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VISUALIZING THE OLYMPIC DIDACTIC SITUATION (ODS): TEACHING MATHEMATICS WITH SUPPORT OF THE GEOGEBRA SOFTWARE

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Abstract: The teaching of Mathematics in the context of the Mathematical Olympiads in Brazil constitutes a context of research still little considered, with particular interest in the teaching and the learning phenomena. In Brazil, we identified a large number of teachers and students who do not participate in official math competitions, such as the Brazilian Olympiads of Mathematics of Public Schools (OBMEP). On the other hand, it is important to implement didactic resources that allow other forms of didactic transposition to the mathematical knowledge addressed for the Olympic Problems (OP). In this way, the present work presents and introduces the notion of Olympic Didactical Situation (ODS), from a perspective of didactic transposition necessary for the teacher, who must transmit the mathematical knowledge characteristic of the Mathematical Olympiads to the greatest possible number of students. Thus, visualization becomes a primordial element to approximate and transmit intuitive and heuristic notions present in a set of three Olympic Problems (OP) discussed throughout the text.

Key words: Teaching of Mathematics, Mathematical Olympiads, Teacher training, Visualization.

1. Introduction

The style, characteristics, and particular type of dissemination of mathematical knowledge, through the national circulation characteristic of the Mathematical Olympics, unquestionably provides and produces a differentiated possibility in order to involve, attract and "seduce" young Brazilians talents, and young students or more old students, to an environment that, in a prosaic way, is constituted as a tournament of competitions, marked by the intellectual activity of competitors and whose main point is revealed through the social distinction acquired by obtaining (silver, bronze, gold) medals or other forms of awards, through the laureation and backing of scientific brazilien societies.

Still in the previous context, we can not avoid pointing out some aspects that we propose to act as obstacles and reductions in the area. In fact, the number of young students who participate annually in the Brazilian Public School Mathematics Olympiad (OBMEP), reveals important indexes and indicators, and which, gradually, tend to make visible and identifiable, more and more, a small group of students in several regions of the country who can achieve progressive success in the gradual phases and increasing levels of complexity present in the (OBMEP) tests.

Nonetheless, problem-solving activity in Mathematics constitutes the "Ariadne thread" and original and capital motivation, which has been producing mathematical knowledge for centuries. In this way, the problem solving activity of Olympic Problems (OPs), addressed in the competitions, in its essence, involves an intrinsic bias of mathematical knowledge that can not be separated and distinguished from didactic approach and transposition to Mathematics in ordinary situations and not official competition. However, another aspect that is little discussed in the literature concerns the elaboration and production of mathematical problems, aiming at the indispensable promotion of the student research profile, in spite of its greater or less inclination to participate in the Olympic competitions. Thus, the mathematical problem-solving activity is undoubtedly a skill that must be cultivated and strongly stimulated in the initial and continuous formation of Mathematics teachers in Brazil (Alves, 2016).

Consequently, teachers who do not act directly in circles or competitions can not be agents or 'passive consumers' of a deep-rooted competition culture and thus should stimulate and 'seduce' as many

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students as possible or a collective and group construction of mathematical knowledge mobilized through interactions with Olympic Problems (OP) from the (OBMEP) tests.

Through the above arguments, in the following sections, we will introduce the notion of an Olympic Didactic Situation (ODS). Thus, one Olympic Didactic Situation (ODS) will be validated and substantiated from the theoretical contribution extracted from the Theory of Didactical Situations (TSD), with emphasis on the visualization component, supported by software GeoGebra. This theory allows to model situations for the teaching of Mathematics (Artigue, 2012; 2013) and, in a special way and emphasized here, for the teaching of the Mathematical Olympiads, through the visualization and interpretation of geometric and dynamics properties extracted from three Olympic Problems (OP).

Thus, in the following sections, we will show that GeoGebra software can provide different ways to approach Olympic Problems (OP) in Mathematics.

2. Olympic Didactic Situation (SDO) and problem-solving activity

The tradition of the Olympic Games has a secular inheritance, originated from the peoples of ancient Greece and has since been a legitimate representative of the dedication, the effort of the athletes in overcoming limits, strength, preparation and achievement of goals, above all, the victory and overcoming of opponents, according to well-defined ethical rules.

Now, from the previous reasoning, when we put in the background the physical effort of preparation of the athletes and we restrict to the intellective effort and the acquisition of abilities derived from the same nature. In the case of the Mathematical Olympiads, we also aim to define and achieve precise goals, to overcome and to achieve high performance, through the sometimes solitary and individual progress of the competitors (students) who are, in a practical way, instrumented and trained to the greatest possible oppositionist efficiency, in the scope of mathematical reasoning and problem solving, whose increasing gradation of logical-mathematical complexity constitutes its main defining role of the challenge (Martins, 2015).

Much is already known about the "selection" of mathematical knowledge (Ernest, 1991). But in the case of the Mathematical Olympiads, undoubtedly, the main target consists of the identification and production of young talents, the "selection" of young prodigies. The visible part of the previous history culminates with the social (and academic) distinction of the medalists and that, in certain cases, they should subsequently embark on and confirm their inclination towards academic research in the framework of strictu senso postgraduate studies. The disregarded part of this plot relates to students who did not achieve representative success in competitions and, above all, to students who, for general reasons that we do not want to discuss here, were either out of the process or lacked the motivation and encouragement to persevere in the process of the study of Mathematics, according to the outstanding characteristics of its standard presentation form through (OP's).

When we restrict our eyes to the case of the participation of Brazilian public schools in mathematical activities of competition (ALVES, 2010), it is worth remembering the following fact:

> The Brazilian Mathematics Olympiad of Public Schools (OBMEP) was created with the aim of stimulating the study of mathematics in public schools, enabling a hypothetical expansion of access to scientific and mathematical information. In 2005, in the official year of the launch of (OBMEP), approximately R $ 7.7 million were budgeted for the preparation and dissemination of the Olympiad. (Neto, 2012, page 3)

In the section above, we understand a governmental action aimed at strengthening the dissemination of a scientific and mathematical culture, specifically, addressed to public school students. The motivation is to encourage young Brazilians, with distinguished skills, to participate in the program and, in a way, to influence the other young people, although, as we have mentioned in the previous paragraphs, little is known or little is done concretely, regarding the actions of inclusion and approximation of a mathematical culture that can attract and provide the greatest contact with this peculiar form of manifestation of mathematical knowledge without, necessarily, not constituting an official participation in tournaments or the contribution of a greater amount of teachers of math.

Historical motivation in competitions and intellectual challenges has always been reported in the History of Mathematics books (Debnath, 2011, Koshy, 2014). For example, Carneiro (2004) explains:

> The books tell us that in earlier times mathematicians challenged each other by proposing complicated questions, and often met in public squares for tournaments where they would have to solve difficult equations. What was born, perhaps, by a whim of the ego of these people, took a more salutary form with the realization of the 1st Mathematical Olympiad in Hungary in 1896. (Carneiro, 2004, p. 3).

For example, Debnath (2011, p. 339) recalls that Fibonacci or Lernarno Pisano, when participating in a tournament, solved three mathematical problems, while none of the other participants achieved the same success. The first, according to Debnath, was to find a rational number 'x', so that the expressions (x^2-5) and (x^2+5) represent the square of a rational number $x=\frac{41}{12}$. He found the next rational number number $x^2-5=\left(\frac{41}{12}\right)-5=\left(\frac{31}{12}\right)^2$, so that and that $x^2+5=\left(\frac{41}{12}\right)-5=\left(\frac{49}{12}\right)^2$. His solution was published in the book *Liber Quadratorum* (Debnath, 2011).

In another problem, Fibonacci showed that the numbers $a^2-2ab-b^2, a^2+b^2, a^2+2ab-b^2$ are in arithmetic progression, which can be easily verified. And in another situation, he was challenged to find a root of the cubic equation $x^3+2x^2+10x-20=0$. Fibonacci showed that no root of it can be expressed as an irrational form $\sqrt{a+\sqrt{b}}$. Hence, it obtained a decimal approximation of the root, with the following value (Debnath, 2011, p.339). And in 1225 he published his final solution $x=1,3688081075$ in a book entitled *Flos* (Flower), without elaborating any formal proof or explanation of his ingenious method for a more attentive reader.

Debnath (2011, p.339) stresses that "it was a real mystery as Fibonacci got a correct solution" to the problem of the cubic equation $x^3+2x^2+10x-20=0$. This author further emphasizes the relevant fact that Fibonacci presented another point of view for the classification suggested in the book of Elements, Book X, which did not include irrational numbers.

However, in spite of Leonardo Fibonacci's successful strategies mentioned above, it is worth noting the space provided by the authors of the Mathematical History books, the examples of great mathematicians competing and that, according to the subjective motivation indicated Carneiro (2004), stimulated the accomplishment of feats and results that have been recorded in the literature. These examples reveal important teachings when we stick to our current context of mathematics teaching (Alves, 2012; Silva, 2016).

