Waikato Journal of Education

Te Hautaka Mātauranga o Waikato

Volume 21, Issue 2: 2016

Language challenges in mathematics education: A literature review

Laura Jourdain and Sashi Sharma

Te Kura Toi Tangata Faculty of Education

The University of Waikato

New Zealand

Abstract

Traditionally, the difficulties associated with mathematics were largely seen as coming from the cognitive demands of mathematics itself. It is now accepted that language and mathematics are connected in mathematics learning and teaching, and, the potential challenges of language in mathematics have been investigated by a number of researchers. This paper reviews research by applied linguists and mathematics educators to highlight the linguistic challenges of mathematics and suggests pedagogical strategies to help learners in mathematics classrooms. The linguistic challenges include the linguistic features that may make mathematical texts hard to understand, vocabulary in academic mathematics, and reading and writing in facilitating the learning of mathematics. Research on pedagogical practices supports developing mathematics knowledge through attention to the way language is used, suggesting strategies for moving students from informal, everyday ways of talking about mathematics into the registers that construe more technical and precise meanings.

Keywords

Mathematics education; linguistic challenges; English language learners; every day language; mathematical register; connecting language and mathematics; implications for practice

Introduction

Imagine a teacher running her fingers across the pages of the textbook and telling her students, "A sample is a subset of a population, chosen freely without any rule or any obvious bias". The students listen quietly, but one of them is thinking, 'I thought it was a small amount of something like blood to test whether a person has some disease or not'.

Commonly, mathematics is viewed as universal and free of cultural influences (Brown, Cady, & Taylor, 2009; Hoffert, 2009; Meaney, 2006). It uses a variety of symbols that are common across cultures and is therefore easily accessible to language learners. However, as the above example illustrates, the language of mathematics can sometimes be confusing and challenging when students bring their prior knowledge to it (Barwell 2011; Boero, Douek, & Ferrari, 2008; Lavy & Mashiach-Eizenberg, 2009).

Mathematics is strongly connected with language and to succeed in mathematics, a student must be able to competently understand and use mathematical language (Anthony & Walshaw, 2007; Boero et

Wilf Malcolm Institute of Educational Research, Faculty of Education, University of Waikato, Hamilton, New Zealand

ISSN: 2382-0373

(pp. 43–56)

Author Contact: Laura Jourdain: lauravjourdain@gmail.com

al., 2008; Kazima, 2006; Kotsopoulos, 2007; Schleppegrell, 2011; Xi & Yeping, 2008). Boero et al. for example, wrote about the role of natural and symbolic languages in mathematics, asserting that "only if students reach a sufficient level of familiarity with the use of natural language in the proposed mathematical activities can they perform in a satisfactory way" (2008, p. 262). They discussed language as a mediator between mathematical objects, properties, and concepts and the development of theoretical systems. It is undeniable that language and mathematics are connected in mathematics learning, and this is particularly important when we realise that there is a steady increase in multi-lingual learners in New Zealand classrooms. This means that teachers need to be aware of the issues surrounding teaching mathematics to English language learners and plan accordingly. Not being aware of these issues can have detrimental consequences for all students.

This paper has three major sections. The first section draws on mathematics education research to discuss the challenges faced by English language learners. The second section discusses strategies teachers can use to support learners. The final section considers some issues arising out of the discussions. It offers suggestions for meeting these challenges.

Problems faced by English language learners in mathematics

There is a growing demand on students' linguistic skills in mathematics lessons (Anthony & Walshaw, 2007; Cobb & McClain, 2004; Moschkovich, 2005; National Council of Teachers of English, 2008). Students at all levels are not only expected to listen, talk and read using mathematical language, but also to write about their work in mathematical language (National Council of Teachers of Mathematics. 2000). Students must not only understand mathematical concepts but also demonstrate that they are able to connect their mathematical knowledge with their everyday life and communicate their knowledge to others (Altieri, 2009; Franke, Kazemi, & Battey, 2007). However, reasoning at complex cognitive levels through mathematical discourse is not something many students are able to achieve easily. This is often due to interference from everyday language and within the mathematical register (Anthony & Walshaw, 2007; Cobb & McClain, 2004; Ferrari, 2004; Kotsopoulos, 2007; Schleppegrell, 2011).

Mathematical registers

One of the key challenges that impacts on English language learners in their ability to learn mathematics in an English-medium environment, is the complexity of working within English language registers. Halliday's (1978, p. 195) seminal work provided a definition of a register: "A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings". We can refer to a 'mathematics register', in the sense that meanings belong to the language of mathematics (the mathematical use of natural language, that is, not mathematics itself), and that a language must express if it is being used for mathematical purposes.

In the mathematics classroom, multiple registers are used. For students to succeed in a mathematics classroom, they need to not only be familiar with and competent in their ordinary English register, but must also have fluency in what can be termed multiple mathematical registers (Boero et al., 2008; Kazima, 2006; Moschkovich, 2005; Schleppegrell, 2011). The mastery of the mathematical registers, and the strong ability to switch between them, requires strong linguistic and metalinguistic skills. These skills are necessary for students to be able to communicate with their classmates, and cope with more advanced mathematics (Boero et al., 2008; Mandy & Garbati, 2014; Planas & Setati-Phakeng, 2014; Setati & Adler, 2001).

