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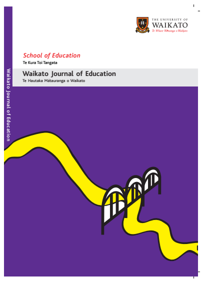

Embedding problem-solving in a primary mathematics programme

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Abstract

New Zealand curriculum documents have long referred to mathematics as a problem-solving endeavour. Although this has been an intended focus for more than 25 years, problem-solving has often been an aspect of mathematics teaching that has been overlooked (Holton, 2009). This research explores the experiences of one teacher who is committed to enhancing children's learning by embedding a problem-solving approach within her mathematics programme. The teacher reports that while students respond positively to mathematics as problem-solving, a number of constraints have been encountered. The teacher's ongoing efforts responding to these are shared.

Keywords

Primary mathematics; problem-solving; teacher resilience and creativity

Introduction and literature review

New Zealand curriculum documents position mathematics as problem-solving

For more than 25 years New Zealand curriculum documents have positioned mathematics as a problem-solving endeavour, and encouraged the teaching of mathematics through problem-solving. One of the seven aims of the Mathematics in the New Zealand Curriculum (MiNZC) document published in 1992 was to "help students to develop a variety of approaches to solving problems involving mathematics …" (Ministry of Education, 1992, p. 8). This curriculum, guiding the teaching and learning of mathematics until 2007, was divided into six strands including mathematical processes, number, measurement, geometry, algebra and statistics. The mathematical processes strand included problem-solving, developing logic and reasoning, and communicating mathematical ideas, and was expected to be embedded within all other strands. The use of real-life, open-ended problems with multiple solutions was encouraged. Characteristics of good problem-solving techniques were listed in the curriculum document, and additional resources to support teachers were published (see Ministry of Education, 1999). While there was a clear emphasis on problem-solving, the document also pointed to the need for balanced mathematics programmes referring to the development and maintenance of skills. It cautioned though that "while fluency with basic techniques is very important,

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such routines only become useful tools when students can apply them to realistic problems" (Ministry of Education, 1992, p. 11).

In the most recent New Zealand Curriculum (Ministry of Education, 2007), the mathematical processes strand no longer appears but there emerges an expectation for key competencies, such as 'thinking' and 'using language, symbols and texts', to be an integral part of all learning areas, including mathematics and statistics. In the statement describing this learning area, mathematics and statistics are presented as "related but different ways of thinking and solving problems" (p. 26). Portrayed as an active endeavour, learners are expected to be creating, exploring, investigating, justifying, explaining, communicating and making sense (Ministry of Education, 2007). Continuing the almost 20 years of MiNZC's emphasis on mathematics as a problem-solving endeavour, the lists of achievement objectives in the 2007 curriculum are prefaced with the statement, "In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations …." (Ministry of Education, 2007). This statement is found at all levels of the curriculum, positioning all students from Years 0 to 13 as mathematical problem-solvers.

What is mathematical problem-solving?

Problem-solving refers to "the solution of problems, the method of which is not immediately obvious to the potential solver" (Holton, Anderson, & Thomas, 1997, p. 3). Problems are tasks that are solved not just by direct effort but also by play or some creative insight (Holton, 2009; Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016). Problem-solving has been regarded as an important aspect of mathematics, and the teaching and learning of mathematics for a long time (Liljedahl et al., 2016; Schoenfeld, 2007). The 1992 New Zealand mathematics curriculum document (Ministry of Education, 1992) refers to problem-solving as involving the use of interesting and realistic contexts that are personally meaningful to the learners; open-ended problems that do not necessarily have one solution; problems that yield solutions that require the use of more than one mathematical skill; problems that require searching for information; thinking; and "the systematic collection of data or evidence, experimentation (trial and error followed by improvement), flexibility and creativity, and reflection" (Ministry of Education, 1992, p. 11). A list of problem-solving strategies includes "simple strategies such as guessing and checking, drawing a diagram, making lists, looking for patterns, classifying, substituting, re-arranging, putting observations into words, making predictions, and developing proofs" (Ministry of Education, 1992, p. 11).