On the other hand, when we consider (OBMEP) competitions as the possibility of promoting and improving educational quality indices in our country, we can not develop an isolationist perception. In fact, we do not know the indexes of mathematics teachers that do not directly involve the environment of competition and selection of young talents. For this representative group of teachers, they can be interpreted as non-consumers or "passive consumers" of an inexorable mathematical culture that is installed in the middle of the competitions between different schools in Brazil.

In addition, we can not allow the evolution of two dichotomous styles of presentation and ways of approaching mathematics. The first style, dedicated to the systematic fulfillment of the specific contents and the official programs in the public schools. Whereas, in the second style, we record the classic form of approach to Mathematics, through the problems and challenges, typical and characteristic of secular problem-solving activity.

Consequently, the essential character of giving students (non-competitors) the contact with a characteristically striking mathematical and reproductive of the primary and original subjective motivations that competed for the evolution of mathematical thought, that is, the activity of resolution and overcoming problems, whose content or motivation did not always originate from day to day.

Thus, we advocate a premise that involves the importance of the mathematical problem builder / builder activity, aiming at providing a learning scenario that stimulates and promotes the active participation of not only the most prominent students in mathematical activity, but also students who do not participate in competitions and that need to be exposed, also, to an adequate culture of Mathematics, aiming at its integral formation. Thus, in order to mark and demarcate some foundational assumptions in the teaching and the role of the Mathematics teacher, in the following section, we will discuss some elements that intend to constitute a methodology for the teaching of Mathematics in the context of Mathematical Olympiads.

3. Theories of Didactic Situations (TSD) and the initial teacher training

In his doctoral thesis entitled *Théorisation des Phénomènes d'Enseignement des Mathématiques*, Brousseau (1986) exalts, emphasizes and indicates a perspective for teaching that we can not forget:

> The modern conception of teaching will require the teacher to provoke in his students the desirable adaptations, by means of a judicious choice, of the problems that he proposes. Such problems, chosen so that students can accept them, should make them act, speak, reflect, and evolve in their own movement. Between the moment the student accepts the problem as his own and, from the same, produces his answer, the teacher refuses to intervene as a producer of knowledge that he wants to make appear. The student knows very well that the problem was chosen to make him acquire a new knowledge, but he must also know that this knowledge is fully justified by an internal logic of the situation and that he can construct it without appeal for didactic reasons. (Brousseau, 1986, p. 297)

The dialectic indicated above requires a detailed and careful expedient of attention and analysis, insofar as we have the interest of perspective and understanding the action of the teacher mediated and supported by fundamentals that substantiate and reveal the adoption of a teaching methodology for Mathematics. Brousseau (2012) adopts a metaphorical perspective of student activity, in view of problem-solving activity, considering a recurring metaphor of "play". Therefore, he still comments that:

> Players' knowledge is manifested in strategies as a means of winning some matches or improving results. A player's decision may be interpreted by an observer: as the result of an earlier strategy, even if it has been learned spontaneously, or has been taught to the player; or as a new improvisation, which is random, resulting from reflection or external information, contemporary to its decision. (Brousseau, 2012, p. 314).

We record an important point of view that, despite referring to mathematical knowledge in the very early series, a scenario considered in his doctoral thesis, almost all of the elements mentioned above can have a natural repercussion in the context of investigation of Olympic problems. Thus, we assume the following axiom.

Axiom 1: For all knowledge, it is possible to build at least one formal game, communicable and without using such knowledge. And, for which, it determines an optimal strategy. (Brousseau, 2012, 314).

Now, from the axiom 1, we can deduce that, for any proposed problem situation, the mathematic teacher can communicate to the players (students) a task that must be assumed by them (devolution) and will always be able to determine and identify a more successful strategy. It is worth noting that in French culture the distinction between the nature and function of knowledge (*connaissance*) and mathematical knowledge (*savoir*) occurs. In fact, for didactic and methodological purposes, Brousseau (2012) further adds about this subject:

> With regard to the same mathematical notion, we can then target a family of situations where such a notion functions as knowledge (action situation). A family of situations where it figures as mathematical knowledge (eg, validation situation). A family of situations where the identification of a need for knowledge arises and the possibility of satisfying it through the communication of corresponding knowledge. (Brousseau, 2012, p. 314).

Therefore, it is urgent the understanding, on the part of the teacher, as elaborator and constructor of problem situations, whose mathematical knowledge acquires and assumes a double character. As we noted in the previous excerpt, in the action situation, knowledge works and plays the role as an instrument, a technical construct for the purpose of solving specific problems. But in the case of a validation situation, for example, the very status of knowledge is objectified. Therefore, the interest in the mathematical knowledge employed and mobilized in the situation, now seen as a theoretical-conceptual object, constituting a broader scientific knowledge. However, the examination of analysis and appreciation of the teacher of mathematics needs this flexibility of change of perspective and, thus, cause the expected changes in the students.

An example of modeling or simplification of the earlier dialectical game is summarized by Douady (1984). In fact, Doaudy (1984, p.6) commented and discussed a trajectory or course capable of organizing and discriminating the changes provoked by the students, through the medium and from the teacher action, in a productive way, when he described three different dialectical forms, taking as reference the action of the teacher, namely:

Situation of action: The student is confronted with a situation that produces a problem. In the search for the solution, it produces actions that can lead to the creation of knowledge in action. He can, more or less, explain or validate his actions, however, the situation does not require.

Situation of formulation: Different conditions allow a change of information and the creation of a language to ensure change. In the formulation situation, the student can justify his propositions, but, the situation does not require.

Situation of Validation: Changes are not just about information but about statements. It is necessary to prove what has been affirmed, by means of the action. It is the goal of a validation situation.

Before discussing the last dialectic phase introduced by the French didactics, some recent considerations of Margolinas & Drijvers (2015), when explaining an important and inescapable movement of transformations and modifications undergone by the mathematical knowledge.

> During the social construction of Mathematics, knowledge is formulated, formalized and written. The initial utility that was the meaning of a specific situation becomes generalized and becomes less explicit or even hidden, a mathematical knowledge becomes a kind of formal knowledge. This process of institutionalization can not be avoided; and it will serve to strengthen and simplify original mathematical knowledge, which is an aspect of didactic transposition (Chevallard, 1985). The process that connects knowledge into situation (connaissance) and institutional knowledge (savoir), work in both directions. (Margolinas & Drijvers, 2015, p.899).

Thus, from the explanations of a movement or set of final transformations that must focus on mathematical knowledge, we will finally have the last situation of institutionalization that is characterized by:

> It is a situation in which the passage from a knowledge and its role of resolution to action, of formulation and of proof, to a new role, as a reference for future personal and collective achievements, is unveiled. Institutionalization therefore entails a change of convention between the actors, a knowledge (justified or otherwise) of its validity and the usefulness of a knowledge and modification of a knowledge. (Brousseau, 2010, p.11).

Also related to the knowledge involved, mobilized and constructed by the teacher, we point out the respective knowledge that contributed to the establishment of the mathematical properties evidenced in each Olympic Problem (OP). In this sense, we agree with Margolinas (2015), when he recalls one of the pioneering distinctions made in the Mathematics Didactics (Alves, 2016), involving the distinction between knowledge (*connaisance*) and knowledge (*savoir*). In this sense, Margolinas (2012) explains that a knowledge has a balancing role between the subject and the environment, constitutes what can be mobilized when an investment occurs in a situation, whose nature can occur as a knowledge of action, a knowledge of interaction or memorization knowledge. Whereas, when we objectify the knowledge (*le savoir*) provided to the group the teachers in formation, we prioritize a component of social and cultural construction, which is revealed in an institution or, according to Margolinas (2015, 34), which is embodied by a text or scientific work, possibly materially written or book.

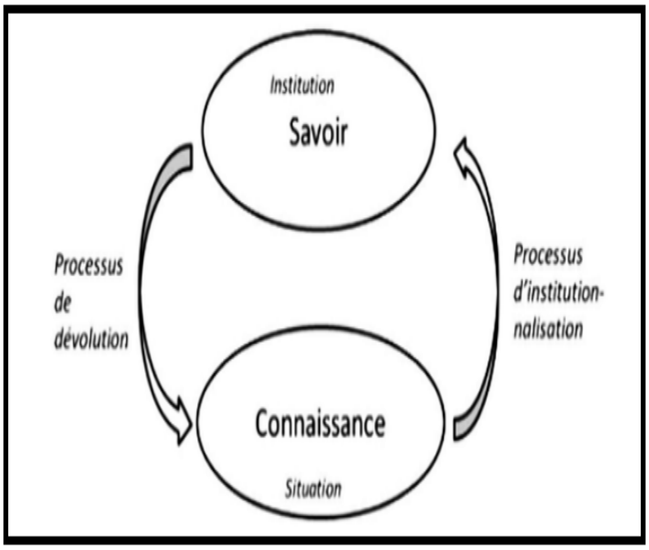


Figure 1. *Margolinas (2015) describes the dialectic between savoir / connaissance and its important role for the Didactics of Mathematics.*

Undoubtedly, we do not wish to discuss in detail the broader foundations of (TSD) (Brousseau, 1986, 2011). On the other hand, the above mentioned elements have an immediate repercussion aiming at the action and improvement of the role of the Mathematics teacher, above all, to the dialectical moments of approach of a certain mathematical subject, as well as the character assumed of its presentation that, here, we seek to emphasize in the ambit of the solving activity of problems on the part of the student, and of the elaboration and construction activity of Olympic Problems (OP). Therefore, in order to extract an understanding of the teaching of the Olympiad supported by the fundamentals of a teaching methodology, in the following section, we will define, with a certain content of novelty, the notion of Olympic Didactical Situation (ODS).

