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Students' Understanding of Calculus Based Kinematics and the Arguments They Generated for Problem Solving: The Case of Understanding Physics

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Abstract

The present study explores students' understanding of calculus-based kinematics (henceforth, CBK), in which argumentation is taken as the sequence of the modes of fostering reasoning and problem-solving. The investigation stresses the importance of arguments students bring to the learning situation of CBK and recognizes the active construction of meaning which goes on constantly as they interact with their learning environment. The study adopted mixed-methods in which one-group pre-posttest design with supplemental questionnaires and interviews were used. Data included video recordings of pre-posttests activities, as well as students' handwritten work. The pre-posttests evaluated the dependent variables (understanding CBK concepts and solving problems inspired by CBK), the treatment was argumentation. No strong causal relationship is assumed but the analyses of the results tend to show that the treatment improves slightly students' understanding of CBK concepts and their ability to identify relevant knowledge of these concepts for problem-solving. Implications of the findings for educational practice are discussed.

Introduction

Physics is about understanding; it is one of the ways we attempt to answer the perennial questions about the physical world. To understand something is to know and to find (or construct) a plausible schema, which allows one to assimilate it to what one already knows (Voss, 2006). A schema is a mental representation of a category. It is thus a concept, some information in memory that allows a person to sort various stimuli into members and non-members of that category. Research on student cognition has shown that learning is to be understood as a process in which the student actively builds-up a personal knowledge structure (Shekoyan & Etkina, 2007). This process can only be studied in relation to some particular subject-specific knowledge. Neither intelligence nor strategies will suffice for the solution of complex and interrelated problems, for example, calculus-based kinematic problems, if the domain-specific knowledge is not applied (Rhoneck, Grab, Schaitmann & Volker, 1998). This implies that physics teachers need to take into consideration the subject-specific knowledge of their students so as to provide them with the optimum condition for grasping concepts, laws and theories needed to interpret and predict natural phenomena (Iwuanyanwu, 2019b). From this perspective, situating argumentation as a central element of instruction in the design of a study aimed at investigating students' arguments and understanding of calculus-based kinematics is a critically important epistemic task and discourse process in science (Belland, Glazewski & Richardson, 2011; Jonassen, 2007; Ozdem, Cakiroglu, Ertepinar & Erduran, 2017; Walker and Sampson, 2013). A literature review of work in this area of physics has shown gaps in both the instructional approach and the combination of outcome variables that the current study is investigating (understanding CBK concepts and solving problems inspired by CBK).

The foregoing leads, for instance, to clarify the deep interplay of subject-specific knowledge necessary for students of this study to understand the fundamental concepts, laws and theories related to kinematic-calculus. At the core of such exposition is the requirement to consider the use of maths-in-physics. The effort to put the use of maths-in-physics on a rigorous footing has inspired many developments, such as the theory of partial differential equations, relativity and quantum relativistic theories, statistical mechanics (and the related areas of variation calculus, hydrodynamics, celestial mechanics, symplectic geometry, thermodynamics, electricity, aerodynamics, and so on). This has gradually supplemented by topology and the use of geometry that play an important role in string theory. In this sense, the use of maths-in-physics covers a very broad academic realm and is sometimes used to denote problem solving to describe quantitative and spatial relationships of the physical environment (Redish & Kuo, 2015). Of direct relevance of physics to the conceptualization of mathematics, physics seeks through the use of inquiry to describe and explain generalized patterns of events in the physical world (Meli, Zacharos & Koliopoulos, 2015). However, a common learning objective in both mathematics and physics is to engage students in developing and evaluating arguments (Belland et al., 2011; Redish & Kuo, 2015). In the present study students must, at the very least understand the interplay of heuristics, intuitive, and arguments suitable for the applications of calculus in kinematics problems.

Most written accounts of application of calculus in vector-kinematics make the point that students should be guided to acquire genuine explanatory knowledge through explicit teaching of argumentation (Bing & Redish, 2009). Here to explain is also to understand. For physics students to increase their understanding of application of calculus in vector-kinematics is just for them to make explanatory connections, and to see the blending of maths-in-physics connections. This is a point often missed by some science and mathematics educators. A student may know that a lot of rules hold when deriving calculus-kinematic equations using constant acceleration, but understand nothing in the sense that they make no links between the various calculus variables they know. As an example, students may know by definition that acceleration is the first derivative of velocity with respect to time $a = \frac{dv}{dt}$ and that $dv = adt$, relates to this by integration $\int_{v_0}^{v} dv = \int_{0}^{t} a\,dt$, but not see that each side of the equation is explanatorily linked to the other. As such their understanding of the interconnectedness provided by the tenets of calculus-kinematic equations is missing. Both fundamental and derived calculus-kinematic equations can potentially support and complement each other (Hashemi, Abu, Kashefi & Rahimi, 2014) and allow fuller understanding of problems inspired by mathematical physics (Bing & Redish, 2009). The level of understanding of application of calculus in vector-kinematics and the extent, to which they are applied, of course, vary according to the type of instruction given and the students' abilities to use relevant knowledge resources to construct sound arguments.