An approach similar to the teaching and learning of mathematics through problem-solving is the use of mathematical investigations. These are open-ended tasks that enable a variety of mathematical avenues to be explored. Frankcom (2009) refers to problems becoming investigations when learners seek to generalise by asking 'what if' questions. Drawing on the thinking of Nathan (2001), Frankcom refers to open-ended tasks or activities that have multiple answers and solution paths as marking the essence of investigative work. Like problems, investigations often require a sustained period of time to undertake and there is an embedded emphasis on the development of a conceptual mathematical understanding (Frankcom, 2009). Choice and autonomy characterise the investigative classroom. Often, a variety of projects (investigations) are offered, providing "different access points for different students and enabl(ing) students to work on them at different mathematical levels" (Boaler, 2002, p. 57). Once learners have been introduced to starting questions or themes there is considerable flexibility about what mathematics may be encountered and explored.

In recent years research focused on 'problem-solving' has been seen less frequently as attention has turned to other areas (Schoenfeld, 2007), such as rich mathematical tasks (Boaler, 2016) or challenging tasks (e.g., see Bicknell, Cramond, & van der Schans, 2015). While these researchers present slightly different variations on ways to teach mathematics, they share an emphasis on deep engagement, thinking and reasoning, multiple solution strategies and the possibility of there being more than one answer (Bicknell et al., 2015; Boaler, 2016; Ministry of Education, 2007). There is an emphasis on high level thinking (Breyfogle & Williams, 2008), and like problem-solving and investigations, explaining, justifying, and communicating and supporting others are expected and valued (Bicknell et al., 2015; Boaler, 2002, 2008, 2016).

Within these approaches to the learning of mathematics there is a focus on the learner's development of conceptual rather than procedural mathematical understanding. Such understanding is aligned with Skemp's (1991) notion of relational mathematics which is described as "knowing both what to do and why" (p. 5). What it means to 'do mathematics' and the social norms that characterise the problem-solving or investigative classroom are different to those of more traditional, didactic mathematics classrooms. In the problem-solving or investigative classroom mathematics is regarded as a human endeavour, involving creativity, imagination and the communication of emerging ideas and concepts (Boaler, 2016; Ministry of Education, 1992, 2007).

New Zealand children's mathematical achievement

Recent international assessments indicate the challenge and complexity of supporting New Zealand students' achievement in mathematics. Most recent 'Trends in International Mathematics and Science Study' (TIMSS) data (TIMSS assesses mathematics and science achievement of Year 5 and Year 9 students every four years since 1994) gathered in 2014/15, found that under half of Year 5 and Year 9 New Zealand students were working at the desired level of mathematics according to the New Zealand Curriculum (Caygill, Hanlar, & Harris-Miller, 2016). New Zealand's primary aged children ranked lower than 33 other countries (out of a total of 49 participating countries) whereas Year 9 students' average achievement was slightly higher, approximately in the middle of 39 countries (Ministry of Education, 2017). Both groups of students are achieving below the TIMSS scale centre-point (Ministry of Education, 2017).

Moreover, the Programme for International Student Assessment (PISA) data gathered in 2015 shows a decline in the average mathematics score of New Zealand students between 2003 and 2015 with much of the decline occurring between 2009 and 2012 (Ministry of Education, 2017). PISA occurs every three years and focuses on the reading, mathematics and science literacy of 15-year-olds, with the mathematics component focusing on applying mathematical literacy for solving everyday problems (It is concerning that 22 percent of New Zealand 15-year-olds performed below 'level two proficiency', a level regarded as a baseline where students can demonstrate mathematical competencies enabling them to actively participate in maths-related life situations (Ministry of Education, 2017). Also of note is the range of New Zealand student achievement evidenced in PISA, a range wider than many other countries (Ministry of Education, 2017). Unlike TIMSS results, "New Zealand 15-year-old students performed above the OECD average, and above 41 (out of 60) other countries" (Ministry of Education, 2017).