4. Olympic Didactic Situation (ODS) and the activity of problem solving

In the previous sections, we have observed some elements of the context of the Mathematical Olympiads of Brazilian public schools. In addition, we point out some problems that arise from a natural process of exclusion of students who do not participate directly and officially in competitions, but who need to preserve contact with the Mathematics conveyed according to a particular bias of competition and overcoming obstacles. Thus, in order to demarcate some essential components in our discussion, we present the following definitions that constitute applications of the (TSD), when we objectify teaching in the context of Mathematical Olympiads. In view of this, we will define an Olympic Problem (OP) as follows.

Olympic Problem (OP): A set of mathematical problems situations, addressed in an official competitive context or marathons, with participation only (and restrictively) of the students, whose approach and characteristics of individual student action involves only objective of achieving the goals (and medals) defined in each competition, through the use of standard strategies, reasoning and efficient mathematical arguments, previously instrumented by mathematics teachers acting in competition cycles.

Olympic Didactic Situation (ODS): A set of relationships established implicitly or explicitly, marked by a teaching methodology (TSD), between a student or group (s) of students, a certain medium (and also understand the mathematical knowledge approached through competition and olympic problems) and an educational system, with the purpose of allowing the appropriation, by these students, a knowledge constituted or in the process of constitution, coming from an environment of collective competition in group and Olympic Problems (OP) or set of characteristic problems of the Mathematical Olympiads.

Below we present a characteristic equation $ODS = OP + TSD$ (see figure 1) that seeks to convey meaning to the notion of the Olympic Didactic Situation (ODS). In this sense, we observe that the notion of Olympic Problem (OP) involves a form of transmission of mathematical knowledge, in a standard and official way, in a distant style and not related to a methodological interest for the teaching of Mathematics and, therefore, restricted and indicated only to the students. On the other hand, the notion of Olympic Didactic Situation (ODS) assumes and involves the incorporation of certain assumptions of a teaching methodology and, consequently, objectify a didactic transposition to the mathematical knowledge present in a (OP).

We can also interpret, from a vectorial point of view $\overrightarrow{ODS} = (OP, TSD)$, that the notion of Olympic Didactic Situation (ODS) has two components. The first vector component OP concerns the standard presentation aspect of problems in Mathematics Olympiads which are predominantly dedicated to a smaller audience of competing students. The second component TSD seeks to emphasize the role of the Theory of Didactic Situations (TSD) with the function of providing a methodological perspective aimed at a didactic transposition that stimulates to broaden the approach of an Olympic problem aiming at a student audience not restricted to the Mathematical Olympiad cycle. (see figure 2).

$$ODS = OP + TSD \qquad \overrightarrow{ODS} = (OP, TSD)$$

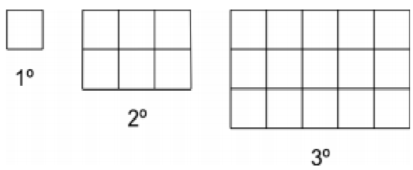
Figure 2. *Mnemonic description of the components of an Olympic Didactic Situation (SDO)*

4. Visualizing Olympic didactic situations with Geogebra software

In this section, we will present three Olympic Problems (OPs) that have been extracted from (OBMEP) exams. In an initial and systematic way, we can verify the original statements and their corresponding translation into the English language (see figure 3). Next, we will discuss the possibilities of building three (ODSs) that involve, in our case, emphasizing the use of GeoGebra software and the corresponding visualization, and the description of a significant scenario for the learning of Mathematics.

4. Com quadradinhos de lado 1 cm, constrói-se uma seqüência de retângulos acrescentando-se, a cada etapa, uma linha e duas colunas ao retângulo anterior. A figura mostra os três primeiros retângulos dessa seqüência. Qual é o perímetro do 100º retângulo dessa seqüência?

(A) 402 cm
(B) 472 cm
(C) 512 cm
(D) 598 cm
(E) 634 cm



With small squares of side 1cm, a sequence of rectangles is constructed adding, at each step, a line and two columns to the previous rectangle. The figure shows the first three rectangles of this sequence. What is the perimeter of the 50th rectangle of this sequence?

Figure 3. *Example of an Olympic Problem (OP) of the year 2008 in Brazil.*

Situation of Action: In the initial phase, the teacher should encourage the students to explore some particular cases from the figures present in the above statement. You can see that the numbers $1, 6, 15$ have certain properties and seek to increase the amount of elements, determining the fourth, fifth and sixth element figural (see Figure 2).

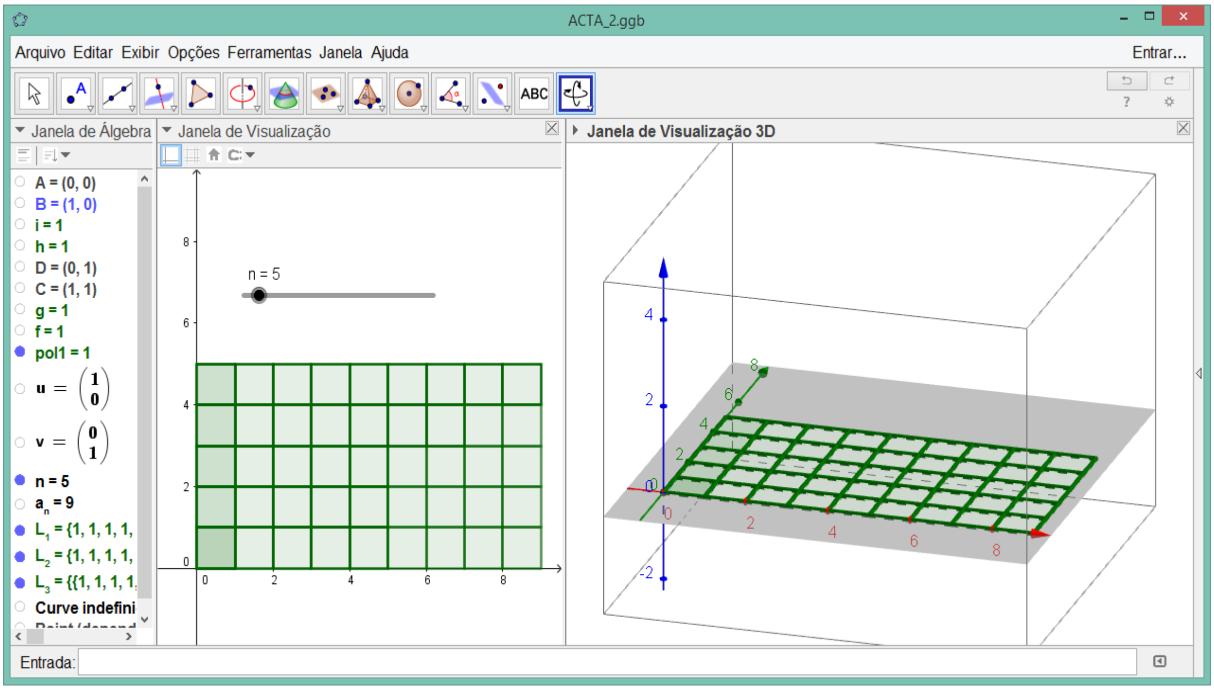


Figure 4. *Exploration of the Olympic Problem (OP) with the compuational resource and the possibilities of 2D and 3D visualization provided by the software. (Author's elaboration)*

The mathematics teacher presents the construction with the GeoGebra software that we display in figures 3 and 4. Students can develop a computational exploration expedient and develop an extended numerical analysis of the static data, initially presented in the Olympic Problem (OP) and in the statement that we see in the figure 3. In figure 4 the students can explore different configurations, for the following corresponding variation of the selector created with the software GeoGebra.