For a student from an English speaking background, mathematical registers can pose a significant challenge, as a new form of language must be learned and mastered (Campbell, Adams, & Davis, 2007; Mandy & Garbati, 2014; Moschkovich, 2005; National Council of Teachers' of Mathematics, 2008; Schleppegrell, 2011). Not only must an English language learner try to learn in English whilst concurrently learning to speak English, she or he must also be working within the English mathematical registers without yet having mastery of ordinary English (Lager, 2006; Mandy &

Garbati, 2014). Hence, to understand mathematics in an English medium classroom, English language learners may require more time for processing than native English speakers (Barwell, 2005; Clarkson, 2007; Latu, 2005; Meaney, 2006). These students can miss out on mathematical learning because they may be spending too much time trying to understand the questions and navigate using these registers.

Furthermore, to be able to perform competently, students must understand the highly technical language used specifically in mathematics (Brown, et al., 2005; Moschkovich, 2005). This language is not used within the ordinary English registers, and therefore is less likely to be familiar or understood by English language learners. The technical language and vocabulary mathematics has is not only essential for students to be able to understand and access the mathematics they are learning now, but has a significant influence on their future mathematical development (Hoffert, 2009; Neville-Barton & Barton, 2005; Xi & Yeping, 2008).

Having to work within multiple registers can therefore result in extra challenges for English language learners. This is especially relevant when it comes to learning vocabulary, where the meaning of words may change between registers.

Ordinary English versus mathematical English

Students come to mathematics with an existing vocabulary. However, some words they are familiar with may hold different mathematical meanings and may be used in mathematics to express different or very specific concepts (Anthony & Walshaw, 2007; Clarkson, 2007). Kotsopoulos (2007) discusses this issue, stating that for a student coming across familiar words in mathematics with different meanings, mathematics can seem like a foreign language. Examples of these words include: *mean*, *operation, element,* and *similar* (Kenney, 2005). It is important that students are taught these words in a mathematical context because they will differ from their everyday English use.

In addition, mathematics contains a number of new words, outside of those used in everyday English (Rangecroft, 2002; Winsor, 2007). This requires students to learn these words and their specific mathematical meanings and how they are applied, if they are to experience success in mathematics. Frequently, due to word origin, such words do not follow a mathematically logical pattern that is obvious in English. For example, in English the number 12 is "twelve", a word that has no obvious relationship to other numbers, whilst in Chinese the number 12 is "ten and two", which shows the relationship between numbers (Kenney, 2005).

Furthermore, statistics has a different set of words compared to regular mathematics. As well as words that are used in everyday and mathematical English, there are also words that are specific to statistics (Lesser & Winsor, 2009; Kaplan, Fisher, & Rogness, 2009). There is also the added confusion with words that differ in ordinary mathematics and statistics, or ordinary English and statistics (Rangecroft, 2002). One example of this is the word 'significant'. It has one meaning in ordinary English, a second in mathematical English (when thinking of *significant numbers*), and a third meaning in statistics (thinking of statistically *significant data*). As a result, grappling with the language of statistics can be difficult for a number of students when their prior knowledge has them believing the word means one thing, but in a specific mathematics context, it means another (Lesser & Winsor, 2009).

Language translation

Not only do English language learners have to contend with a variety of vocabulary within mathematical registers and changing meanings between registers, they also need to deal with concepts that do not exist in their home language vocabulary. Latu (2005) noted that English words are sometimes phonetically translated into Pasifika languages to express mathematical ideas when no suitable vocabulary is available in the home language. Kazima (2007) and Fasi (1999) independently made the same point. Fasi, in his study with Tongan students, found that concepts such as "absolute value", "standard deviation" and comparative terms like "very likely", "probable" and "almost certain" have no equivalents in Tongan language. Latu (2005) and Fasi (1999) also suggested that special courses in English mathematical discourse be delivered with the intent of connecting the underlying meaning of mathematical concepts in English with the students' home language.

Syntax

English is a complex language with a complex syntax (Kazima, 2006; Padula, Lam, & Schmidtke, 2001; Schleppegrell, 2011). Sometimes, the structure of natural English is at odds with the conventions of mathematical language structures (Dawe, 1983; Kaplan et al., 2009).

Issues surrounding syntax were discussed by Neville-Barton and Barton (2005), who looked at tensions as experienced by Chinese Mandarin-speaking students in New Zealand schools. Specifically, their study focused on the difficulties that could be attributed to limited proficiency with the English language. It also sought to identify language features that might have created difficulties for students through administering two tests, seven weeks apart. In each test, one half sat the English version and the other half sat the Mandarin version. Across both tests, each student sat both versions. The results showed there were noticeable differences in their test performances over the two versions. On average, students were disadvantaged in the English test by 15 percent and Neville-Barton and Barton reported that the syntax of mathematics discourse had created problems for them. In particular, prepositions, word order and interpretations of difficulties arising out of the contexts. The researchers found, in addition, that the students' teachers were not aware of their students' misunderstandings. These findings are consistent with the claims made by Dawe (1983) and Mousley and Marks (1991). The latter authors remark that, in relation to students from low socio-economic background, "Logical connectives are commonly misunderstood by students from lower socio-economic classes, those not so familiar with the 'secondary school' language of argument, justification and context specific materials" (p. 62). If this problem of prepositions, word order and interpretations is true of second language learners, *and* lower socio-economic groups, then there is a serious problem around mathematical language teaching and learning.

