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MATHEMATICS TEACHERS' OPINION ABOUT THE HISTORY OF MATHEMATICS

Kani BAŞIBÜYÜK, Ömer ŞAHIN

Abstract: In this study, we used the case study to determine the views of mathematics teachers towards history of mathematics. The participants consisted of 21 mathematics teachers who worked in different provinces in Turkey and were determined by purposeful sampling. A structured interview form was used as a data collection tool. The data obtained from interview were analyzed through the content analysis technique. As a result, the participants usually stated that, considering the history of mathematics, they mostly thought about the famous mathematicians and the emergence of mathematics. It was found that they felt inadequate in utilizing the history of mathematics, and as a result, they did not sufficiently include it in their classes. The participants also stated that there is no sufficient amount of materials that would allow them to utilize the history of mathematics. Moreover, they reported that they used history of mathematics usually in developing the affective characteristics of their students.

Key words: Mathematics teacher, History of mathematics, Mathematics, Opinion

1. Introduction

Pythagoras argued that the most important tool in presenting philosophical thinking is mathematics (Ernest, 1989). There is a need for mathematics to have a profound construct of thinking and be able to understand the universe. Likewise, Galileo stated that the universe is coded in the language of mathematics, and to understand it, there is a need for mathematics (Ernest, 1985). The thought system that pioneered the development of mathematics and its usage in ever part of life is the philosophical thought system in the aesthetical sense (Baki, 2014; Ernest, 2004). It is seen that mathematics, which used to only have abstract and philosophical approaches, is the power that facilitated the science and technology that are used today (Baker, 2017). In this context, there is a need for a mathematical area of interest that may explain the nature of mathematical knowledge and promote thinking in this direction. One of such areas is obviously history of mathematics. Based on this, Lakatos claimed that mathematics, which emerged as an activity of humanity, cannot be considered separately from its history (Davis, Hersh, & Marchisotto, 2011). Topdemir and Unat (2012) claimed that the history of a science is important in terms of being the most fundamental component of intellectual culture and containing the historical journey of all mental activities of humanity. What is the purpose of mathematics? What is the human factor in mathematics? How is mathematical knowledge formed? What kind of a relationship is there between mathematics and human experiences? Why do theories and proofs need to be applied on practical problems so that they could gain strength? History of mathematics is an effective way to look for answers to these questions and understand the inner world of mathematics (Ernest, 2004).

History of mathematics is a scientific field which explains the journey of mathematical knowledge and the people who formed this knowledge throughout civilizations (Baki, 2014). This field is not characterized by only focusing on certain subjects and discussing life stories. History of mathematics also helps formation of a certain philosophy in the mathematical thought system and achievement of a depth in this issue. While history of mathematics tells us the knowledge in the past periods, it is also guiding in how the existing knowledge may be improved (Marshall & Rich, 2000). A. de Morgan said, "*The thought structure of those who dealt with mathematics in the ancient times makes us face our wrongs today, and therefore, the history of mathematics should be focused on carefully*" (Cajori, 1999). It is clear that history of mathematics, which expresses the development of mental processes that are explained in terms of mathematics, will open new horizons for those who learn it.

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Using history of mathematics in teaching mathematics is important in terms of enriching the learning environment. Mathematics educators have recently drawn attention to this topic. The International Congress on Mathematics Education (ICME) that was held in 2000 gathered with the main theme of history of mathematics. Likewise, there are assessments that aim to encourage usage of history of mathematics in the curricula of different countries like Portugal, China, Germany and Poland, as well as the relevant literature (Fauvel & Maanen, 2002). For students to be able to learn mathematics effectively they should enjoy the course and their motivation in the learning environment should be high (Skemp, 1986). For this, students need approaches that reveal their different points of view and different aspects of mathematics and help students look mathematics positively. It is clear that one of such approaches is using history of mathematics while teaching mathematics (Carter, 2006; Fauvel & Maanen, 2002). History of mathematics has significance in terms of revealing different aspects of mathematics, usage of the lives of famous mathematicians as an inspirational element and being able to discover the philosophical side of mathematics (Ponce Campuzano, Matthews, & Adams, 2018; Schubring, 2018). In general, in the light of the relevant literature (Baki, 2014; National Council of Teachers of Mathematics, [NCTM], 2000) the contribution of history of mathematics on teaching mathematics could be modelled as described below.

Figure 1. *Benefits of history of mathematics*

Considering that Pythagoras, who contributed to the historical journey of mathematics, travelled to far away countries to learn mathematics, ancient civilizations built structures that would even make today's technology jealous by using mathematics, al-Khwarizmi presented algebraic systems that reveal very high-level thinking skills and there are mathematical claims about number systems that are surprisingly amazing thinking about that period, this will raise a different and respectable perception of mathematics in students. History of mathematics may be used as an eye-opening assistive element for mathematics teachers, too. It may be used in problem-solving and the introduction parts of topics. Nevertheless, mathematics stays shallow when it is discussed solely based on numerical concepts (Furinghetti, 1997). There should be a cycle in the learning environment that will answer questions of why, how, when, how much and who. Students should be introduced to al-Khwarizmi when they are taught algebra, Pythagoras, Euclid and Thales when they are taught geometry.