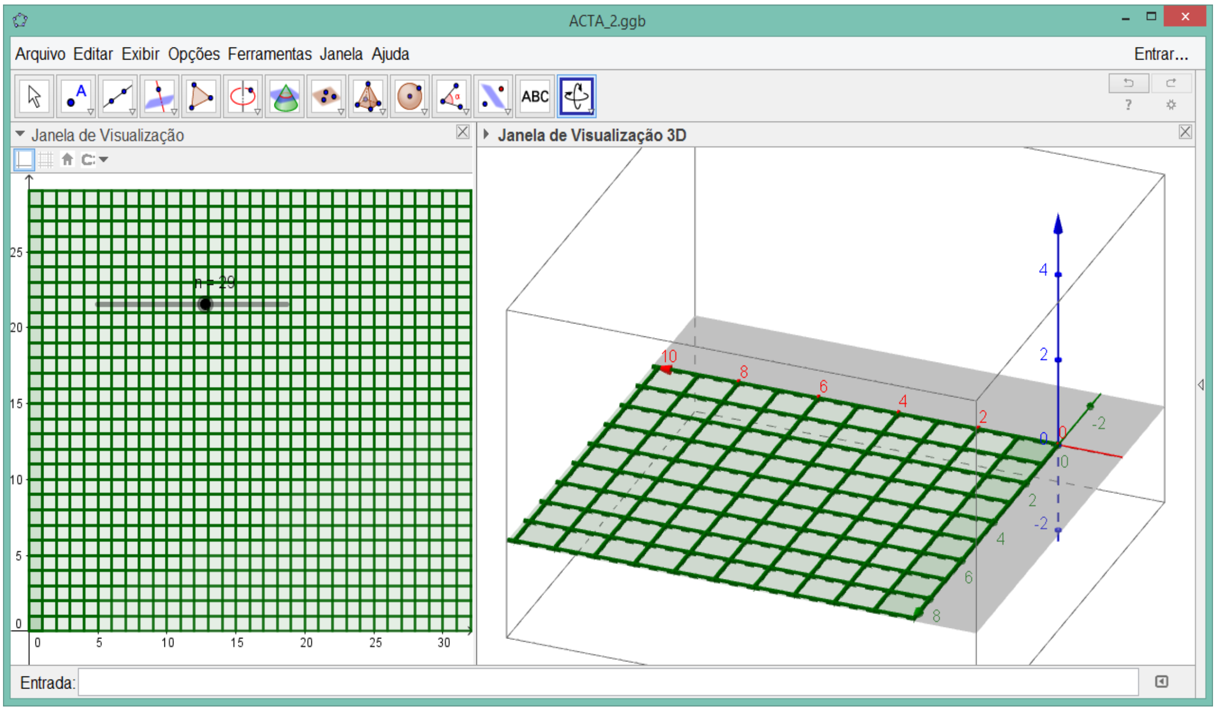


Figure 5. *Exploration of the Olympic Problem (OP) with the compuational resource and the possibilities of 2D and 3D visualization provided by the software. (Author's elaboration)*

In Table 1 we observe some numerical values extracted directly from the construction with GeoGebra software that we observed in figures 4 and 5.

Table 1. Numerical description of values determined by *software Geogebra*.

n	a_n	n	a_n
1	1	**10**	19
2	3	**11**	21
3	5	**12**	23
4	7	**13**	25
5	9	**14**	27
6	11	**15**	39
7	13	**16**	31
8	15	**17**	33
9	17	**18**	35

Situation of Formulation: At this point, students should be encouraged to produce a set of conjectures derived from inferences and manipulation developed directly on the software. According to Almouloud (2007, p. 38), in this phase "the student exchanges information with one or more persons, who will be the transmitters and receivers, exchanging written or oral messages".

Thus, with this exchange of information, the student can conclude that since the sequence will prevail for the nth rectangle, this difference will be the reason for an arithmetic progression.

Hence, the student must establish a model, by which it is possible to calculate the base measure of any figure that is in that sequence, to then calculate the perimeter of the 50th rectangle. Using his knowledge of arithmetic progression and his general term, the apprentice will observe that to calculate the measure of the base of the nth rectangle will be valid the formula $a_n = a_1 + (n-1) \cdot r$.

Situation of Validation: At this point, students should be encouraged to produce and adopt a symbolic representation system, which allows the generalization of certain numerical properties recorded in the past stages. $a_n = a_1 + (n-1) \cdot r$.

Some particular numerical values can be analyzed in the cases we list below, for some values of ‘n’. The teacher should encourage the examination of some of the values listed below.

$n=1 \therefore a_n = 1+(1-1)\cdot r = 1$, $n=2 \therefore a_2 = 1+(2-1)\cdot 2 = 1+2=3$, $n=3 \therefore a_3 = 1+(3-1)\cdot 2 = 1+4=5$, $n=4 \therefore a_4 = 1+(4-1)\cdot 2 = 7$, $n=5 \therefore a_5 = 1+(5-1)\cdot 2 = 9$, $n=6 \therefore a_6 = 1+(6-1)\cdot 2 = 11$, etc.

The general term formula for arithmetic progressions can be demonstrated by mathematical induction if the teacher registers students' interest. In this case, for some initial values, we have $n=1 \therefore a_1 = a_1 + (1-1)\cdot r$, $n=2 \therefore a_2 = a_1 + r = a_1 + (2-1)r$. In the inductive step, we can observe that $a_{n+1} = a_n + r = (a_1 + (n-1)\cdot r) + r = a_1 + n\cdot r - r + r = a_1 + n \cdot r$, c. q. d.

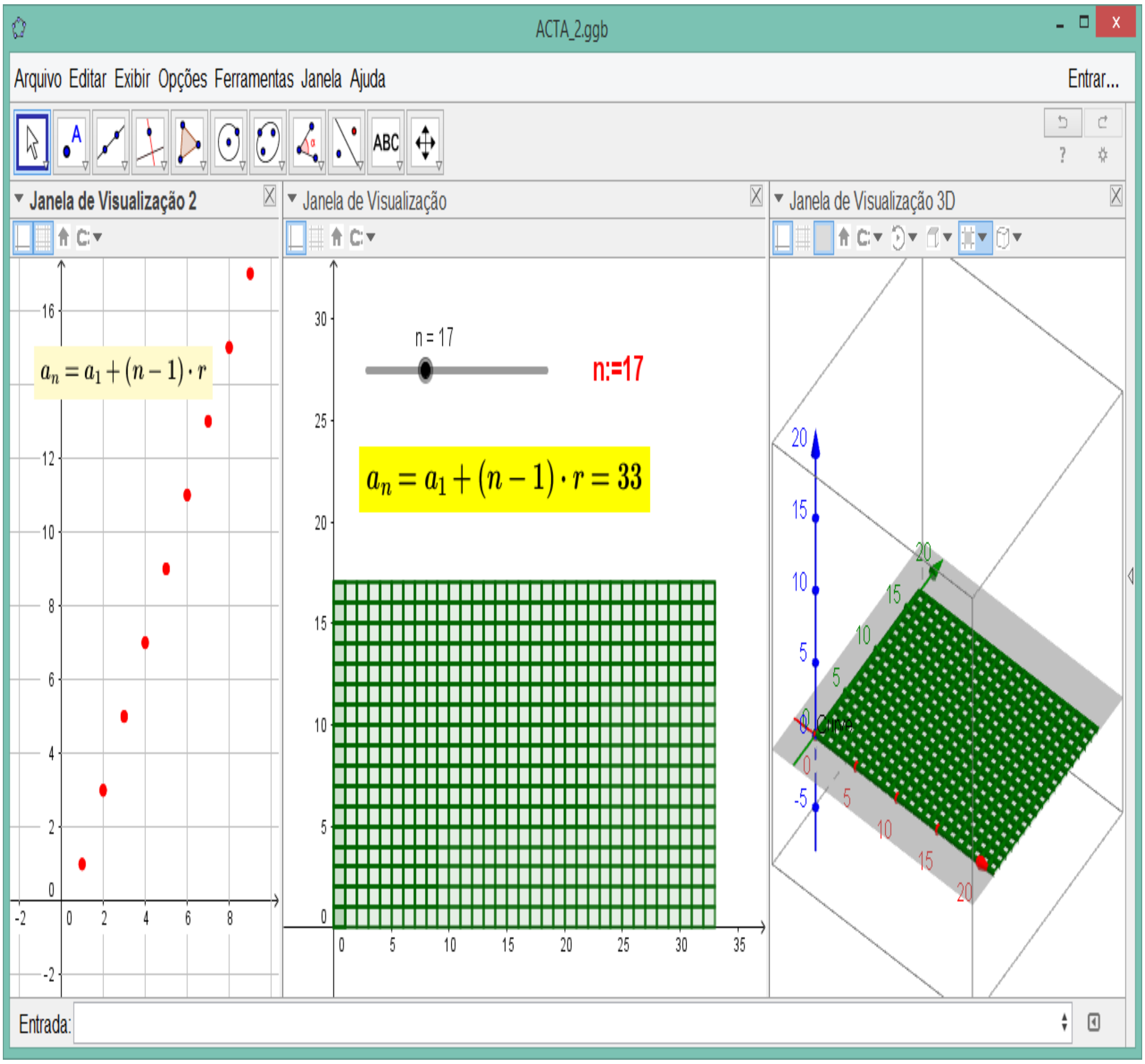


Figure 6. *Exploration of the Olympic Problem (OP) with the compuational resource and the possibilities of 2D and 3D visualization provided by the software. (Author's elaboration)*

Situation of Institutionalization: In the last dialectical phase, the teacher resumes the direct conduction of the investigation inside the (ODS), with the intention of revealing to the teachers in initial formation that in fact, the Olympic Problem (OP) was extracted from a test of the (OBMEP) tests and that, with the software GeoGebra, can identify other elements methodologically exploited with the technological resource. In the figure below the teacher can explore three windows of view of the software.

