A FIRST LAW THOUGHT EXPERIMENT WITH A SECOND LAW RESULT FOR HEAT ENGINES BASED ON A SELF-CONTAINED MODIFIED CARNOT CYCLE

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Abstract

The first law of thermodynamics does not forbid heat engines with 100% thermodynamic efficiency; it is the second law of thermodynamics which states that heat engines cannot obtain 100% efficiency. Can the first law ever show this second law result? Yes. We show that the first law alone can show that some heat engines cannot achieve efficiencies of 100% when self-contained cycles (which we define) are considered for some temperature reservoirs. Self-contained cycles do not rely on the surroundings for anything but heat transfer to the cycle. Self-contained cycles require that any work input required by the cycle must come from previously stored energy from the cycle (such as, by potential energy in a weight previously lifted by the engine when performing work). Also, any regenerative heat output must occur before it is used as regenerative heat input. Using an example of a self-contained modified Carnot cycle that utilizes an ideal gas, the first law shows that efficiencies of 100% are not possible for $T_{\mathrm{H}} < 2\,T_{\mathrm{C}}$, where T_{H} is at a hotter temperature than T_{C}, a colder temperature. These cycles, which could operate through a finite temperature difference ΔT in some regions of a PV diagram, are not possible in other regions.

Keywords: Thermodynamics; first law; second law; independence.

INTRODUCTION

Imagine that a scientist performs a thought experiment for a new engine cycle. The scientist has knowledge of the first law, but not the second, and wants to develop a 100% efficient heat cycle whereby all heat input is converted to work output. The first law will be used for an accounting of energy in forms such as work and heat, and the transfer of energy. Apart from the first law, the only other requirement on the cycle will be that it is *self-contained*, which only allows heat input (not work) into the cycle from the surroundings (and no other interaction with the surroundings). This will be the entire basis for attempting to derive the 100% efficient cycle. This first law cycle will show that a 100% efficient heat engine is *not* possible after all, at least for the temperature region $T_{\mathrm{H}} < 2\ T_{\mathrm{C}}$, using reasonable temperatures $T > 0\mathrm{K}$. Therefore, the first law cycle leads to a second law result for a heat engine. For the remainder of this paper, we will follow the scientist's thought process for this first law thought experiment. [Although the scientist will not actually build the engine, if the reader were to take any first law issues with any steps/processes of the derived cycle, it would only support the argument that a 100% efficient engine is not possible on this "first law only" basis.]

Although historically the second law of thermodynamics was postulated before the first (Zemansky and Dittman, 1997), the first and second laws are normally taught and developed in sequential order (Baierlein, 1994). This is understandable since easier concepts normally precede harder ones. Sources such as Reese (2000) state that the first law is about conservation of energy and that the first law allows for the possibility of a 100% efficient heat engine. It is not until the second law that it is stated that heat engines cannot achieve 100% efficiency (p. 670). The unwitting scientist starts out with only knowledge of the first law but will end up with this second law result for heat engines.

Since the purpose for a heat engine is to convert externally supplied heat transfer to work, the desired cycle will not rely on energy from a previous cycle or work from an outside process. In this way, the cycle is *self-contained* and does not rely on its surroundings for anything but heat input.

In order for the cycle to be self-contained, any work input required by the cycle must come from previously stored energy from the cycle (such as, by potential energy in a weight previously lifted by the engine when performing work). Also, any regenerative heat output must occur before it is used as regenerative heat input later in the cycle. A review of the literature has not turned up the concept of a self-contained cycle, but it may well have been anticipated or approximated by others previously.

In order to derive any result based on the first law alone, the second law must be ignored. This statement must be considered carefully. It simply means that a *derivation* that relies on the first law alone cannot assume any prior knowledge of the second law. This does not mean that the derived cycle is physically realizable; the second law cannot be ignored in reality. Nevertheless, the cycle is perfectly valid for a first law analysis that shows a result typically associated with the second law.

The reader may find it difficult to ignore the second law. The situation is analogous to a paper by Lemons and Penner (2008) where the first law is suspended instead of the second law. They state that their *"… paper may tax the imaginations of its readers because we ask them to suspend their belief in the first law of thermodynamics. We realize that such a suspension is extremely difficult*" (p. 21). Similarly, in this paper, it may be taxing for the reader to suspend belief in the second law.