So, how is history of mathematics integrated into the teaching environment? Tzanakis et al. (2002) discussed the integration of history of mathematics into the teaching environment under the following titles.

- Historical passages.
- Research projects based on historical sources.
- Primary sources.
- Worksheets.

- History packages.
- Utilizing mistakes, alternative concepts, changing points of view, heuristic arguments, reviewing implicit assumption.
- Historical problems.
- Mechanical materials.
- Experimental mathematics activities.
- Games.
- Films and other visuals.
- Open air experiences.
- The internet.

Historical passages are pictures about the history of mathematics, chapters from old books, biographies of famous mathematicians and social and cultural passages (Ho, 2008). Research projects are studies that aim to reveal the ways of using mathematics in past periods and different points of view and allow students to see different points of view related to mathematics. Primary sources are works that contain historical perspectives. These works are important in terms of observation of expression systems that were used in ancient times and making various comparisons and assessments. Examination of these works by students is important for motivation. Worksheets are used in several steps of teaching mathematics. Worksheets related to history of mathematics are instruction tools that have the quality of making mathematical comprehension easier and revealing historical perspectives. History packages are guiding works that contain activities related to history of mathematics and aim for these activities to be used in the classroom environment (Fernandes & Garnica, 2015). Using historical problems is a method that expresses different ways and historical patterns of solving mathematics problems and additionally teaches students that mathematical problems have existed for centuries. Mechanical materials are mathematics-related historical materials. These materials are important in terms of their revelation of the pioneering nature of mathematics for the development of new technologies that emerged with today's perspective (Ernest, 1998; Maschietto & Trouche, 2010). Empirical mathematics activities are those that help discussion of numbers and methods based on history of mathematics and students' access to a certain depth in this issue. Games include those that are played with contents on the history of mathematics. Films and other visuals include those that have contents related to history, open air experiences refer to examination of the mathematical shapes and symbols found in nature and mathematical studies from the past, and the internet is used for communicating with various people related to a topic and accessing some programs (Tzanakis et al., 2002).

History of mathematics is a significant option for making mathematics a more liked and cherished subject (Gazit, 2013). Integrating situations related to daily life and social life into classroom environments through history of mathematics will have positive reflections in students (Lewis, 2016). Considering the relevant literature about history of mathematics, it is seen that there has recently been an increase in studies on the topic. It is seen that studies (Horton, 2011; Mcbride, 1974) are usually on the effect of history of mathematics on student success and attitudes. However, some other studies (Horton, 2011) focused on different variable such as mathematics image, mathematics perception and scientific development levels. Additionally, it is noteworthy that some studies (Lim & Chapman, 2015; Siu, 2007; Yevdokimov, 2007) worked on large samples and duration of their implementation was long. Some studies (Clark, 2012; Fadlelmula, 2015; Guillemette, 2017) reported that teachers and prospective teachers provide positive views about usage of history of mathematics in teaching mathematics. It was understood in some studies (Clark, 2012; Haile, 2008; Lit, Siu, & Wong, 2001) that people in the sample did not encounter or use activities related to history of mathematics in classes much. History of mathematics cannot find a sufficient place in primary sources and curriculum (Başıbüyük, 2018). In this context, the biggest role in integrating history of mathematics into the teaching environment falls onto teachers. However, it was reported that teachers do not have sufficient knowledge about history of mathematics (Haile, 2008; Horton, 2011). For a more effective mathematics education, it is needed to reveal the factors that lead to this negative situation and conduct an in-depth examination of such factors. Therefore, this study aimed to determine the opinion

of secondary school mathematics teachers towards history of mathematics based on different dimensions of history of mathematics.

2. Method

This study used the qualitative research method of case study with the purpose of determining the opinion of secondary school mathematics teachers towards history of mathematics. In the case study method; an event, situation, relationship or process is subjected to an in-depth analysis with a limited sample (Denscombe, 2010). The cases in a case study may be persons, books, curricula, communities, behaviors or events (Creswell, 2011). The case in this study was the opinion of mathematics teachers towards history of mathematics.

2.1. Participants

The participants consisted of 21 secondary school mathematics teachers who worked in 11 different provinces in Turkey. The participants were selected by using purposeful sampling method which is among non-random sampling methods. The criterion for selecting the participants, based on the objective of the study, was being a mathematics teacher who has the opportunity to utilize history of mathematics in their classes (Patton, 1987). The participants volunteered to participate in the study, and their names were coded as T_1, T_2, T_3 … T_{21} to keep their identities protected. Table 1 shows the demographic characteristics of the participants.