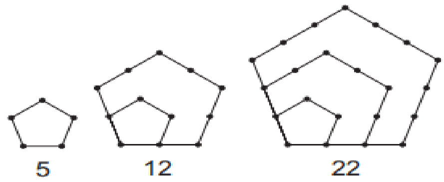
In the 3D windows we observe, for each natural 'n', the corresponding variation of the quantity of rectangles produced by the formula $a_n = a_1 + (n-1) \cdot r$.

In the institutionalization phase, based on the visualization and understanding of the properties displayed in the three windows of the GeoGebra software, the teacher must establish the proper functional relations with a function similar to the type, whose restriction to the field of natural numbers produces the behavior of the points (in red color), to the left in the figure 5.

Now let's look at our second Olympic Problem (OP). The teacher should present the construction below so that students can explore and produce their first guess. In this case, we notice, we deal with pentagonal figures.

14. Abaixo temos três figuras pentagonais: a primeira com 5 pontos, a segunda com 12 pontos e a terceira com 22 pontos. Continuando esse processo de construção, a vigésima figura pentagonal terá 651 pontos. Quantos pontos terá a vigésima primeira figura?

A) 656
B) 695
C) 715
D) 756
E) 769



Below we have three pentagonal figures: the first with 5 points, the second with 12 points and the third with 22 points. Continuing this process of construction, the twentieth pentagonal figure will have 651 points. How many points will the twenty-first figure have?

Figure 7. *Example of an Olympic Problem (OP) of the year 2008 in Brazil.*

Situation of Action: In the initial phase, the teacher should encourage the students to explore some particular cases from the figures present in the above statement. Below, students can manipulate the constructions shown in Figures 7, 8, and 9.

Situation of Formulation: At this point, students should be encouraged to produce a set of conjectures derived from inferences and manipulation developed directly on the software. Let us consider the numerical behavior of some particular cases. Preliminarily, we can see that $1=\frac{1\cdot 2}{2}=\frac{1\cdot(3-1)}{2}=\frac{1\cdot(3\cdot 1-1)}{2}$, $5=\frac{2\cdot 5}{2}=\frac{2\cdot(6-1)}{2}=\frac{2\cdot(3\cdot 2-1)}{2}$, $12=3\cdot 4=\frac{2\cdot 3\cdot 4}{2}=\frac{3\cdot(9-1)}{2}=\frac{3\cdot(3\cdot 3-1)}{2}$, $22=\frac{2\cdot 2\cdot 11}{2}=\frac{4\cdot(12-1)}{2}=\frac{4\cdot(3\cdot 4-1)}{2}$, $35=\frac{2\cdot(5\cdot 7)}{2}=\frac{5\cdot(15-1)}{2}=\frac{5\cdot(3\cdot 5-1)}{2}$, $51=\frac{2\cdot 51}{2}=\frac{2\cdot(3\cdot 17)}{2}=\frac{6\cdot 17}{2}=\frac{6\cdot(3\cdot 6-1)}{2}$. The study of these particular cases and other numeric values made possible by the use of the software should constitute data that need to be formulated and validated in the subsequent section.

Situation of Validation: At this point, students should be encouraged to produce and adopt a symbolic representation system, which allows the generalization of certain numerical properties recorded in the past stages. On the other hand, the teacher should stimulate the exploration of the following numerical sum and fractions that we indicate by $P_1=1=\frac{1\cdot(3\cdot 1-1)}{2}$, $P_2=5=1+4=1+(3\cdot 2-2)=\frac{2\cdot(3\cdot 2-1)}{2}$, $P_3=12=1+4+7=1+4+(3\cdot 3-2)=\frac{3\cdot(3\cdot 3-1)}{2}$, $P_4=22=1+4+7+(3\cdot 4-2)=\frac{4\cdot(3\cdot 4-1)}{2}$

$P_5=1+4+7+\cdots+(3\cdot 5-2)=\frac{5\cdot(3\cdot 5-1)}{2}$, $P_6=1+4+7+\cdots+(3\cdot 6-2)=\frac{6\cdot(3\cdot 6-1)}{2}$.

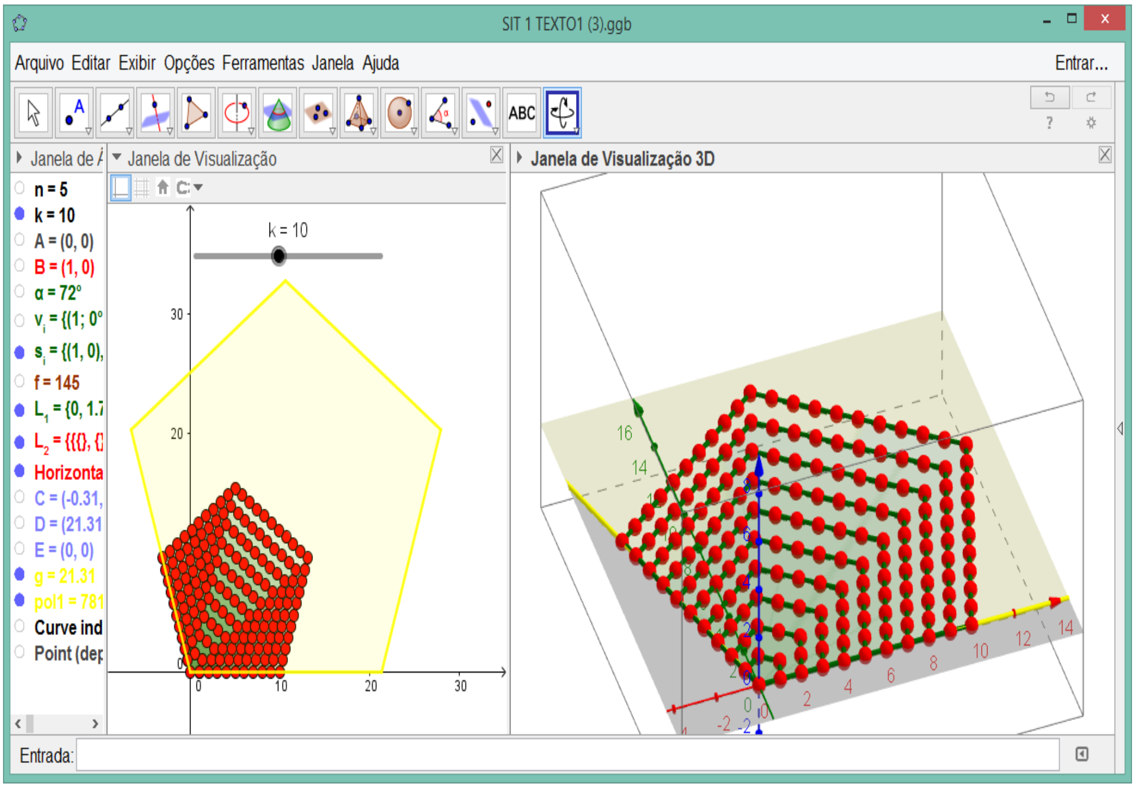


Figure 8. *Exploration of the Olympic Problem (OP) with the compuational resource and the possibilities of 2D and 3D visualization provided by the software. (Author's elaboration)*

Situation of Institutionalization: Through mathematical induction the teacher should propose to the students the following equality $P_n = 1+4+7+\cdots+(3n-2) = \frac{n\cdot(3n-1)}{2}$. In previous phases some particular behaviors were verified and, it must take the inductive step for index 'n' and check for the subsequent step. In fact, we note that $P_{n+1} = 1+4+7+\cdots+(3n-2)+(3(n+1)-2)$. Then we will make the following substitution $P_{n+1} = 1+4+7+\cdots+(3n-2)+(3(n+1)-2) = \frac{n\cdot(3n-1)}{2}+(3n+1)$. The mathematic teacher should also perform the following algebraic calculations $\frac{n\cdot(3n-1)}{2}+(3n+1) = \frac{3n^2-n+6n+2}{2} = \frac{3n^2+5n+2}{2} = \frac{(n+1)\cdot(3n+2)}{2} = \frac{(n+1)\cdot(3(n+1)-1)}{2}$.

Finally, we can see that the following explicit formula for the terms $P_{n+1} = \frac{(n+1)\cdot(3(n+1)-1)}{2}$, for every positive integer n.

Finally, in the context of the investigative process, it is important for the teacher to reveal at least one theorem that allows us to validate the mathematical processes that were used and to confront the veracity of the conjectures elaborated in the previous phases. It is fundamental to understand the relationships and numerical properties extracted directly from the constructions made possible by Geogebra software, as we can see in Figures 8 and 9.