Reading mathematics

The language of mathematics is expressed in mathematical words, graphic representations and symbols (Kenney, 2005; Lampert & Cobb, 2003; National Council of Teachers of Mathematics, (2000). Mathematical texts provide learners with an extra challenge while reading English. Learners must simultaneously comprehend and process information in both the language of English and the language of mathematics (Kester-Phillips, Bardsley, Bach, & Gibb-Brown, 2009; MacGregor, 2002; Raiker, 2002) and then undertake the required mathematical task.

Redundancy is one characteristic of ordinary English that has a significant influence on how students (mis-)read mathematical English (MacGregor, 2002) because ordinary English has a high degree of it. Consequently, students learn to skim read, sampling key words to get to key points, such as when reading a novel. In comparison, mathematical English is concise; each word has purpose with little redundancy, and a large amount of information is contained in each dense sentence (Padula, et al., 2001). Students who transfer the same reading skills from the expectations of reading ordinary English to mathematical English texts, may therefore be disadvantaged by the tendency to overlook key information (MacGregor, 2002; Raiker, 2002). Cultures with less redundancy in their languages are more likely to pay attention to every word and therefore understand better some forms of mathematical English, despite this being an additional language (Latu, 2005; Padula et al., 2001).

Barwell (2005) argues that students need to learn how to read and solve word problems. He claims that simply decoding words or extracting arithmetic operations may not be sufficient; students must learn to read between the lines to understand what they are expected to do mathematically. Mousley and Marks (1991) argue that the way students read the systematical structure of sentences, relational statements, order of sentences and logical connections can hinder conceptual understanding, while Adams (2003), in advocating a more active role of reading in mathematics learning, listed the following five points as possible areas of difficulty:

- formal definitions;
- multiple meanings of words (particularly those that are both used in everyday interactions and in mathematics discourse with different meanings);

- homophones and similar sounding words;
- the interaction between words, numerals, and symbols; and
- the significance of order in math.

According to Latu (2005), students' ability to read in the language of instruction is vital to their performance in most academic disciplines, adding that if students aim for success in mathematics, but are continually hampered by reading problems, then frustration and reduced self-expectation is likely to occur.

Code switching

Code switching involves the movement between languages in a single speech act and may involve switching a word, a phrase, a sentence or several sentences (Adler, 1998). Students' use of code switching and teachers' attitudes towards it can also have implications for English language learners. English language learners may code switch for various reasons, including to seek clarification and to provide an explanation (Moschkovich, 2002). Code switching can promote both student-student and student-teacher interactions in classrooms involving ESL students (Setati & Adler, 2001). Code switching can be used strategically and advantageously by teachers as a way of utilising a students' home language as a resource (Kasmer, 2013; Parvanehnezhad & Clarkson, 2008; Planas & Setati-Phakeng, 2014; Winsor, 2007).

In the mathematics classroom, English language learners often use code switching to clarify their understanding and to express their arguments and ideas (Clarkson, 2007; Moschkovich, 2005; Parvanehnezhad & Clarkson, 2008; Planas & Setati-Phakeng, 2014). Code switching not only occurs between languages but also between registers potentially adding an extra layer of challenge to English language learners as they find themselves working between a multitude of registers in both English and their home language (Barwell, 2005; Lager, 2006; Moschkovich, 2002). In a study of Australian Vietnamese learning mathematics in Australia for example, Clarkson (2007) found that some of these students switched between languages when solving mathematics problems because solving problems in their first language "gave them more confidence" (p. 211) or switched their languages because they found the problem difficult to solve in English. Such linguistic complexity for English language learners further demonstrates the need for instruction specifically geared toward meeting the needs of English language learners.

Perhaps this links to a common misconception teachers hold about code switching. Some believe that code switching has negative implications for students or that students should be required to work exclusively within the language of instruction. This assumption about code switching is frequently a reflection of wider societal attitudes (Planas & Setati-Phakeng, 2014; Walker, Shafer, & Liams, 2004) and reflect wide variations in teacher attitudes towards the use of a students' home language in classrooms. Walker, Shafer, and Iiams' (2004) study of teacher attitudes towards English language learners in an area with an historically low but rapidly increasing English language learner population found that just over half (51%) of the teachers responded either favourably (15%) or neutrally (46%) to the idea that students learn English best when prohibited from using their home language. There were also examples of principals issuing blanket bans on any language but English throughout entire schools.

Word problems

Word problems often use words found outside mathematical registers and this causes problems for some students. Barwell (2005) for example, stated that students from minority cultural or linguistic backgrounds will often have difficulties with word problems because they are unlikely to be familiar with these words. When students grapple with vocabulary or syntax they are unfamiliar with before being able to attempt the mathematical problem, can negatively impact on their learning. This may not only have an effect on their ability to learn in class, but can also invalidate assessment measures. In a California study with eighth grade students for example, Abedi and Lord (2001) found that the gap in mathematics achievement between English language learners and students from English speaking backgrounds widened when students were assessed using word problems. When the word problems were rewritten in such a way that the content was the same, however, but the grammar, syntax and vocabulary used were simpler; the gap between English language learners and students from English speaking backgrounds was reduced. Examples such as these have implications for mathematics teaching in New Zealand schools.

