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School-based mathematics teacher professional learning: A theoretical position on the lesson study approach

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This theoretical paper focuses on how school-based continuous professional development (CPD) for mathematics teachers in schools located in disadvantaged areas can be carried out using the lesson study approach. School-based CPD is based on the notion that teachers need real-time and on-site professional training tailored to improve the instructional practices unique to their school and classroom contexts. The paper seeks to address the research question: How can a lesson study [or research lesson] be used in the school-based CPD of mathematics teachers in schools located in disadvantaged areas? Empirical evidence from lesson study research suggests that it is an effective tool for CPD activities because it is school-centred, focuses on learner learning, and draws on the collective and collaborative experiences of teachers working in the same school mathematics department. Therefore, this paper significantly contributes to theoretical and practical debates on school-based CPD activities in schools located in disadvantaged areas, using lesson study. Further research would necessitate a focus on understanding how the social learning processes within school-based CPD can be linked to school contexts.

Keywords: continuous professional development; lesson study; mathematics noticing; research lesson

Introduction

This position paper focuses and discusses school-based mathematics teacher CPD learning in schools located in disadvantaged areas in the South African mathematics education context. In the context of this paper, "disadvantaged areas" refers to geographical locations where low-income earners reside. Schools located in these areas are usually under-resourced. The paper is based on the premise that mathematics teachers need new just-in-time and job-embedded instructional practices that are unique to their school and classroom contexts (Althauser, 2015).

The purpose of this paper is twofold: to significantly contribute to deeper theoretical understanding and debates about school-based CPD for mathematics teachers, as well as identifying ways in which the lesson-study approach can be used for teacher learning in schools located in disadvantaged areas. Research on lesson study as a form of CPD in the South African mathematics context is scarce. Researchers outside South Africa would be interested in the impact of lesson study as a form of CPD from a South African mathematics education perspective. Research in the past three decades reports that, on the one hand, teachers who are exposed to "frequent, rich learning opportunities have in turn been helped to teach in more ambitious and effective ways" (Little, 2012:1), while, on the other hand, teachers who experience sporadic, superficial, and disconnected CPD activities are left disappointed by limitations in learning opportunities (DeMonte, 2013).

The school-based CPD discussed in this paper, though not new, heralds a departure from traditional CPD, which depends almost exclusively on outside-school expertise from teacher educators and subject advisors – thus largely rendering the CPD inadequate, because it hardly addresses systemic challenges pertinent to individual teachers, and not within their school contexts (Dunne, Nave & Lewis, 2000). The other premise of this paper is that for a school to support high levels of learners learning effectively, the school must be accountable for the way in which teacher learning takes place within the school (Shaffer & Thomas-Brown, 2015).

Literature Review

In South Africa, and in South African Education Departments, most of the arranged CPD activities take place outside schools, where subject advisors and teacher education experts from institutions of higher learning take the lead. Part of the argument in this paper is that school-based CPD activities should be at the core of teacher learning and should also be linked purposefully and coherently to other external CPD activities (Little, 2012). Thus, what is the rationale behind focusing on school-based CPD or job-embedded professional development activities of mathematics teachers? Firstly, the school is at the centre of teaching and learning and this is where problems such as pressure to achieve quality teaching and learning are felt directly and evaluated as having failed or succeeded (Darling-Hammond, 2017; Woodland & Mazur, 2015). Secondly, school-based CPD connotes a direct association between a teacher's teaching in the classroom and the CPD that the teacher receives, and that the understanding and use of a school's learning culture can make a difference to the teacher's learning, job satisfaction and well-being (Postholm & Wæge, 2016). Thirdly, as a social learning process, lesson study represents a strong relationship between practice and teachers' needs (Koellner & Jacobs, 2015). Hadfield and Jopling (2016:205) posit that the nature of the lesson study "cycle makes it sensitive to how practice issues are constructed, and improvement is defined in a specific context."

There has been significant concern about the state of South African learners' academic achievements in the high-stakes National Senior Certificate (NSC) examinations, particularly the performance of learners whose schools are located in low socio-economic areas. One of the concerns faced by the South African government in the post-apartheid era is its failure to provide quality mathematics education for its diverse population of 55 million. According to Heneveld and Craig (1996:13), the word "quality" in relation to education, refers to "both changes in the environment in which education takes place and the detachable gains in students' knowledge, skills and values." In general, South African children are exposed to poorer quality of teaching than is found in its poorer neighbouring countries, particularly in Mathematics in the Foundation, Senior, and Further Education and Training phases (Taylor, 2008).

