

Adults Learning Mathematics: transcending boundaries and barriers in an uncertain world

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Abstract

Adults return to study mathematics for a range of varied and complex reasons. This article addresses the boundaries and barriers that adult learners of mathematics may have faced in the past, or in the present and the future. Taking into account the impact of our era of rapid technological change, I argue for the importance of developing structural understanding of disciplinary mathematics together with the equally important contextual knowledge and skills necessary for understanding and engaging with mathematics in often vastly different circumstances from the past. The goal is for adult learners to feel confident, socially included, and able to participate in the mathematical activities with which they are confronted over a lifetime.

Introduction

Adults return to study mathematics for a range of varied and complex reasons. Although some programs focus on preparing for entrance into specific professions or vocations, other more general programs often aim at meeting economic and/or social inclusion goals. In many countries formal mathematics curricula have given way to sets of learning outcomes where the focus is on what learners can do rather than what they know and how they know it. One consequence is that adult numeracy education (as it is commonly known) consists of collections of mathematical tasks which necessarily have obvious links to practical applications. In vocational education this can often take the form of solely those tasks that are deemed to be of immediate use and easy to demonstrate. Often for lack of available time, this is likely to come at the expense of developing depth of mathematical knowledge and understanding.

A second major issue globally is that curricula and learning outcomes in general tend to focus on past and present economic and societal demands, even though these are both undergoing massive change as exponential rates of technological advance reconfigure how people live and how they will work, or not, as the case may be. In this article my aim is argue the case for ensuring that the learners (who have often missed out in earlier years) develop meaningful structural understandings of number, space, and shape that will enable them to continue to build on their mathematical (and other) learning throughout a lifetime, and to understand the mathematical communications of others at work and in civic life generally.

In this article, I will focus on adults, on learning, and on mathematics, separately and collectively. Focusing on adults, I will address briefly the possible interests that they might have in learning mathematics, particularly in a fractured and fragmented world with constantly changing horizons in terms of politics, economics, technology, the environment, and so on. Focusing on mathematics, I will identify three different perspectives on mathematics, briefly summarise what doing mathematics means at work, and consider the importance of mathematics education in relation to democracy. Focusing on learning (set in the context of the adult world of work), I will address the need for workplace based innovation (as well as standardisation), and the importance of informal learning at

work. I will draw on Bernstein's theories to stress the importance of understanding the big ideas of mathematics, and hence its underlying structures and relationships, as well as the crucial importance of contextual knowledge. Finally, in order to support the kind of mathematics education necessary in this era of globalisation and rapid change, bringing about uncertainties to a degree not previously experienced in human history, I will draw parallels between knowing in the discipline of mathematics and in the discipline of music.

Adults: what are their interests in learning mathematics?

In this section I will survey major reasons, past, present, and future, for adults returning to study mathematics, highlighting some of the barriers and boundaries they have confronted in the past, and outlining those which the future might hold.

The past: unfinished business

The first two parts of this section draw largely on FitzSimons (1994). For adults, unfinished business may arise from restricted access to mathematics education, or education more generally, due to a variety of reasons which may be broadly classified as socio-economic, socio-political, and cultural-historic, but which intersect and overlap, and are in a constant state of flux.

In the formative years, a variety of contextual situations may lead to a child missing school or changing schools frequently, or leaving school at or below the minimum age, perhaps even prematurely to their desires and/or abilities. These include relative poverty, chronic illness or unplanned pregnancy during the school years, social and cultural traditions and expectations (particularly in the case of girls and women), belonging to a social or cultural minority group, and family immigration choices. Deliberate acts of human aggression and/or destruction and cultural discrimination (race, gender, religion, etc.) also have potentially negative effects on children's education. Such acts include views of individual worth vs. political expediency (at all levels of education), outdated beliefs in conservatism (back-to-basics) and control in education (curriculum, pedagogy, & assessment), political preferences for funding favoured groups (classes, cultures, religions, etc.) while de-funding others, individual teachers' negative discrimination/mistreatment (mental, physical, racial, sexual, etc. abuse). On a more global scale, children's education may be disrupted or non-existent due to the impact of war, ethnic cleansing, and forced relocations; disruptions also occur as a result of environmental disasters, natural as well as due to human activity.