Table 1. *Demographic information of participants*

Category	**Sub-category**	**Participants**
Gender	Male	T_1, T_3, T_4, T_6, T_{10}, T_{11}, T_{13}, T_{14}, T_{15}, T_{17}, T_{19}, T_{21}
	Female	T_2, T_5, T_7, T_8, T_9, T_{12}, T_{16}, T_{18}, T_{20}
Taking Education for The History of Mathematics	To take education	T_1, T_2, T_4, T_7, T_9, T_{14}, T_{18}, T_{19}, T_{20}, T_{21}
	Not taking education	T_3, T_5, T_6, T_8, T_{10}, T_{11}, T_{12}, T_{13}, T_{15}, T_{16}, T_{17},
Professional Experiences	0-5	T_2, T_9, T_{14}, T_{18}, T_{21}
	5-10	T_1, T_4, T_7, T_{12}, T_{20}
	10-15	T_3, T_6, T_{11}, T_{17}
	15-20	T_5, T_8, T_{10}, T_{15}, T_{16}
	20 and over	T_{13}, T_{19}

2.2. Data Collection

The data were collected using the "*History of Mathematics Views Form*" which consisted of seven questions and contained the comments of the participants' on history of mathematics. While developing the form, a question pool consisting of 12 questions was created based on the relevant literature. The interview form consisting of 12 questions was examined by two experts of the field. The experts asked for the questions that did not serve the objective of the study and those with the same meanings to be removed from the form. As a result of the expert opinions, the number of questions in the form was reduced to seven. Before using the interview form for the purpose of collecting data from the participants, a pilot study was conducted with three mathematics teachers who were not included among the participants. Necessary adjustments were made in the examinations that were made after the pilot study, and the form took its final shape. The questions on the form aimed to obtain the views of the participants about issues such as their adequacy in terms of history of

mathematics, their levels of knowledge, whether or not they used history of mathematics in their classes, their purpose and method if they used it, what kind of contributions it made on the class, and the state of curriculum and textbooks regarding history of mathematics.

2.3. Data Analysis

The qualitative data analysis technique of content analysis was used to analyze the data obtained from the responses of the participants to the interview form. Content analysis is a systematic and repeatable method where a text is summarized by codes and categories based on certain rules in order to determine the presence of words or concepts in the text and their relationships (Büyüköztürk, Kılıç - Çakmak, Akgün, Karadeniz, & Demirel, 2011). The main purpose of content analysis is to reach concepts and relationships that may explain the data that are collected (Yıldırım & Şimşek, 2011). In this study, codes and themes were separately created for the responses of the mathematics teachers to each question in the form. For example, the categories and codes for the responses of the participants to the question on the purpose for which they used history of mathematics are summarized in Table 2.

Table 2. *The sample of framework of data analysis*

Categories	Affective Domains	The Nature of Mathematics	Cognitive Domain	No Purpose
Codes	*Drawing attention*	*Transferring mathematics culture*	*Permanent learning*	*I have no purpose*
	Make students curious	*To relate mathematics to daily life*		
	Motivating	*Teaching the emergence of mathematics*		
	Making students like mathematics	*Introducing the famous mathematicians*		

To guarantee reliability, the data were analyzed by two researchers. Coding reliability should be examined in studies where two researchers analyze the data separately. In this case, the researchers code the same data set independently of each other. Then the similarities and differences between the coding processes are numerically compared to determine a percentage of reliability. In this type of studies, the percentage of reliability should at least be 70% [39]. For this purpose, the data obtained in the study were coded by the two researchers, and the percentage of coding reliability was found as 89%. The researchers then gathered and reached an agreement on the differences.

3. Findings

This section of the study presents the responses of the participants to the interview questions in the form of tables. Direct quotes from the statements of the teachers are also included.

3. 1. First Question: What comes to your mind considering history of mathematics? What does history of mathematics mean to you?

Table 3. *Findings of first question*

Category	**Code**	**Participants**	**Statement**
The Historical Process of Mathematics	The emergence of mathematics	T_1, T_7, T_8, T_{13}, T_{16}, T_{20}	**T_8:** *The process of the existence of mathematics comes to my mind.*
	The existence of humanity	T_{15}	**T_{15}:** *I see it to be the same as the beginning of humanity…*

	The development process of mathematics	T_4, T_6, T_7, T_9, T_{18}	**T_6:** *The emergence and the development process of mathematics comes to my mind.*
	Development of science	T_{12}	**T_{12}:** *I recall the history of people's advancement in the field of science.*
	Infinity	T_2	**T_2:** *I consider a world which has no limit. It is like a ocean. A world is constantly evolving due to needs.*
The Nature of Mathematics	Old mathematical methods	T_3 ,T_5	**T_5:** *I think the methods used in the history of mathematics.*
	Mathematics	T_{19}	**T_{19}:** *Everything about mathematics come to my mind while considering history of mathematics.*
	Famous mathematicians	T_7, T_8, T_9, T_{13},T_{16}, T_{18}, T_{20}, T_{21}	**T_7:** *I consider famous mathematicans*
Ancient Civilization	Andalusia	T_{10}	**T_{14}:** *Egypt and Greece are coming to my mind. Because mathematics first appeared in these two societies.*
	Babylonians	T_5	
	Mathematics in European societies	T_5	
	Mathematics in Islamic societies	T_5	
	Indians	T_5	
	Egyptian mathematics	T_{14}	
	Greek mathematics	T_{11}, T_{14},T_{17}	

