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Beauty, Bees, and God: The Fibonacci Sequence as a Theological Springboard in Secondary Mathematics

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A Catholic school is called to be much more than a school that prepares students for college and career readiness. While this is part of its mission, a Catholic school should be in the business of making saints. As a high school math and theology teacher, I seek to use the limited contact time I have with students to engage them not only in academic pursuits but also in a discussion of our divine mission as humans. Since this can be difficult to meaningfully do in a math class, I am continually searching for meaningful, engaging points of contact between mathematics and theology. These points of contact can serve as launching points for engaging students in dialogue about the Catholic faith, thereby giving me an opportunity to intentionally accompany students on their own faith journeys and aid them in this quest to become saints.

I have found the Fibonacci Sequence to be a topic that not only piques students' interest about sequences and series but also serves as a springboard to theological discussion. In this article, I will provide a brief historical background to the Fibonacci Sequence, an explanation of how it can be used in a high school math classroom, and an exploration of three different theological touchpoints that the Fibonacci Sequence offers.

The Fibonacci Sequence: Historical and Mathematical Background

The sequence now commonly referred to as the Fibonacci Sequence was formally introduced to Europe in 1202 by an Italian mathematician named Leonardo de Pisa in *Liber Abaci*. In this book, de Pisa proposed the string of values to solve the rabbit problem:

> A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive? (Trefil, 2003, p. 169)?

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The problem, which assumes that rabbits are immortal, can be solved by modeling the behavior exhibited by pairs of rabbits over the course of the first several months and finding the pattern to their reproduction. The sequence below shows the number of pairs of rabbits present in a specific month; the rabbit problem can be solved using it.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, …

Leonardo de Pisa, now typically called Fibonacci, proposed this sequence (absent the first value) and noted its repetitive nature.

In the centuries that followed, countless mathematicians would explore the shocking mathematical patterns found in this simple sequence and would discover applications of the sequence throughout creation. One finding is worthy of mention.

In 1608, Johannes Kepler discovered a connection between the Fibonacci Sequence and the Divine Proportion, a name that a 16th century Franciscan friar named Luca Pacioli had given to the irrational quantity commonly known now as the golden ratio. This value, approximately 1.618, had long been considered an aesthetically pleasing ratio and a value that mysteriously and seemingly supernaturally appeared throughout creation. For example, the golden ratio has been observed in measurements of human arm and hand bones, human facial structures, and other anatomical features (Wang, Ma, Jin, & Yu, 2017). The earliest precise definition of the golden ratio is found in Euclid's *Elements* in approximately 300 BC. Since then, mathematicians, scientists, architects, artists, and

Fibonacci Sequence	Ratio of Consecutive Fibonacci Numbers
1	-
1	1
2	2
3	1.5
5	1.666666667
8	1.6
13	1.625
21	1.615384615
34	1.619047619
55	1.617647059
89	1.618181818
144	1.617977528
233	1.618055556
377	1.618025751
610	1.618037135
987	1.618032787
1597	1.618034448
2584	1.618033813
4181	1.618034056
6765	1.618033963
…	…

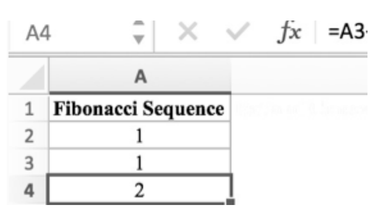
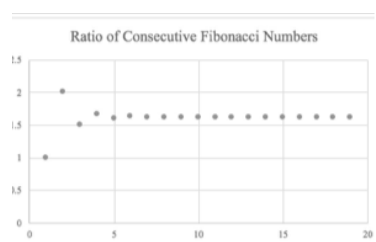
Figure 1. Fibonacci sequence.

many others would be fascinated by this value. Kepler, a man of faith, "was searching for ultimate causes, the mathematical harmonies in the mind of the Creator" (Dampier, 1929/1989, p. 127). His 17th century discovery noted that the ratio of consecutive Fibonacci numbers formed another sequence which converged to the mysterious Divine Proportion, as shown in Figure 1. For Kepler, this discovery was a sign of the Divine Mathematician's intentional, rational, and beautiful creation (Livio, 2008).

Teaching the Fibonacci Sequence and the Divine Proportion

I have used the Fibonacci Sequence in units related to Sequences and Series in both Algebra II and Pre-Calculus classes. The lesson detailed below serves as an introduction to sequences, sequence notation, recursive equations, and limits (when appropriate in a Pre-Calculus class). Typically, I have allotted approximately 90 minutes for this lesson. As a result, I have typically extended this lesson progression over multiple days of class.

Brief	**Prayer:** I open with St. Thomas Aquinas' "Prayer Before Study."
15 Minutes	**Small Group Work:** I pose "the rabbit problem" to students, who work in small groups of three or four to model the month-by-month progression using a set of manipulatives I provide to them. Students use the assigned resource (coins, crayons, poker chips, etc) to model the maturation and reproduction pattern. I monitor small group work until students have created a model that resembles the image below. In this model, 'heads' represents an immature pair of rabbits and 'tails' represents a mature pair of rabbits. 
5 Minutes	**Gallery Walk:** I facilitate a brief gallery walk, where students see the other groups' solutions to the problem.
5 Minutes	**Writing:** By this point, most students have begun to grasp the pattern of the Fibonacci Sequence, so I ask students to write a sentence describing the pattern. I further refine this question so that students are required to not only describe the pattern but also the initial value.