The present: more contemporary reasons

In addition to the desire to overcome and ameliorate prior disadvantage, as discussed above, adults may find themselves motivated to return to study mathematics for a variety of reasons, according to the circumstances they face at the time or see as likely in the future. As adults they are likely to have more agency and control over some things, but not all. The broad classifications listed above (social, cultural, economic, and political) may still apply.

From personal experience, one of the most important reasons for adults choosing to return to study mathematics is likely to be curiosity: the joy of wanting to learn (more) mathematics — its big ideas, techniques, puzzles, history, aesthetics, etc. — for its own sake, and now finding it actually making sense! Seeking personal enrichment and new friendships, and being among a group of like-minded adults who are sharing this journey, affectively (Kelly, 2018) and cognitively, in a respectful environment where no questions are belittled or ignored — no matter how seemingly oblique or “dumb,” and encouragement is pervasive — are motivations among adult learners. The fact that there are alternative, meaningful, and respectful adult teaching approaches, accepting that there are many points of view and ways of looking at problems which are frequently derived from the adults' own experiences, also encourages people to join and remain with the class. Helping others, such as children,

and other family members, also community groups (e.g., children's homework, a family business, social or political groups) is often given as a reason for joining a class. There might also be an expressed need to keep up with technology, especially new mathematical and other learning technologies, or to (re)learn mathematics in English or other new home language (see, e.g., Safford-Ramus, Maaß, & Süß-Stepancik, 2018).

Very often, the desire to study mathematics arises from a re-evaluation of the learner's current situation. It could be finding a new direction, often after a significant life change, including relationship breakup, children all left school/home, release from carer's duties, etc. It could be due to the dawning realization of having had limited options over a lifetime or in current work, for example, and attempting to overcome these. For many people at this point, the need (but not necessarily the desire) to study mathematics could arise as a compulsory requirement: as a condition for receiving government or other benefits, or as an entry test or hurdle requirement for employment and/or further education. Sometimes awareness of the importance of being able to do more mathematics, and to feel comfortable in using it, arises part-way through, or even after, programs delivered in or for the workplace, as people feel more empowered to negotiate, to answer back to employers' claims of poor work practice based on incorrect or misleading statistical claims. On a more positive note, workers feel more empowered to ask intelligent questions about work practice with a view to improvement or the avoidance of serious errors (FitzSimons, 2014, 2015; FitzSimons & Boistrup, 2017; Kelly, 2018).

Contemporary and future issues: globalisation and technology

Many of the issues listed above are related to the phenomenon of globalisation which has, by its very nature, changed the conditions of living and working (in the industrialised world, at least) over a relatively short space of time.

The phenomenon of globalisation has been characterised as a series of inter-related flows, including knowledge, capital, jobs, people and cultures, ecological systems. It is also associated with increasingly rapid developments in information and communication technologies [ICTs] as well as transport, facilitating cultural and economic exchange. (CIEAEM, 2010)

Underlining the massive changes in the way that people have lived and worked, Gray (2016) reminds us that the first Industrial Revolution used water and steam power to mechanize production, the second used electric power to create mass production, and the third used electronics and information technology to automate production. He claims that the fourth Industrial Revolution (where we now find ourselves) builds on the third, the digital revolution that has been occurring since the middle of the last century.

Discussing the fourth Industrial Revolution from a human capital perspective, Gray (2016) argued that there will be major changes to the skills that have, until recently, been considered important in the workforce. Technological developments which transcend national boundaries are transforming the way people live and the way they work. This means that some jobs will disappear, others will grow, and jobs that don't even exist today will become commonplace. As a consequence, creativity will become one of the top three skills workers will need to accommodate new products, new technologies, and new ways of working. People working in sales and manufacturing will need new skills, such as technological literacy. The likelihood is that there will be several changes of jobs and even occupations, in different geographical locations, periods of unemployment or under-employment, and either more casualised/contract labour or relatively more secure employment for the fortunate, with a hollowing out at the centre. Automation and the technologically mediated global movement of people, knowledge, and capital, is profoundly changing work and working lives. The demand for highly skilled workers has increased while the demand for workers with less education and lower skills has decreased.