It is seen in Table 3 that the the first impressions of the participants regarding history of mathematics were observed to be the historical process of mathematics, the nature of mathematics and ancient civilizations. The participants who stated that the historical process of mathematics came to their mind regarding history of mathematics usually emphasized the emergence of mathematics and its development process. The participants who emphasized the nature of mathematics usually stated that famous mathematicians appeared in their minds. The participants who mentioned ancient civilizations usually referred to the mathematics in the Ancient Greek civilization. Some examples of the statements of the participants are given below.

> **T_{13}**: *The process of the first emergence of mathematics, what was done in these processes and who did what come to mind.*
>
> **T_{20}**: *Things like the first emergence of mathematics and who pioneered it come to my mind.*
>
> **T_{18}**: *When someone says history of mathematics, how mathematics emerged and the processes it experienced come to my mind. I recall famous mathematicians who served mathematics.*
>
> **T_{17}**: *The Greeks come to my mind, as we always use their methods.*

T_{13}, T_{18} and T_{20} stated that, considering history of mathematics, they recalled the emergence of mathematics and who contributed to this birth in this process. It is seen that T_{13} with the statement "*who did what*" and T_{20} with the statement "*who pioneered it*" referred to famous mathematicians. T_{17} used the expression "*the Greeks*" as the methods we are using now come from the Ancient Greek civilization. Clearly, the Ancient Greek civilization may have contributions to the methods we are using today, but we cannot argue that all of these methods were created by this civilization.

Some participants stated that, considering history of mathematics, existence of humanity and development of science came to their minds.

> **T_{15}**: *I see it to be the same as the beginning of humanity. Mathematics has a history which goes back to the first days of humanity. People have always needed it.*
>
> **T_{12}**: *I recall the history of people's advancement in the field of science. What was done in the name of science, who did it, such things…*

T_{15} emphasized the existence of humanity to point out how important history of mathematics is. They stated that humanity began with mathematics, because humans have always used mathematics in their lives. T_{12} associated history of mathematics and history of science.

3. 2. Second Question: Who do you know among famous mathematicians? Who are the mathematicians whose work you have a knowledge on?

Table 4. *Findings of second question*

Category	Code	Participants	Statement
Turkish Mathematicians	Ali Kuşçu	T_1, T_2, T_5, T_9, T_{14},T_{19}	**T_9**: *The famous mathematicans I remembered Euclid, Pythagoras, Al-Khwarizmi, Ali Kuşçu ,Kerim Erim.*
	Avicenna	T_7, T_8, T_{10}	
	Cahit Arf	T_1, T_4, T_6, T_{11}, T_{14}, T_{15}	
	Sinan Sertöz	T_1	
	Ali Nesin	T_1	
	Kerim Erim	T_9	
Persian mathematicians	Nasîrüddin Tûsî	T_{12}	
	Al-Farabi	T_{14}, T_{16}	
	Omar Khayyam	T_1, T_2, T_3, T_4,T_7, T_8 ,T_{10} ,T_{12}, T_{15}, T_{16}, T_{18}, T_{19}, T_{20}	
	Al-Khwarizmi	T_2, T_3, T_5, T_7, T_9,T_{10} ,T_{12}, T_{13}, T_{16}, T_{18}, T_{20}, T_{21}	
Arabic Mathematicians	Al-Battani	T_1	
Ancient Greek mathematicians	Euclid	T_1, T_3, T_9, T_{12}, T_{13}, T_{17}, T_{18}, T_{20}	
	Pythagoras	T_1, T_3, T_4, T_5, T_7, T_9 ,T_{11}, T_{12}, T_{13}, T_{17}, T_{18}, T_{19}, T_{20}, T_{21}	
	Thales	T_3, T_4, T_5, T_6, T_7, T_{11}, T_{13}, T_{17} ,T_{18}, T_{20}, T_{21}	
	Archimedes	T_5	
Indian Mathematicians	Brahmagupta	T_2, T_{20}	
	Ramanujan	T_{20}	
European Mathematicians	Carl Friedrich Gauss	T_5, T_{11}, T_{12},	
	Fermat	T_{12}	
	Euler	T_5, T_{12}	
	Riemann	T_5, T_{12}	
	Abel	T_5	
	Leibniz	T_5, T_{12}	
Non -mathematician	Jean Piaget	T_6	
	Aristotle	T_{21}	
	Gregor Mendel	T_{11}	

