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PRE-SERVICE ELEMENTARY TEACHERS' DIFFICULTIES IN SOLVING REALISTIC DIVISION PROBLEMS

Sumeyra DOGAN COSKUN, Emre EV CIMEN

Abstract: The purpose of this study is to investigate pre-service elementary teachers' realistic considerations and difficulties in solving "division with remainder" type of problems which focus on the measurement meaning of division operation. The qualitative research method of case study design was used for the study. The purposive sampling method was used to determine the participants. A total of 60 senior pre-service teachers who had already taken mathematics teaching courses participated in the study. They were enrolled in the Elementary Teacher Education Program of the Faculty of Education in Turkey in 2018-2019 academic year. The data collection tool included a problem based questionnaire and semi-structured interviews to obtain a detailed description of participants' difficulties related to division with remainder problems. The questionnaire included four groups of problems according to the types of the remainder: Remainder Divisible, Remainder Not Divisible, Remainder as the Result, and Readjusted Quotient by Partial Increments. Based on Verschaffel, De Corte and Lasure's (1994) framework, participants' answers to the problems were analyzed and categorized into four groups as "realistic answer, expected answer, arithmetic error, and other error". The findings showed that the pre-service teachers' answers were mostly coded under the Realistic Answer category compared to the other categories.

Key words: division with remainder; realistic problems, measurement

1. Introduction

One of the goals of mathematics education is to help students understand the connection between mathematics and the real world (Niss, Blum, & Galbraith, 2007; Verschaffel & De Corte, 1997). Mathematical problems can be used to show students how mathematics is related to the real world (Verschaffel, Greer, & De Corte, 2000). However, if a problem requires students to apply their previously learned procedures rather than to analyze context given in the problem and interpret what it means in the real world, then students would see mathematics as "cold, detached, (and) remote body of knowledge" (Boaler, 1994, p. 552). As a result, students would ask themselves and their teachers such questions as "Why do I have to learn this topic?" or "Where will I use this knowledge?".

There are different kinds of definitions or explanations made for the term "problem". In this study, a problem refers to a situation presented in realistic contexts that require students to consider the real world rather than a typical textbook problem (Gerofsky, 2004). That is, situations given in problems can be experienced by students in their daily life and hence, these problems would be meaningful enough for the students (van den Heuvel-Panhuizen, 2005). In the literature, it is criticized that textbook problems cause two issues for students: They prevent students to develop their own solutions and to solve problems outside of school (Lesh & Caylor, 2007). Specifically, Bottge and Hasselbring (1993) explain that since textbook problems are different from daily life situations, students do not see them relevant to their own lives. The most important response expected from students is to find a numerical answer to textbook problems. Furthermore, presenting textbook problems as a separate part at the end of the unit or topic expands the gap between problem solving and mathematics (Verschaffel et al., 2000). On the other hand, providing students with meaningful and engaging problems and encouraging them to discuss their thinking and solutions would foster them to use their real-world knowledge and help them "cope with natural situations they will have to face out of school" (Bonotto, 2013, p. 53).

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One of the topics that offers opportunities for students to make connections between mathematics and the real world is division operation. For example, a problem related to the topic of division was as follows: “If there are 14 balloons for 4 children at a party, how should they be shared out?” (Verschaffel, De Corte, & Lasure, 1994, p. 17). Students would possibly try to solve the problem using the equation of 14:4=4.5. However, if they delve into the problem by questioning what they are trying to find, they can discover that their answers are not reasonable in terms of real life. If students are given similar opportunities to help them understand that problem solving is more than applying the necessary algorithms, they would start to question their answers considering the real world (Sarrazy & Novotná, 2013). To provide students with these opportunities, first of all, teachers need to know that solving problem goes beyond numerical answers. In this study, we focus on division problems in which the remainder is not zero. While there are two different meanings, measurement and partition focused on in division problems, we specifically focus on the measurement meaning of division problems. These problems require teachers to not only find numerical answers but also consider whether or not these answers are meaningful for the real world. In this context, the purpose of this study is to examine whether or not pre-service elementary teachers fail to notice (interpret) the real-world context of division with remainder problems.

2. Theoretical Framework

2. 1. Division

Division, one of the four operations introduced in the elementary school level, is the most difficult one for students compared to the other operations. The reason for this difficulty may be the presenting of division operation as the last of the four operations and teaching division operation as the inverse of the multiplication operation (Beckmann, 2008). Also, Squire and Bryant (2002) assert that since students are not able to distinguish and know the role of division terms which are dividend, divisor, and quotient, they may not be able to write the number sentences correctly. Furthermore, although they are able to distinguish these terms, they may not know the connections among them (Correa, Nunes, & Bryant, 1998). In order to overcome this difficulty, Squire and Bryant (2002) state that students need to “…be exposed to different problem representations and problem contexts” (p. 464). However, another difficulty may result from the contexts of division problems, as division operation has two different meanings: partition and measurement (Haylock & Cockburn, 1997).