Theorem: For every positive integer n, we have a pentagonal number $P_n = 1+4+7+\cdots+(3n-2) = \frac{n\cdot(3n-1)}{2}$.

Proof. Mathematical induction.

To conclude, after studying the elements that can be visualized in figures 8 and 9, we will approach our last Olympic didactic situation (ODS) from the OBMEP tests in the year 2012.

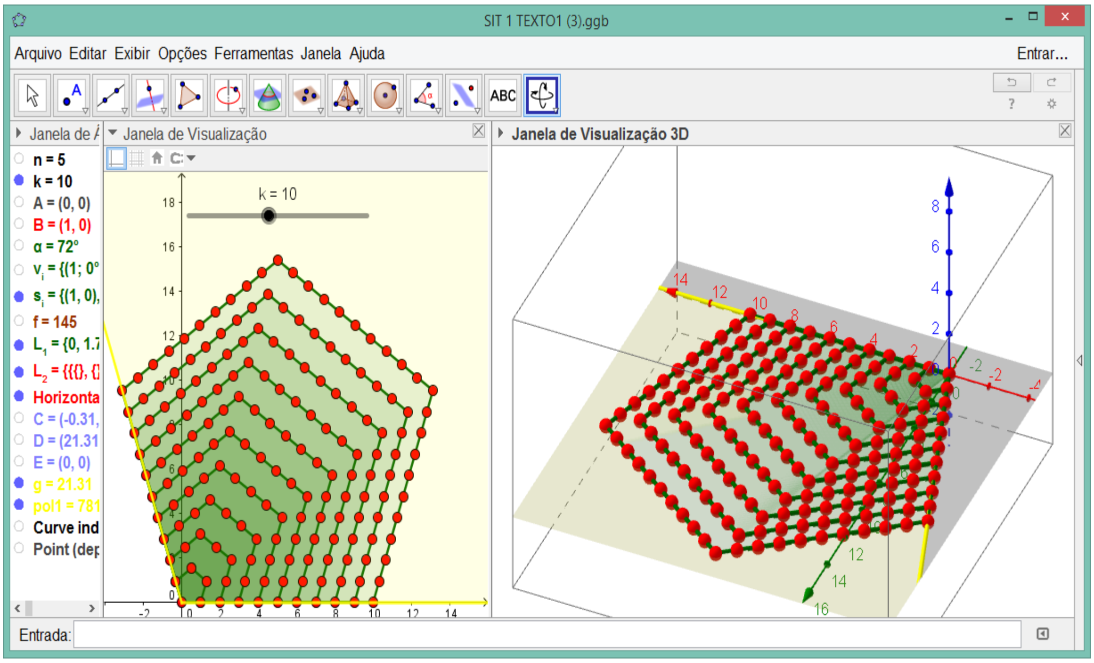


Figure 9. *Visualization of arithmetic and geometric properties (2D and 3D) with support in Geogebra software. (Author's elaboration)*

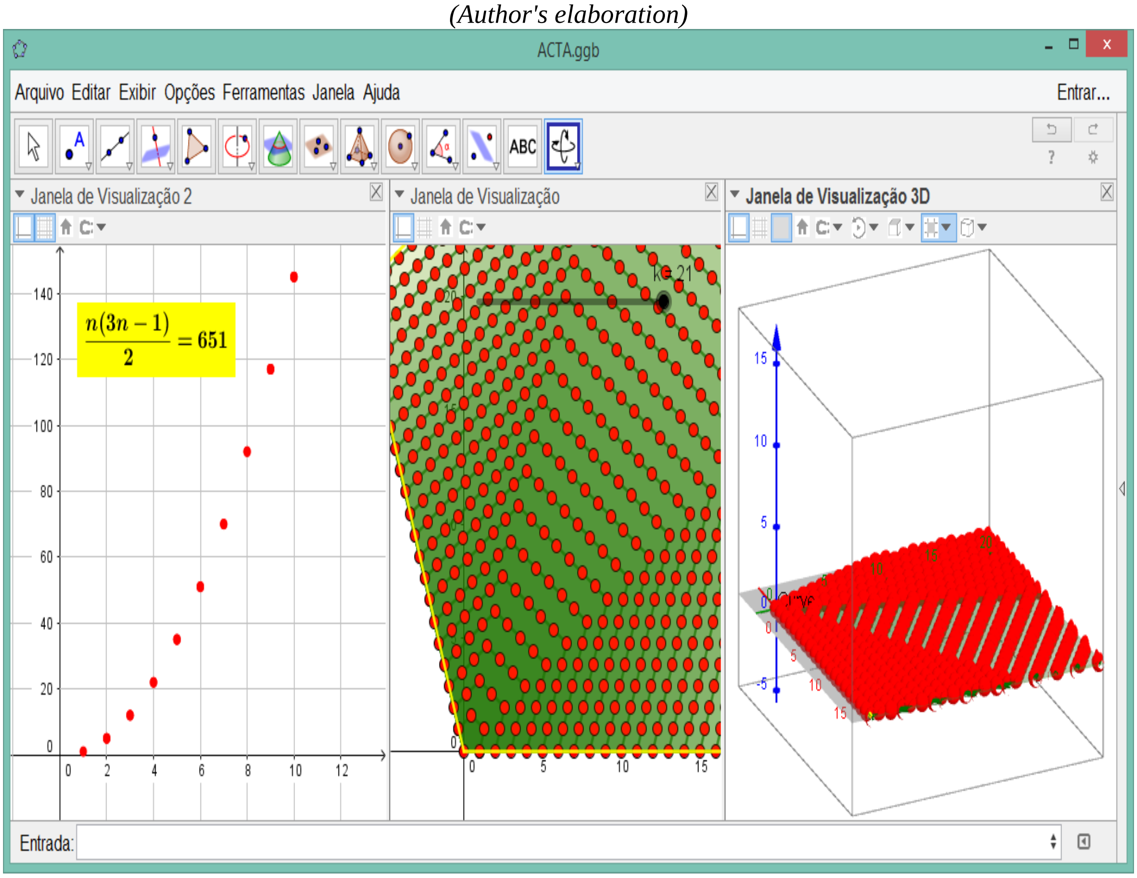
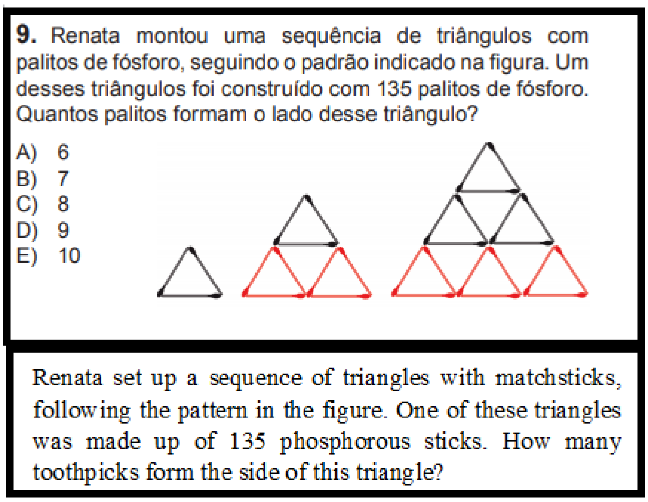


Figure 10. *Visualization of arithmetic and geometric properties (2D and 3D) with support in Geogebra software. (Author's elaboration)*

With GeoGebra software, students can develop a global and local analysis ability of numerical properties extracted from the computational and geometric environment, as we can observe in the manipulations and different configurations of Figures 8 and 9.


9. Renata montou uma sequência de triângulos com palitos de fósforo, seguindo o padrão indicado na figura. Um desses triângulos foi construído com 135 palitos de fósforo. Quantos palitos formam o lado desse triângulo?

A) 6
B) 7
C) 8
D) 9
E) 10

Renata set up a sequence of triangles with matchsticks, following the pattern in the figure. One of these triangles was made up of 135 phosphorous sticks. How many toothpicks form the side of this triangle?

Figure 11. *Example of an Olympic Problem (OP) of the year 2012 in Brazil.*

Situaton of Action: In the initial dialectical phase, students should be stimulated in an investigative process and must confront the static numerical data provided by the preliminary statement of the Olympic Problem with the other research scenario, originating from the exploration of the numerical and geometric properties that we observe in the figures that follow. From Figure 12 students should be stimulated in the investigation and determination of numerical properties extracted from the geometric configurations of the construction. The teacher should encourage the exploration of properties derived from the software that allow the exploration of 3D properties, as we observe in the window on the right side in the figure 12.