Since none of the equivalent statements of the second law will restrict the derivation, we can presume that 100% efficient engines are possible, and we can presume that heat transfer can occur spontaneously from a cooler to a hotter temperature. Without the second law, we can also presume that a maximally efficient engine could have heat transfer through a finite temperature difference. We can also presume that an engine cycle could perform work by extracting the same amount of energy from a single temperature reservoir, contrary to the Kelvin-Plank statement (Zemansky and Dittman, 1997, p. 153). It will also be assumed that all cycle processes can occur where the temperature and pressure of an ideal gas are quasi-static (*i.e.*, states and transitions can be depicted on a PV diagram). Quasi-static changes in a gas along a path from an initial state to a final state allow for work to be determined geometrically on a P-V diagram as the area under the path (curve), as shown by Reese (2000, p. 620).

Furthermore, the derivation of the first law cycle will not consider reversibility. This is because the concept of reversibility is associated with the second law and entropy. Together these processes, laws, and concepts dictate what heat engines can or cannot do. Zemansky and Dittman (1997, p. 158) describe and define a reversible process (which need not be repeated here) in a chapter on the second law. Marcella (1992) ties the second law with the concept of reversibility with the statement, *"…the second law states that all real processes are irreversible, generate an increase in entropy, and cause a degradation in energy"* (p. 890). Furthermore, Samiullah (2007) states that the second law should be discussed before reversibility:

> *The key to the idea of reversibility lies in the second law of thermodynamics which forbids any real process from being reversible. Therefore, we should discuss the second law of thermodynamics before or along with the definition of a reversible process. It is a mistake to introduce the idea of reversibility well before the second law has been discussed.* (p. 608)

The second law is related to reversibility; the first law says nothing about reversibility. Therefore, reversibility will not be considered for the derived first law cycle.

For a heat engine cycle, we desire to extract the most amount of work for the least amount of heat input. In order for a cycle to have positive net-work, the cycle starts at an initial state on a Pressure-Volume (*PV*) diagram and undergoes a clockwise cycle to return to the initial state. The net-work is equal to the area bounded by the clockwise cycle (Reese, 2000, p. 622). Since the initial and final states are the same, then any energy added to the system must be removed. For a heat engine, this energy is heat transfer and work.

For simplicity, when heat transfer input is meant, the term *heat input* or Q_{in} will be used: when heat transfer output is meant, the term *heat output* or Q_{out} will be used.

The first law of thermodynamics for any (general) closed cycle states:

$$W_{out} - W_{in} = Q_{in} - Q_{out} \qquad (1)$$

where: $W_{\text{out}} - W_{\text{in}}$ is the work output from the cycle minus the work input to the cycle; $Q_{\text{in}} - Q_{\text{out}}$ is the heat input to the cycle minus the heat output from the cycle

Note that we will use all work and heat terms with their absolute value, such as $W_{in} = |W_{in}|$, to avoid any confusion with signs. Also, only temperatures on an absolute scale (*i.e.,* Kelvin) are considered.

The cycle's thermal efficiency is:

$$\eta = \frac{W_{out} - W_{in}}{Q_{in}} \qquad (2)$$

Equation 1 shows that if it were possible to eliminate heat output (*i.e.*, $Q_{\text{out}} = 0$), then the thermal efficiency would be 1 (100%). The scientist's thought experiment will do just that. The next section shows that heat output from the cycle must be regenerated back into the cycle as heat input in order to obtain 100% efficiency.

HEAT OUTPUT AND REGENERATION FOR A CYCLE

Let us use only the first law to see what it can tell us about the feasibility of some cycles. We will assume that reasonable temperatures are used in the cycle (*i.e.*, temperatures above absolute zero, as the third law would dictate (Reese, 2000, p. 674-675)). As shown in Appendix A, all clockwise heat cycles (at least those without heat regeneration) must have heat output.

In order to eliminate heat output, we will consider a regenerative heat cycle that regenerates any possible heat output back into the cycle as heat input; there is no heat output from the cycle to the surroundings. Heat regeneration is heat energy transfer from one process of a cycle to another.