The National Planning Commission, The Presidency, Republic of South Africa (2012) acknowledges that in the main, black children receive poor quality education in South Africa; this denies these children access to higher education, and access to decent employment, leading to reduced earnings and limited career mobility for those who get jobs. International comparisons of educational achievement in Mathematics, for example, the Trends in International Mathematics and Science Study (TIMSS) conducted in 1995, 1999 and 2002, at Grade 8 level, have shown that South African learners underperformed significantly compared to those of other participating countries, and that the improvements achieved in the years 1995 to 2002 were statistically insignificant (Spaull, 2013). Although TIMSS shows that between 2002 and 2011 there was a huge improvement in Mathematics scores for Grades 8 and 9, South African learners' performance continued to lag behind those of learners in other middle-income countries.

While a number of factors have been reported pertaining to the performance of learners in Mathematics, this paper argues that the learners' backgrounds and the quality of the teachers and the teaching are the factors most intimately related to quality of education. According to the Organisation for Economic Co-operation and Development ([OECD], 2005:2), "factors to do with teachers and teaching are the most important influences on pupil learning" – a view shared by Barber and Mourshed (2007:12) who state that "the main driver of the variation in pupil learning at school is the quality of the teachers," and "the quality of the education system cannot exceed the quality of its teachers" (p. 41).

Recent lines of research concerning high-school mathematics teachers in South Africa have shown that these teachers possess inadequate common content knowledge (CCK) and specialised content knowledge (SCK), and that frequent and quality professional development activities are lacking, to name a few problems (Fleisch, 2008). While the notion is true that teachers can learn from social gatherings such as workshops and seminars outside the school (traditional CPD of teachers) (Meirink, Meijer, Verloop & Bergen, 2009), the reality in the South African mathematics education context is that the quality of mathematics teaching in disadvantaged areas remains poor (Taylor & Reddi, 2013). Bearing in mind the necessity for change in the CPD activities of mathematics teachers from schools located in disadvantaged areas, this paper's position is that school-based teacher learning can contribute to teachers' professional learning in these schools. Therefore, this paper seeks to theoretically address the research question: How can a lesson study [research lesson] be used in school-based CPD of mathematics teachers in schools located in disadvantaged areas?

The paper first discusses lesson study as a theoretical framework that can be used for school-based mathematics teachers' CPD. In addition, we briefly discuss two theories that underpin lesson study – variation pedagogy and learning trajectories. This is followed by an explanation of the context of teaching and learning Mathematics in disadvantaged schools. Following this, a discussion about operationalising lesson study is presented, before the paper concludes with some discussion.

Theoretical Framework

Lesson study

The paper addresses ways in which schools located in disadvantaged areas seek to improve the quality of the teaching and learning of Mathematics through mathematics teachers' CPD. To achieve this CPD, this paper proposes to use lesson study as theoretical framework. According to Lewis (2009), lesson study is a CPD approach:

> … in which teachers work together to: formulate goals for student learning and long-term development; collaboratively plan a 'research lesson' designed to bring to life these goals; conduct the lesson in a classroom, with a team member teaching and others gathering evidence on learner learning and development; and discuss the evidence gathered during the lesson, using it to improve the lesson, the unit, and instruction more generally. (p. 95)

Lesson study is a collaborative teacher-inquiry CPD with specific emphasis on reflection on practice and learners' cognition, leading to the development of a teacher's expertise and learning within the context of their work environment (Lee & Choy, 2017). As a form of CPD, lesson study is firstly concerned with the development of key aspects of mathematical knowledge for teaching (MKT) (Ball, Thames & Phelps, 2008). While being mindful of the inherent challenges associated with identifying teacher knowledge – the domains of teacher knowledge are neither simple nor uncontested (Leavy & Hourigan,

2018) – the paper focuses on SCK for teachers' CPD.

MKT is divided into two main sub-categories: subject matter knowledge (SMK) and pedagogical content knowledge (PCK). This paper does not intend to describe SMK and PCK in detail, except to mention that SMK is further subdivided into three categories: common content knowledge (CCK), specialised content knowledge (SCK), and horizon content knowledge (HCK). SCK is the essential knowledge for teaching, and it "allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, [and] provide explanations for common rules and procedures" (Hill, Ball & Schilling, 2008:377–378). For the purposes of this paper SCK is the most important, because it empowers teachers with the ability to interpret contexts and recognise common errors and misconceptions by learners; to justify, understand and assess mathematical productions; and to respond to learners' "why" questions. In addition, having SCK allows teachers to develop usable definitions, know alternative solutions, and analyse mathematics treatments in learning resources such as textbooks (Zembat, 2013).