It is seen in Table 4 that the participants mostly used Persian and Ancient Greek mathematicians as examples. Most of them provided examples of famous mathematicians they know about as the Persian mathematician al-Khwarizmi, who was the founder of algebra and the first discoverer of the number zero, and Omar Khayyam, who is claimed to be the first mathematician who discovered Pascal's triangle. Among the examples of Ancient Greek mathematicians, the participants usually included Euclid, Pythagoras and Thales. As opposed to these, European mathematicians, Indian

mathematicians, Arabic or Turkish mathematicians were not mentioned much by the participants. Additionally, some participants mentioned scientists who were not mathematicians as if they were. For example, T_6 mentioned the famous Swiss psychologist Jean Piaget as a famous mathematician. While Piaget worked on the development of mathematical concepts in small children, he never studied something that produced purely mathematical knowledge. T_{11} stated the famous Austrian biologist Gregor Mendel who is one of the pioneers of the science of genetics as a famous mathematician. They also included the Ancient Greek philosopher Aristoteles as a famous mathematician.

3. 3. Third Question: Do you include history of mathematics in your classes?

Table 5. *Findings of third question*

Category	Code	Participants	Statement
Using History of Mathematics	Interesting	T_5, T_9	**T_5:** *I used it too much. My students are interested in history of mathematics.*
	No explanation	T_1, T_{14}	**T_{14}:** *Yes, I used history of mathematics.*
Sometimes Using History of Mathematics	Not compatible with the curriculum	T_2, T_{11}, T_{17}	**T_2:** *I sometimes use it,not too much. Because there is no information about history of mathematics in the curriculum.*
	As used in the textbook	$T_4, T_{10}, T_{11}, T_{12}, T_{18}$	**T_{11}:** *I sometimes include it, but only because it is in the textbooks, not because I feel such a need. …*
	Not enough source	T_{17}	**T_{17}:**... *there is not enough source we can obtain knowledge about history of math*
	Not enough time	T_{15}, T_{16},	**T_{15}:** *I do not include it much, because there is not enough time and we have to keep up with the curriculum….*
	No explanation	T_7, T_{20}	**T_{20}:** *Yes I sometimes used.*
Not Using History of Mathematics	Not compatible with the curriculum	T_3, T_{13}, T_{21}	**T_{13}:** *No, I do not… I do not have a purpose of using history of mathematics, because there is nothing about the history of mathematics in the curriculum…*
	No information about history of mathematics	T_6, T_8, T_{21}	**T_6:** *I do not include it as I am not very knowledgeable.*
	Not enough time	T_{16}	**T_{16}:** *No, I do not. I cannot include it as the curriculum does not mention this topic and I do not have enough time. I do not use it.*
	No question in the public exam	T_3, T_{13}, T_{19}	**T_{19}:** *No, I do not. I do not have enough time. Also there is no question regarding history of mathematics in the public exam.*

When Table 5 is analyzed, many participants stated that they did not utilize history of mathematics in their classes. Some examples of the statements of the participants are given below.

> **T_3:** *No, I do not. There is no instruction in the curriculum anyway. History of mathematics is not within the exams we prepare and the mathematics content.*
> **T_6:** *I do not include it as I am not very knowledgeable.*
> **T_{13}:** *No, I do not include it much, we discuss topics for exams in the class. I do not have a purpose of using history of mathematics, because there is nothing about the history of mathematics in the curriculum or the exams the students will take.*
> **T_{16}:** *No, I do not. I cannot include it as the curriculum does not mention this topic and I do not have enough time. I do not use it. There is no explanation about history of mathematics in the targeted outcomes. Besides, we are not historians of mathematics, that is, we do not have knowledge.*

As seen in the statements of the participants, they reported that they did not include history of mathematics in their classes because it is not included in the curriculum, they did not have knowledge

on the topic, no questions are asked on the topic in the public exams that their students take, and they thought that history of mathematics would take too much time.

Some teachers who participated in the study stated that they sometimes included history of mathematics. Some examples of the statements of the participants are given below.

> **T_{10}:** *Not much. We only include cross-sections from the lives of famous mathematicians, and that because it is in the textbooks. We do not have such a job. Such a thing is not asked from us.*
> **T_{11}:** *I sometimes include it, but only because it is in the textbooks, not because I feel such a need. … and that is all biographies, nothing else. There is no history of mathematics in the curriculum anyway.*
> **T_{15}:** *I do not include it much, because there is not enough time and we have to keep up with the curriculum. We only talk about the lives of mathematicians sometimes.*

As seen in the statements of the participants who reported that they sometimes included history of mathematics, they said they had to include history of mathematics because it is in the textbooks. They also stated that they could not include it much as they were worried about keeping up with the topics targeted in the curriculum. Additionally, T_{17} stated that there are not many activities related to history of mathematics in the curriculum and the textbooks.

The mathematics teachers who stated that they used history of mathematics in their classes often though that history of mathematics is useful in terms of drawing students' attention to the subject.

3. 4. Fourth Question: Which characteristics of history of mathematics do you utilize in your classes?