Moreover, putting problems in unfamiliar contexts can hinder student progress. Brown, Cady, and Taylor (2009) give the example of a young Mexican girl in an American classroom being given a word problem involving hanging streamers for a party. The girl was unaware of the word streamer, and in her culture streamers weren't necessarily as ubiquitous at parties as they were in the area of the United States that she was living in. As a result, this child tapped into her prior knowledge, knowing that a stream was a moving body of water. She thus assumed that a streamer was related to a stream. She then spent a significant amount of time figuring out how streams could be measured and quantified in a room. If the language context had been accessible to this student, these issues could have been avoided. She would have been able to focus on the mathematics, rather than deal with the complexities the language context presented her with.

However, this does not negate the value of word problems. Instead, it highlights the need to design problems that are appropriate and meet the learning needs. A contextual image might help reduce misunderstandings with a word problem for example. An image of streamers for a party springs to mind. Using word problems in classrooms has a number of benefits so they are worth using because students can gain a deeper understanding of specific concepts through being applied in a real world context. Word problems should be designed to enable students to make sense and meaning of a problem, rather than as a means to demonstrate mastery and understanding of a computational property (Barwell, 2005; Moscardini, 2010).

Strategies for supporting English language learners

There are many strategies teachers can use to address language challenges discussed in the previous section and some are discussed next.

Mathematical reasoning

According to Moschkovich (2010), mathematical reasoning should not be restricted to the specific reasoning taught in class. Instead, all students, including English language learners, should be encouraged to problem solve using the techniques and strategies they know and are most familiar with (Brown et al., 2009). Through encouraging and accepting diverse strategies and techniques for solving mathematical problems, students' mathematical understanding of mathematical processes is enriched. (Brown et al., 2009). Furthermore, through teachers embracing diverse strategies that English language learners for example might bring to class, these learners can be seen as having expertise among their peers. This would happen alongside teaching everyone in a class the mathematical language and vocabulary needed for each topic.

Mathematics requires students to use a high degree of language. It is important, therefore, that teachers recognise their dual role as both mathematics teachers and language teachers when teaching English language learners (Brown et al., 2009). This helps students to meet the language demands of mathematics and therefore to successfully reason mathematically in an English speaking context.

Home language

A key strategy to support English language learners' mathematical development is accepting and encouraging the use of students' home languages in the mathematics classroom. To best support students, home languages should be recognised as resources, not as hindrances, much as their different mathematical strategies might be respected (Moschkovich, 2010; Planas & Setati-Phakeng, 2014).

When students can access and use their home language, their development of mathematical vocabulary is faster (Xi & Yeping, 2008).

Deliberate strategies from students' home languages can facilitate their mathematical learning and language development. One of these is a mathematical word bank, where students are given mathematical words with translations in their home language (Lee, Lee, & Amaro-Jiménez, 2011). These also facilitate relationships with parents, who they can then understand the language used in their child's mathematics homework and other work (Lee et al., 2011).

As well as integrating students' home languages, it is also important to ensure that familiar English words are used. English language learners typically develop their proficiency in everyday, conversational English before developing academic language proficiency, including proficiency in the academic language used in mathematics (Lee et al., 2011). As a result, it can often be beneficial to use everyday English terms that the learner may be familiar with to prompt their understanding of academic English terms. Lee et al., in illustrating this point, used the word 'difference' to prompt a student's understanding of the word 'subtraction'.

Visual aids/diagrams/manipulatives

Visual strategies are helpful in scaffolding students who may not have language skills that match their mathematical ability (Altieri, 2009; Nguyen & Cortes, 2013; Lee et al., 2011). When visual aids such as diagrams and posters are used, students who may not have the ability to pose their questions in English or who do not have the confidence to approach their teachers, are enabled to find answers (Nguyen & Cortes, 2013). Furthermore, physical items can assist English language learners; for example, props to support their comprehension of word problems (Brown, Cady, & Taylor, 2009; Nguyen & Cortes, 2013). Using tools such as visual aids and manipulatives can help to contextualise mathematics. This is important because academic mathematical language is frequently abstract, which is challenging for English language learners (Lee, et al., 2011; Moschkovich, 2002).

Using manipulatives can be an effective strategy for teaching all learners, especially English language learners (Altieri, 2009; Nguyen & Cortes, 2013). When manipulatives are used with English language learners, the language load of mathematics is reduced because learners are able to learn through seeing and physically manipulating resources (Nguyen & Cortes, 2013).

Collaborative learning

Collaborative learning is a powerful tool for all students, but especially for English language learners. When English language learners are able to work alongside a partner, they are given the opportunity for interaction and support, enhancing their learning (Brown, Cady, & Taylor, 2009). Collaboration affords English language learners the chance to ask questions and make mistakes in a safe setting, where they can receive direct and immediate feedback. Furthermore, when students are engaged in authentic conversation and interaction, it fosters language development. This is especially true when English language learners are partnered with a peer who has a higher degree of English language proficiency. Strategic use of flexible grouping to meet specific needs, can support English language learners' mathematical understanding and development (Ngyuen & Cortes, 2013).

Word origins

The origins of words can help older students make connections between mathematical terms. Rubenstein (2000) claims that teaching the derivation of a word can offer students insights into the construction and meaning of the word. This in turn increases the likelihood that the word will be remembered. For example, *trigonometry* may sound new, but when broken down into its components can be linked to existing ideas: *tri-* for three, *-gon* for form and *metr-* for measurement. So when a student finds out it is about triangles and measurements they can make sense of it more clearly than just the word itself, especially if they have had experience with those three aspects before—triangles, shapes and measurement (Benjamin, 2011).