By 2020 the top three skills needed by workers will be:

- Complex problem solving
- Critical thinking
- Creativity (Gray, 2016).

This global shift towards the ongoing development of advanced technologies in work and elsewhere has implications for education, particularly for adults returning to study mathematics for work related reasons. They are likely to be involved in various forms of education throughout their lifetimes, while having to adapt to ongoing technological change in the mathematics classroom and beyond. No longer is it sufficient to equip adults with just the so-called “basic skills,” important as they are. Adults need to understand the structural properties of number and quantities, right from the beginning (Tuominen, Andersson, Boistrup, & Eriksson, 2018), as I will discuss later.

On a personal note, I grew up in an age of apparent certainty, change happened at much slower rate, jobs were for life, and vocational qualifications or a basic university education was for most people considered the completion of an education. Lifelong learning was not even considered as a possibility or necessity, technology was in its infancy, and school subjects were compartmentalised; learning mathematics had little or no obvious relation to any other school subject or everyday life, apart from number skills and measurement.

As a mathematics teacher over many decades, and across different sectors of education, I have had to learn new content, new pedagogies, and new techniques. In terms of mathematical content, I was able to do this relatively easily because I had a sound understanding of the basic structures of mathematics. Any teacher or researcher starting in a new position will also realize just how much they need to learn about the context: “how things are done around here.” These ideas about the need for mathematical structures and contextual knowledge will be elaborated on below.

Considering that the fourth Industrial Revolution is already upon us, and likely to intensify in ways we cannot as yet conceive, how can we as members of ALM, as mathematics (& statistics) teachers in adult, vocational, and further education, and as researchers, support our students to prepare for learning new ways of living and working (just as people of my generation have had to learn about and to participate with some degree of confidence in the rapidly evolving technological world)? This means not only preparing for participation in the economic sphere, but also in the aesthetic and cultural spheres where mathematics plays an integral role as part of our humanity.

Perspectives on mathematics

The word mathematics is often taken for granted. In this section I offer a brief selection from the literature to illustrate the complexity of the concept of mathematics, going beyond the relatively narrow impression given by most school mathematics texts and public discourse, thereby setting implicit boundaries as to what is valued in society.

A cognitive perspective

According to David Tall (2013, see chapter 6), mathematical thinking develops through three worlds of mathematics:

- the practical mathematics of shape and number, with experiences in shape, space, and arithmetic (embodied operations);
- theoretical mathematics with a focus on properties, leading to Euclidean proof and algebra, and definitions based on known objects and operations; &
- formal mathematics, working with properties proved from given axioms and definitions, and formal objects based on formal definitions.

Tall stresses that each world is interlinked with the others and valuable in its own right.

Although invisible in most workplaces, the logic and mathematical power of formal mathematics underpin technologies of management and production: for example, extending human capabilities in the realms micro- and nano-technology, extreme speeds, temperatures, distances, etc. The more easily visible theoretical mathematics, with its symbolic algebra and geometry, can be seen in the functioning and design of spreadsheets, three-dimensional machining, and quality control statistics, for example. Finally, the conceptual embodiment of shape and space, the operational symbolism of arithmetic, with ubiquitous forms of measurement (formal and informal) are the most easily visible forms of practical mathematics (FitzSimons & Boistrup, 2017). Although practical mathematics is where we begin as humans, it is also an essential part of much mathematical activity at work and elsewhere to ensure that desired outcomes may actually be achieved. One important aspect of doing mathematics is accurate communication.

A sociocultural perspective

Anna Sfard (2017, p. 42), in her conceptualisation of mathematics “as a certain well-defined form of communication,” asserts that: “to think mathematically means communicating — with others or with oneself” — in the special way generally endorsed by the mathematical community. Her major contribution to the field is that cognition and communication are fully integrated, and she coined the word commognition to capture this.