Table 6. *Findings of fourth question*

Category	Code	Participants	Statement
History of Mathematics	Old methods	T_1, T_5 T_{12},	**T_1:** *I teach old mathematics methods*
	Historical developments of concepts	T_2, T_5, T_9, T_{20}	**T_9:** *I use the historical development of concepts.*
	Famous mathematicians	T_2, T_4, T_5, T_9, T_{10}, T_{11}, T_{12}, T_{14}, T_{15}, T_{17}, T_{18}, T_{20}	**T_4:** *I'm just telling about the lives of famous mathematicians.*
	Mathematics in ancient civilizations	T_5	**T_5:** *Acording to lesson topic I can tell about mathematics in ancient civilizations…*

The participants stated that, while using history of mathematics in their classes, the utilized old methods, historical developments of concepts, lives of famous mathematicians and how older civilizations did mathematics (Table 6). Additionally, the component that was preferred the most was the stories of famous mathematicians' lives. While there is a lot of information in several sources about history of mathematics on how mathematics developed in old civilizations, only T_5 stated that they provided information about the mathematics that was used in old civilization to their students.

3. 5. Fifth Question: What is your purpose for using history of mathematics in your classes?

Table 7. *Findings of fifth question*

Category	Code	Participants	Statement
Affective Domain	Drawing attention	T_1, T_2, T_5, T_7, T_{14}, T_{17}, T_{18}, T_{20}	**T_5:** *I used history of mathematics in order to draw students' attention to the lesson, to increase their motivation and to make them like mathematics.*

	Make students curious	T_5	
	Motivating	T_5, T_{17}	
	Making students like mathematics	T_5, T_{17}, T_{18}	
Nature of mathematics	Transferring mathematics culture	T_1	**T_1**: *… to ensure the accumulation of mathematics culture in children*
	To relate mathematics to daily life	T_1, T_2	**T_2**: *Firstly, I use it to help them make the connection between mathematics and life…*
	Teaching the emergence of mathematics	T_4, T_9, T_{11}, T_{12}	**T_9**: *I use it to describe how mathematical concepts are formed.*
	Introducing the Famous Mathematicians	T_1, T_2, T_{10}, T_{11}	**T_{10}**: *We give homework to the children about the lives of famous mathematicians for the mathematics newspaper project …*
Cognitive Domain	Permanent learning	T_7	**T_7**: *I use it to improve the permanence of the course.*
No purpose	I have no purpose	T_{15}	**T_{15}**: *I did not have a purpose while including history of mathematics in my classes.*

The statements of the participants about their purposes for using history of mathematics in their classes were categorized under affective, cognitive and nature of mathematics categories (Table 7). Some responses of the participants about supporting the affective characteristics of students are given below.

> **T_{14}**: *I use it to draw attention to the class. Students are interested in biographies.*
>
> **T_{17}**: *It is interesting for students. It gets their attention. I think they are motivated for the class and like mathematics more when they read life stories.*
>
> **T_{18}**: *It will lead students to like mathematics more in the long-run. I include history of mathematics to increase the students' interest in the class.*

As seen in their statements, the participants said they utilized history of mathematics for attracting students' attention to the class, increasing their motivation levels, raising their sense of curiosity and getting them to like mathematics.

Some participants stated that they used history of mathematics to help students understand the nature of mathematics. Some statements were as the following.

> **T_2**: *Firstly, I use it to help them make the connection between mathematics and life. I also use it to raise curiosity and introduce them to famous mathematicians.*
>
> **T_4**: *I want them to know how mathematics first started.*
>
> **T_{10}**: *We give homework to the children about the lives of famous mathematicians for the mathematics newspaper project so that they know about these mathematicians.*

It is seen in their statements that they used history of mathematics with the purposes of introducing famous mathematicians, helping students understand how mathematics emerged, helping hem make the connection between mathematics and daily life and transferring the culture of mathematics. In addition to these, while T_{17} stated that they used history of mathematics to help permanent learning, T_{15} stated that I did not have a purpose while including history of mathematics in my classes.

3. 6. Sixth Question: Do you find yourself adequate in using history of mathematics in mathematics classes?

Table 8. *Findings of sixth question*

Category	Code	Participants	Statement
Sufficient	To take a course in undergraduate education	T_1	**T_1:** *Yes. I took a course during my undergraduate teacher education. History of mathematics does not require highly detailed knowledge in the classes.*
	To make research	T_5	**T_5:** *Yes I do. I'm interested in the history of mathematics. Also I have made research about it.*
	Interested in	T_5	
Partially Sufficient	To take a course in undergraduate education	T_7, T_{20}	**T_7:** *I feel partially sufficient. Altough I took a course, I don't have enough knowledge about how to use it.*
	Insufficient knowledge	T_7, T_{14}	
Insufficient	Not interested in	T_6, T_{15}, T_{16}	**T_{15}:** *No, I do not…I am not interested in history of mathematics.*
	Insufficient sources	T_{10}, T_{15}	
	Not taking education	T_6, T_8, T_{10}, T_{15}	
	Insufficient knowledge	T_2, T_3, T_4, T_8, T_9, T_{10},T_{11}, T_{12},T_{13},T_{17},T_{18}, T_{19}, T_{21}	**T_4:** *I do not have sufficient knowledge….*

The vast majority of the participants stated that they felt inadequate in using history of mathematics in their classes (Table 8). Some statements were as the following.