Purpose of the study

The driving aim of this study is to investigate second year physics students' understanding of calculus-based kinematics and the nature of arguments they generate while solving problems. To achieve this, the study examined two phases: a) arguing to learn i.e. treatment for understanding the concepts of calculus-based kinematics, and b) solving calculus-based kinematics problems.

Theoretical Framework

A fundamental characteristic of physics or any field of inquiry is the use of a paradigm or a library of knowledge (i.e. laws, theories, concepts, models, skills and associated processes) which serve as a framework of reference (Feynman, 1992). In light of this, the present study is situated within the argumentation framework. The evidence that exists suggests that argumentation is fostered by a context in which student-student interaction is permitted and fostered (Erduran & Jimenez-Aleixandre, 2012; Evagorou & Osborne, 2013; Ghebru & Ogunniyi, 2017). The practical point, of course, is that argumentation has three generally recognized forms: *analytical*, which is grounded in the theory of logic and proceeds inductively or deductively from a set of premises to a conclusion; *dialectical*, which occurs during discussion or debate; and *rhetorical*, which is employed to persuade an audience (Toulmin, 2003). In this study, argumentation developed by students to clearly demonstrate an understanding of calculus-based kinematics (CBK) and solve problems, which involve CBK concepts is a combination of analytical and dialectical elements. If these two forms are successful, the rhetorical form of argumentation occurs and can be documented. In most studies with this focus, the analysis of argumentation practices is based on Toulmin's (2003) Argumentation framework with the following elements: claims, data, warrants, qualifiers, and rebuttals. Claim is an assertion in need of evidential support. Data is evidence used to support a claim. Warrant or justification - connects the evidence with the claim. Backings support the validity of the warrants. Qualifier refers to conditions in which the claim is valid. Rebuttals are statements which show the claim to be invalid. From this perspective, any argument relies on an evidential base which consists of supporting data whose relationship to the claim is elaborated through the warrant, which in turn, may be dependent on a set of underlying theoretical presumptions or backings (Simon, Erduran, & Osborne, 2006).

In the literature, learning about the application of calculus-based kinematics equally requires formal modes of understanding, using precisely defined symbolic notations and explicit rules for their manipulation (Hashemi et al., 2014; Mason, Stacey, & Burton, 2010; Tall, 2012). From maths-in-physics perspective, formal modes of understanding involve exploiting mathematics and logic (or arguments) that are widely used in physics (Govender, 2007; Kiray & Kaptan, 2012; Reif, 1995). This is because mathematics is the science of valid arguments (Redish & Gupta, 2010). This implies arguments where if the assumptions are true, then the conclusion, reached by accepted rules of logic and known mathematical facts, are also true (Belland et al., 2011; Voss, 2006). Furthermore, formal modes of understanding requires the ability to assess students' reasoning at each stage in the process of developing argumentation skills and the understanding of any particular concepts, such as calculus-based kinematics as well as solving problems. However, what is currently known or believed is that the use of calculus-based kinematics problem solving to enrich students' learning and understanding of physics concepts requires teachers' awareness, understanding, and support in their teaching (Kuo, Hull, Gupta & Elby, 2013; Meli et al., 2015; Tall, 2012). An important issue flowing from this is that there are many different calculus symbolic notations to denote vector-kinematics, and students often get confused in using them in physics (Govender, 2007). Previous studies have shown that students also experience difficulties with kinematics concepts of displacement, velocity and acceleration, and in applying the concepts of calculus within problem contexts (Stacey, 2006; Tall, 2010). Although some new methods have been invented to support students' understanding of calculus, especially derivation (Ghanbari, 2012; Mason et al., 2010), but according to Hashemi et al. (2014), Roknabadi (2007), and Azarang (2012) undergraduate students have serious difficulties such as learning calculus-based courses in undergraduate level. However, science and mathematics education reformers are of the view that using argumentation as a form of discourse and explicitly teach about task structuring and modeling could provide a useful anchor for new learning (Bing & Redish, 2009; Erduran et al., 2004; Evagorou & Osborne, 2013; Kuo et al., 2013).