In the problems focusing on the partition meaning of division operation, the number of subsets is provided and the size of these subsets is asked. On the other hand, the size of the subsets is provided and the number of these subsets is asked in the problems emphasizing the measurement meaning of division operation. For example, while a problem given as “8 apples will be separated into 4 plates with the same number of apples in each plate. How many apples will there be in each plate?” is a partition division problem, a problem presented as “8 apples will be separated into plates with 2 apples in each plate, how many plates will be used?” is a measurement division problem. Since the number of objects (dividend) is equally shared among a certain number of recipients (divisor) in the partition problems, these problems are also known as sharing problems. Although the difference between these two meanings is not usually explained or taught in schools, teachers prefer to start to teach division operation by means of the partition problems as students experience sharing activities throughout their games saying like “one for me and one for you” (Squire & Bryant, 2002). That is, before learning division operation in formal classrooms, students perform division operation informally, but consciously. Therefore, the partition problems are more common compared to the measurement problems (Rodríguez, Lago, Hernández, Jıménez, Guerrero, & Caballero, 2009). Whether or not it results from the above-mentioned facts, students are more successful in division problems focusing on the partition meaning. However, presenting only partition problems would not be enough to improve, in fact, it may even inhibit students’ understanding of division operation (Correa et al., 1998). Actually, as mentioned above, since early grade students solve the partition problems using one to one correspondence, they may not notice the relationship between the divisor and quotient (Bryant, 1997). For example, they may focus on sharing 8 apples equally in 4 plates; however, they may not notice how the size of the quotient is related to the number of the subsets. That is, when they are asked how many apples would have been in each plate, if there were 2 plates, they may not see the inverse relationship between the number of plates and the number of apples in each plate. At this point, to help students discover how the divisor and quotient are related to each other, measurement problems are important (Bryant, 1997).

In spite of its importance, Simon (1993) emphasizes that pre-service elementary teachers' knowledge regarding division operation is weak as they do not consider "the relationship between symbolic division and real-world problems" (p. 233). Division with remainder problem is one of the opportunities that helps teachers to make connections between mathematics and the real-world. Division with remainder problem is a problem in which a division operation does not produce a whole number and there is a number left over. To interpret the number, teachers need to make sense of the problem. Otherwise, although they are able to write the correct number sentence and to compute this sentence correctly, their results may not be true as an answer to the problem (Cai & Silver, 1995). Division with remainder problems are also categorized into four different groups according to the types of the remainder: Remainder Divisible, Remainder Not Divisible, Remainder as the Result, and Readjusted Quotient by Partial Increments (Silver, Shapiro, & Deutsch, 1993; Rodríguez et al., 2009). While a Remainder Divisible problem (RD) refers to a problem in which a number left over is divisible, a Remainder Not Divisible problem (RND) refers to a problem in which a number left over is not divisible considering the real-world. If a number left over is an answer to a problem, then the problem is called as Remainder as the Result Problem. Finally, in a Readjusted Quotient by Partial Increments problem, the number left over is to make an adjustment on the quotient to arrive at the solution, the remainder objects is partially distributed over the quotient. In this study, we use division with remainder problems focusing on the measurement problems to examine whether or not pre-service elementary teachers make realistic considerations when solving these problems.

3. Metodology

3. 1. Participants

Since the purpose is to examine pre-service elementary teachers' difficulties for division with remainder problems focusing on the measurement meaning of division operation, purposive sampling method, one of the non-probabilistic sampling methods, was used (Merriam, 1998). By means of the purposive sampling method, researchers can select the participants who can provide the data needed (Fraenkel & Wallen, 2006). In order to get detailed data about pre-service elementary teachers' difficulties, the senior pre-service teachers who had already taken both mathematics teaching courses were decided to be the participants of this study. A total of 60 pre-service elementary teachers enrolled in an Elementary Teacher Education Program during the fall semester of 2018-2019 academic year volunteered to participate in this study.

3. 2. Data Collection and Analysis

The data collection tool included a problem based questionnaire and semi-structured interviews to reach a detailed description of pre-service elementary teachers' difficulties related to division with remainder problems. The problem based questionnaire was developed by the authors of this study considering the objectives of division operation of the elementary mathematics curriculum of Turkey. The questionnaire included four problems focusing on the measurement meaning of division operation. Furthermore, these problems differed in the types of remainder: Remainder Divisible, Remainder Not Divisible, Remainder as the Result, and Readjusted Quotient by Partial Increments (Silver et al., 1993; Rodríguez et al., 2009). Specifically, one problem was administered to the pre-service elementary teachers for each of the above-mentioned types. To determine the content validity of the questionnaire, the prepared problems were examined by mathematics educators and the necessary modifications were completed considering their suggestions. The problems and the types of the remainders in these problems are given in Table 1.

Table 1. *The problems and types of these problems considering the remainder*

Type of remainder	Problem
Remainder Divisible	An athlete runs an average of 2 km per hour. How many hours does this athlete run in 15 km?
Remainder Not Divisible	A teacher wants to give three cookies to each of her students. If the teacher prepares a total of 32 cookies, what is the maximum number of students that would be given cookies?
Remainder as the Result	110 apples will be placed in the boxes equally provided that placing 20 apples in each box. So, how many apples will there be left not placed in the boxes?
Readjusted Quotient by Partial Increments	A group of 31 friends have booked a restaurant for dinner. Since each table has a minimum of 4 and a maximum of 5 persons, how many tables are needed to be reserved for this group of friends?

To get a complete picture of pre-service elementary teachers' difficulties for division with remainder problems, semi-structured interviews were also conducted after administering the problem based questionnaire. The interview questions were prepared according to the pre-service elementary teachers' responses given to the questionnaire. That is, these questions may be different for each of the pre-service elementary teachers. During the interviews, the pre-service elementary teachers were given their papers to help them remember what they did in the questionnaire. Then, the specific questions were asked to understand their interpretations of the problems. Throughout the interviews, pre-service elementary teachers were not given any feedback or explanations about whether their answers were right or not. These interviews were audio-recorded and transcribed.