Similarly, PCK is divided into three categories. Knowledge of content and students (KCS) refers to teachers' knowledge of their individual learners. KCS can manifest in the way that teachers' knowledge of their specific practices is seen to be changed or influenced by the way their learners learn. The other two categories are knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC).

Secondly, lesson study is also concerned with teachers' interpersonal relations. Planning, consultations among teachers, shared consciousness and visions on issues of learning and teaching Mathematics, among others, are shared beyond the participating teachers – in effect, they are shared throughout the school community and with other schools (Lewis, Perry & Murata, 2006).

Lastly, lesson study greatly influences teachers' personal qualities and dispositions, including their positive beliefs about their ability to implement desired instructional practice changes (Nelson, Slavit, Perkins & Hathorn, 2008). The question is: What makes lesson study as a form of a CPD approach appropriate for this study? I have alluded to the notion that this paper is about CPD in under-resourced schools. This means that the learning and teaching contexts in these schools are different from those of other, well-resourced schools. In addition, these schools are also different from other schools located in low socio-economic areas – each school is unique. Therefore, the use of lesson study as a CPD approach offers opportunities that do not usually form part of typical CPD projects, such as the top-down approach where experts take centre stage in CPD activities.

Firstly, CPD activities, through the use of the lesson study, are embedded in the school and classroom contexts (Chong & Kong, 2012), giving teachers the opportunity to see teaching and learning as it happens in real time in their school contexts. Secondly, lesson study affords the opportunity to observe teachers and learners interacting during lessons – meaning that learners are at the centre of CPD activities. Lastly, lesson study, as a form of CPD, is teacher-driven, meaning that teachers have a say in issues concerning the processes of instructional practice change and curriculum development.

Theories underpinning lesson study

The activities of lesson study are guided by notions of learning trajectory (Sztajn, Confrey, Wilson & Edgington, 2012) and variation pedagogy (Marton & Pang, 2006).

A **learning trajectory** provides a foundation for classroom learning as it allows for "descriptions of children's thinking and learning in a specific mathematical domain and a related, conjectured route through a set of instructional tasks" (Clements & Sarama, 2004:83). Researchers have stated that learning trajectory supports the growth of teachers' MKT, improves mathematical noticing during instruction, and allows learners' thinking to be visible to the teachers – leading to improved mathematical achievements by learners (Clements, Sarama, Spitler, Lange & Wolfe, 2011; Sztajn et al., 2012).

Variation pedagogy implies that learning is about developing ways of discerning and comparing critical features of the objects or concepts being studied; for example, discerning the similarities between and comparing squares, rhombi, and parallelograms, in geometry. In this example, while they are comparing and discerning the key features of geometrical shapes, learners are supposed to also discern the variant and invariant features of these shapes. The in-depth analysis of these varying shapes is necessary to allow the development of the learners' conceptual understanding of and conjecture about these geometrical shapes (Huang, Gong & Han, 2016; Ling & Marton, 2011). Ling and Marton (2011:21) posit that the variation theory provides "potential gains to the lesson study in the sense that it provides an additional theoretical component to guide decisions about teaching."

Understanding the context of teaching and learning in disadvantaged schools

Before indicating how lesson study can be operationalised as a CPD approach, it is important to understand what type of data can be collected. This will result in a clearer understanding of the teaching and learning contexts of Mathematics in schools located in disadvantaged areas. In other words, before implementing CPD through the use of lesson study, existing teaching and learning practices at the selected schools should be documented and analysed. This

analysis would include documented analyses of the school's vision and mission statements; the Mathematics curriculum statements; samples of learners' written assessments (projects, homework and assignments); and samples of learners' written work from classroom activities. Furthermore, it would also include focus-group interviews with teachers, learners and former learners, to gain a deep understanding of the teaching and learning of Mathematics and general instructional practices at the school. In addition, classroom observations would also be conducted.

A narrative approach is used to complement the document analysis and focus-group interview approach. Through narratives, rich, nuanced stories are captured of what teachers do and do not value and work within classrooms and schools. Learners' narratives are also important, in the sense that they reveal the lived experiences of learners, as they are shaped by teachers' instructional practices and the school learning environment. Equally important is the notion of how, while pursuing quality instruction, teachers deal with diversity among learners in terms of learning mathematics. For instances, in disadvantaged schools, like in other schools, mathematics teachers also deal with learners with different levels of mathematics achievement – these include gifted learners, average learners, and low-achieving learners.