Concept circle

This is a categorisation strategy (Altieri, 2009) encouraging students to study words critically, relating them conceptually to one another. The strategy involves writing a word or phrase in each section of a circle and asking students to describe the common attributes they ascribe to the word or phrase or label. For example, in Figure 1, the first circle identifies angles formed when a transversal crosses a pair of parallel lines. Alternatively, the name or the label of the concept circle can be provided, where one section of the circle is blank, and students are to provide other examples to complete the concept circle.

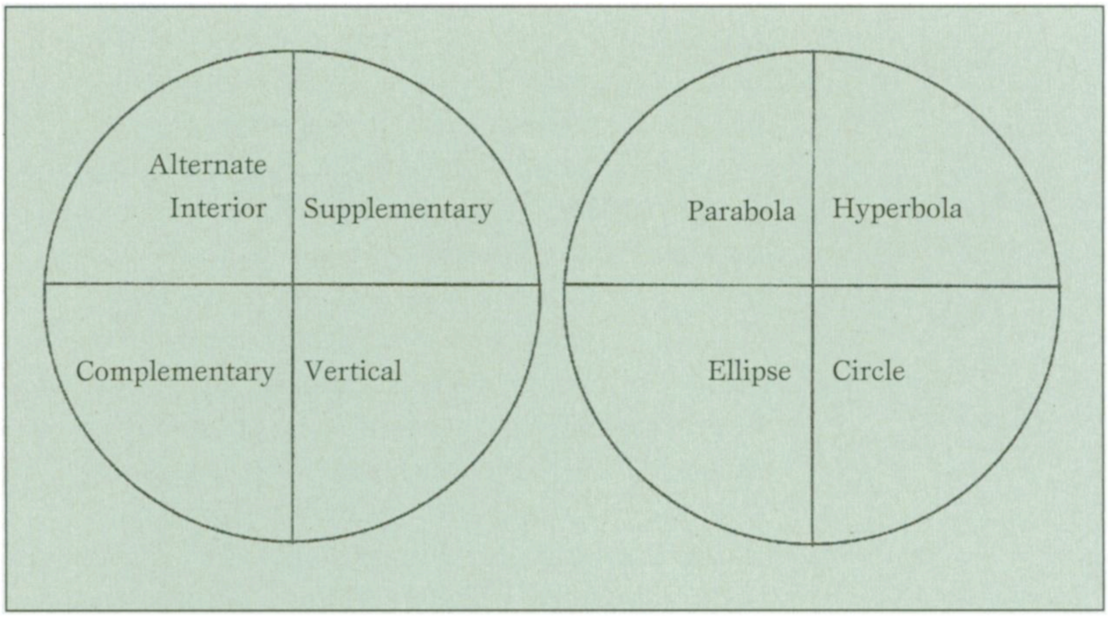


Figure 1. Examples of categorisation strategy for types of angles and non-linear graphs (Altieri, 2009)

Puzzles and games

Crosswords are excellent for developing links between mathematical words and meanings, as they challenge students to make the connections in reverse (Altieri, 2009; Benjamin, 2011).

Card games can be a fun way of helping students to link verbal, symbolic, graphic or story representations of the same values and concepts. Figure 2 shows playing cards with four sets of equivalent geometry concepts. The cards can be used in traditional games like go fish, or rummy. The cards can be expanded to include a variety of strands such as number, measurement and algebra. For a richer learning experience, students can create their own set of cards for new terms and symbols they have learnt.

A set of cards using terminology, symbols and pictures can be presented for the "I have … Who has …" game format. The teacher could demonstrate first then students could play this in groups, following the usual practice in card games of shuffling the deck then handing out the cards first. For example, "I have an isosceles triangle. Who has a figure with all sides congruent and all right angles?" Or, "I have a square. Who has an equilateral triangle?" One student begins by reading the second line of the card, the student who has the same word in the first space of the card reads next. This game can easily be tied into mathematics classroom of multiple levels and takes minimal time while supporting the review of facts and concepts.

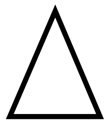
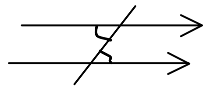
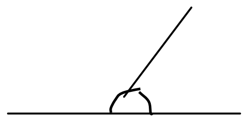
Isosceles triangle	A triangle with two sides of equal length and two equal angles at the vertices		A foldout ladder makes this shape when opened up and placed on the ground
Rectangle	A quadrilateral with two pairs of parallel sides and right angles at each vertex.	Area equals Length x width	
Alternate angles	Internal angles of a Z drawn by two parallel lines and a transversal	Angles formed on alternate sides of a transversal and a pair of parallel lines	
Supplementary angles		Two angles that add up to 180°	Adjacent angles on a straight line

Figure 2. Examples for a card game (Each row represents an equivalent value), (Rubenstein & Thompson, 2002)

Rich tasks/word problems

Worthwhile tasks bring mathematical vocabulary and context together. In order to be successful, students must understand both. Understanding and solving problems become more important at senior levels of high school. Providing non-linguistic cues such as visual diagrams and drawings can make more complex language accessible for all (Brown, Cady, & Taylor, 2009).

A process for solving word problems proposed by George Polya (1945, as cited in Adams, 2003) follows:

1. **Read the problem**. Read the problem in its entirety to ensure an understanding of the context and problem in full.
2. **Understand the problem**. This second phase is about applying the understanding of vocabulary and thinking about the demands of a problem. What information is needed, or extraneous? Can students restate the problem in their own words and translate the English into mathematics?
3. **Solve the problem**. This is about selecting and using appropriate strategies to respond to problems
4. **Look back**. Revise, check validity and accuracy of solutions.