Based on Verschaffel and his colleagues' (1994) framework, pre-service elementary teachers' answers to the problems in the questionnaire were analyzed and categorized into four groups as realistic answer, expected answer, arithmetic error, and other error. If the pre-service elementary teachers considered their answers' reality after they applied the necessary algorithms, then their answers were coded as a realistic answer. That is, the pre-service elementary teachers whose answers were coded as a realistic answer, checked their answers were realistic or not. On the other hand, the answers given without considering real-life refer to an expected answer. The third one, arithmetic error, refers to the answers in which the pre-service elementary teachers wrote a correct number sentence; however, they made calculation errors while solving the problem. Finally, if the pre-service elementary teachers could not even write the number sentence correctly, then, their answers were coded as other error. The frequencies of these categories were gathered for each task. To ensure the reliability of the study, both authors of this study coded the pre-service elementary teachers' answers throughout the analysis process. The transcriptions were also shared with the pre-service elementary teachers to comment on whether or not they agreed what was written. For the purpose of the study, these transcriptions were translated into English by the authors of this study and were reviewed by an expert. Throughout the findings part of the study, direct quotations for both correct and incorrect answers were used to eliminate the possible inferences that we might have made.

4. Findings

The purpose of this study is to examine pre-service elementary teachers' realistic considerations when solving division with remainder problems focusing on the measurement meaning of division operation. When the pre-service elementary teachers' answers to these problems were examined, it was seen that these answers could be coded under four categories as realistic answer, expected answer, arithmetic error, and other error. In some of the answers that were coded as a realistic answer or an expected answer, the pre-service elementary teachers interpreted the remainder correctly considering the reality of the problem statement. The distribution of the answers to the problems was given in Table 2.

Table 2. *Pre-service Elementary Teachers' Answers Given to the Problems Focusing on the Measurement Meaning of Division*

Answers	Measurement Problems			
	Remainder Divisible	Remainder Not Divisible	Remainder as the Result	Readjusted Quotient by Partial Increments
Realistic Answer (+)	0	0	0	4
Realistic Answer (-)	51	51	54	38
Expected Answer (+)	2	8	4	5
Expected Answer (-)	7	1	0	5
Arithmetic Error	0	0	2	7
Other Error	0	0	0	1

In this study, "An athlete runs an average of 2 km per hour. How many hours does this athlete run in 15 km?" was given for the problem whose remainder was divisible. When the answers for the problem were examined, it was seen that the majority of them (51/60) did not provide any explanations indicating that they noticed the word of "hour" given in the problem statement was a divisible situation in the real-world. Specifically, they solved the problem using division operation or counting forward by twos and an example for each of these solutions was given in Figure 1 and Figure 2, respectively.

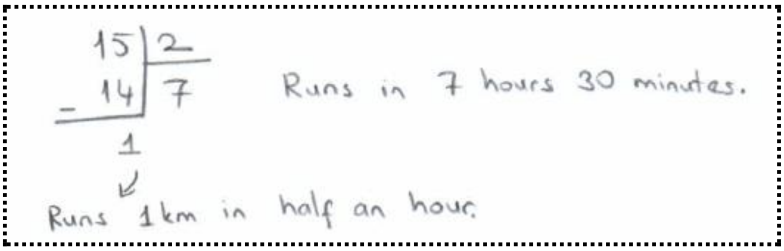

Figure 1. *PT_5's answer to the problem*

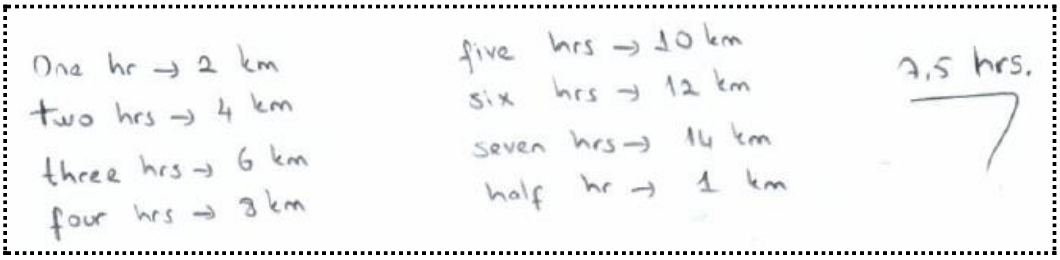

Figure 2. *PT_7's answer to the problem*

When two of the pre-service elementary teachers whose answers were coded as Expected Answer (+) were examined, one of these teachers (PT_1) used division operation to solve the problem; however, she did not continue her operation and responded to the problem as shown in Figure 3.

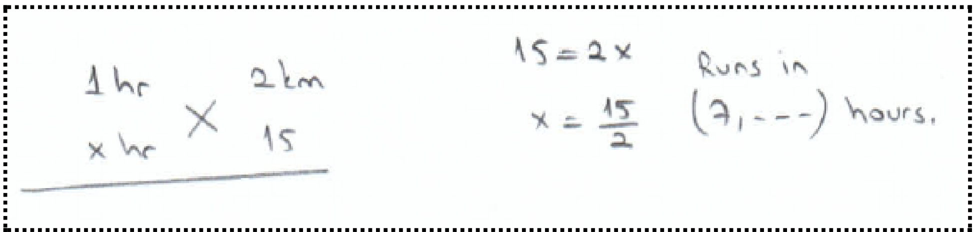

Figure 3. *PT_{10}'s answer to the problem*

Although the other teacher (PT_{48}) performed division operation similar to the above-mentioned teacher, she stated that the athlete would run 15 km in 7 or 7.5 hours as can be seen in Figure 4.