Initially, let's consider the standard Carnot cycle before any modifications. Ideally, the Carnot Cycle uses reversible processes, including isothermal and adiabatic processes, to achieve the well-known maximal Carnot efficiency $(1 - (T_C/T_H))$, which is shown from sources such as Wu (2015). Isothermal heating processes require the gas to maintain the same temperature while being heated.

The ideal Carnot cycle is shown in Figure 1. The cycle starts at state 1 via an isothermal expansion with external heat input Q_{in} at high-temperature, T_{H}, to state 2, followed by an adiabatic expansion process to state 3, then an isothermal compression with heat output Q_{out} at cold-temperature, T_{C}, to state 4, and finally an adiabatic compression process back to the initial state 1. Heat transfer with the reservoirs occurs only at the reservoir temperatures.

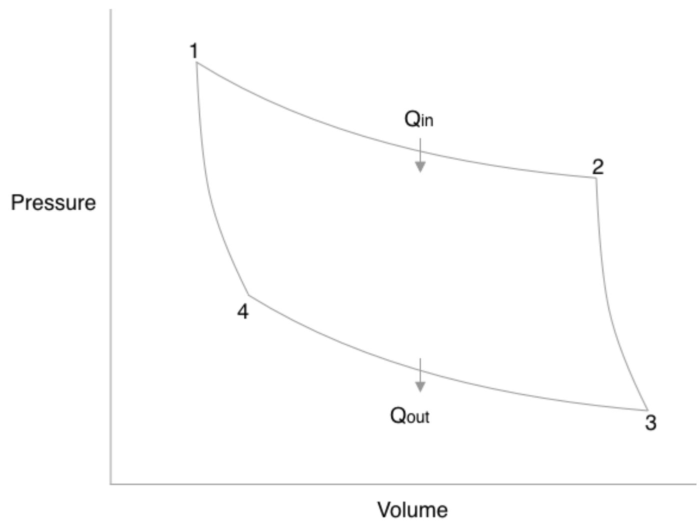

Figure 1. *The Carnot Cycle*

In order for the cycle to provide for a reasonable amount of work (and in keeping with how the cycle is almost always depicted), it is assumed that:

$$V_2 > V_4 \tag{3}$$

Now we come up with the volume ratios of the Carnot Cycle, which will be used later. For an adiabatic process using an ideal gas with γ, the ratio of molar specific heats, it is well known from sources such as Reese (2000, p. 656) that:

$$TV^{\gamma-1} = constant. \qquad (4)$$

Therefore, for the adiabatic process from T_{H} to T_{C}, we have that $T_H {V_H}^{\gamma-1} = T_C {V_C}^{\gamma-1}$. The Carnot cycle uses two adiabatic processes, which provides for two equations: $T_H {V_1}^{\gamma-1} = T_C {V_4}^{\gamma-1}$ and $T_H {V_2}^{\gamma-1} = T_C {V_3}^{\gamma-1}$. From these two equations, it is easy to show the volume ratios:

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}. \qquad (5)$$

It is also well known from sources such as Reese (2000, p.656) that adiabatic work is a function of the temperature difference only ($|W_{adiabatic}| = |nC_v \Delta T|$). Therefore, for the Carnot cycle which operates between two temperatures, the absolute values of the work done during the adiabatic expansion process and the adiabatic compression process are the same:

$$W_{adiabatic\_expansion} = W_{adiabatic\_compression}. \qquad (6)$$

In order for the Carnot cycle to have 100% efficiency, the cycle will be modified. This modified Carnot cycle is shown in Figure 2. The cycle operates with one temperature reservoir (not shown) which could be at any temperature below T_{H} (and greater than 0 K). External heat input from the temperature reservoir to the isothermal expansion process is shown as Q_{in}. Spontaneous heat transfer from a cold temperature to a hot temperature is allowed – and it is the only recognized heat energy flow for the cycle. (Again, the second law says that this is impossible, but we are only considering the first law here.)

The cycle also uses a regenerator. Cengel and Boles (1994) describe regeneration as "*a process during which heat is transferred to a thermal energy storage device (called a regenerator) during one part of the cycle and is transferred back to the working fluid during another part of the cycle*" (p. 469). A regenerator is used so that the heat output of the isothermal compression process at T_{C} is later transferred to the isothermal expansion process at T_{H} as heat input, Q_{isot}, so that there is no heat output from the cycle. Again, we will take liberty in ignoring such impossible scenarios imposed by the second law (which would otherwise show that 100% efficiency could not be obtained).