A philosophical perspective

Roland Fischer (1993) provides an explanation for the apparent alienation of people from mathematics on a personal level when he outlines the duality of mathematics as a means and a system:

Mathematics provides a means for individuals to explain and control complex situations of the natural and of the artificial environment and to communicate about those situations.

Mathematics is also a system of concepts, algorithms and rules, embodied in us, in our thinking and doing; we are subject to this system, it determines parts of our identity. This system runs from everyday quantifications to elaborated patterns of natural phenomena to complex mechanisms of the modern economy. (pp. 113-114)

It is the systemic, embodied, side of mathematics which is often overlooked in public discourse. However, both aspects have a role to play in people’s lives, and mathematics education plays a crucial role in helping learners of all ages to develop agency. (See FitzSimons, 2002, for further discussion on mathematics and mathematics education.)

Doing mathematics at work (paid & unpaid)

Doing mathematics at school and doing mathematics at work are two very different activities, epistemologically and socioculturally (FitzSimons, 2013). Most mathematics teachers and students who visit work sites, and even workers themselves, find it very difficult to recognize any activity they are able to judge as being mathematical beyond number and measurement. At work, according to FitzSimons and Boistrup (2017):

people are required to use, develop, and communicate mathematical ideas and techniques in a diversity of ways with others who have differing expertise, experience, and interests, including in mathematics itself. Problems requiring mathematical reasoning and calculations are usually embedded in physical and/or intellectual tasks, rich in context, with a range of constraints that are oftentimes mutually contradictory, but always need a workable answer & within a short space of time. ... In many jobs problem solving is an expected and routine part of the day’s work: Every new request or order requires an original or customised solution within given parameters. Whether using mathematics explicitly or implicitly in these processes, no matter how trivial, the worker must also take into account all of the relevant contextual knowledge in their decision making. Crucially, the kinds and complexity of problems that occur at work contrast sharply with those found in formal mathematics education texts and assessment tasks. (pp. 229-230)

One of the most important aspects of working life is the continuous learning that takes place, mostly unconsciously and generally unremarked. First, however, I briefly address pedagogic rights.

Democracy and mathematics education

Bernstein (2000) advocates three pedagogic rights which enable democracy to function: enhancement, inclusion, and participation. At an individual level, the right to enhancement is the right to the means of critical understanding and to new possibilities and a condition for confidence. At the social level, is the right to be included, socially, intellectually, culturally, and personally (inclusion). At the political level, the right to participate must be about practice which has outcomes (participation). How can adult mathematics education position itself to support students in asserting these three rights, so essential in this uncertain world of rapid economic, social, and technological change?

Bernstein (2000) discusses formal mathematical pedagogic situations, where arbitrary selections are made from the discipline of mathematics which provide different forms of consciousness to different social groups, with differential access to what he terms unthinkable knowledge in the possibility of new knowledge creation (e.g., curiosity, creativity, innovation), as distinct from thinkable knowledge which takes the form of official knowledge (e.g., rule following). Given that both forms of mathematical knowledge, thinkable and unthinkable, are likely to support adults developing agency in work and life generally, in line with the future trends outlined above, I turn to the kind of mathematics education which encompasses both kinds of knowledge development.

Perspectives on learning

In this section I will focus on the kinds of learning that can take place in adults' lives, where their mathematical knowledge and skills can play a crucial role in supporting others as well as in enhancing their own agency in our era of rapid change and uncertainty.