Figure 4. *PT_{48}'s answer to the problem*

When the pre-service elementary teachers' answers coded as Expected Answer (-) were examined, it was seen that these teachers performed division operation correctly; however, they wrote 7 hours as an answer given in Figure 5 or 8 hours as an answer given in Figure 6.

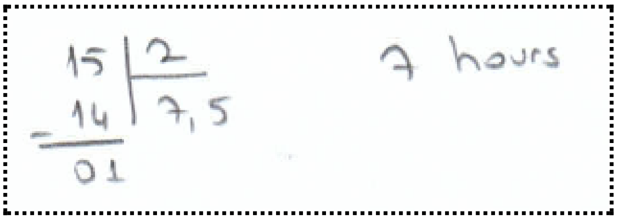

Figure 5. *PT_4's answer to the problem*

Figure 6. *PT_{55}'s answer to the problem*

A problem, "A teacher wants to give three cookies to each of her students. If the teacher prepares a total of 32 cookies, what is the maximum number of students that would be given cookies?", focusing on the measurement meaning of division operation and of which the remainder is not divisible was given in the problem based questionnaire. Similar to the problem in which the remainder is divisible, it was not seen that most of the pre-service elementary teachers (51/60) took the word of "students" in the problem statement into account. To state differently, the pre-service elementary teachers performed the necessary division operation to solve the problem correctly as shown in Figure 7 and stated that the teacher would give three cookies to ten of her students and two cookies would be left to herself.

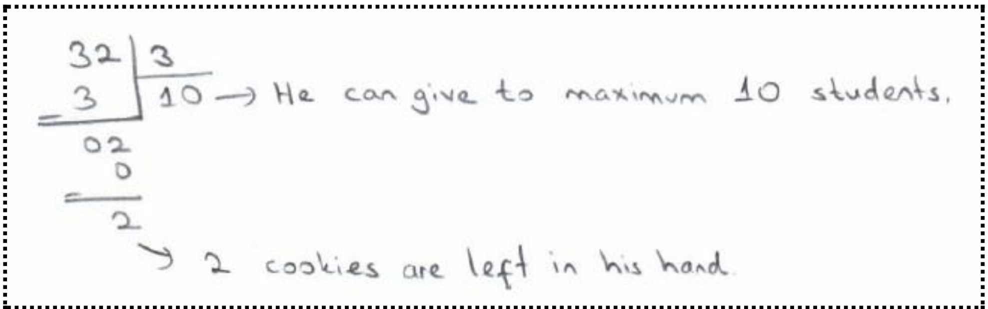

Figure 7. *PT_5's answer to the problem*

The pre-service elementary teacher (PT_{45}) whose answer was coded as Realistic Answer (-), solved the problem in a similar way to her previous solution given in the problem based questionnaire during the interview and stated that ten students would be given three cookies and two would be left. Although PT_{45} explained that "I could have shared the cookies that were left as they were divisible", when she reread the problem statement and reapplied division algorithm, she noticed that the answer, 10, referred to the students. Therefore, she explained that she cannot use division algorithm.

Contrary to the previous problem statement, it was found that there were more answers coded under the Expected Answer (+). When these answers were examined, we found that it was common to perform division algorithm and to find the remainder as 2 cookies, then to give these cookies to another student. Therefore, these pre-service teachers identified the maximum number of students as 11. Similarly, some other pre-service teachers distributed these two cookies one by one and found the number of students as 12. An example for both of these answers was given in Figure 8 and Figure 9, respectively.

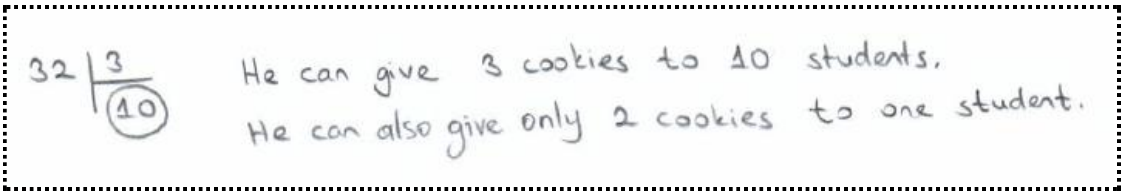

Figure 8. *PT_{21}'s answer to the problem*

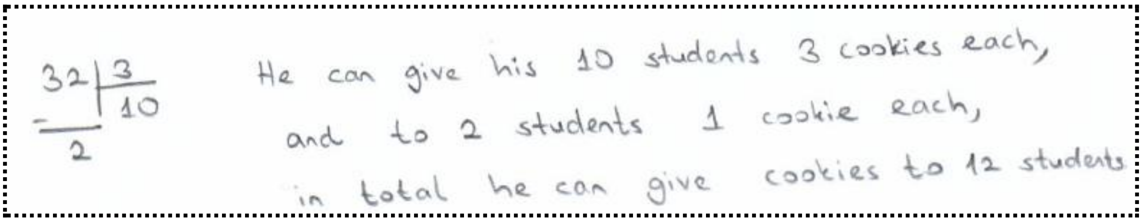

Figure 9. *PT_{25}'s answer to the problem*

The pre-service elementary teacher whose answer was coded under Expected Answer (-) did not take the word "student" written in the problem statement into account and continued to division algorithm and produced the following answer given in Figure 10.