Furthermore, using English language learners' contexts and stories can help bridge some language challenges (Lee et al., 2011). Providing opportunities for learners to share their stories and use them within mathematics problem solving not only means that they will understand the contexts used, but also will show these learners that their cultural resources are valid and valued. Successful teachers of such learners connect to their experiences and cultures, showing them in real ways that they are valued (Lee et al., 2011).

Giving learners opportunities to write their own word problems can benefit those learning English, for it facilitates both mathematical and language development. Self-written problems also guarantee that familiar contexts are used.

Journal writing

Winsor (2007) found journal writing helped English language learners with their mathematics and mathematical language simultaneously. While writing in their most comfortable language, students also had to write mathematical terms in English. Using mathematical terms in English helped students associate English terms with mathematical concepts already in their minds. An integral part of journal entries was evaluation. At the end of each week, students evaluated their peers' work using a simple three-point rubric where they exchanged journals with a partner, assigning a score to specific journal entries and give written rationales for scores. The journal writer then read and responded to the journal evaluation, using the same rubric. Winsor claims that students benefited in a number of ways. First, writing about the mathematics forced them to decide what they did and did not understand and to put those thoughts on paper. Next, they became more proficient in communicating mathematically. Although the first journal entries were unsophisticated and relied on non-mathematical terms to express their ideas, as the year progressed, their journal entries became mathematically precise. Equally, students used English more frequently in their writing. Students put more effort into journal entries because they knew that their peers were reading them and would often discuss one another's evaluation when they did not agree with or understand the evaluation. The discussions were often mathematical in nature because students had to explain their reasoning to their peers.

Another simple writing strategy integrates writing and problem solving. On take-home problem-solving assignments, students fold the paper vertically down the middle (Rubenstein & Thompson, 2002). On the left they record their mathematical problem-solving work and on the other side record explanations of their thinking. Thompson and Thompson (2002) found although students initially remarked that these assignments Rubenstein took a long time to complete, they also noted that the writing strengthened students understanding. In particular, English language learners showed marked improvement with this strategy.

Implications for classroom practice

Mathematics does not exist in a language-free vacuum. This means that when teaching English language learners, mathematics teachers must not only be looking to teach mathematics in and of itself, but recognise their dual role as mathematics teachers and language teachers (Brown, Cady, & Taylor, 2009). Teachers need to be aware of, and sensitive to, issues of mathematical language acquisition and to be creative and persistent in findings ways to support students' learning. Planning for learning is easier if teachers are aware of common language difficulties, such as those discussed in this paper.

Although language may present a barrier to participating in mathematics, it also provides a solution. The key to success in mathematics is being able to own the language and concepts that make mathematical language different from ordinary English. Making connections between mathematics terms and their own language and experiences, and to use their mathematics language through discussion is important to foster in classrooms.

Dealing with multiple languages in classrooms is challenging for teachers. There are challenges in finding resources in the students' home language, and ascertaining a learner's mathematical understanding when there is a language barrier. To only allow answers in English limits what a student is able to communicate. When teachers do not speak the language of all learners in the class, they have trouble with understanding students' English responses. Without outside assistance, this limits the ability of students to answer in anything but English.

A number of mathematics education researchers have advocated using students' first languages as resources for learning and teaching mathematics (Adler, 1998; Mady & Garbati, 2014; Moschkovich, 2002; Planas & Setati-Phakeng, 2014). These researchers see students' first language(s), as a tool for thinking and communication. Some authors state that students' first languages provide the support these students need as they simultaneously develop proficiency in the language of instruction *and* learn mathematics (Mandy & Garbati, 2014; Winsor, 2007). We have established that sole use of a target language can neglect the influence of a students' prior knowledge on making mathematical meaning.

Students' cognitive load increases when exposed to unfamiliar contexts in grappling with a foreign or unfamiliar language (Campbell, Adams, & Davis, 2007; Goldenberg, 2008). This means that giving an English language learner a problem with an unfamiliar context makes the problem more difficult compared with peers who are already immersed in the context (Barwell, 2005; Lubienski, 2002). This means teachers pre-teach word problem contexts or provide visual cues to help. Reading and discussing a book with the class or using a shared experience can help this process. Also, using one context throughout lessons, rather than multiple contexts, can make it easier for English language learners to understand the word problems they are faced with.

Views about mathematics teaching and learning have shifted considerably in New Zealand and internationally over recent decades, and it is important for teachers to be kept informed about changes in the ways that mathematical processes and language are being emphasised (Anthony & Walshaw, 2007; Ministry of Education, 2007). Schools making a point of highlighting the importance of language in mathematics and other learning areas in their interactions with teachers and families would help their learners immensely. Not only will this help with mathematics learning, but is applicable across the curriculum. Furthermore, teachers could use parents and family as resources to contribute to students' learning. Teachers that keep parents informed of the topics (perhaps by sending home learning grids) (Anthony & Walshaw, 2007), can also make connections more meaningful and inclusive, benefitting mathematics learning. This can enable parents share their own mathematical knowledge through their first language and provide the students about how mathematics is situated within their wider experiences.

Overt and deliberate practices that support all learning in mathematics will help learners avoid the problems indicated by prior knowledge such as the stream/streamer incident and the sample of blood/sample of population scenarios.