These analyses of the teaching and learning context at the selected schools raise important questions about "what counts as mathematical knowledge and productive mathematical activity" (Nasir, Hand & Taylor, 2008:190). In other words, an understanding of different features of the social contexts and the interfacing of proclivities and disposition of learners will determine what and how mathematics is learned in the selected schools. Research in mathematics education has shifted towards the conceptualisation of knowledge as socially constructed, signalling the role of nature and culture in learning (Nasir & Hand, 2006; Nasir et al., 2008; Vygotsky, 1978). This means that the cultural practices that learners engage in, in their everyday and school contexts, will shape and influence what and how they learn mathematics in schools. Bourdieu (1977) argues, using the notion of **cultural capital**, that school mathematics as cultural practice is historically and socially reified through broader societal structures that privilege some or marginalise other groups (Saxe & Esmonde, 2005).

To summarise: research in mathematics education acknowledges the existence of boundaries between **cultural** and **domain knowing** and **coming** to know in three different ways. Firstly, the interface between the everyday use of mathematics and school Mathematics – also known as mathematics knowing, is cultural activity. Secondly, mathematics learning as cultural enterprise is represented by classroom, home and local community discourses. The third way refers to accessing and success in mathematics education – it is worthwhile looking at mathematics education as a cultural system (Nasir et al., 2008; Saxe & Esmonde, 2005). These three ways of perceiving mathematics education are positioned in this paper as critical features that inform the CPD of mathematics teachers at schools located in disadvantaged areas.

Operationalising lesson study during school-based CPD

As an effective CPD strategy for mathematics teachers, lesson study has many characteristics that make it an effective tool – "it is site-based, practice-oriented, focused on learner learning, collaboration-based, and research-oriented" (Murata, 2011:2). The construct of lesson study is simple: a group of teachers come together organically to share and address pertinent questions regarding learner learning – the teachers consider the learning goals where the learning challenge or challenges of the learners are located, plan a lesson or "research lesson," teach and observe the lesson, and reflect on the lesson (Bradshaw & Hazell, 2017; Dudley, 2015; Huang et al., 2016).

In the main a lesson plan consists of four stage activities: set goals; plan; implement and debrief. When **setting goals**, teachers identify a clear set of mathematics learning goals, the aim of which is to improve their learners' learning. For example, improving learners' learning could be about helping unmotivated learners to engage in learning of specific concepts such as percentage change. As a way to motivate learners, teachers can decide to embed the teaching and learning in everyday contexts. The goal can also be in the introduction of a specific mathematical concept; for example, the characterisation of quadratic functions using everyday life situations (Dudley, 2015).

When **planning,** teachers identify and design learning tasks using learning trajectory and variation pedagogy theories. The teachers plan the research lesson with three learners (or case learners) in mind – these learners could all be struggling with mathematics, or they could be higher, middle, or lower achievers. The idea here is that, as teachers plan the research lesson, they should think about how each of these learners would respond to each of the stages of the lesson – essentially, the teachers must predict the learning behaviours of each of the three learners (Cajkler, Wood, Norton & Pedder, 2014). Dudley (2015) cautions that research has shown that teachers often get their choices of case learners wrong – whatever they predict about these learners is often incorrect, as evidenced by the observations of the research study.

The strict planning discipline required of a research lesson has the potential to show that the predicted learning behaviours of learners can differ

from the behaviours observed by teachers at the beginning of the research lesson. In the absence of the research lesson, some of these differences between observed and predicted differences may not be noticed. Also, if the research lesson is not the first one in the series, then teachers are expected to focus on "responding to learners' thinking, based on their observations (attending) and interpretations of learners' thinking from the previous lesson(s)" (Amador & Carter, 2018:11). In other words, lesson study supports the development of professional mathematics-teacher noticing (Louie, 2018; Ulusoy & Çakiroğlu, 2018). Recent lines of research have revealed that teachers increase their SCK when attending to learners' thinking during the professional noticing phases of attending, analysing, and responding to learners' mathematical thinking (Amador & Weiland; 2015; Weiland, Hudson & Amador, 2014).

The planning stage is followed by an **implementation** stage, in which one of the teachers leads the teaching, while the others act as observers. In certain instances, all four stages of the research lessons are audio- and video-recorded, with the aim of sharing the clips in the future (Vrikki, Warwick, Vermunt, Mercer & Van Halem, 2017). During research lesson observation, the focus is observing learners' learning in the context of the content being taught, rather than the teaching (Schipper, Goei, De Vries & Van Veen, 2017). Observing learners' learning is at the heart of lesson study; it provides a rich narrative or "detailed picture of the effect of the research lesson on pupils' thinking and cognition" (Dudley, 2015:10). Teachers are encouraged to create a learning environment where learners interact and are able to share their thoughts with other learners when solving tasks, in addition to initiating whole-class discussions of learners' works, and helping learners build connections between mathematical concepts and topics.