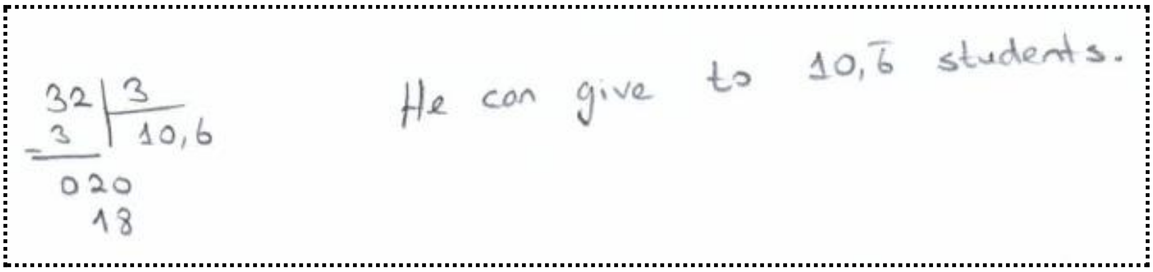

Figure 10. *PT_3's answer to the problem*

For the problem focusing on the measurement meaning and whose remainder is the result of the problem, *"*110 apples will be placed in the boxes equally prvided that placing 20 apples in each box. So, how many apples will there be left not placed in the boxes?*"* was given in the problem based questionnaire. Among the problems focusing on the measurement meaning, highest number of realistic answers were given to this problem. Some of the pre-service elementary teachers solved the problem similar to PT_5 whose solution given in Figure 11 stated that the remaining ten apples cannot be placed in any boxes. Other pre-service elementary teachers used the repeated addition algorithm instead of division algorithm given in Figure 12 and mentioned that the number of the apples which would not be placed in any of the boxes would be 10.

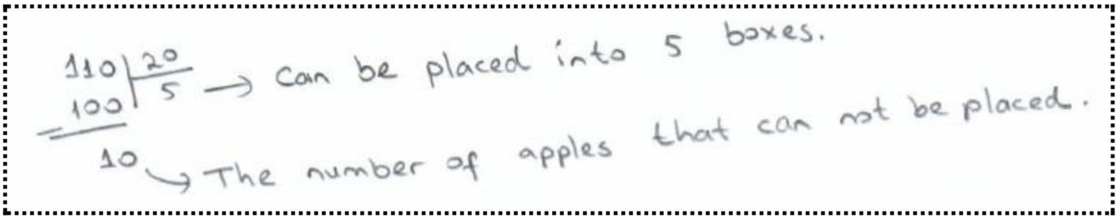

Figure 11. *PT_5's answer to the problem*

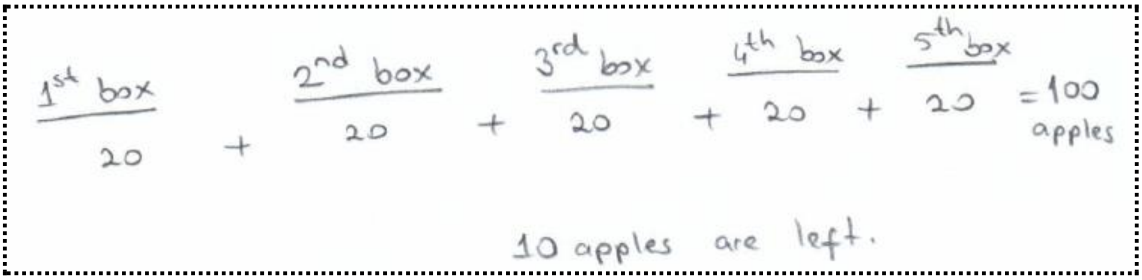

Figure 12. *PT_{11}'s answer to the problem*

When the answers of the four pre-service elementary teachers whose answers were coded under Expected Answer (+) were examined, it was seen that two of these teachers applied division algorithm and found the quotient as 5.5 given in Figure 13. However, in spite of their answers, these teachers stated that the number of apples that cannot be placed would be 10. The other two of the pre-service elementary teachers found 5 as the quotient of division algorithm and 10 as the remainder of the problem. On the other hand, they mentioned that these apples would be placed into the sixth box.

