Total		1.60

Source: Test scores.

The Table 2 shows the descriptive statistics of mathematical abilities that recent high school graduates achieved in different mathematical domains. The arithmetical mean of the responses to the instrument was 2.06 for procedural knowledge questions, 1.01 for conceptual understanding questions, 1.60 for problem-solving questions, and 1.60 overall. As expected, the procedural knowledge had the highest rank, while the conceptual understanding had the lowest.

5.2. What are Students' Misconceptions while Solving Conceptually-Orientated Tasks Involving Different Mathematical Domains?

As per Table 1, averages in conceptual understanding in the measurement and data analysis domains are in the normal range of 50 percent, and at the same time much higher than those in the other three domains. This is mainly because in these two domains the visualization and the sound of the problem make it easier to answer the question. On the other hand, the numbers, algebra, and geometry are more abstract concepts. Three main misconceptions were found:

1. In multiplying two decimal numbers, students could not reason that by comparing the integral part they can estimate the product.
2. In comparing 1/3 and 1/6, many students just compared the denominator and reasoned that 1/6 is larger.
3. Estimation of the height of a building, where the students do not have an awareness of the height or could make a reasonable estimate.

5.3. Is there a Correlation between the Conceptual Understanding and Problem Solving in Mathematics?

Table-3. Pearson correlation value between the conceptual understanding and problem-solving skills.

	Pearson correlation value	Sig. (2-tailed)
Correlation between the conceptual understanding and problem-solving skills	0.283**	0.000

**. Correlation is significant at the 0.01 level (2-tailed).

Table 3 shows that the conceptual understanding is positively and significantly correlated to problem-solving skills. We apply the Pearson's correlation test as it does not take into account the dependent or independent variables, i.e., that conceptual understanding and problem-solving skills are positively correlated to each other, i.e., increase in one yields an increase in the other.

5.4. Are there any Differences between the Mathematical Abilities Test Score and High School Students' Specializations and GPA?

Table-4. ANOVA test between the results in the three abilities and the students' specialization.

ANOVA

		Sum of squares	df	Mean square	F	Sig.
Procedural knowledge	Between groups	1138.303	4	284.576	6.898	.000
	Within groups	13573.482	329	41.257		
	Total	14711.784	333			
Conceptual understanding	Between groups	517.122	4	129.281	2.839	.024
	Within groups	14984.234	329	45.545		
	Total	15501.356	333			
Problem-solving skills	Between groups	276.068	4	69.017	.818	.514
	Within groups	27510.118	326	84.387		
	Total	27786.186	330			
Total	Between groups	4643.239	4	1160.810	4.455	.002
	Within groups	85720.752	329	260.549		
	Total	90363.992	333			

Source: Test scores.

The results in Table 4 shed light on the following:

- Significant differences at statistical level 0.001 in the aspect related to procedural knowledge domain only between the specializations of foundation students at BTC.
- Significant differences at statistical level 0.05 in the aspect related to conceptual understanding domain only between the specializations of foundation students at BTC.
- No significant differences at statistical level 0.05 in the aspect related to problem-solving skills domain only between the specializations of foundation students at BTC.
- Significant differences at statistical level 0.01 in the aspect related to mathematical abilities only between the specializations of foundation students at BTC.

In order to determine which specialization of the study sample (science, arts, commercial, industrial, and vocational) these differences may be attributed to, the researchers applied Scheffe's test on the study sample responses. The results are in Table 5.

Table-5. Results of Scheffe test according to the specialization.

Dependent variable	(I) specialization	(J) specialization	Mean difference (I-J)	Sig.
Procedural knowledge	Arts	Science	-4.65*	.000
		Commercial	-3.80*	.013
Overall	Arts	Science	-9.50*	0.003
		Commercial	-8.62*	0.036

*. The mean difference is significant at the 0.05 level.

From Table 5, the trend of differences according to the aspects in the questionnaire for each domain is as follows:

i. Procedural knowledge, the trends of the differences found were:
- Differences between arts specialization and science and commercial specialization for the benefit of science and commercial specialization.

ii. Conceptual understanding, the trends of the differences found were:
- Differences within specialization groups.

iii. Overall, the trends of the differences found were:
- Differences between arts specialization and science and commercial specialization for the benefit of science and commercial specialization.

Table-6. ANOVA test between the results in the three abilities and the students' specialization in GPA in high school.

ANOVA

		Sum of squares	df	Mean square	F	Sig.
Procedural knowledge	Between groups	1553.981	3	517.994	13.013	.000
	Within groups	13175.673	331	39.806		
	Total	14729.654	334			
Conceptual understanding	Between groups	550.844	3	183.615	4.051	.008
	Within groups	15002.809	331	45.326		
	Total	15553.654	334			
Problem-solving skills	Between groups	313.560	3	104.520	1.237	.296
	Within groups	27704.557	328	84.465		
	Total	28018.117	331			
Total	Between groups	6681.259	3	2227.086	8.807	.000
	Within groups	83699.178	331	252.868		
	Total	90380.437	334			

Source: Test scores.