Bind and Redish's (2009) study similar to the present study have analyzed students problem solving using math in physics. From videotaped observations of intermediate level students solving problems in groups, they note that students often get stuck using a limited group of skills or reasoning and fail to notice that a different set of tools (which they possess and know how to use effectively) could quickly and easily solve their problem. For the context of mathematics use in physics problem solving, they presented four dimensions (calculation, physical mapping, invoking authority, and math consistency) for classifying physics students' epistemological framing via warrants. Regarding the four dimensions, there is a more general point about distinguishing a calculation framing from physical mapping framing. The distinction between them concerns a student's in-the-moment awareness of the physical referents of his/her math-in-physics. The idea of invoking authority is another aspect which illustrates how a student can forage his/her knowledge resources during physical referents. An invoking authority stems from framing ideas, laws and theories which are closely tied to finding the right level of detail to go into during a problem solving. The fulfillment or the attainment of the latter is referred to as math consistency by Bing and Redish (2009). A lesson learned from their study is that using the four dimensions they identified to classify warrants could provide a useful lens on the development of students' problem solving skills. This is worthy of consideration in the present study. Thus, two issues are relevant here. The first issue is the nature of arguments students generate to develop criteria for establishing a reasonable solution pathway. Here their thinking is dynamic. They need to identify or recognize instances of valid and invalid arguments pertaining to CBK, for example, the class of warrants such as computational steps of algorithm relevant to set of relations and inferences. The second issue concerns the identification of specific warrants used, like all the other warrants, especially those that are shared because both educator and student frame the discussion as calculation. Again, it is through identifying these (relatively explicit) warrants that the researcher can get information about relatively implicit "epistemological framing" process in the student's mind (Bing & Redish, 2009, p. 9).

Method

This study followed a mixed-methods approach in which one-group pre-posttest design was supplemented with interviews and questionnaires. The pretest evaluated the previously mentioned dependent variables. The treatment was argumentation (i.e. arguing to learn). The posttest evaluated the dependent variables and interviews.

Participants

An intact second year undergraduate physics class of 86 students (49 males, 37 females) served as the research participants. Students ranged between 19 and 23 years old. Most of these students were from middle-class families. The students learning majors are physics, chemistry and biology. During the first year of their study they are required to do calculus-based modules for use in the previously mentioned majors. Less expected of the students in their second year is to demonstrate poor understanding of calculus-based kinematics.

Procedure

This 13week (two class periods of 180 min per week: a double class period of 120 min on Monday and a single class period of 60 min on Wednesday) study included two phases: arguing to learn (treatment for understanding the concepts of calculus-based kinematics, and solving calculus-based kinematics problems. A week before treatment commenced, a handout was provided to the students including information about different phases of the study, differences between the procedures, the role of the educator and expectations from students, purpose of the study, and instruments and instructional materials to be used during treatment. Normatively, students receive instruction through conventional method of teaching. As such, they have not been exposed to argumentation instruction before this study. The intention of selecting *arguing to learn* (treatment) was to offer students new learning opportunity to nurture their understanding of calculus-based kinematics, and (if successful), then it will lead to transforming the physics teaching practices in the context of calculus-based kinematics. The experienced educator had 12 years of physics teaching and practical experiences in argumentation teaching. The instructional context followed students' lecture timetable for physics. As such, no instructional time was wasted. The physics educator taught the calculus-based kinematics, a unit section of the physics course that most students find difficult to understand. The pretest and posttest took two class periods each (120 min in weeks 1 and 12). The interviews took one class period (60 min in week 13). Week 2 and the remaining class time in weeks 3 – 11were focused on the treatment.