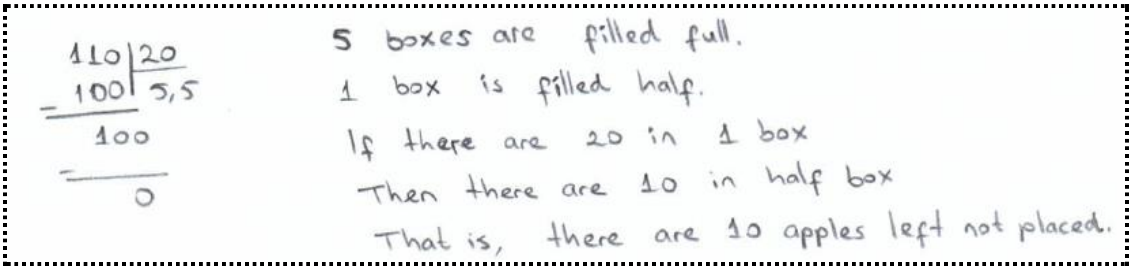

Figure 13. *PT_{20}'s answer to the problem*

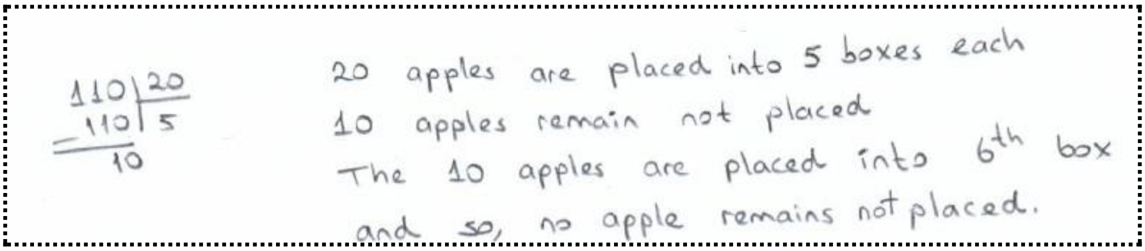

Figure 14. *PT_{49}'s answer to the problem*

Although two pre-service elementary teachers knew that they had to use "110 ÷ 20" to find the answer of the problem, since they deleted the zeros in ones digit place of the dividend and divisor, they found 1 as the remainder of division algorithm shown in Figure 15. Therefore, they stated that the number of apples not placed in any of the boxes is 1.

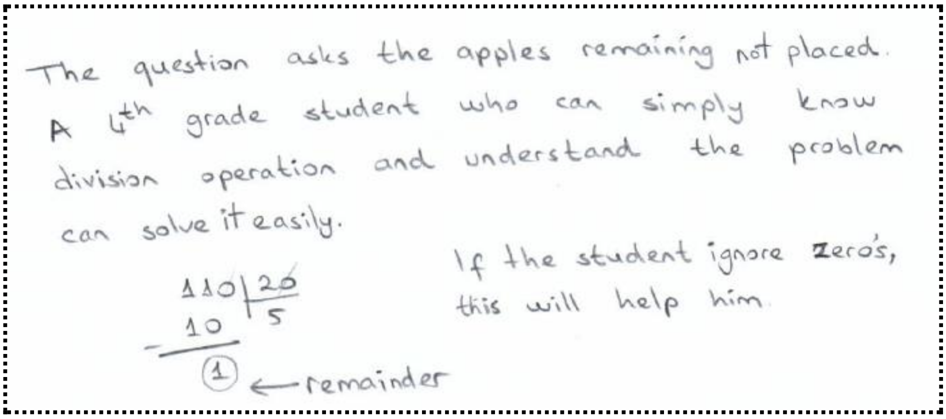

Figure 15. *PT₂₈'s answer to the problem*

During the interview with PT_{28}, one of these two pre-service elementary teachers, explained that "in order to find the number of apples, I have to divide 110 by 20. To make this algorithm easier, I can delete these zeros". As can be seen from her quotation, she solved the problem similar to her previous solution. Then, she explained that multiplying 20 by 5 is equal to 100, therefore, the number of apples that cannot be placed into any of the boxes is equal to 10. When she was asked to check her division algorithm, she stated "the remainder was 1and this was correct; however, it was not equal to the number of apples. Therefore, I cannot use division algorithm for the solution of this problem".

A problem in which the type of remainder was the Readjusted Quotient by Partial Increments in the problem based questionnaire was as follows: "A group of 31 friends have booked a restaurant for dinner. Since each table has a minimum of 4 and a maximum of 5 persons, how many tables are needed to be reserved for this group of friends?" As can be seen in Table 2, the types of pre-service elementary teachers' answers varied compared to the other three problems. Similar to the other types of remainders, it was determined that the majority of the pre-service elementary teachers (42/60) gave correct answers for this type of remainder. When these answers were examined, since four of the pre-service elementary teachers made explanations considering the real-world, their answers were coded as Realistic Answer (+). On the other hand, 38 of the pre-service elementary teachers' answers were coded under the Realistic Answer (-) as they did not make any explanations. Two examples for both of these answers were given below in Figure 16 and Figure 17, respectively.