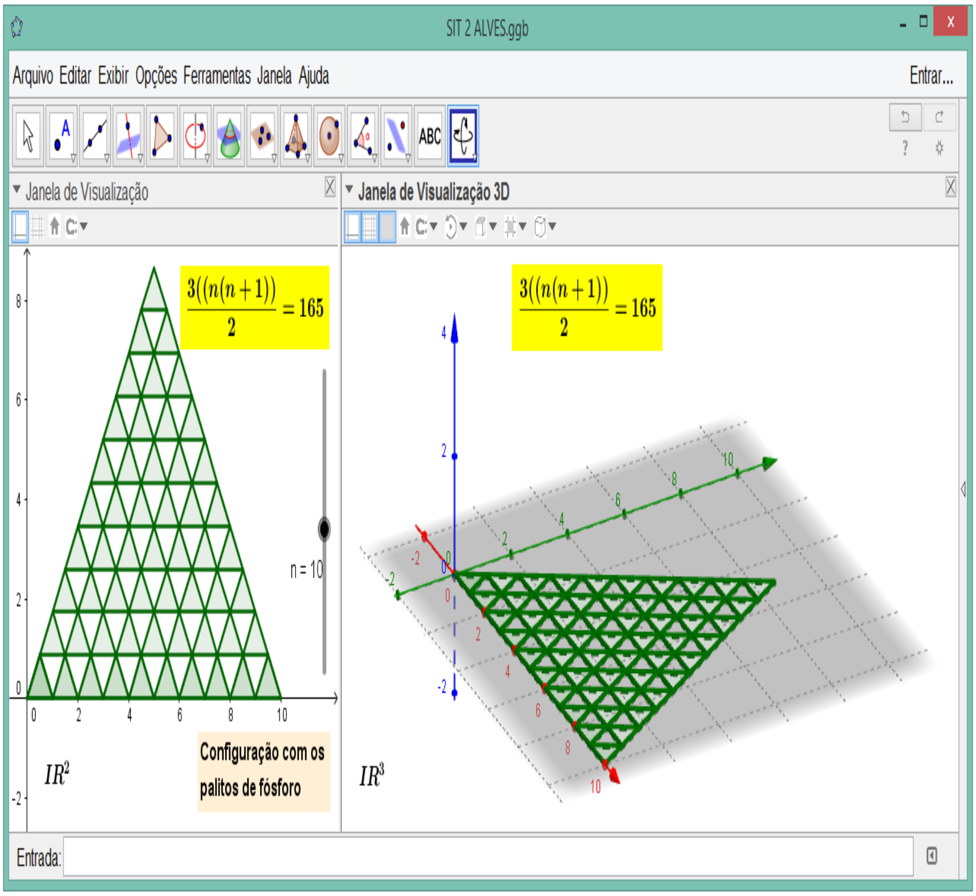


Figure 12. *Visualization of arithmetic and geometric properties (2D and 3D) with support in Geogebra software. (Author's elaboration)*

Table 2. Numerical data extracted from the construction corresponding to the Olympic problem.

n	$\frac{3(n(n+1))}{2}$
1	3
2	9
3	18
4	30
5	45
6	63
7	84
8	108
9	135
10	165
……	…………
19	570

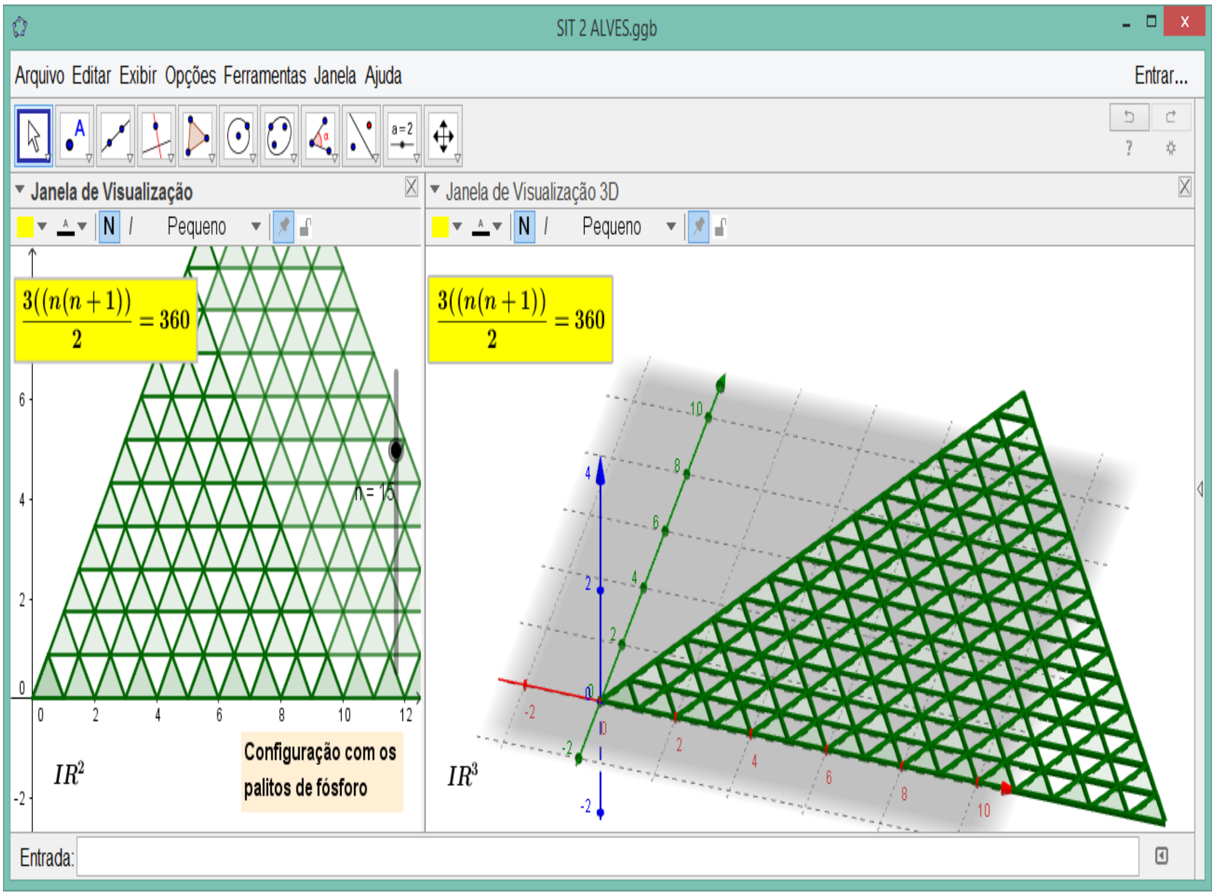


Figure 13. *Visualization of arithmetic and geometric properties (2D and 3D) with support in Geogebra software. (Author's elaboration)*

Situation of Formulation: From some particular numerical values extracted from the GeoGebra software and the construction corresponding to the Olympic problem, the students can organize, in groups, a investigative process of verification of properties confronted with the visualization of the figures. In the second phase, the teacher should stimulate the introduction of a particular notational system that allows the generalization of certain erected properties of the data produced in the action situation.

Situation of Validation: At this point, students should be encouraged to produce and adopt a symbolic representation system, which allows the generalization of certain numerical properties recorded in the past stages.

Teachers should be encouraged to search for and identify a general formula $\frac{3(n(n+1))}{2}$ that allows the counting and determination for each natural number n corresponding to the total quantity of match sticks that constitute a 2D triangular arrangement and, with the support of the Geogebra software, a configuration which can be manipulated in 3D space.

Situation of Institutionalization: In the final phase, the teacher will act to establish the official status of the mathematical knowledge involved, and all formal mathematical relationships. Thus, in Figure 12, the teacher should stimulate the multiple mathematical relationships involving the three windows of GeoGebra software. On the left side, we mark a dotted line, in the red color that constitute the particular points belonging to the graph of a quadratic function of the type $f(x)=\frac{3x^2+3x}{2}$.