Learning Material: Calculus-based kinematics (CBK)

In designing the learning material used in this study several issues were considered. For example, most calculus students tend to use analytical strategies to compute derivatives and integrals (Eisenberg & Dreyfus, 1991), and this tendency makes it more difficult to infer students' strategies when they are solving procedural tasks (Haciomeroglu, Aspinwall & Presmeg, 2010). Thus, the learning material CBK was designed to develop information-rich cases of students' arguments and understanding of calculus-based kinematics. The CBK was enriched with activities related to kinematics concepts of displacement, velocity, time and acceleration, and integrated the practices of arguing to learn (Walker & Sampson, 2013). The content of the CBK activity was obtained from three physics textbooks that students were using at the time of the study (e.g. Introduction to physics by Cutnell & Johnson, 2013, Fundamentals of physics by Walker, 2008, and Physics principles with application by Giancoli, 2005). CBK expects students to focus on important parts of nurturing their understanding of symbolic notations and reflecting on various aspects of sentential calculus. It makes deriving equations of motions much simpler for the students in the study. For instance, in deriving the second equation of motion, by definition, velocity is the first derivative of position with respect to time. Instead of differentiating position to find velocity, the operation in the definition was reversed such that the integration of velocity was used to find position. Doing so offers the students in the study to grasp the kinematic-calculus concepts easily than was previously the case. The following step-by-step description of kinematic-calculus derivation is an example of how the learning material portrays teaching for understanding to the students. In deriving the second equation of motion inspired by calculus, it started with $v = \frac{ds}{dt}$, followed by $ds = v\,dt$, which can be expressed as $ds = (v_0 + at)dt$. The latter follows the integration of velocity to find position $\int_{s_0}^{s} ds = \int_0^t (v_0 + at)dt$. From this we get ($s - s_0 = v_0 t + \frac{1}{2}at^2$), which gives us the position-time equation for constant acceleration ($s = s_0 + v_0 t + \frac{1}{2}at^2$), known as the second equation of motion.

Furthermore, the CBK activity, which took about 180 min per week during the previously mentioned class periods, taught the students to actively engage in arguing to learn. It was hoped that the CBK learning material would help students nurture their arguments pertaining to understanding of kinematic-calculus concepts, as well as solving problems involving the concepts. For the pretest and posttest items pertaining to quantitative data, there were ten questions testing for students' understanding of kinematic-calculus concepts. The ten questions placed heavy emphasis on both dialectic and analytic forms of learning. Another four questions of the same concepts tested students' problem solving skills within the context of arguing to learn. The four items placed emphasis on elements of analytic solution, whereby a student would critique or examine an equation-based problem to help him/her develop criteria for establishing a reasonable solution pathway. An example of these questions (see Q3, p.7) and how it composes dialectic and analytic utility of learning as well as how students blended auxiliary information for problem-solving organization and solution is presented in the findings and discussion section. Thus, analytic solution to a calculus-based kinematic task may involve translation of sentential calculus to an equation (or symbolic representation), computing the integral part of the equation, and then using this new equation to solve follow-up problems (Haciometroglu et al., 2014; Tall, 2012). Analytic solution also includes numeric strategies in which the students substituted values for x to find y or any other variable. In fact, analytic solution resonates well with students' analytical understanding. This is to say, however, that analytical understanding denotes mental process employed by individuals using their previous experience and mental abilities to argue, infer and understand problems and difficulties encountered in order to reach conclusions and make decisions (Zimmerman, 2007).

Moreover, data quality considerations involving the content validity of the CBK worksheet were reviewed by two physics educators who read, discussed, and reversed the worksheet to reach consensus. After attending to the revisions recommended by the previously mentioned reviewers, the CBK worksheet was resubmitted to them for final checkup. To establish the degree to which the two sections of the CBK worksheet measures: 1) students' understanding of calculus-based kinematics concepts, and 2) solving CBK problems through arguing to learn, the reliability of the CBK items was explored using internal consistency. Cronbach's alphas were computed for each section of the CBK, 1 and 2. Cronbach's α was found to be $.77$ for section 1, and $.74$ for CBK section 2. Study of Bing and Redish (2009) was taken as reference for scoring students' framing of arguments while completing the CBK worksheet.

Phase 1: Arguing to learn: Treatment for understanding the concepts of calculus-based kinematics

Phase 1 focused on two conditions related to when a student has fully understood a given kinematic-calculus concept, s/he is reasonably expected to: a) define the concept in symbolic notation, and b) recognize instances and non-instances of valid of kinematic-calculus arguments. A warm-up practice of setting out an argument was used as an introductory CBK activity that would familiarize the students with argumentation as a form of discourse and explicitly teach about task structuring and modeling (Erduran et al., 2004; Evagorou & Osborne, 2013; Ghebru & Ogunniyi, 2017; Walker and Sampson, 2013). Thus, the educator guided the students to learn and practice setting out an argument related to CBK concepts. In order to make argumentation instruction as explicit as possible, argumentative elements (e.g., claim, evidence, reasoning, counterclaim, and rebuttal) and their application within the scope of the study were explained to students. Important terms were clarified; especially those elements with overlapping meaning, for example, claim and warrant. In this regard, the overlapping elements were discriminated using words such as "because", "so" or "since" as cues to indicate data/evidence, claim and warrant, respectively (Erduran, Simon, & Osborne, 2004). That way, instances of ambiguities were relatively resolved, hence the use of the operative word "so" which itself is implied in Toulmin's definition for reaching conclusions.