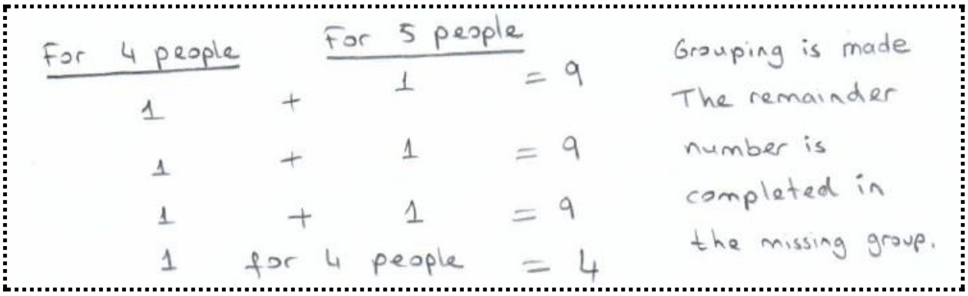

Figure 16. *PT₃₂'s answer to the problem*

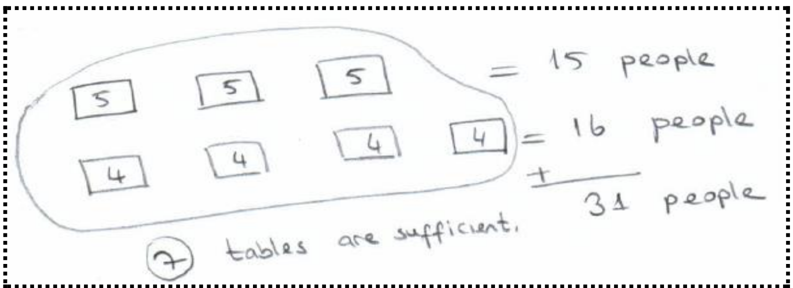

Figure 17. *PT₄₃'s answer to the problem*

When the answers of pre-service elementary teachers which were coded under Expected Answer (+) were examined, it was seen that although these teachers noticed that the tables in the restaurant accommodate 4 or 5 people, they did not notice that these numbers for minimum and maximum. In other words, the teachers who accepted the tables were for 4 people solved the problem using "31 ÷ 4" or for 5 people solved the problem using "31÷5" and found the quotient as 8 tables or 7 tables for the above algorithms, respectively.

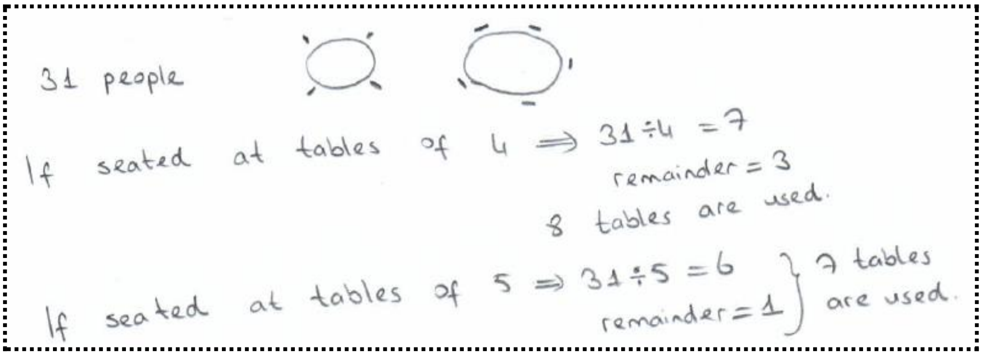

Figure 18. *PT$_{56}$'s answer to the problem*

Pre-service elementary teachers whose answers were coded under Expected Answer (-) category, accepted that all the tables were for 4 people and found that 7 tables were necessary for the reservation. As can be seen in Figure 19, these pre-service elementary teachers only focused on the quotient and ignored the remainder.

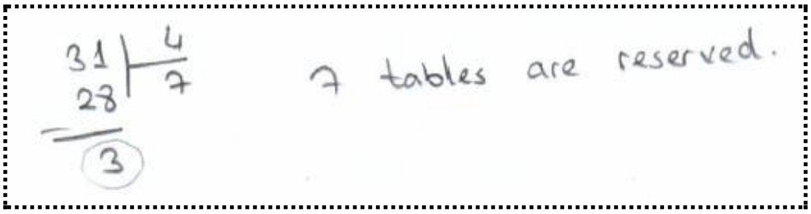

Figure 19. *PT$_{14}$'s answer to the problem*

Seven pre-service elementary teachers whose answers were coded as Arithmetic Error made calculation errors in the multiplication and division algorithms. Two of these answers were given in Figure 20 and Figure 21, respectively.