Debriefing after a research lesson should consist of gathering evidence about learners' learning from research lesson observations (Bradshaw & Hazell, 2017; Lomibao, 2016; Skott & Møller, 2017). Post-research lesson observation should not focus on aspects of teaching unless they are specifically related to learners' learning or mis-learning. Teachers can use learners' written work, and focus-group interviews to inform post-research lesson discussions, in addition to their observation narratives (Dudley, 2015).

The advantage of the lesson plan is its simplicity of implementation in small groups of teachers; and that no technology or prior experience is required from the group of teachers (Dudley, 2015). In the context of teaching Mathematics in disadvantaged schools in South Africa, this paper proposes that a research lesson as a form of professional development could initially be co-implemented by mathematics teachers, teacher educators, and subject advisors – though this is not a requirement. At the beginning of the research lesson cycle, the group of teachers should have a clear focus on what they need to improve as far as their learners' learning is concerned (Dudley, 2015; Huang et al., 2016; Leavy & Hourigan, 2018).

Discussion and Conclusion

This position paper supports and acknowledges the role that lesson study can play in the CPD of mathematics teachers in schools located in disadvantaged areas in South Africa. Currently, learner performance in schools located in disadvantaged areas is poor and is likely to remain so for the foreseeable future, unless there is a shift in terms of the MKT of teachers – lesson study as CPD can help to achieve the goal of developing teachers' MKT in these schools. International readers could be interested in reading on how lesson study can be used as a form of CPD in schools located in disadvantaged areas. While the construct of lesson study is not new, recent lines of research have shown that, as an example of school-based CPD, it affords teachers the opportunity to improve their SCK within their work environment. The premise of this paper is that mathematics teachers in disadvantaged schools have been exposed to other forms of CPD activity, and the majority of these CPD activities have largely been driven by external experts – people who do not know these schools' learning cultures and contexts. Teachers from disadvantaged schools attend generic CPD workshops facilitated by these experts, together with teachers from other schools – a one-size-fits-all scenario. External experts include teacher educators, subject advisors, and non-governmental organisations, to name a few.

This paper advocates that school-based CPD through the use of lesson study can provide a solution to the CPD of mathematics teachers in disadvantaged schools. Research has shown that teachers learn when engaged in CPD activities that are "embedded within their school and classroom contexts" (Chong & Kong, 2012:264). In other words, lesson study as a social co-learning process, as a form of CPD, is sensitive to the instructional practices and teachers' needs within these disadvantaged schools (Koellner & Jacobs, 2015). CPD for teachers should focus clearly on defined learning goals, thereby providing opportunities for deepening their learning. If the CPD is school-based, it becomes the responsibility of the school, at an organisational level, to create a conducive environment for teacher professional learning in the form of processes that support instructional change – for example, freeing blocks of time in the school calendar for teachers to meet regularly (Chong & Kong, 2012; Hadfield & Jopling, 2016; Little, 2012). In addition, school administrators are compelled to set aside the necessary resources to support teacher skills development and remove any bureaucratic structural obstacles that may hinder the change processes (Chong & Kong,

2012). Kieran, Krainer and Shaughnessy (2013) argue that treating mathematics teachers as stakeholders during lesson study empowers them to build linkages between theories and practice through "critical features of research … [such as] a significant action-research component, and dynamic duality of research and professional development" (Huang et al., 2016:436–437).

This paper recommends and supports the notion that lesson study has the potential to improve the SCK of mathematics teachers in disadvantaged schools, and that this potential can only be realised when teachers acquire the skills to examine their lessons critically (Lee & Choy, 2017; Takahashi & McDougal, 2016).

According to Fernandez, Cannon and Chokshi (2003), three lenses may be used to examine mathematics lessons within the lesson study CPD approach. **The researcher lens** encourages teachers acting as researchers to identify problems of practice, design appropriate strategies to solve them, and use the findings to inform the success of their interventions. **The curriculum development lens** refers to how teachers sequence learning activities and align them to the learners' learning and cognition during lessons. **The learner lens** is about how teachers predict possible solutions and challenges to learners' learning tasks and use these predictions to inform further learner engagement.

In conclusion, we present lesson study as an alternative CPD paradigm for mathematics teachers in schools located in disadvantaged areas. The study significantly contributes to ongoing debates on school-based CPD for mathematics teachers. Further research should focus on understanding how the social learning processes within school-based CPD are linked to the contextual environments in which they are enacted in selected schools.

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Notes

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