Learning through practice-based innovation

Learning takes place throughout a lifetime, consciously or otherwise. As discussed above, adults today will have a much greater need to learn than in generations of the past, especially in relation to the need for innovation. In the workplace, Ellström (2010) identified practice-based innovation as a cyclical process of learning emanating from the problems that need to be addressed and solved on a daily basis. The need for innovation may occur within a logic of production, where speed, accuracy, and consistency are needed and valued in routine procedures and processes — for example in standardised medical and scientific procedures or manufacturing production processes. Starting from the explicit work process, officially prescribed and based on existing codified knowledge (theories, models, etc.), following the logic of production there is a process of adaptive learning and reproduction which results in the implicit work process, where the work is subjectively interpreted and performed on the basis of tacit knowledge. However, where possible legally and in practice, variation and improvisation play a role leading to developmental learning, and the task is transformed through what Ellström termed the logic of development, where questioning, creativity and innovation are more highly valued. As these innovations are accommodated into explicit work practice, the cycle continues. The relationship with Gray's (2016) top three skills of Complex problem solving, Critical thinking, and Creativity is clearly apparent here. In many jobs, problem solving is an expected and routine part of the day's work, where each new "task" demands individualised attention to specific contextual setting and "customer" request, whether arising from within or beyond the actual worksite.

Informal learning at work

As noted above, many people do not consciously recognize that learning takes place at work. Learning via non-formal education may occur via specific activities such as training courses in new procedures or new skills development. Informal learning at work is an ongoing process that encompasses personal, social, and cultural knowledges and skills (Eraut, 2004). Importantly, there is an ongoing need for communication of an educative kind between stakeholders where information, often mathematical, is sought and shared (see, e.g., FitzSimons, 2013). Such communication can be considered as a pedagogic relation (Bernstein, 2000). People's mathematics-related knowledge is, or can be, transformed from the academic discipline of mathematics to address the specific context of the problem at hand. Pedagogic relations occur frequently in communications at work, for example between: (a) co-workers, (b) managers or team leaders and workers, (c) clients and their suppliers of services or goods, etc. Workplace mathematics is not commonly recognized as a pedagogic activity, unlike school mathematics where the activity takes place within recognized spaces, at particular times, for given periods, with a recognizable hierarchy between the teacher and the taught. School mathematics curriculum and assessment is generally controlled by external authorities, and conducted via a limited range of activities, mostly verbal and written, sometimes online, and sometimes with specific mathematical tools such as rulers and protractors. By contrast, mathematics at work may be partially or completely structured by the available artefacts and employing a range of verbal and non-verbal communication modalities (Björklund Boistrup & Gustafsson, 2014), and takes place within a wide range of possible constraints, such as time, money, specific industrial legal requirements, and occupational health and safety laws. Most important, mathematical solutions at work need to be fit for the particular purpose and error free.

Pedagogic relations at work

How do pedagogic relations at work actually function? Following Hordern (2014), recontextualisation offers a powerful means of “bridging the gap” between theoretical and experiential knowledge. Recontextualisation combines the disciplinary knowledge of mathematics with contextual knowledge of the specific situation, including personal and sociocultural knowledges, in the form of communication (with self or others; cf. Sfard, 2017). Adult students in formal education also need to learn these skills to communicate effectively with other stakeholders. Doing mathematics at work (& elsewhere) involves: learning to find out what to do, how to do it, when, where, how often, and sometimes why, ... until there are no more questions. However, the learning and teaching process is two-way, depending on who has information and/or authority to speak, to ask, and to answer (FitzSimons & Boistrup, 2017). Communicating at work can take place through symbols, diagrams, speech, gestures, and so on — often simultaneously, described by Björklund Boistrup and Gustafsson (2014) as multimodal. In summary, (mathematical) recontextualisation at work involves transforming what you know, both mathematically and contextually, to make the most appropriate decision; or perhaps even to ask further questions.

The importance of knowledge structures

In order for adults to be able communicate effectively at work and elsewhere, what kind of mathematical knowledge structures do they need? Is it enough to teach them only “the skills they need” — as seems to be common practice in many English-speaking countries around the world where competency-based frameworks hold sway over adult literacy and numeracy as well as vocational education? Random collections of mathematical rules and ‘tricks’ needed for particular contexts will not be sufficient for learners/workers to develop new knowledge to meet local conditions unless they have access to the integrating structures of the discipline of mathematics as a foundation on which to build.