Professional mathematics development in New Zealand schools since 2000

Considerable effort has been made in New Zealand to support teachers' understanding of the teaching and learning of mathematics. From the year 2000 and by 2008 almost every Year 1–6 teacher and the majority of Year 7–8 teachers in New Zealand had an opportunity to be involved in the Numeracy Professional Development Projects (Ministry of Education, 2008a). The focus of these projects was to improve student performance in mathematics through improving the professional capability of teachers. Teachers were regarded as key figures for changing the way mathematics and statistics were taught and learned in schools, and teacher subject matter and pedagogical knowledge were considered to be critical factors (Ministry of Education, 2008a). Central to the project is The Number Framework (Ministry of Education, 2008a), which sets out, as a series of nine stages, increasingly sophisticated ways of mathematical thinking, showing a progression from counting to part-whole thinking (Higgins & Parsons, 2009).

Much of the intended emphasis within the Numeracy Professional Development Projects is consistent with the literature surrounding the nature of mathematics as a creative human endeavour. Students are expected to "create models and predict outcomes, conjecture, justify and verify, and seek patterns and generalisations" (Ministry of Education, 2008b, p. 0). In the teacher resource materials there is a focus on conceptual thinking, and dimensions of quality teaching are listed, including an inclusive classroom climate, focused planning, problem-centred activities, responsive lessons, and connections, including the use of realistic contexts, high expectations and equity (Ministry of Education, 2008b). Although the project offers many worthwhile activities designed to support children's learning and understanding, it could be argued that many of these would be more closely aligned with the development of mathematical skills rather than being centred on the solving of complex problems.

Problem-solving as a way forward for the teaching of mathematics

Despite a problem-solving emphasis in curriculum documents this vision of mathematics, and mathematics teaching and learning, has not been consistently adopted (Holton, 2009). Holton (2009) suggests much of the time spent on mathematics in schools and universities is spent on skill rather than the excitement of discovery encountered during problem-solving. This is also the case for many American schools (Schoenfeld, 2007). An emphasis on skill development is worrying, given that it has been shown that learning and teaching mathematics through problem-solving improves children's learning (Boaler, 2016; Holton et al., 1997; Schoenfeld, 2007). It is interesting to note that when students taught with curricula centred on problem-solving are tested on mathematics skills, they perform at a similar level to students taught with traditional curricula (Schoenfeld, 2007). However, when tested on conceptual understanding and problem solving, they significantly outperform students who have studied more traditional curricula. In New Zealand, Holton et al. tell of the significant gains made by a Year 3–4 class, and a 'low to average ability fourth form [Year 10] class' in both "their mathematics and problem solving" (1997, p. 99).

One of the benefits of a problem-solving approach to the teaching and learning of mathematics is its ability to cater for a wide range of learners (Boaler, 2016), a pressing issue for New Zealand as demonstrated by recent international test results. Holton states "problem-solving, as well as being good for all students so that they can learn more about mathematics, is accessible for pretty well all students" (2009, p. 50). Boaler adds further argument to the need for focusing on a problem-solving approach by demanding a change in the 'role' of mathematics in our schools. She states, "There is an imperative need for mathematics to change from an elitist, performance subject used to rank and sort students (and teachers) to an open, learning subject, for both high-achieving students, who are currently turning away from mathematics in record numbers, as well as the low-achieving students who are being denied access to ideas that they are fully capable of learning" (2016, pp. 101–102).

Teaching mathematics and learning to teach mathematics are complex endeavours, and recently gathered assessment data shows we are still on a pathway to effectively support children's mathematical learning in Aotearoa New Zealand. Research indicates considerable time is needed to effect changes in mathematics teaching (Schoenfeld, 2007), and the challenge of bringing about deep and lasting change in teachers' understanding of mathematics learning should not be underestimated (Young-Loveridge, 2010). These are not new dilemmas, and nor are attempts to embed problem-solving within mathematics curricula. Problem-solving research in the 1970s and 1980s led to the development of the controversial reform-based mathematics curricula (advocating for the teaching of mathematics as problem-solving) of the 1990s in the United States. This, in turn, resulted in "the maths wars, the most vicious public battle over mathematics curricula in US history" (Schoenfeld, 2007, p. 545). As of 2007, Schoenfeld explains that the anti-reform forces have been successful in focusing the teaching of mathematics on teaching for mastery (a 'back to basics' movement) rather than teaching for understanding, a premise underpinning problem-solving. More hopefully, Schoenfeld also suggests that, given the likely results that will be achieved by a skills focus, the 'back to basics movement' will pass, clearing the way for the next round of curriculum development that can incorporate what has been learned about problem-solving and mathematical thinking.