> **T_6:** *No, I do not. I did not receive education on the subject and I am also not interested.*
>
> **T_{10}:** *No, there is no efficient source. Additionally, I did not receive education on the subject.*
>
> **T_{17}:** *No, I do not find myself adequate. I do not have any qualification for this subject, I need to do some reading.*

The participants explained their inadequacy about history of mathematics by not having an education on history of mathematics, indifference, lack of sources and lack of sufficient knowledge.

Only two participants (T_1, T_5) stated that they felt adequate in using history of mathematics in their classes. These participants explained their high self-efficacy by being interested in the history of mathematics, taking a course in their undergraduate education, doing research and that history of mathematics does not require highly detailed knowledge in the classes.

3. 7. Seventh Question: Could you please assess the curriculum and the textbooks in terms of using history of mathematics effectively?

Table 9. *Findings of seventh question*

Category	Code	Participants	Statement
Insufficient	No sufficient information in the curriculum or the textbooks	T_1, T_4, T_5, T_6, T_7, T_8, T_9, T_{10}, T_{11}, T_{13}, T_{14}, T_{18}, T_{20}, T_{21}	**T_{18}:** *There is no sufficient information in the curriculum or the textbooks, almost nothing.*
	Just biographies of famous mathematicians	T_2, T_8, T_{10}, T_{12}, T_{14}, T_{15}, T_{16}, T_{17}, T_{19}	**T_8:** *….. There are just biographies at some parts in the textbooks.*

	Only used in the introduction sections	T_3	**T_3:** *History of mathematics is only used in the introduction sections…..*

All participants stated that there is no sufficient information in the curriculum or the textbooks on history of mathematics (Table 9). Some statements were as the following.

> **T_3:** *History of mathematics is only used in the introduction sections. There is no sufficient information on ways of using it in the curriculum or the textbooks.*
> **T_4:** *I believe it is inadequate. There is no sufficient content or instruction in the curriculum or the textbooks.*
> **T_8:** *History of mathematics is not included much in the textbooks or the curriculum. There are just biographies at some parts in the textbooks.*
> **T_{12}:** *The curriculum and other sources are inadequate in using history of mathematics. There are just biographies at some parts in the textbooks. It should be used more.*
> **T_{21}:** *There is not much on history of mathematics in the curriculum or the textbooks.*

As seen in their statements, the participants stated that history of mathematics was not included much in the curriculum or the textbooks of mathematics. The participants reported that the textbooks included just some information on the lives of famous mathematicians, such information was usually included in the introduction part of the class, or there was no information.

4. Conclusion

As a result of the study, it was seen that the first images that came to the minds of the mathematics teachers regarding history of mathematics were the historical process of mathematics, the nature of mathematics and ancient civilizations. The participants who stated the historical process of mathematics usually emphasized the emergence of mathematics and the development process of mathematics. Those who stated the nature of mathematics usually emphasized famous mathematicians. Those who mentioned old civilizations usually referred to the mathematics in the Ancient Greek civilization. The results of the study were in agreement with the components of history of mathematics reported in the literature. The curriculum that was published by the Turkish Ministry of National Education (2018) included the historical sub-components of mathematics in different cultures, the historical development of mathematics, the lives of famous mathematicians and mathematical games. Tzanakis et al. (2002) included several components such as the lives of famous mathematicians, primary sources, historical problems, games and historical passages.

The participants mostly used Persian and Ancient Greek mathematicians as examples. Most of them provided examples of famous mathematicians they know about as the Persian mathematician al-Khwarizmi, who was the founder of algebra and the first discoverer of the number zero, and Omar Khayyam, who is claimed to be the first mathematician who discovered Pascal's triangle. Among the examples of Ancient Greek mathematicians, the participants usually included Euclid, Pythagoras and Thales. As opposed to these, European mathematicians, Indian mathematicians, Arabic or Turkish mathematicians were not mentioned much by the participants. Besides, some participants mentioned scientists who were not mathematicians such as Jean Piaget, Gregor Mendel and Aristoteles as famous mathematicians. As a result, it may be argued that the participants were, though partly, successful in exemplifying famous mathematicians.

It was seen that most of the participants did not include history of mathematics in their classes sufficiently. As their reasons for this, they stated the unsuitability of the curriculum, lack of sufficient information in the textbooks, that it is time-consuming, not having knowledge about the history of mathematics and exclusion of the topic of history of mathematics from central examinations. In parallel to the results in this study, Siu (2007) while listing the factors that prevent utilization of history of mathematics included that it is time-consuming, it is not used in exams, teachers' lack of knowledge and material-related problems. Başıbüyük (2012) stated the obstacles against usage of history of mathematics as insufficient time, insufficient resources, and inadequacy of teachers and lack of appreciation of the history of mathematics.