The importance of both vertical discourse and horizontal discourse

Bernstein (2000) describes vertical discourses such as mathematics as being comprised of disciplinary knowledge (albeit fallible), theoretical, conceptual, and generalizable knowledge, coherent, explicit, and systematically principled. Because the procedures of vertical discourse are linked hierarchically, they allow the integration of meanings beyond relevance to specific contexts. In other words, once you understand a mathematical principle, you can use it in a range of contexts. However, although a working understanding of the vertical discourse of mathematics (at whatever level the person has reached) is essential, it is not enough to guarantee that a person will be able to communicate and participate fully at work or as a citizen. Knowledge of the horizontal discourse within the particular context is also absolutely necessary to make sense of the task at hand.

According to Bernstein (2000), horizontal discourse refers to contextual knowledge, which is likely to be oral, ... tacit, multi-layered, and contradictory across but not within contexts (p. 157). Individual skills may have nothing in common with other skills, but may be related to specific contexts or to everyday life, and pedagogic practices for these may well vary with each activity. Examples include learning to tie up one's own shoe laces, brush one's teeth, ride a bicycle, drive a car, make a cup of tea, ... Such knowledges are "culturally localised, and evoked by contexts whose reading is unproblematic" (p. 159). (As an aside, these kinds of skills may be lost with the degeneration that occurs with old age!) A person may build up an extensive repertoire of strategies which may vary according to the context; a group may likewise build up a reservoir of strategies of operational knowledges. In horizontal discourse, there is not necessarily one best strategy relevant to any particular context. (See FitzSimons & Boistrup, 2017, for further explanation in relation to mathematics at work.)

Being competent to work within the vertical discourse of mathematics is necessary but not sufficient. When employers complain that new graduates cannot use their mathematical knowledge and skills, this is because they have not yet comprehended the horizontal discourse at the local worksite. There are specific ways of being a research scientist, a mathematics teacher, a plumber, etc., and it is also crucial for learners and workers to develop the necessary situational and contextual knowledge and skills (practical, personal, social, structural, etc).

In order to illustrate the claims made above in relation to knowledge structures, I turn to the theoretical discipline and practice of music which in many ways bears a parallel to mathematics but may offer insights through the potential lack of familiarity (or "otherness") to many readers belonging to the adult mathematics education community.

Mathematics and music

Structured bodies of knowledge are important. They allow us to account for and explain the natural and social world in systematic ways, and to participate in, and reflect on key human experiences (see Bernstein & Democracy, above) such as in the literary, visual, or musical domains. Disciplines such as mathematics and music enable abstraction, reflection, prediction, and application across time and contexts.

At ALM conferences, music has played a big role over the last 25 years in supporting social cohesion and in breaking down potential boundaries between the many international participants in an enjoyable and instructive way. Many conference delegates have revealed hitherto unknown talents in musical and theatrical performance. (This was particularly in evidence at the ALM 25 conference dinner!) The question is: How is it that people can have agency to learn and to create new music beyond merely learning by imitation — important as that is in passing on cultural knowledges and skills?

Music and mathematics have many similarities. Music performance, like mathematics, requires mastery of simple pieces, gradually building up to a reflective, nuanced, and more original practice.

Formal music education, theoretical and practical, offers insight into musical conventions and boundaries, even if only in order to violate them! Mathematics and music are both pan cultural, with historical and cultural variations, including ethno-mathematics or ethno-music, and are available to all of us as part of our humanity, in theory at least: for example, rhythmic clapping and community singing. Mathematics and music each have: (a) a structured body of knowledge which is ever evolving, (b) a codified set of symbols with universally accepted meanings within the discipline, and (c) several different sub-fields (instrumental & vocal specialisations, genres etc. in music; specialisations such as algebra, geometry, statistics, etc. in mathematics), each with their own rules, but within the framework of a vertical discourse. In an era where integrated STEM [Science, Technology, Engineering, & Mathematics] education is widely promoted, in some form of interdisciplinary education, there is no reason why music should not be included — perhaps as a form of technology. In FitzSimons (2001) I offer an example of my own experience of integrating disparate disciplinary structures in workplace mathematics and statistics education.