Resistance to change in mathematics education does not only occur at the level of national politics, as evidenced by the above brief history of mathematics education in the United States of America, but also amongst parents and in the classroom (Dossey, 1992; Schoenfeld, 2007; Thomas & Cooper, 2016). Dossey (1992) refers to research that followed a first-year teacher who attempted to initiate a problem-solving approach in his mathematics teaching. The students' were threatened by this change and their reactions were such that the teacher returned to a mode of teaching akin to traditional teaching with a focus on 'presenting'. Parents can also have a significant effect on mathematics curricula, as was evidenced in the parental backlash in response to the 'new math' movement of the early 1960s in America. This resulted in a move to "a rote, 'basics' approach" that then dominated much of mathematics teaching throughout the 1970s (Schoenfeld, 2007, p. 542). In more recent research Thomas and Cooper (2016) refer to parents not being privy to new pedagogical developments within education systems, and suggest changes are needed in teacher education enabling teachers to work with parents so they are more disposed to change.

In New Zealand, in the late 1990s, Derek Holton led a research project specifically investigating teachers' learning about teaching mathematics by problem-solving. He comments within the final report, that with more widespread use of problem solving there will be challenges for teachers and a need for more professional development. He found there were nine requirements for teachers teaching mathematical problem solving: common sense, imagination, openness, problem solving experience, thinking like a student, patience, a store of good problems, reflective practice and help (Holton et al., 1997). This research seeks to build on from this, some 20 years later, by investigating what challenges are encountered and what form of support or professional development might be needed by teachers who are interested in embedding a problem-solving approach within their mathematics programme. This article reports on initial findings from the efforts of one teacher who is committed to embedding the use of problems, investigations, and rich, challenging tasks within her mathematics programme. This is part of a larger project exploring a range of teachers' (beginning and fully certificated) efforts across a variety of New Zealand classrooms. The research question guiding this particular aspect of the project is: *What affordances and constraints do teachers encounter when they adopt a problem-solving approach for the teaching and learning of mathematics?*

Methods and Methodology

Context and research participants

Knowledge needed to understand and improve educational situations cannot be located outside of teachers' contexts and merely transported in for direct implementation and use. Rather collaboration between teachers and teacher educators can generate improved practice in particular contexts (Cochran-Smith & Donnell, 2006). O'Neill (2008) states that such research needs to be built on "honesty, extensive dialogue and the active, respectful involvement of teachers" (p. 61) in order to develop deep understandings of teaching. These aspirations underpin this research.

In this small qualitative research study one primary school teacher, Katherine, and the author, a mathematics education researcher, are working together to explore and record the teacher's self-reported experiences as she embeds a problem-solving approach within her mathematics programme. As previously mentioned, a problem-solving approach for the teaching and learning of mathematics can be closely aligned with investigations and/or tasks that are denoted as 'rich' or 'challenging'. In this research the teacher has used a variety of tasks, some of which are presented as problems, some as investigations and others that might more accurately be called 'rich' tasks. Hereafter, for the sake of simplicity, 'problem-solving' will be the term used, as this closely exemplifies the teacher's overall approach, given there has been an inclusion of problems or tasks that need sustained engagement, considerable time (more than one lesson), critical thinking and an expectation of children gaining a conceptual understanding of mathematics.

Katherine works in a small rural decile seven full primary (Year 0–8) co-educational school. The author is a mathematics education researcher who teaches pre-service primary mathematics education, and has experience of teaching primary and junior secondary mathematics at schools in New Zealand. Katherine has been teaching for six years, and is in her third year teaching Year 7–8 children the full range of curriculum learning areas, including a problem-solving approach within her mathematics teaching, for at least part of her mathematics programme. Prior to this Katherine had responsibility for teaching technology within a middle school.