The regenerator could operate at temperature T_{R} initially, where $T_{\mathrm{C}} \lesssim T_{\mathrm{R}} < T_{\mathrm{H}}$. Heat from the isothermal compression process raises the temperature of the regenerator above T_{R}. Later heat is transferred from the regenerator to the isothermal expansion process and the regenerator temperature returns to T_{R}. The regenerator may have a high thermal capacity to do this. One way in which the regenerator could operate is to transfer heat from an isothermal compression process or to an isothermal expansion process at a rate which keeps the gas temperature constant for the process. [Recall that for this first law only analysis, we do not consider reversibility and that heat transfer is allowed through a finite temperature difference.] The total heat transfer to the isothermal expansion process is a combination of Q_{isot}, which is transferred *within* the cycle by the regenerator (not with any temperature reservoirs), and Q_{in}, which is transferred to the cycle externally from the temperature reservoir.

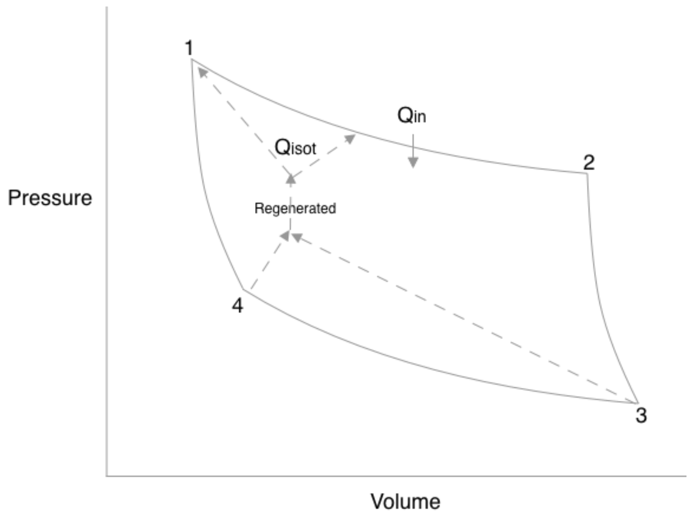

Figure 2. *Modified Carnot cycle with regenerative heat transfer, low temperature to high temperature*

(*The second law is not considered: There is one temperature reservoir, not shown)

It is important to note that regenerated heat does not appear as a heat term in the thermodynamic efficiency of Equation 2 since regenerated heat is not transferred between the surroundings and the cycle (regenerated heat is all internal to the engine cycle). In other words, heat regeneration within the cycle is not heat transfer *to* or *from* the surroundings. The only heat transfer between the surroundings and the engine occurs during part of the isothermal expansion process of the modified Carnot cycle.

SELF-CONTAINED CYCLE USING THE MODIFIED CARNOT CYCLE WITH AN IDEAL GAS

In our case of the self-contained modified Carnot cycle, all of the regenerated heat output Q_{isot} from the isothermal compression process at T_C must occur before it is used as regenerated heat input to the first part of the isothermal expansion process at T_H. Therefore, the entire isothermal compression process at T_c from state 3 to state 4 must occur before any regenerated heat input to the isothermal expansion process occurs at T_H. Also, the isothermal compression process from state 3 to state 4 relies upon work input that must be provided by stored energy from work output generated previously by the cycle. This work output occurs as part of the isothermal expansion at T_H, a process that involves external heat transfer from the temperature reservoir (and not any regenerated heat transfer). Therefore, it is necessary for some processes to occur before others. In order to meet these conditions, a self-contained cycle cannot be initiated at any arbitrary point in the cycle.

A self-contained modified Carnot cycle utilizing an ideal gas can be accomplished as shown in Figure 3, where it proceeds through the states, 0, 2, 3, 5, 6, 4, 1, and then back to 0. The cycle starts at state 0 along the isothermal expansion at T_{H} to state 2; this process is where all of the external heat input into the cycle occurs (later the cycle will proceed from state 1 to state 0 using regenerated heat input). State 0 cannot be chosen arbitrarily along the isothermal expansion since it must exist at a location to meet the conditions of the cycle for heat (regeneration from the isothermal compression, etc.) and work.