References

Abedi, J., & Lord, C. (2001). The language factor in mathematics tests. *Applied Measurement in Education, 14*(3), 219–234. https://doi.org/10.1207/S15324818AME1403_2

Adams, T. L. (2003). Reading mathematics: More than words can say. *The Reading Teacher, 56*(8), 786–795.

Adler, J. (1998). A language of teaching dilemmas: Unlocking the complex multilingual secondary mathematics classroom. *For the Learning of Mathematics, 18*(1), 24–33.

Altieri, J. L. (2009) Strengthening connections between elementary classroom mathematics and literacy. *Teaching Children Mathematics, 15*(6), 346–351.

Anthony, A., & Walshaw, M. (2007). Effective pedagogy in mathematics/pangarau. Best Evidence Synthesis Iteration [BES]. Wellington, New Zealand: Ministry of Education.

Barwell, R. (2005). Working on arithmetic word problems when English is an additional language. *British Educational Research Journal, 31*(3), 329–348. https://doi.org/10.1080/01411920500082177

Barwell, R. (2011). What works? Research into practice. A research-into practice series produced by a partnership between the Literacy and Numeracy Secretariat and the Ontario Association of Deans of Education. *Research Monogram,* 34, 1–4.

Benjamin, A. (2011). *Math in plain English, literacy strategies for the mathematics classroom*. New York, NY: Eye on Education.

Boero, P., Douek, N., & Ferrari, J. L. (2008). Developing mastery of natural language: Approaches to some theoretical aspects of mathematics. In L. D. English (Ed.), *Handbook of international research in mathematics education* (2nd ed., pp. 262–297). New York, NY: Routledge.

Brown, C. L., Cady, J. A., & Taylor, P. M. (2009). Problem solving and the English language learner. *Mathematics Teaching in the Middle School, 14*(9), 532–539.

Campbell, A. E., Adams, V. M., & Davis, G. E. (2007). Cognitive demands and second-language learners: A framework for analyzing mathematics instructional contexts. *Mathematical Thinking and Learning, 9*(1), 3–30. https://doi.org/10.1080/10986060709336603

Clarkson, P. C. (2007). Australian Vietnamese students learning mathematics: High ability bilinguals and their use of their languages. *Educational Studies in Mathematics*, *64*, 195–215.

Cobb, P., & McClain, K. (2004). Principles of instructional design for supporting the development of students' statistical reasoning. In D. Ben-Zvi & J. B. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 375–395). Dordrecht, The Netherlands: Kluwer. https://doi.org/10.1007/1-4020-2278-6_16

Dawe, L (1983) Bilingualism and mathematical reasoning in English as a second language. *Educational Studies in Mathematics*, *14*, 325–353. https://doi.org/10.1007/BF00368233

Fasi, U. M. L. (1999). *Bilingualism and learning mathematics in English as a second language in Tonga.* (Unpublished doctoral thesis). University of Reading, Berkshire, England.

Ferrari, P. L. (2004). Mathematical language and advanced mathematics thinking. In M. Haines & A. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (pp. 383–390). Bergen, Norway: Bergen University College.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225–256). Charlotte, NC: Information Age.

Goldenberg, C. (2008). Teaching English language learners: What the research does-and does not say. *American Educator, 33*(2), 8–19.

Halliday, M. A. K. (1978). *Language as social semiotic.* London, England: Edward Arnold.

Hoffert, S. B. (2009). Mathematics: The universal language? A teacher enumerates. The challenges, strategies, and rewards of teaching mathematics to English language learners. *Mathematics Teacher, 4*(2), 130–139.

Kaplan, J. J., Fisher, D., & Rogness, N. (2009) Lexical ambiguity in statistics: What do students know about the words: Association, average, confidence, random and spread? *Journal of Statistics Education, 17*(3). Retrieved from http://www.amstat.org/publications/jse/v17n3/kaplan.html

Kasmer, L. (2013). Pre-service teachers' experiences teaching secondary mathematics in English-medium schools in Tanzania. *Mathematics Education Research Journal*, *25*(3), 399–413. https://doi.org/10.1007/s13394-013-0078-3

Kazima, M. (2007). Malawian students meaning for probability vocabulary. *Educational Studies in Mathematics, 64*(2), 169–189. https://doi.org/10.1007/s10649-006-9032-6

Kenney, J. M. (2005). Mathematics as language. In J. M. Kenney (Ed.), *Literacy strategies for improving mathematics instruction* (pp. 1–8). Alexandria, VA: Association for Supervision and Curriculum Development.

Kester-Phillips, D. C., Bardsley, M., Bach, T., & Gibb-Brown, K. (2009). "But I teach math!": The journey of middle school mathematics teachers and literacy coaches learning to integrate literacy strategies into the math instruction. *Education*, *129*(3), 467–472.

Kotsopoulos, D. (2007). Mathematics discourse: "It's like hearing a foreign language". *Mathematics Teacher*, *101*(4), 301–305.

Lager, C. (2006). Types of mathematics-language reading interactions that unnecessarily hinder algebra learning and assessment. *Reading Psychology*, *27*(2/3), 165–204. https://doi.org/10.1080/02702710600642475

Lampert, M., & Cobb, P. (2003). Communication and language. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 237–249). Reston, VA: National Council of Teachers of Mathematics

Latu, V. F. (2005). Language factors that affect mathematics teaching and learning of Pasifika students. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche(Eds.), *Building connections: Theory, research and practice: proceedings of the 28th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 483-490). Sydney, NSW, Australia: MERGA.