The results in Table 6 show:

- Significant differences at statistical level 0.001 in the aspect related to procedural knowledge domain only between the GPA of foundation students at BTC.
- Significant differences at statistical level 0.01 in the aspect related to conceptual understanding domain only between the GPA of foundation students at BTC.
- No significant differences at statistical level 0.05 in the aspect related to problem-solving skills domain only between the GPA of foundation students at BTC.
- Significant differences at statistical level 0.001 in the aspect related to mathematical abilities only between the GPA of foundation students at BTC.

In order to determine which GPA range of the study sample (Less than 85, From 85 to 89.99, From 90 to 94.99, 95 and Higher) these differences may be attributed to, the researchers applied the Scheffe test to the study sample responses. The results are in Table 7.

Table-7. Results of Scheffe test according to the GPA.

Dependent variable	(I) specialization	(J) specialization	Mean difference (I-J)	Sig.
Procedural knowledge	95 and higher	Less than 85	8.76*	.000
		From 85 to 89.99	6.89*	.000
		From 90 to 94.99	5.23*	.015
Conceptual understanding	95 and higher	Less than 85	5.34*	.014
Overall	95 and higher	Less than 85	18.88*	.000
		From 85 to 89.99	14.73*	.002
		From 90 to 94.99	12.75*	.021

*. The mean difference is significant at the 0.05 level.

From Table 7, the trend of the differences according to the aspects in the questionnaire for each domain is as follows:

- Differences between a GPA of 95 and higher; of a GPA less than 85; of a GPA from 85 to 89.99; and of a GPA from 90 to 95.

5.5. Students' Interviews

The general observation obtained from grading the tests are:

1. **Procedural Knowledge:** Students scored an average of 66 percent. This is based on the fact that this type of problem is familiar to the students from their K-12 education in schools in Bahrain. These are problems that can be solved easily once the procedure is known.

2. **Conceptual understanding:** In this category, where the students need to understand the concepts that were learnt in school and apply them to different type of problems. Here the students scored average of 33 percent, which is a very low score and almost half that of the procedural knowledge problems. This shows significant issues with the reasoning of the procedures. In addition, the gap between this percentage and the percentage of the procedural knowledge is alarming and shows that students are not competent in mathematical reasoning.
3. **Problem Solving:** These types of problems depend on the first two categories and the average results of the student reflected that fact. In particular, Table 3 shows that problem-solving skills are positively correlated to conceptual understanding. However, it was observed that students mostly struggled in translation of a word problem into a proper mathematical statement. In some cases, once the translation was done, the students used their procedural knowledge to find a proper solution. However, students struggled most in those problems where conceptual understanding is required.

In order to identify the misconception and to rationalize the results, a group interview was conducted for around 100 randomly chosen students with following the results. There was clear support for the above observation as the first problem (D1-c1, D1-p1). In the conceptual understanding part, students were asked to reason why multiplying 8.45 times 5.98 cannot equal 505.31 (this is D1-c1); while in the problems solving skills, students were asked through a word problem to carry the multiplication of 14.5 and 15 (this is D1-p1).

In the first problem (D1-c1) an average of five percent of students gave a valid reason, while in the latter, 86 percent carried the multiplication correctly after translating the word problem into a number multiplication problem.

An interesting case of the clear difference between the two categories was found. '8.45 multiplied by 5.98 is not 505.31' can be reasoned by rounding the number to the nearest whole number and finding the product, or performing a normal multiplication and comparing it to the given. But the students did not think in that way even though it is evident that 75 percent of them could solve a direct multiplication problem as in D1-k1.

During the interview with the students, they were asked the same question. Only a few students thought of carrying the normal multiplication procedure. The other dominant reason was that the integral part of the multiplication would not be equal, and only 29 percent of the students actually gave that reason.

Another interesting case to support this observation was in problem four, comparing fractions from D1: number and operations. In this problem, students were asked to compare two fractions using greater than or less than.

In the procedural knowledge problem (D1-k4) students were asked explicitly to compare two given fractions by either < or >. This is a familiar question and once the students understand how to compare the fraction, it can be easily accomplished. We have found that the average was 58 percent which is the same average for the same problem in the problem-solving category (D1-p4).

On the other hand, in the conceptual understanding problem (D1-c4), a comparison was made between 1/3 and 1/6 and students are asked to explain why 1/3 is greater than 1/6. On average 53 percent give a correct explanation. This percentage is slightly off since these problems can be easily interrupted similar to the procedural knowledge problem, and thus the percentage should be close to the percentage of the procedural knowledge, if not more given the comparison between 1/3 and 1/6 is much easier.