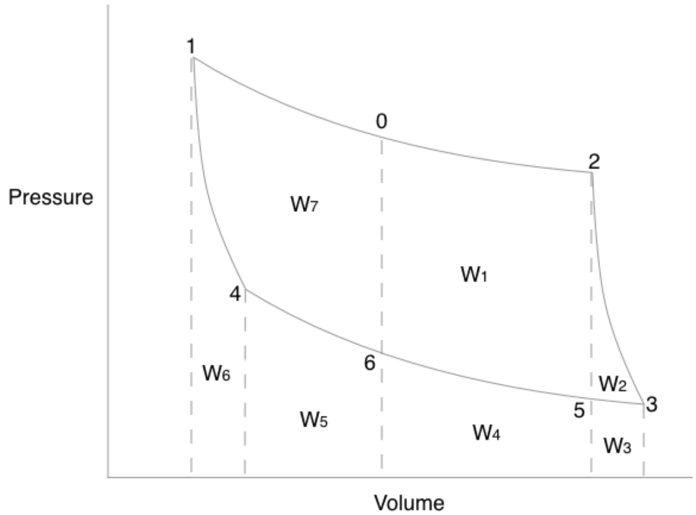

Figure 3. *Modified Carnot cycle: Points of Interest*$(V_4 < V_0 < V_2)$

It is clear that state 0 cannot occur after state 2 or there would be no heat input to the cycle (*i.e.*, the self-contained cycle could not be completed since it does not have enough work energy available to complete the isothermal compression process and the adiabatic compression process per Eq. 6). In Figure 3, a cycle is considered where V_0 is between V_4 and V_2. In Appendix B, the other two possibilities are considered: 1) $V_0 = V_4$, or 2) V_0 is between V_1 and V_4. In all cases, the same result will be shown: $T_H \geq 2T_C$ for efficiency 1.

Recall that the work of a process is equal to the area under a process curve on a PV (Pressure-Volume) diagram and that all work values are given as absolute values.

Table 1. *Cycle Transitions*

State Transition	Work
0-2	$W_1 + W_4$
2-3	$W_2 + W_3$
3-5	W_3
5-6	W_4
6-4	W_5
4-1	W_6
1-0	$W_5 + W_6 + W_7$

The cycle starts at state 0. At state 6, the only work still available to the cycle is $W_1 + W_2$ (which is the area enclosed by the preceding states). In other words, the work output from state 0 to state 3 is $W_1 + W_2 + W_3 + W_4$, but the work input from state 3 to state 6 is $W_3 + W_4$, leaving $W_1 + W_2$ available as work output.

This work output $W_1 + W_2$ must be sufficient to provide the work input required for the self-contained cycle from state 6 to state 1, therefore

$$W_1 + W_2 \geq W_5 + W_6. \qquad (7)$$

All of the heat output from the isothermal compression from state 3 to state 4 is regenerated as heat input to the isothermal expansion from state 1 to state 0. Since $W = Q$ for isothermal processes (*i.e.*, from the first law, $Q = \Delta U + W$ and the change in internal energy, ΔU, is zero for an isothermal process with no temperature change), then:

$$W_3 + W_4 + W_5 = W_5 + W_6 + W_7. \qquad (8)$$

For an ideal gas undergoing an isothermal process from an initial volume to a final volume, then $W = nRT\ ln\left(\frac{V_f}{V_i}\right)$, where n is the number of moles and R is the gas constant.

Therefore, for the isothermal process at T_C:

$$W_3 + W_4 + W_5 = nRT_C\ ln\left(\frac{V_3}{V_4}\right). \qquad (9)$$

For the isothermal process at T_H:

$$W_1 + W_4 + W_5 + W_6 + W_7 = nRT_H\ ln\left(\frac{V_2}{V_1}\right). \qquad (10)$$

Combining the last three numbered equations, then $W_1 + W_4 + nRT_C ln\left(\frac{V_3}{V_4}\right) = nRT_H ln\left(\frac{V_2}{V_1}\right)$, or:

$$W_1 + W_4 = nRT_H\ ln\left(\frac{V_2}{V_1}\right) - nRT_C\ ln\left(\frac{V_3}{V_4}\right). \qquad (11)$$