Music, like mathematics, also has the possibility of knowledge and skills leading to performance being passed on through an oral tradition, often culturally specific, and displayed/performed in contextually appropriate settings. In music, popular social and cultural songs are sung on appropriate occasions, and are a valuable means of social cohesion, for example: Happy Birthday, Auld Lang Syne, Danny Boy (Londonderry Air), Will Ye Go Lassie Go? In mathematics, most children have the opportunity to learn counting rhymes, songs, and related stories. Both vertical and horizontal discourses exist in music and mathematics: knowing what to do, how to do it, where, why, and when.

As in mathematics, there are obvious differences between professional, amateur, and novice musicians. The more skilled and knowledgeable the musician, the easier it is for them to confront and solve unexpected problems that may arise in the form of being asked to sight-read and perform an unseen piece of music, or even to perform it in a different key (transpose it up or down). When you are confronted with a piece of sheet music, what do you see? Does it have any meaning for you? What meaning you make depends on where you are on the continuum of expertise. Is it too much of a stretch to imagine that the task of interpreting an unfamiliar page of music is parallel to how mathematics appears to many adults? What are the consequences for people with a limited, interrupted, or discontinuous mathematics education? Do they feel confident, feel included, and are they able to participate in this human activity? How will learning only “the maths you need” for a certain job at a certain moment in time help people to live in the world in our uncertain times?

A brief example

In work I did recently to support young adult learners returning to study mathematics and their tutors and in a numeracy program situated in a learning outcomes framework, the Measurement outcome stated:

Can use straightforward measurement and the metric system to estimate and measure for the purpose of interpreting, making or purchasing materials in familiar practical situations.

It contained three Elements: Mathematical knowledge and techniques, Language, and Interpretation. These were elaborated using active verbs such as estimate, choose, use, read, convert, calculate, interpret, and decide on reasonableness.

At first glance, these sound reasonable, and the document offered possibilities for holistic assessment. However, there are several things to observe. First, the language implies rule following, and clearly measurement should be appropriate, and accurate to appropriate levels. Although it is important that these learners should be familiar with parts of the metric system listed, the document did

not invite them to pursue related mathematical questions of their own, and it offered no intimation of a holistic system of measurement, the *Système Internationale* or SI. It is likely that some learners in the group had not yet developed a secure understanding of the decimal number system, or even the basics of whole numbers, and none of the small number of professional staff had any post-school mathematics qualifications. There was no underpinning structure of the mathematics available to assist the teaching staff in identifying predictable knowledge gaps or to demonstrate to staff and learners how the various mathematical outcomes fit together. The selection of a collection of well known, but not necessarily well understood, metric units and prefixes limits the ability of learners to see the bigger picture and develop their own knowledge in this area. As such, it is not clear how learning outcomes such as this could engender the creativity, problem solving, and critical thinking (Gray, 2016) in relation to the learners' mathematical knowledge.

In the limited space here it is not possible to outline a whole teaching program, but here are some possible early approaches in classes of learners with diverse and intermittent histories of schooling (in no particular order). Ideally, the learners should be working in small groups.

- Explore the history of the basic metric units of length and mass, and the use of water in the latter
- Construct a portable “metre ruler” (e.g., from foldable soft cardboard) and, when appropriate, subdivide it into decimetres and centimetres; similarly with common & decimal fractions, and percentages, in order to anchor meanings and build cross-connections
- Explore (together) the origin, etymology, and use of language in the (invariant) metric prefixes, and gradually build up a metric table for each learner or group of learners
- Extend the meanings of prefixes to introduce the concept of powers of 10, eventually using exponential notation when appropriate (e.g., 10²)
- Use the internet to investigate very large and very small numbers
- Collect food packaging (e.g., rectangular boxes, cans), and analyse the mathematical information on the labels; also draw out relations between lengths, surface areas, volumes, and capacity
- Problem solve how to deal with irregular shapes
- Develop embodied knowledge about the approximate length of 1mm, 1 cm, 1 metre, etc., also the mass of various common items weighing a gram, 100 gram, 500 gram, a kilogram etc.