Data collection

During the first six months of this year-long study, the mathematics education researcher met with Katherine three times, with the school principal also present for one meeting. This research with Katherine builds on a previous research relationship and an initial lecturer-student association. Through semi-structured interviews Katherine's thinking about and recollections of using a problem-solving approach in her mathematics programme have been discussed and audio-recorded. Occasionally the researcher has shared ideas that Katherine might then choose to act upon within her teaching. Each of these discussions has taken place for approximately one and a half hours to two and a half hours at the teacher's school at a time nominated by and suitable for the teacher. Later in the research study observations of problem-solving lessons will be included. Some planning documents have also been collected. Contact is maintained between these meetings via e-mails and texts. After discussions, audio-recordings were listened to by the researcher, and notes taken to provide a written record of each conversation. Parts of the discussions that were judged as particularly noteworthy (e.g., because of particular significance to the research question or the evident importance to the teacher or the emotion being expressed) were transcribed word for word.

Qualitative semi-structured interviews characterised by 'informal conversational' and 'interview guide' approaches (Patton, 1980, cited in Cohen, Manion, & Morrison, 2013) underpin these meetings with Katherine. Interviews such as these provide flexibility with the interviewer able to follow-up responses as relevant and appropriate. A list of questions is prepared by the researcher to guide each discussion, both inquiring after the teacher's current thinking and experiences, and building on and forward from prior conversations and issues that have previously been to the fore. A concerted effort is made during each interview to enable the teacher's stories or narratives to emerge. This requires attributes of trust and curiosity (Woods, 1986 cited in Cohen et al., 2013), which in part have been established by the relatively long association (eight years) between the teacher and researcher. The mathematics education researcher also maintained an electronic journal throughout the research, periodically noting down her thinking about the discussions, research literature and possible next steps.

Data analysis

Data analysis has occurred in two ways. An emergent analytical approach (Borko, Liston, & Whitcomb, 2007; Strauss & Corbin, 1994) was employed to analyse the notes and transcribed parts of each discussion. These were read and re-read with notes taken as particular issues emerged. These notes were recorded in a column alongside the original notes/transcripts and not only constituted data anlysis but also provided the impetus for questions and discussion in subsequent interviews. Ongoing reading of literature about teachers using a problem-solving approach for the teaching of mathematics also informed the analysis. This process reflects the thinking of St. Pierre (2011) who writes, "If we don't read the theoretical and philosophical literature, we have nothing much to think with during analysis except normalised discourses that seldom explain the way things are" (p. 614).

In order to closely represent Katherine's perspective, findings are presented in the form of quotes. Consistent with Clandinin and Rosiek's (2007) call for researchers to listen to people's stories about everyday experiences, the presenting of the teacher's 'voice' is a deliberate attempt to present and acknowledge her experience and perspective. Ethical consent for the research was obtained from the researcher's university ethics committee, and gained from the teacher and school principal(s) (there has been two changes of principal during the research period). Psuedonyms have been used throughout for confidentiality.

Findings

Katherine reported that although the majority of students responded positively to this approach for learning mathematics, a number of issues and constraints have been encountered. Key findings discussed in this paper include a need for teacher resilience and collegial support to counter resistance from parents; and an intermediate, temporary step developing an alternative mathematics programme has been needed. Data will first be presented to paint a picture of the students' experiences of engaging with a programme rich in problem-solving.

Problems and rich tasks engage and support children's learning in mathematics

Katherine has consistently commented that student engagement in mathematics is enhanced by the use of problems and/or challenging tasks. Katherine notes that this includes students who previously held negative mindsets towards mathematics. She explained, "I found the kids with negative mindsets towards maths thrived, and I saw them make breakthroughs, them have A-ha moments." Katherine described the students as "engaged, challenged", and she said she knew they were learning. Katherine recalled one student who moved from being well below in the mathematics curriculum standards (see Ministry of Education, 2009) to achieving 'at' the standard, and going on to do well in mathematics at secondary school the following year. Katherine recalled one specific teaching incident where she had posed a question in relation to a well-known children's book called Counting on Frank (Clement, 1991). Katherine remembered how this student surprised herself by readily answering the question. Katherine said to this student, "that is excellent, that is an excellent maths brain and her [the child's] jaw dropped open … From there we started making progress …" Katherine works hard to challenge all children to consider mathematics as being much more than being fast with basic facts. She recalls telling students, "basic facts are useful … but they don't make you a good mathematician …" Katherine also referred to another student who responded well to problems being set in a context that he could relate to. She explained that this "weaker" student became repositioned as an "expert", saying "he had never had the opportunity to shine in maths".