Since the adiabatic processes have the same work (Eq. 6) then:

$$W_6 = W_2 + W_3. \qquad (12)$$

Equation 7 can now be written as $W_1 + W_2 \geq W_5 + (W_2 + W_3)$, or cancelling W_2 and adding W_4 to both sides:

$$W_1 + W_4 \geq W_5 + W_4 + W_3 \qquad (13)$$

Combining Eq. 9, 11 and 13:

$$nRT_H \, ln\left(\frac{V_2}{V_1}\right) - nRT_C \, ln\left(\frac{V_3}{V_4}\right) \geq nRT_C \, ln\left(\frac{V_3}{V_4}\right). \quad (14)$$

Finally, rewriting this equation and using Eq. 5, we have that

$$T_H \geq 2T_C \quad (15)$$

in order for the cycle to exist with efficiency = 1.

If $T_H < 2\,T_C$, then the efficiency cannot be 1. In this case, all heat output cannot be regenerated in the self-contained cycle, or said differently, the cycle does not provide enough work output that is stored as potential energy to provide for the required work input of a 100% efficient cycle. Only the first law of thermodynamics was used in this analysis of a self-contained cycle.

FINAL REMARKS

The first and second laws are typically described as independent laws. Zemansky and Dittman (1997) state:

> *There is nothing in the first law to preclude the possibility of converting […] heat completely into work. The second law, therefore, is not a deduction from the first law, but stands by itself as a separate law of nature, referring to an aspect of nature different from that described by the first law. The first law denies the possibility of creating or destroying energy; the second law denies the possibility of utilizing energy in a particular way*. (p. 153-154)

The independence of the first and second laws is also suggested by Lemons and Penner's (2008) statement, *"… there is no necessary linkage between the first and second laws of thermodynamics*" (p. 21).

In spite of statements like this, a review of the literature has not turned up any *proof* of the independence of the first and second laws in the literature. Nevertheless, this paper does not claim that the second law is a strict deduction from the first law alone. Specifically, the 100% efficient cycle (zero heat output) developed in this paper required the use of a self-contained cycle and specific temperature regions ($T_H \geq 2\,T_C$).

We have defined self-contained cycles as those which do not rely on the surroundings for anything but heat transfer to the cycle. Self-contained cycles require that any work input required by the cycle must come from previously stored energy from the cycle (such as, by potential energy in a weight previously lifted by the engine when performing work). Also, any regenerative heat output must occur before it is used as regenerative heat input.

In practice, some heat engines use repeating cycles whereby the work output of one cycle is used for (some) work input to the next cycle. Therefore, they are not self-contained cycles. (They could not be self-contained cycles anyway since heat output to the surroundings is required.) These engines cannot normally start at any desired position on the PV diagram and may require starter motors for work input to the cycle before the cyclic process can begin.

Using only the first law of thermodynamics, a self-contained modified Carnot cycle utilizing an ideal gas cannot achieve 100% efficiency where $T_H < 2\,T_C$. Typically, we appeal to the second law of thermodynamics to show that an engine cycle cannot achieve 100% efficiency; however, we have shown this using only the first law and a self-contained cycle operating between some temperature reservoirs.

A literature search has not provided any previous examples, context, or limited applications in which a second law result was based on only the first law. Certainly, it is interesting to consider that there is one restricted example, even though there is no claim for a fully general dependence between the laws (especially for any type of heat engine utilizing an arbitrary working fluid between unrestricted temperature regions). It is hoped that this thought study will provide interested readers with a more in-depth understanding and consideration of the first and second laws of thermodynamics.

APPENDIX A

Any cyclic process can be bounded by separate leftmost and rightmost adiabatic processes. This is shown in Figure 4 where a clockwise cycle, C_1, is shown on top. The bottom portion of this cycle can be used along with the intersecting adiabatic processes to form a separate clockwise cycle, C_2, with an isothermal compression process at a lower temperature. The bottom portion of C_1 and the top portion of C_2 follow the same path, s, in opposite directions.