The same question to students was repeated, and it was found that 46 percent again got the wrong answers. A student volunteered to explain their reasoning for the wrong answer on the board, and did so by comparing two diagrams representing the fractions but with different partitions. The same question was then repeated to all students and 56 percent voted for the volunteer's solution to be correct (compared to only 46% at the beginning). Students were asked how many of them knew why the solution by the diagram was wrong. Only 33 percent (almost the same percentage as the conceptual understanding) knew why it was wrong, and that because the region should be divided into the same parts in both diagrams.

What we have found from that particular question is that using the procedural knowledge by direct computing according to a learnt formula is much easier and more comfortable for the students to compare using the idea of conceptual understanding and reasoning.

(Problem solving into equations) Another example to justify our observation is from the domain of algebra (D2) and specifically from the problem solving question (D2-p1) where students were asked to find how many from 120 computers did "Ahmed" sell given that he sold three times as many as "Hassan". In this question, the average was only 44 percent. The main problem was that students knew how to solve linear equations but arriving at the correct linear equation from the word problem was problematic. This observation is confirmed by looking at responses to the same question in the knowledge understanding section (D2-k1). Students were asked to solve a given linear equation. Here the average was high, around 86 percent. The other way to confirm this observation was through interviews with the students, where only 34 percent of them said that they could translate the word problem into the correct equation.

Area and circumference in the geometry domain (D3) produced poor results. Most students did not know the difference between area and circumference. Either it was a direct question of finding the area/circumference/volume of a standard shape, or through a word problem that required more analysis. This was reflected by the overall score. For example, in knowledge understanding, the average was 66 percent, and conceptual understanding 20 percent. In problem solving it was 52 percent. During the interview with the students it was found that 34 percent were able to tell the difference between the area and circumference.

Measurement: A similar situation occurred in the measurement domain, in particular with clock arithmetic (D4-k2,c2,p2). 91 percent of students were able to find the difference between two times (in hours and minutes), while 66 percent could not where the times were given as a real-life problem. During the interview, it was asked how many students thought that adding five hours to 10:30 am made the time 15:50 pm. A very high 45 percent thought so.

Measurement: Another interesting question regarding the measure is D4-c4 where students were asked to give a reasonable estimate for the height of the building. There were a wide variety of conclusions - some apparently random - however 1500m, 3500m, 7500m were the most frequent. By asking individual students who arrived at these answers and why, the surprising response in most instances was that they had answered by means of their imaginations, and were incapable of linking it to a specific height or distance.

5.6. Some Odd Cases (Very Low Scores)

Here the least two questions in terms of the average score are given and analyzed:

For the conceptual procedure question D1-c3 in the numbers and operations domain, students were asked to explain why it is impossible to divide by zero. The average score for this question was the lowest in the study at around one percent. This confirms the result that even for a very well-known fact such as this; students cannot determine the reason behind it, although they can still perform long division accurately (94 percent on average in question D1-k3). Surprisingly, this percentage was the highest average score in the entire study.

For question D5-k4 in the data analysis and probability domain, the lowest average score in the knowledge category and the second lowest score were in the of statistical graphs represented as bar charts. The average score was 2.7 percent. That contributed to the conclusion that students could with high proficiency solve knowledge problems once they had seen the procedure before. In the statistical case, the knowledge part is very close to the procedural part as these two require thinking and overall understanding given every graph is different.

6. SUGGESTIONS AND CONCLUSION

The authors recommend a change to the learning approach for mathematics in Bahrain, where the focus should be shifted from 'how' and 'what' to do into 'why' and 'when'. Educators should focus more on conceptual understanding than emphasizing the mere procedure required to perform a calculation. This can be done through shifting from the traditional learning approach (the role approach) tom a more interactive and student-centered method like contextual learning (Jazuli *et al.*, 2017).

Since Mathematics is valuable in everyday life, it is recommended that mathematics in early primary school be introduced through real life problems where the teachers can explain both the facts and the concepts to the children. This will enhance both abilities simultaneously from an early stage. Presenting mathematical concepts from simple to complex using concrete models in the early stages will help in the abstraction of the concept. Actively engaging students in the learning process will help them in making connections which help to achieve a greater understanding of mathematical concepts.

Introducing concepts by multiple representations that include using manipulatives, showing a picture, drawing out the problem, and offering a symbolic representation to address different learning styles will help students to understand the concepts in a better way. To give students an opportunity to communicate their reasoning, teachers can ask them to explain how the concept works and how to solve problems using that concept to other students. In the study by Cummings (2015) a small group being tutored with the emphasis on understanding the concepts behind of topic improved their overall conceptual understanding level in a post-test compared to a pre-test. It is recommended that at the high school level, small tutorial groups should be implemented to focus on the conceptual. These groups could be led by high achieving students. It is important that the mathematics instructors should focus more on conceptual based classroom environments, but without eliminating procedure-based learning.

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