The participants in the study stated that, while they used history of mathematics in their classes, they utilized old methods, historical developments of concepts, the lives of famous mathematicians and how mathematics was done in old civilizations. The most frequently preferred component of history of mathematics was the biographies of famous mathematicians. While there is a lot of information in several sources about history of mathematics on how mathematics developed in old civilizations, only one participant stated that they provided information about the mathematics that was used in old civilization to their students.

It was seen that the participants utilized history of mathematics to contribute to their students' development of affective and cognitive skills and help them understand the nature of mathematics. Those who emphasized affective characteristics stated that they used history of mathematics as it increased the interest of students on the class, attracted their attention, awakened their sense of curiosity and helped them develop positive attitudes towards mathematics. Some stated that they utilized history of mathematics to introduce famous mathematicians, help students understand how mathematics was born, help them establish a relationship between mathematics and daily life and transfer the culture of mathematics. Additionally, one participant stated that utilization of history of mathematics made learning processes more permanent. History of mathematics was utilized similarly as reported in the relevant literature. Yevdokimov (2007) stated that teachers may utilize history of mathematics to draw the attention of students in the learning process. Marshall and Rich (2000) reported that the interest of students in mathematics, their motivations and positive attitudes increased, and their anxiety decreased in mathematics classes where problems on history of mathematics were integrated. In the study by Dündar and Çakıroğlu (2014), prospective teachers stated that history of mathematics may be utilized in mathematics classes for purposes such as increasing the motivations of students, developing positive attitudes towards mathematics, increasing interest in mathematics and making students' knowledge more meaningful. Fried (2007) stated that history of mathematics may be used to reach the essence of knowledge. Awosanya (2001) concluded that usage of history of mathematics in teaching mathematics made it easier to learn mathematical concepts. The relevant literature and the results of this study suggested that history of mathematics would rather contribute to the development of the affective skills of students such as interest, attitudes and motivation. On the other hand, utilizing history of mathematics for the cognitive dimension was not preferred much.

The vast majority of the participants stated that they felt inadequate in terms of utilizing history of mathematics in their classes. The participants explained their inadequacy about history of mathematics by not having an education on history of mathematics, indifference, lack of sources and lack of sufficient knowledge. Only two participants stated that they felt adequate in using history of mathematics in their classes. These participants explained their high self-efficacy by being interested in the history of mathematics, taking a course in their undergraduate studies, doing research and that history of mathematics does not require highly detailed knowledge in the classes

All participants stated that there is no sufficient information in the curriculum or the textbooks on history of mathematics. As seen in their statements, the participants stated that history of mathematics was not included much in the curriculum or the textbooks of mathematics. The participants reported that the textbooks included just some information on the lives of famous mathematicians, such information was usually included in the introduction part of the class, or there was no information. Several studies in the relevant literature reported agreeing results. Şahin, Başıbüyük and Soylu (2016) reported that such activities were included in sixth-grade mathematics textbooks in the introduction part of the class rather for attracting the attention of students to the class and increasing their interest and motivation. However, it was also stated that problem-solving techniques in the field of history of mathematics were not included in the textbooks sufficiently. Erdoğan, Eşmen and Fındık (2015) found that history of mathematics was not sufficiently covered in mathematics textbooks. They also revealed that a large part of the elements of history of mathematics in textbooks were rather related to the affective characteristics of students, but the elements that aim to make sense of and analyze information were more limited. Likewise, Tan-Şişman and Kirez (2018) also reported that history of mathematics was not included in mathematics textbooks and the mathematics curriculum sufficiently. The stated that history of mathematics was usually included in the activities in the introduction part of the course. The results in the relevant literature and those in this study showed that history of

mathematics is not paid enough importance in the mathematics curricula and textbooks in Turkey. It is also seen that history of mathematics is considered only to be a tool for increasing the interests and motivations of students towards the subject. Thus, the cognitive aspect of history of mathematics and its dimension about helping the comprehension of the nature of mathematics are neglected.

5. Recommendations

In the light of the results of this study, the following recommendations may be made:

- While including courses on the history of mathematics in teacher training programs, these courses should be organized in a way that will provide them with skills related to how they could implement history of mathematics in their classes.
- In-service training programs or workshops related to history of mathematics may be organized for teachers.
- More space should be allocated to the history of mathematics in textbooks and curricula.
- Activities that will make it possible to use the cognitive aspect of the history of mathematics should be included.
- Libraries of schools may be enriched by books related to the history of mathematics.

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Authors

Kani BAŞIBÜYÜK, Erzincan University, Refahiye Vocational School, Erzincan, Turkey. Email: kanib_24@hotmail.com

Ömer ŞAHIN, Amasya University, Faculty of Education, Amasya, Turkey. Email: mersahin60@gmail.com