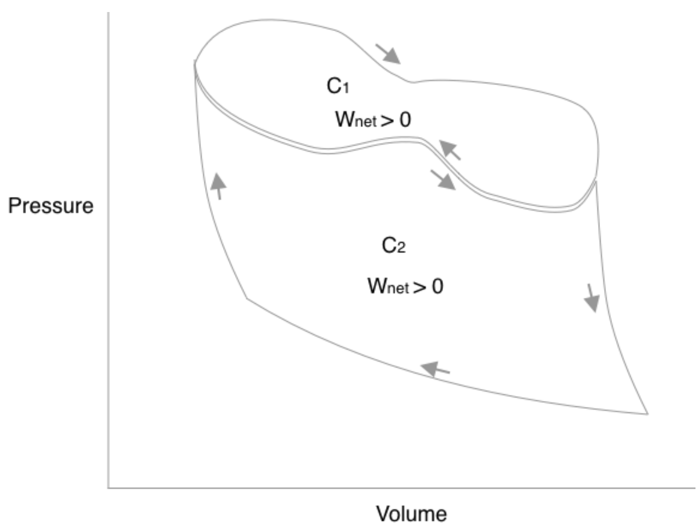

Figure 4. *Cycle (Top) with Leftmost and Rightmost Adiabatics as Part of Another Cycle (Bottom)*

Clockwise cycle C_2 has positive net-work output; the first law says that cycle C_2 must have heat input. Since there is no heat input for the adiabatic processes, and the isothermal process only has heat output, then the net-heat input for C_2 must come from path *s*. Cycle C_1 followed path *s* in the opposite direction, so the path must have net-heat output for C_1.

The same argument can be made for any portion of *s*, whereby adiabatic processes can intersect the segment's ends and an isothermal process can be used at a lower temperature. The resulting clockwise cycle has net-heat input for the segment of *s* when traversed as the top of a clockwise cycle, or net-heat output when traversed as the bottom of a clockwise cycle.

Therefore, for any clockwise cyclic process, the bottom portion which lies between leftmost and rightmost adiabatic processes is always a net-heat output process. Likewise, the top portion which lies between leftmost and rightmost adiabatic processes is always a net-heat input process.

APPENDIX B

Two cases are shown in the appendix for which the same result is obtained: $T_H \geq 2T_C$ for efficiency 1.

Case 1:

The first case is where $V_0 = V_4$, as shown in the following figure.

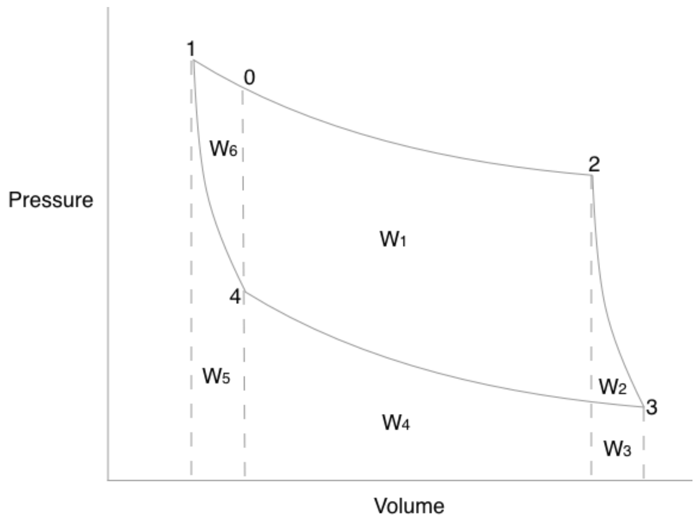

Figure 5. *Modified Carnot cycle: Points of Interest* $(V_0 = V_4)$

By state 4, the work output of the cycle must be greater than the work input to state 1:

$$W_1 + W_2 \geq W_5. \quad (16)$$

Since the adiabatic processes have the same work (Eq. 6) then:

$$W_5 = W_2 + W_3. \quad (17)$$

All of the heat output from the isothermal compression from state 3 to state 4 is regenerated as heat input to the isothermal expansion from state 1 to state 0:

$$W_3 + W_4 = W_5 + W_6. \quad (18)$$

For the isothermal process at T_C:

$$W_3 + W_4 = nRT_C \ln\left(\frac{V_3}{V_4}\right). \quad (19)$$

For the isothermal process at T_H:

$$W_1 + W_4 + W_5 + W_6 = nRT_H \ln\left(\frac{V_2}{V_1}\right). \quad (20)$$

Using the last three equations:

$$W_1 + W_4 = nRT_H \ln\left(\frac{V_2}{V_1}\right) - nRT_C \ln\left(\frac{V_3}{V_4}\right). \quad (21)$$