Katherine was strongly supported in her efforts to teach mathematics with an emphasis on problem-solving by her principal. The principal, Yvonne, remarked on how the approach was resulting in students being enthusiastic about mathematics and the students being able to specifically articulate their mathematics learning. Yvonne said, "They really got it. They had conversations with me." She explained that some of the students who were achieving above the expected curriculum level were "fizzing, fizzing would be the word. They weren't just saying they liked a task, they really got it".

Teachers need resilience and collegial support to counter resistance

Katherine first experienced the teaching of mathematics using a problem-solving approach during the second year of her teacher education. During a compulsory mathematics education paper, she became interested in, and enthusiastic about, teaching mathematics using problems, rich tasks and meaningful contexts. She went onto embed these approaches during two of her practicum experiences, and was delighted to observe this way of teaching mathematics "worked" (see Bailey, 2013). By the time Katherine started teaching at the full primary school she had also studied three post-graduate mathematics education papers, working towards gaining a Masters qualification. She once again began teaching mathematics in the way she had valued during her teacher education, embedding rich tasks and problems set in meaningful contexts within her mathematics programme.

Towards the end of Katherine's first year in the full primary school she became aware that there was some resistance to her mathematics programme from a few parents and students. She explained,

> Some of the students didn't see it as maths. It wasn't in maths time. They didn't require the maths book. They weren't in a group. They didn't have a rotation. [The students thought] we haven't done any real maths … Some of the parents were also concerned because they couldn't see it, and there wasn't [sic] screeds and screeds of calculations done out of a text-book, filling up the book.

Katherine commented that these parents did not view her mathematics programme as "real school maths". Even though there were also some supportive parents, Katherine took the concerns seriously, gathered relevant research together and held a parent meeting where she explained her teaching approach. Katherine recalls that initially this appeared to appease the parents, but as the next year unfolded, concerns were still being discussed amongst some of the parent body.

This marked a difficult period for Katherine, and also in part for the school principal. Both were committed to Katherine's use of a problem-solving approach, had witnessed the benefits in terms of children's learning and engagement and were keen for it to continue. A comment typical of those made by Katherine about this time reveals her feeling:

> Very vulnerable, fragile, I felt really strongly about what I was doing. If I hadn't felt like that it would have been so easy to cave and I'll just do traditional. I'll just pull out [a more traditional mathematics programme with an emphasis on skills and ability grouping]. I'll just chunk it to you and it doesn't matter whether you need it or not, you're going to be learning it that day.

Katherine's principal also remembers this as "a very tense time for me as well", speaking about the need for more parent education, and also for building an understanding about Katherine's teaching approaches with the team of teachers with whom Katherine worked. Yvonne also remarked about the relatively significant impact that parents and the Board of Trustees can have within a small school setting.

An additional concern held by Katherine was that she had not found a way to formally record the mathematics learning embedded within the investigations, problems and rich tasks that were a part of her programme. She explained,

> One of the problems with my contextual approach was that I personally didn't have a good system to track it. Maybe it felt a bit ad hoc to the students … And I didn't say we are going to do geometry … I didn't have flags or hooks for the students to recognise the mathematics.

It appears that Katherine's students were not sure if and/or what mathematics they were learning, and it is likely this contributed to parents' perceptions that mathematics was not being taught, at least not in a manner they could recognise.

Changing mathematics programme: A temporary, intermediate step

After two terms of being aware of the ongoing discontent present in a small section of the school community, Katherine decided to change her mathematics programme and devised what she has termed an Individual Learning Programme (ILP). She recalled,

> I got to the point where I felt I was defending myself all the time, and it's not a nice feeling, and not having a solid model to go on I then went to the next best thing I had seen.