On a larger scale, invite learners to participate in projects involving measurement and data where they can achieve concrete outcomes within given mathematical and other parameters; where possible these should be agreed upon by teacher/tutors and learners alike. These could include designing activities and explaining the reasoning to others. Most importantly, these projects offer the opportunity to confront reality with its inherent constraints, and where the best mathematical solution may not be the best in practice or may not even be feasible at all. For example:

- In the local community space, indoors or outdoors, (re)design or renovate an existing space; or organise a theme-based walking tour or fun run for community members.
- Collect, analyse, interpret, and present data in order to argue for or against a case concerning a locally important issue
- Collect information on health & nutrition, keep diaries of relevant food intake over a given period of time, prepare and serve a meal to others in keeping with recommended food guidelines.

It may also be feasible to involve learners in the assessment of such projects, particularly those of other groups (see, e.g., Hahn & Vignon, 2019); even evaluating the project as a whole. Such projects expose the learners to both the vertical discourse of mathematics and the horizontal discourses of the tasks. They also have the potential for learners to notice serious flaws in their mathematical calculations, reasoning, or assumptions in a much more memorable way than textbook learning could ever do. The logic underpinning such activities is to invite the learners to actively participate in the world of mathematics in ways that have meaning for them, and to see mathematics as both a means towards

achieving an outcome or to support making a decision, and as something that is part of our common humanity (cf. Fischer, 1993).

Conclusion

Although there are obviously many other theoretical and practical approaches to adult education (see, e.g., FitzSimons, 2010), this article does not attempt to reflect directly on any particular teaching practice. I have drawn predominantly on the work of Bernstein (2000) to argue for the importance of developing structural knowledge in the discipline of mathematics, from the earliest educational levels. At the same time, adults need to be as fully aware as possible of the given contextual setting of the situation they are addressing, in order to be able to recontextualise the mathematics they know in ways that are adequate and appropriate to that situation. Communication and recontextualisation are multi-directional processes to understand and inform the thinking of ourselves and others.

As Bernstein (2000) argues, abstract theoretical knowledge enables society to conduct a conversation about itself, and to imagine alternative futures; and access to “unthinkable” knowledge enables the possibility of new knowledge creation (i.e., through curiosity, creativity, innovation). For a person to have agency, to have confidence, to feel able to be included, to have the right to participate in the discourse of the discipline of mathematics, and to be able to learn new things and to create (locally) new knowledge, they must have access to the most relevant, structurally supported and connected mathematical knowledge, which is at the same time flexible enough to meet their contextually specific demands. This means that they should be able to recognise the same or similar mathematical structures in novel contexts and transform the mathematics they already know, creating a potentially innovative and workable solution in that particular context.

Learning outcomes in adult and vocational education often tend to abstract the lived experiences of adults from the power relations of work and life and to negate the possibilities of understanding and criticism. We need to teach adults how to use mathematics to answer back to power at work and in society more generally, going well beyond the mastery of mathematical techniques and the following of sets of instructions without question. For this reason, to paraphrase the words of one reviewer, we need to be alert to adult mathematics education contexts that undervalue structural mathematics.

In stressing the importance of taking into account technological innovation and rapid change globally, rather than a backward looking orientation on the reproduction of assemblages of skills of the past, I raise the following question: Do we, as adult mathematics education teachers and researchers, not have an ethical responsibility to ensure that our students, at whatever level, develop more secure and connected content knowledge in order for learners to have the possibility of transcending the boundaries and barriers at work and elsewhere in an uncertain world. Without access to both vertical and horizontal discourses, often invisible boundaries and barriers will be very difficult to transcend.

Acknowledgements

I would like to acknowledge my colleagues Tine Wedege and Lisa Björklund Boistrup. Thank you to Tine, for “saving” me from premature retirement by inviting me to join her Swedish Research Council project: Adults Mathematics: In Work and for School, and thank you to Lisa for continuing with the project and beyond. I also thank Lisa for her comments on an earlier draft of this article. Thanks also to the thoughtful comments of the reviewers.

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