Lavy, I., & Mashiach-Eizenberg, M. (2009). The interplay between spoken language and informal definitions of statistical concepts. *Journal of Statistics Education, 17*(1). Retrieved from http://www.amstat.org/publications/jse/v17n1/lavy.html

Lee, J., Lee. Y., & Amaro-Jiménez, C. (2011). Teaching English language learners (ELLs) mathematics in early childhood. *Childhood Education*, 87(4), 253–260. https://doi.org/10.1080/00094056.2011.10523187

Lesser, L., & Winsor, M. (2009). English language learners in introductory statistics: Lessons learned from an exploratory case study of two-pre service teachers. *Statistics Education Research Journal, (8)*2, 5–32.

Lubienski, S. (2002). Research, reform, and equity in US mathematics education. *Mathematical Thinking and Learning, 4*(2/3), 103–125. https://doi.org/10.1207/S15327833MTL04023_2

MacGregor, M. (2002). Using words to explain mathematical ideas. *Australian Journal of Language and Literacy, 25*(1) 78–88.

Mandy, C., & Garbati, J. (2014). Calling upon other language skills to enhance second language learning: Talking taboo about first languages in a second language classroom. *Research Monogram*, 51, 1–4.

Meaney, T. (2006). Acquiring the mathematics register in classrooms. *SET: Research Information for Teachers*, *3*, 39–43

Ministry of Education. (2007). *The New Zealand Curriculum*. Wellington, New Zealand: Learning Media.

Moscardini, L. (2010). 'I like it instead of maths': how pupils with moderate learning difficulties in Scottish primary special schools intuitively solved mathematical word problems. *British Journal of Special Education, 37,* 130–138. http://dx.doi.org/10.1111/j.1467-8578.2010.00461.x

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning,* *4*(2/3), 189–212. https://doi.org/10.1207/S15327833MTL04023_5

Moschkovich, J. (2005). Using two languages when learning mathematics. *Educational Studies in Mathematics, 64*, 121–144. https://doi.org/10.1007/s10649-005-9005-1

Moschkovich, J. N. (2010). Recommendations for research and on language and mathematics education. Language(s) in learning mathematics: Resources, challenges and issues for research. In J. Moschkovich (Ed.), *Language and mathematics education: Multiple perspectives and directions for research* (pp. 113–150) Charlotte, NC: Information Age.

Mousley, J., & Marks, G (1991) *Discourses in mathematics* Geelong, Vic, Australia: Deakin University Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers' of English. (2008). *English language learners: A policy research brief produced by the National Council of Teachers of English*. Retrieved from http://www.ncte.org/library/NCTEFiles/Resources/Policy Research/ELLResearchBrief.pdf

Neville-Barton, P., & Barton, B. (2005). *The Relationship between English language and mathematics learning for non-native speakers* (Final Report). Retrieved from http//www.tlri.org.nz/index.html

Nguyen, H. T., & Cortes, M. (2013). Focus on middle school: Teaching mathematics to ELLs: Practical research-based methods and strategies childhood education. *Childhood Education, 89*(6), 392–395. http://dx.doi.org/10.1080/00094056.2013.854130

Padula, J., Lam, S., & Schmidtke, M. (2001). Syntax and word order: Important aspects of mathematical English. *The Australian Mathematics Teacher, 57*(4), 31–35.

Parvanehnezhad, Z., & Clarkson, P. (2008). Iranian bilingual students reported use of language switching when doing mathematics. *Mathematics Education Research Journal, 20*(1), 52–81. https://doi.org/10.1007/BF03217469

Planas, N., & Setati-Phakeng (2014). On the process of gaining language as resource in mathematics education. *ZDM Mathematics Education*, 46, 883–893. https://doi.org/10.1007/s11858-014-0610-2

Raiker, A. (2002). Spoken language and mathematics. *Cambridge Journal of Education, 32*(1), 45–60. https://doi.org/10.1080/03057640220116427

Rangecroft, M. (2002). The language of statistics. *Teaching Statistics*, *24*(2), 34–37. https://doi.org/10.1111/1467-9639.00080

Rubenstein, R. (2000). Word origins: Building communication connections. *Mathematics Teaching in the Middle School, 5*(8), 493–498.

Rubenstein, R., & Thompson, D. R. (2002). Understanding and supporting children's mathematical vocabulary development. *Teaching Children Mathematics*, *9*(2), 107–112.

Schleppegrell, M. J. (2011). Language in mathematics teaching and learning: A research review. In J. Moschkovich (Ed.), *Language and mathematics education: Multiple perspectives and directions for research* (pp. 73–112). Charlotte, NC: Information Age.

Setati, M., & Adler, J. (2001). Code switching in a senior primary class of secondary-language mathematics learners. *For the Learning of Mathematics, 18*(1), 34–42.

Walker, A., Shafer, J., & Liams, M. (2004). “Not in my classroom”: Teacher attitudes toward English language learners in the mainstream classroom. *NABE Journal of Research and Practice, 2*(1), 130–160.

Winsor, M. S. (2007). Bridging the language barrier in mathematics. *Mathematics Teacher, 101*(5), 372–378.

Xi, C., & Yeping, L. (2008). Language proficiency and mathematics learning. *School Science & Mathematics*, *108*(3), 90–93. https://doi.org/10.1111/j.1949-8594.2008.tb17811.x