20 Minutes	**Notes:** At this point, I guide students through a brief set of notes that includes the definition of a sequence, an introduction to sequence notation, and a definition of the Fibonacci Sequence, which requires the use of recursive equations. Additionally, I show students several real-world applications of the Fibonacci Sequence.
10 Minutes	**Reading:** To reinforce students' understanding of recursive equations and to push this lesson beyond mathematics, students read a short excerpt of Aquinas' writing in which he lays out a proof for God's existence. The line of reasoning used by Aquinas is explored later in this article and serves as a theological point of contact.
5 Minutes	**Practice:** Students practice defining equations recursively in groups.
15 Minutes	**Computer Modeling:** At this point, the leap between the Fibonacci Sequence and the Golden Ratio is explored through the use of spreadsheets. Using my projected screen, I guide students through the process of writing a simple function (see image below, left) to model the Fibonacci numbers. I then ask students to work in their small groups to write a function to determine the value of the ratio of consecutive Fibonacci numbers. This formula is applied to the entire column of Fibonacci numbers to produce a new column of values, which can then be graphed to produce a graph similar to the one below, right. I use this figure to show students how the consecutive ratios are beginning to approach a particular value called the Golden Ratio.  
10 Minutes	**Notes:** I formally define the Golden Ratio for students and then give a number of real-world examples. This point in the lesson represents an opportunity to connect mathematics to theology as will be described later in the article.
5 Minutes	**Closing:** At this point, I close the lesson by summarizing the day's findings and assigning any relevant homework.

I have found this lesson to be an effective introductory lesson because I can return to it throughout the unit and use it as a reference point for my students. For example, when students learn about explicit equations for sequences, many are tempted to ask why recursive equations are even necessary. Comparing the explicit (Figure 2) and recursive (Figure 3) equations for the Fibonacci Sequence has been an effective way for me to demonstrate to students that some sequences can be represented much more simply through recursive equations.

$$F_n = \frac{(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$$

Figure 2. Explicit equation

$$F_1 = 1$$
$$F_2 = 1$$
$$F_n = (F_{n-1}) + (F_{n-2})$$

Figure 3. Recursive equation

The remainder of this article will be dedicated to exploring three points of contact that the Fibonacci Sequence has with the Catholic faith.

Theological Point of Contact #1: Beauty and Evangelization

The intrigue generated by looking for examples of the Fibonacci Numbers and the Divine Proportion in nature, human anatomy, art, music, and even the ratio of length to width of a standard credit card quickly captures student interest and can generate moments of amazement for students. During one class period in which students were exploring occurrences of the Divine Proportion, a student raised her hand and said, "Why does this 1.6 number keep coming up? This is getting weird." This is exactly the mindset and attitude of awe that I, as a math teacher, want my students to discover. I want them asking the "Why?" questions to spark their interest and encourage them to dig more deeply into mathematics. More importantly, though, these moments of amazement and "Why?" questions can bring us to an appreciation of the beauty of God's creation.

Mathematics can reveal this beauty, which in turn reveals God. As Galileo Galilei argued in *The Assayer*, the world "is written in the language of mathematics" (1623, p. 4). Thus, as a math teacher, I can help students to use mathematics to unveil the incredible complexity and simultaneous beauty of the world. Like Kepler recognized, this unveiling should lead us, students and teachers alike, to marvel at the beauty of God's handiwork and thereby give Him praise.

Theological Point of Contact #2: Bees and the Exsultet

Inevitably, during this lesson, questions will arise about whether the original rabbit problem is a realistic scenario. When this question arises, I acknowledge the theoretical nature of the problem but note that the Fibonacci Sequence *can* be used to model the ancestry chart of a male honeybee. A male honeybee is not the product of a fertilized egg from a mother and father bee; rather, through a unique biological process, a male honeybee is the product of an unfertilized female egg (Gempe et al., 2009). As such, an ancestry chart for a single male honeybee would resemble Figure 4.

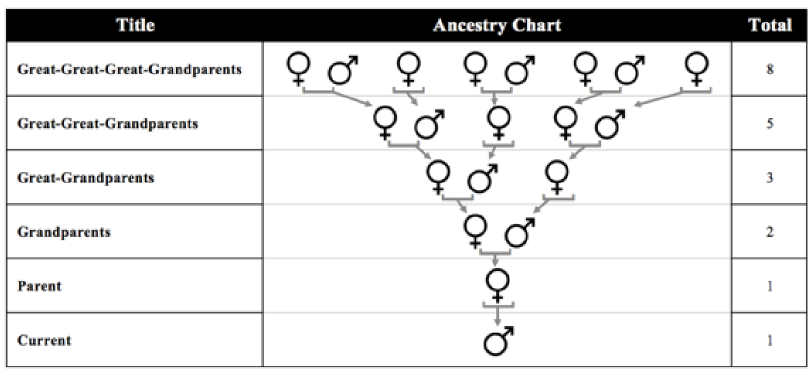


Figure 4. Ancestry chart

As is apparent in the chart, the number of bees in each generation matches the terms of the Fibonacci Sequence. This connection typically appeases this common objection to the rabbit problem, but it also serves a deeper purpose.