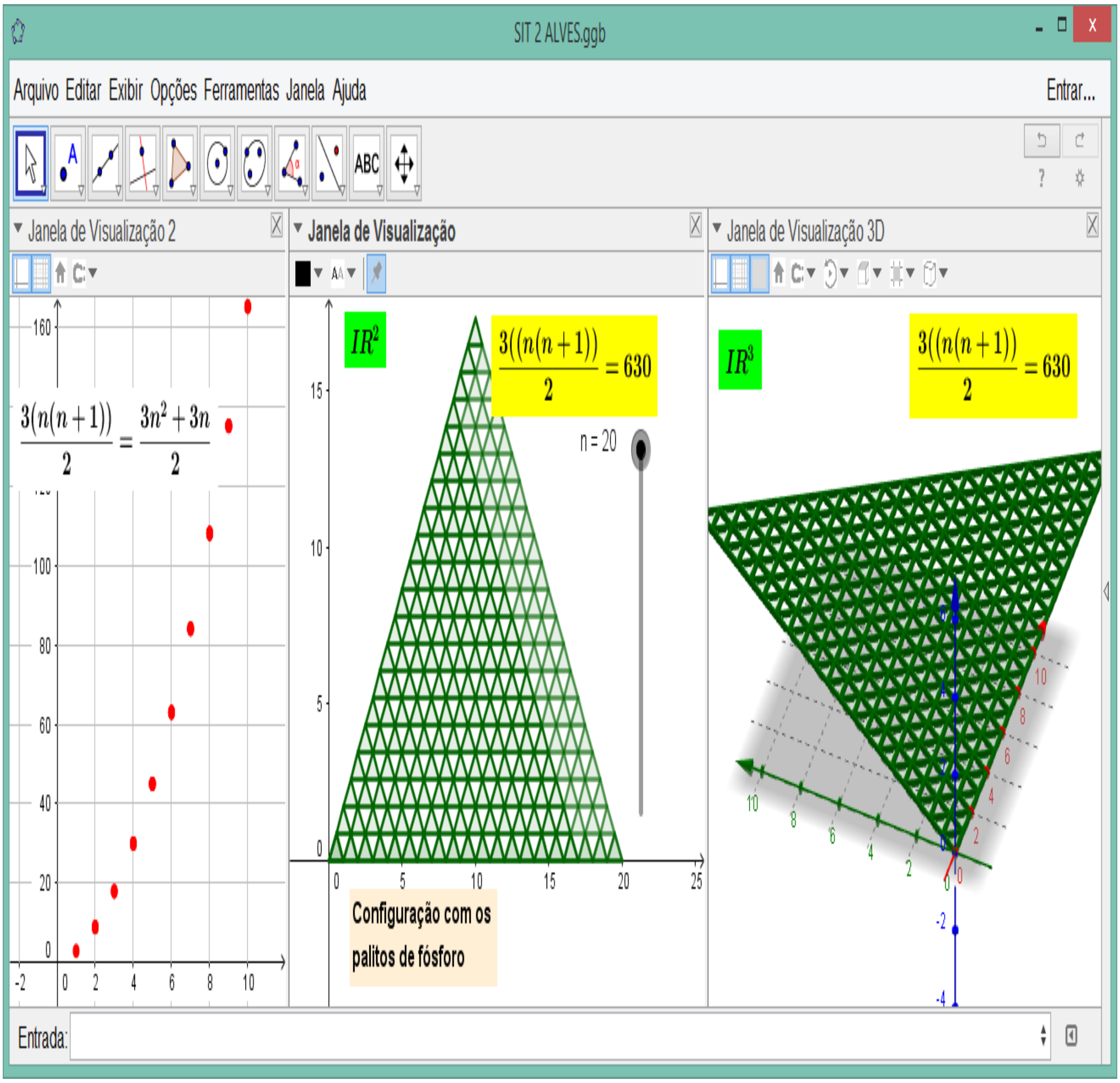


Figure 14. *Visualization of arithmetic and geometric properties (2D and 3D) with support in Geogebra software*

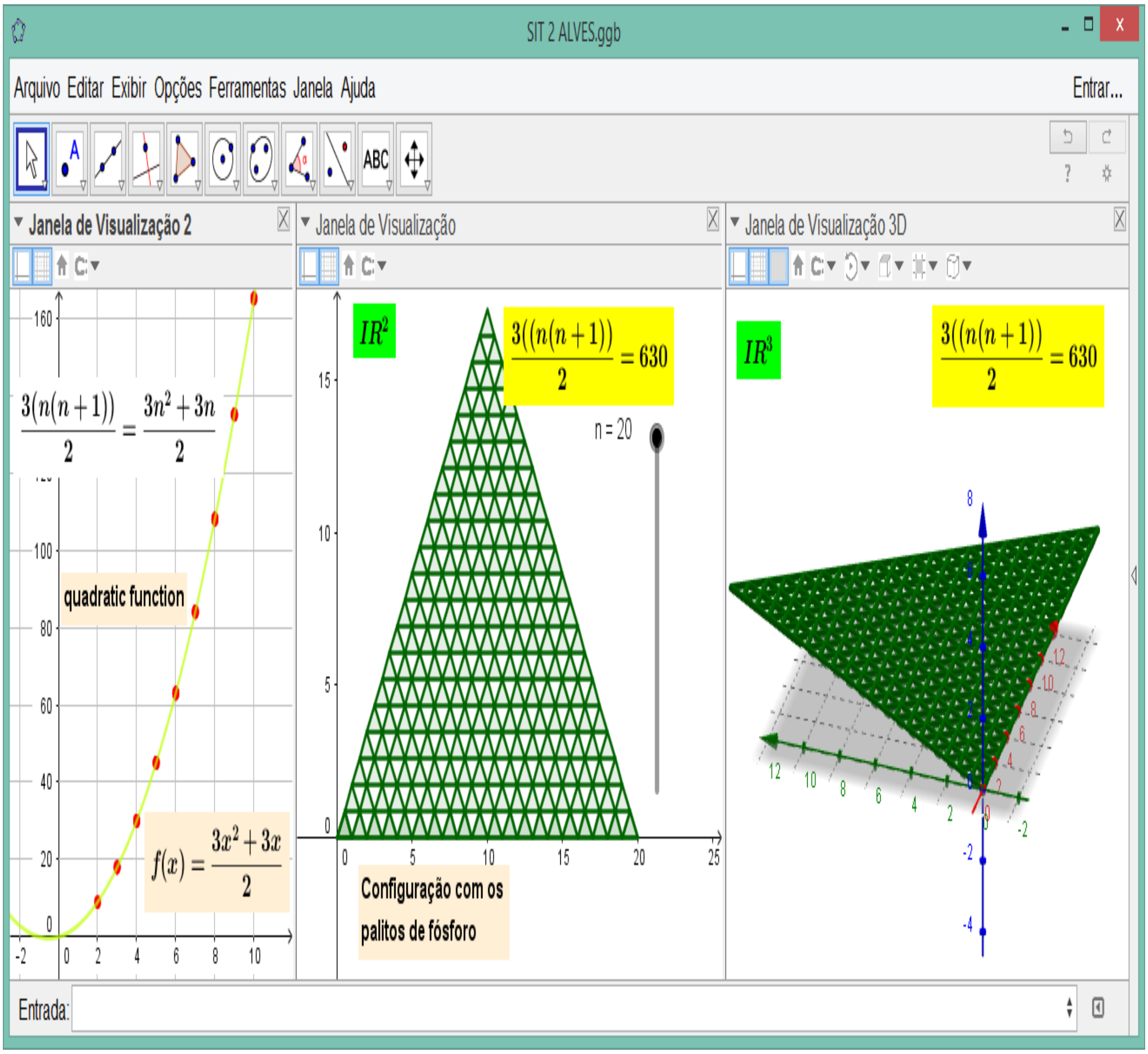


Figure 15. *Visualization of arithmetic and geometric properties (2D and 3D) with support in Geogebra software*

3. Conclusion

In the previous sections, we have approached and defined a new notion of Olympic Didactic Situation – (ODS). We show that, based on some assumptions of the Theory of Didactical Situations (TSD), whose French studies in Didactics of Mathematics represents a strong tendency in the production of academic studies and systematic investigations around the binomial (teaching – learning), we provide a bias of implication and repercussion for the an activity of the mathematics teacher who, despite acting directly or indirectly in Olympic competitions, needs to develop mechanisms for the inclusion and repercussion of an expanded mathematical culture that naturally emanates from the style adopted, for example, by the Brazilian Mathematical Olympiads of Public Schools (OBMEP).

Thus, based on the current observation that the ritual developed in (OBMEP) official competitions tends to gradually increase the character of the "classification" and social distinction of young prodigies, whose mathematical abilities are evidenced in harmony with the which is sought, for example, in the academic context of research in Pure Mathematics, consubstantiated by the systematic search for solutions to non-trivial problems. On the other side of the process, as we seek to emphasize, we find young students whose mathematical abilities are classified as "median" or even incongruent with a context of individual competition and which can not be disregarded in an intrinsically global and educational process.

In the set of three Olympic Problems (OP) explored in the work, with the use of GeoGebra software, we sought to present and stimulate the Mathematics teacher in Brazil an alternative for the teaching of Mathematics contents extracted from the official (OBMEP) tests. In general, we defend a perspective of involving more and more Brazilian students to the type or characteristics of a Mathematics peculiarly treated and presented in the context of mathematical competition, and not only the official competitive students or more skilled students from the point of view of Mathematics.

Significantly, based on the potential of GeoGebra software, the brazilien teacher has the possibility to stimulate student engagement in the dynamic exploration of numerical and geometric properties, so that visualization, perception and intuition play an essential role for evolution of the learning of all involved in each Olympic didactic situation (ODS). As we seek to present to the reader, in all three (ODS) we verified the role of visualization and the important activity of producing conjectures, the collective investigative work as a driving force for the construction of a mathematical social knowledge shared by a group of students. Unlike the individualising style of Olympic competitions!

Finally, we note, however, that all constructions with the GeoGebra software are previously elaborated and the domain of the corresponding computational construction for each Olympic problem is configured with an extra responsibility of the mathematics teacher, however, through the exploitation of the technology, the teacher can experiment teaching situations involving a larger number of students who, in general, are insecure about problems reportedly described as competition problems in (OBMEP).

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