Using the first two equations (Eq. 16 and Eq. 17), cancelling W_2 and adding W_4 to both sides:

$$W_1 + W_4 \geq W_3 + W_4. \quad (22)$$

Using the last two equations along with Eq. 19:

$$nRT_H \ln\left(\frac{V_2}{V_1}\right) - nRT_C \ln\left(\frac{V_3}{V_4}\right) \geq nRT_C \ln\left(\frac{V_3}{V_4}\right). \quad (23)$$

Finally, rewriting this equation and using Eq. 5, we have that

$$T_H \geq 2T_C \quad (24)$$

in order for the cycle to exist with efficiency = 1.

Case 2:

This case is where V_0 is between V_1 and V_4, as shown in the following figure.

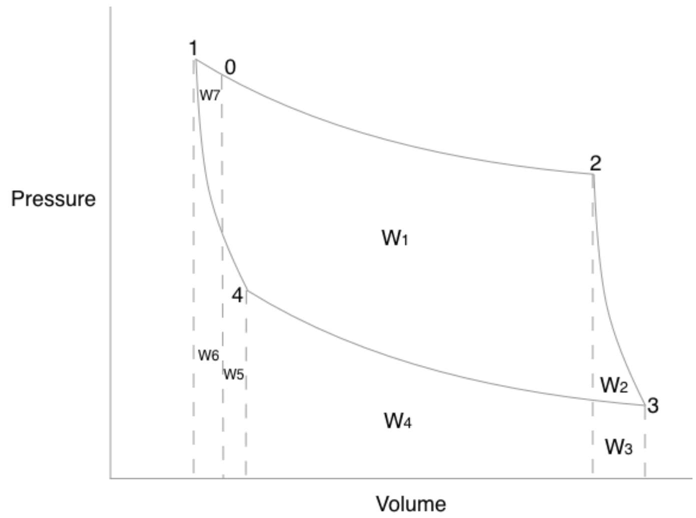

Figure 6. *Modified Carnot cycle: Points of Interest* $(V_1 < V_0 < V_4)$

For the cycle to have an efficiency of 1, then all the heat input to the cycle (equivalent to the work, which is the area under the isothermal) from state 0 to state 2 must be converted into the net work of the cycle:

$$W_1 + W_4 + W_5 = W_1 + W_2 + W_7. \quad (25)$$

The cycle starts at state 0 and returns to the same volume (after state 4). At this point, the work output of the cycle must be greater than the work input to state 1:

$$W_1 + W_2 \geq W_6. \quad (26)$$

For the isothermal process at T_C:

$$W_3 + W_4 = nRT_C \ ln\left(\frac{V_3}{V_4}\right). \quad (27)$$

For the isothermal process at T_H:

$$W_1 + W_4 + W_5 + W_6 + W_7 = nRT_H \ ln\left(\frac{V_2}{V_1}\right). \quad (28)$$

All of the heat output from the isothermal compression from state 3 to state 4 is regenerated as heat input to the isothermal expansion from state 1 to state 0:

$$W_3 + W_4 = W_6 + W_7. \quad (29)$$

Using the last three equations:

$$W_1 + W_4 + W_5 = nRT_H \, ln\left(\frac{V_2}{V_1}\right) - nRT_C \, ln\left(\frac{V_3}{V_4}\right). \quad (30)$$

The first two equations (Eq. 25 and Eq. 26) show that:

$$W_1 + W_4 + W_5 \geq W_6 + W_7. \quad (31)$$

Using the last three equations along with the equation for the isothermal process at T_C:

$$nRT_H \, ln\left(\frac{V_2}{V_1}\right) - nRT_C \, ln\left(\frac{V_3}{V_4}\right) \geq nRT_C \, ln\left(\frac{V_3}{V_4}\right). \quad (32)$$

Finally, rewriting this equation and using Eq. 5, we have that

$$T_H \geq 2T_C \quad (33)$$

in order for the cycle to exist with efficiency = 1.

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