Honeybees[1] have historically been a symbol used by Christians. To understand the symbolism, let us turn to an ancient hymn of the Church. The Exsultet, sung each year at the Easter Vigil, only directly mentions two animals, the lamb and the bee, which is mentioned twice.

> On this, your night of grace, O holy Father,
> accept this candle, a solemn offering,
> *the work of bees* and of your servants' hands,
> an evening sacrifice of praise,
> this gift from your most holy Church.

1 Interestingly, the cover of *The Assayer* featured the crest of the family of Pope Urban VIII, which portrayed three bees.

> But now we know the praises of this pillar,
> which glowing fire ignites for God's honor,
> a fire into many flames divided,
> yet never dimmed by sharing of its light,
> for it is fed by melting wax,
> *drawn out by mother bees*
> to build a torch so precious. (The Exsultet, 2010, *emphasis mine*)

At first glance, it may seem odd that the bee is featured so prominently in this hymn. However, these two phrases indicate two elements of Christian symbolism commonly associated with bees.

First is the idea of the working nature of bees. As Pope Pius XII noted in 1948, "Bees are models of social life and activity, in which each class has its duty to perform and performs it exactly…" (Pius XII, 1948). The analogy Pius XII goes on to draw is to the necessity of humans to seek God's will and use their lives to glorify God. One of the products of the bees, honey, is frequently viewed in Scripture as a symbol of goodness; so, too, is the product of humans when we cooperate with God's will. Even the wax that bees produce can be used to form a candle, which serves a role in providing light. The flame of the Paschal Candle used at the Easter Vigil, which literally lights the darkened church building, is analogous to the light that we are called to provide in this oft-darkened world.

The second element of symbolism present in the Exsultet is the mother bee's connection to Mary. While the analogy is imperfect, the apparent virgin birth of a male bee to a female mother has historically been viewed as a symbol of Mary's virgin birth of Jesus.

Theological Point of Contact #3: Aquinas and the Five Ways

As part of this lesson progression, I challenge students to describe the sequence using words. Most students will write something like "The next term can be found by adding the previous two together." While this description accurately describes the pattern, I ask students to refine their descriptions by posing the following question: "If you were describing this sequence to someone who had never seen it before, what information would you have to give them in addition to the description that you provided?" This question leads to an important insight for defining sequences recursively: one not only needs to describe the process for moving from one term to the next but also must define the first value(s).

The concept of recursive equations requiring a defined beginning point that lies outside the described pattern is like an argument developed by St. Thomas Aquinas to prove the existence of God. He used a philosophical argument originally proposed by Aristotle to write about five ways to prove God's existence in his most well-known text, the *Summa Theologiae*. The second of his arguments is summarized as follows:

> We find that among sensible things there is an ordering of efficient causes, and yet we do not find—nor is it possible to find—anything that is an efficient cause of its own self. For if something were an efficient cause of itself, then it would be prior to itself—which is impossible… But it is impossible to go on to infinity among efficient causes… Therefore, one must posit some first efficient cause – which everyone calls a God. (Aquinas, 1485)

While the philosophical terminology used in this argument may be confusing for an untrained philosopher, the basic notion behind the argument is that there must be an uncaused cause that caused "things" (humans, rocks, paper, etc.) to exist. Without this cause, there is no being itself. In a very similar way to how a sequence cannot be defined without a value uncaused by the pattern, the world cannot exist without God.[2] Though this analogy is imperfect,[3] I have found it to be a useful tool to simultaneously teach stu-

2 Aquinas and other philosophers have shown that not only must God exist but also the necessity for God's definition as ipsum esse, or "being itself." God is not a being among many; He is being itself. When Moses asks for God's name on Mt. Sinai, God simply replies, "I AM." God is.

3 St. Thomas Aquinas did not believe that all types of infinite regressions were impossible; rather, he believed that an infinitely long list of causes was not possible. He thought it possible that a series of temporal events that serve as secondary causes could be infinitely long. In the comparison between Aquinas' second way and recursive definitions for sequence, there is a temporal order to the first, second, third, and infinitely more values. The first value is simply one of many values, unlike God's nature of being itself. Therefore, the sequence analogy is imperfect. Regardless, it can still serve as a helpful introduction to St. Thomas' reasoning.

dents about St. Thomas' second argument for God's existence and remind them of the necessity to identify the first term in a recursive definition of a sequence.

Conclusion

"Faith and reason are like two wings on which the human spirit rises to the contemplation of truth; and God has placed in the human heart a desire to know the truth" (John Paul II, 1998). As teachers in Catholic schools, it is imperative that we invite our students to study God and His Created world so that we can come to love Him more fully. Mathematical concepts such as the Fibonacci Sequence offer unique opportunities for teachers and students to journey on this pathway together in the classrooms of Catholic high schools.

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