Katherine explained she did not want to limit students' learning potential by ability grouping and so devised a system (based on a literacy programme system she was familiar with) where students were diagnostically tested, then provided with a table that listed key mathematical skills (corresponding to those assessed in the diagnostic test) matched with worksheets and exercises from text-books for students to complete. Each child works on what is, in essence, an individualised learning programme as determined by the diagnostic tests. Katherine concurrently conducts small group workshops designed to teach children specific mathematical skills and concepts. Children are expected to self-select or are prompted by Katherine to attend the workshops as necessary. Children are required to provide two pieces of evidence to show their understanding of each skill or concept, and their progress is recorded on a Google spreadsheet.

Katherine explained that most parents and students responded positively to this new programme. Referring to a few students who had previously perceived they were not doing mathematics, Katherine said "this got switched-on mathematicians who needed it to be out of the book, this incorporated them". She also explained that the ILP helped previously reticent and unconfident students realise that they had, during their previous engagement in problems and tasks, "become good at maths". Katherine has now been using her ILP programme for one year. She says,

> There are things I like about each system—I like the tracking of this approach but not how it is compartmentalised. I like the flexibility. I like the contextual ability of the problem-solving way but I need a better way for me and the students to track it and for it to be more transparent for the parents.

When Katherine refers to 'compartmentalising' she is concerned about how each strand (number and algebra, geometry and measurement, and statistics and probability) is considered separately in her ILP system. This is a contrast to the rich tasks she has previously used where the mathematical ideas are naturally entwined and dictated by real-world contexts rather than the artificial nature of the school mathematics curriculum.

Yvonne remained supportive of Katherine as she developed the alternative mathematics programme. As the assessing of children's learning was discussed during the research interview, Yvonne remarked that too much focus on assessing children's learning might not be in the best interests for children's learning. Instead of intense periods of assessment, she would rather see "an ongoing drizzle of monitoring". Throughout the time of challenge and development of the alternative programme Yvonne supported Katherine to maintain her confidence and willingness to try new things. She appreciated Katherine's reflective nature. Making reference to more traditional mathematics programmes, Yvonne said, "If we always stick with what we've done we'll always get what we got … this is about reflecting on your own practice."

Although the ILP is not the way Katherine would like her mathematics programme to be, she regards it as successful in responding to students' individual learning needs, parents' concerns and providing a way to closely and clearly monitor children's learning within each mathematics strand. With her principal's support (which included purchasing a useful text that supports the use of rich mathematical tasks) and encouragement Katherine is now looking to find a way to incorporate problems and 'rich tasks' within or alongside the ILP system. She explains she wanted to get the individual learning programme going first and then merge it with problem-solving. She remains committed to finding a way of merging the best from both systems, commenting, "I haven't changed the passion and drive…" Katherine's enthusiasm for problem-solving resonates with that of Holton, who writes, "I believe we do our students a disservice if we don't show them this side of the subject" (2009, p. 49).

Discussion

Katherine's noticing of enhanced student engagement and understanding in mathematics through the use of problems, investigations and rich mathematical tasks resonates with research linking student engagement and learning with problem-solving and/or rich tasks (e.g., see Boaler, 2016; Hattie, Fisher, & Frey, 2017; Holton et al., 1997). Also regarded as an approach that supports teachers to cater for a diverse range of understanding and engagement (Boaler, 2016; Hattie et al., 2017; Holton et al., 1999), Katherine's experiences suggest that the approach not only enhanced the engagement and learning of those who were achieving above expected curriculum levels but also students who had previously struggled in mathematics. As Katherine explains, "Sometimes with more traditional systems you have them in groups and put a ceiling on their group, based on diagnostic data, whereas if you give them a more open task that accommodates all levels then they can excel."

Based on Katherine's experiences, embedding a problem-solving approach within a mathematics programme while engaging and enhancing children's learning, may incur challenges. This backs up Holton et al's. (1997) contention from twenty years ago that there will be challenges for teachers who seek to embed problem-solving within their mathematics programme. To date the greatest challenge for Katherine has been the resistance of a small group of parents, which compounded her own concerns about not being sure how to record children's learning when using a variety of problems and rich tasks. Katherine's experience, albeit on a very much smaller scale, resonates with the issues encountered by the 'new math' movement in America some 50–60 years ago, and has similarities to the 'back to basics' backlash currently evident in American mathematics education (Boaler, 2016; Schoenfeld, 2007).