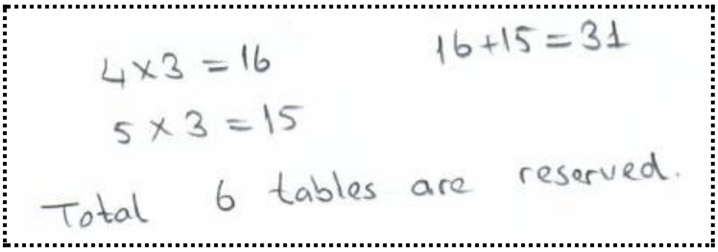

Figure 20. *PT$_{26}$'s answer to the problem*

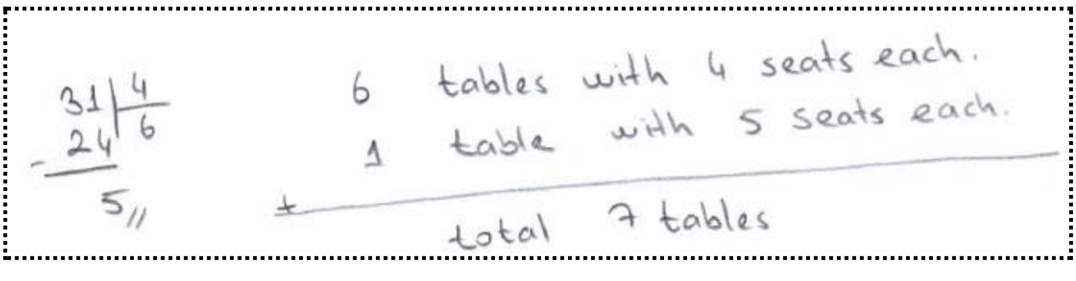

Figure 21. *PT$_{16}$'s answer to the problem*

5. Conclusion, Discussion, and Recommendations

The purpose of this study was to examine pre-service elementary teachers' realistic considerations when solving division with remainder problems. The findings of this study showed that the pre-service elementary teachers' answers were mostly coded under the Realistic Answer category compared to the other categories. When these realistic answers were specifically examined, it was found that most of these answers were given to the problem whose remainder is the result of the problem. On the other hand, it was seen that pre-service elementary teachers could not produce realistic answers to the problem in which the type of remainder was the Readjusted Quotient by Partial Increments. This finding is similar to Rodriquez and his colleagues' study (2009) as they also mentioned that Readjusted Quotient by Partial Increments problems were more difficult than the other three types of problems. Furthermore, this problem was the problem in which the pre-service elementary teachers' answers were coded under different response types. It is also noteworthy that the pre-service elementary teachers did not make realistic considerations throughout the problem solving process similar to the elementary students (Greer, Verschaffel, & De Corte, 2002; Silver et al., 1993). In this context, throughout the mathematics education courses, teacher educators can help pre-service elementary teachers become aware of the steps in the problem-solving process and let them realize that finding a numerical answers is just one of these steps, and the last and the most important step is to check whether or not this numerical answer is meaningful for the real-world. Furthermore, how the type of remainder, which was the subject of this study, would impact the answer to the problem can be shown by means of the problems. A further study can be conducted to examine the reasons for pre-service elementary teachers' difficulties which were determined in this study. A similar study can be conducted with pre-service elementary mathematics teachers to see if these findings are specific to the pre-service elementary teachers or not.

References

Beckmann, S. (2008). *Mathematics for elementary teachers*. Addison-Wesley.

Boaler, J. (1994). When do girls prefer football to fashion? An analysis of female underachievement in relation to "realistic" mathematic contexts. *British Educational Research Journal*, *20*(5), 551-564.

Bonotto, C. (2013). Artifacts as sources for problem-posing activities. *Educational Studies in Mathematics*, *83*(1), 37-55.

Bottge, B. A., & Hasselbring, T. S. (1993). Taking Work Problems off the Page. *Educational Leadership*, *50*(7), 36-38.

Bryant, P. (1997). Mathematical understanding in the nursery school years. In T. Nunes & P. Bryant, (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 53-67). East Sussex, UK: Psychology Press.

Cai, J., & Silver, E. A. (1995). Solution processes and interpretations of solutions in solving a division-with-remainder story problem: Do Chinese and US students have similar difficulties?. *Journal for Research in Mathematics Education*, *26*(5), 491-497.

Correa, J., Nunes, T., & Bryant, P. (1998). Young children's understanding of division: The relationship between division terms in a noncomputational task. *Journal of Educational Psychology*, *90*(2), 321-329.

Fraenkel, J. R., & Wallen, N. E.(2006). *How to design and evaluate research in education*. McGraw-Hill, New York.

Greer, B. Verschaffel, L., & De Corte, E. (2002). The answer is really 4.5: Beliefs about word problems. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education* (pp 271-292). Springer, Dordrecht.

Gerofsky, S. (2004). *A man left Albuquerque heading east: Word problems as genre in mathematics education*. New York: Peter Lang.

Haylock, D., & Cockburn, A. (1997). *Understanding mathematics in the lower primary years*. London: Paul Chapman.

Lesh, R., & Caylor, B. (2007). Introduction to the special issue: Modeling as application versus modeling as a way to create mathematics. *International Journal of computers for mathematical Learning*, *12*(3), 173-194.

Merriam, S.B. (1998). *Qualitative research and case study applications in education*. San Francisco: Jossey-Bass.

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction to modelling and applications in mathematics education. In W. Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education. 14th ICMI Study.* (pp. 3-32). New York: Springer.

Rodríguez, P., Lago, M. O., Hernández, M. L., Jiménez, L., Guerrero, S., & Caballero, S. (2009). How do secondary students approach different types of division with remainder situations? Some evidence from Spain. *European journal of psychology of education, 24*(4), 529-543.

Sarrazy, B., & Novotná, J. (2013). Didactical contract and responsiveness to didactical contract: a theoretical framework for enquiry into students' creativity in mathematics. *ZDM*, *45*(2), 281-293.

Silver, E. A., Shapiro, L. J., & Deutsch, A. (1993). Sense making and the solution of division problems involving remainders: An examination of middle school students' solution processes and their interpretations of solutions. *Journal for Research in Mathematics Education, 24*(2), 117-135.

Simon, M. A. (1993). Prospective elementary teachers' knowledge of division. *Journal for Research in Mathematics Education*, *24*(3), 233-254.

Squire, S., & Bryant, P. (2002). From sharing to dividing: young children's understanding of division. *Developmental Science*, *5*(4), 452-466.

Van Den Heuvel-Panhuizen, M. (2005). The role of contexts in assessment problems in mathematics. *For the learning of mathematics*, *25*(2), 2-23.

Verschaffel, L., & De Corte, E. (1997). Word problems: A vehicle for promoting authentic mathematical understanding and problem solving in the primary school? In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 69-97). Hove, England: Psychology Press/Erlbaum (UK) Taylor & Francis.

Verschaffel, L., De Corte, E., & Lasure, S. (1994). Realistic considerations in mathematical modeling of school arithmetic word problems. *Learning and Instruction, 4*, 273-294.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse: Swets & Zeitlinger.

Authors

Sumeyra DOGAN COSKUN, Eskisehir Osmangazi University, Department of Elementary and Early Childhood Education, Eskisehir, (Turkey). E-mail: s-dogan@ogu.edu.tr

Emre EV CIMEN, Eskisehir Osmangazi University, Department of Mathematics and Science Education, Eskisehir, (Turkey). E-mail: evcimen@ogu.edu.tr

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