While disappointing for Katherine, and despite having full support of her school principal, this turn of events might not be considered surprising given the strong beliefs held by many, including parents, about mathematics. It is interesting, however, to consider the significant impact of what Katherine reports to be a relatively small group of parents, on her decision to at least temporarily change her mathematics programme. It appears that in this context more than a single parent meeting was needed to educate and allay parent concerns about a different and likely unfamiliar way of mathematics teaching and learning. The timing of such a meeting(s) also needs to be considered, with it being possible that such meeting(s) would have been valuable as Katherine started her mathematics programmes based on problem-solving. This is consistent with the findings of Thomas and Cooper (2016) who explain "while teachers and administrators have had considerable access to resources to help them understand new-reform teaching pedagogy and practice, parents have been left out of the loop" (p. 40). They highlight the need to create a school climate where parents understand and embrace new teaching pedagogies.

Katherine's comment that she did "not hav[e] a solid model to go on" and not knowing anyone else teaching mathematics in this way suggests that when teachers work to change their practice they are best doing this working within a community of practitioners. A community of support beyond the principal, preferably those who are also trialling and/or have experience with teaching mathematics through problem-solving is needed. This would have benefitted Katherine at a time of considerable self-doubt and vulnerability. A challenge may be finding such teachers. The idea of collegial support resonates with the findings of Holton et al., who write "help" was "a very important part of teaching problem solving at the moment, because of its novelty to so many teachers" (1997, p. 92). It appears that almost several decades later collegial support and help are still relevant but absent.

The alternative independent learning programme has supported Katherine's development of manageable systems for recording individual children's learning of specific mathematical skills and concepts. As the research continues it is hoped these systems of recording learning will be generalised to the use of problems, investigations and rich tasks as these are re-introduced within or alongside the ILP. Perhaps evaluating children's understanding of problem-solving stages and strategies will also be considered. Furthermore, Holton et al. (1997) explain that assessment of problem-solving cannot be limited to analysing a child's written work but needs to be complemented by observing and questioning children as they solve problems. These broader notions of what constitutes evidence of children's mathematical learning will also need to be communicated to parents to support their understandings.

Katherine's alternative programme maintained a commitment to meeting individual needs and avoiding ability grouping. The interesting idea here is that if teachers try and change a mathematics programme and find that the change has been too rapid (e.g., for parents), it is possible to create an intermediate step/programme that maintains some aspects of what is being aimed for (e.g., catering for a diversity of learning needs), while temporarily compromising on other aspects (e.g., the use of problems and rich tasks). Preliminary data suggests that it will be possible to weave together aspects of both approaches in a way that responds to concerns while also moving towards a way of teaching mathematics that is known to support children's chances in life (Boaler & Selling, 2017).

Conclusion

Given the inequities prevalent in New Zealand's education system (Cochran-Smith et al., 2016) evident in the most recent PISA data (Ministry of Education, 2017), and the research validated claims that problem-solving enhances children's learning of mathematics (Boaler, 2016; Hattie et al., 2017; Holton, 1997), we need to learn more about what will enable and constrain teachers' efforts to embed problem-solving within their mathematics programmes. It is acknowledged that the background and experiences of all teachers are varied, as are the nature of the schools and communities where they teach. However, as we share Katherine's experiences via an in-depth case study approach, it is hoped that insights can be gained that may support other teachers (and teacher educators) interested in implementing this approach for the teaching and learning of mathematics in primary classrooms in Aotearoa New Zealand.

Katherine's experiences to date confirm that embedding problem-solving within a mathematics programme engages children in mathematics and supports their learning. It seems, however, that adopting a problem-solving mathematics programme is a process that may take time, needing teacher resilience, a determination to persevere and creativity in the face of resistance. Increased education and communication for and with parents, at least in a small rural school, is required. A willingness to change to an alternative individualised learning programme that avoids ability grouping, has been an intermediate step enabling an initial response to parent concerns and providing support for teacher learning about recording children's progress. Establishing communities of support with other teachers committed to and ideally with some experience of problem-solving is suggested as a potential pathway forward.

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