Other important conditions were also explained. A reason was termed as a set of claims grouped together with cue such as "because". For example, since students were expected to provide claims for solution of the CBK problems and support their claims with evidences, a reason in this case, aims to raise a student's confidence in a conclusion. The claims comprising each reason are unified when it is plausible to support the conclusion. Further to this, to facilitate students' understanding and use of those strategies in their learning, conditions of what counts as a good argument were explained to them. The arguing to learn practice was implemented as follow: 1) Intra-argumentation i.e. the brain-storming or self-conversation stage. At this first stage, students performed individual tasks depicted in the CBK learning material which required them to define and construct valid kinematic-calculus expressions, 2). Inter-argumentation or small group discussion stage, at this point each small group of 4-6 students received tasks that required the transfer of knowledge resources of kinematic-calculus to different contexts which required different levels or types of arguments, and 3) trans-argumentation or whole-group discussion and reflection. This is the stage where collaborative consensus is reached (Ogunniyi, 2009). Students were encouraged to discuss with each other, share co-constructed knowledge and tacit understanding emanating from their learning. Thus, students reflect on and learn from what they are doing. To ascertain how well the treatment condition was implemented as planned, the average and binary complier fidelity indexes were considered to ensure a measure of the reliability of the administration of the intervention. The average fidelity index was considered by using mean scores of students' response to CBK in each test conditions (pre-test in week 1 and posttest in week 12). On the other hand, the binary complier index involved examining the distribution of students' responses in relation to achieving the minimum conditions set for learning CBK through arguing to learn. Regarding this, the baseline data were considered as a cut-point against which the following conditions were observed between weeks 2 - 11. The conditions include what Ogunniyi (2007a) described as: the ability to follow an argument (clearly a good grasp of the language used and mental alertness are critical for this to happen); a willingness to submit to the force of a better argument; the aptitude to treat each other as equal and reasonable arguers; and a willingness to learn something new. In doing so, students learn from each and had the opportunity to interrogate sentential symbolic notations of kinematic-calculus.

Phase 2: Solving calculus-based kinematics problems

After phase 1, the students were assigned problem sections of the calculus-based kinematics (CBK) learning material. They were arranged to use their learned knowledge and skills in phase 1 to complete assigned tasks. The process focused on students using the substances of arguments they had learned to solve CBK problems. The completion of the problem solving tasks took about 60 min of the single class period. Some sections of the tasks were done collaboratively as share activity in groups of 4-6 students. The educator encouraged students to discuss the instances of valid and invalid arguments of applications of kinematic-calculus concepts. He encouraged students to actively use the learned skills, but did not provide any answers (solutions) to tasks meant for use in the result section of the study.

The posttest (120 min distributed in two class periods in week 12) was administered using the same CBK learning material. For the qualitative data, the interview item was constructed to collect all students' viewpoints and learning experiences on argumentation as an instructional practice in the context of calculus-based kinematics. Since students were arranged in groups, a student was taken from each group of fourteen groups and was interviewed by the researcher. The data gathering took 60 min (in week 13), the last week of the study. The interview protocol included an introduction, rapport-building, free narrative, and closing. Students were encouraged to share their opinions about the combination of arguing to learn i.e. treatment for understanding the concepts of calculus-based kinematics, and solving calculus-based kinematics problems. The interviews were audio recorded and transcribed and transcripts were sent back to the interviewees to check their responses and the tentative qualitative outcome. In addition to this, the interview transcripts were again reviewed by the previously mentioned reviewers of the CBK worksheet. They made judgments about coding, following which they met to reach consensus regarding the codes and emergent trends.

Data analysis

Overall data treatments involved established scoring and coding techniques. The ten questions in the pre-posttests related to understanding calculus-based kinematics concepts were scored 2 points for right and 0 for wrong answers for a total possible score of twenty. The CBK scoring adopted well-developed common framings and their primary (i.e. warrants) and secondary indicators (Bing & Redish, 2009) that included four dimensions (calculation, physical mapping, invoking authority and math consistency). Each dimension was scored based on a class of warrant used and other common indicators. Scoring of the argument also depended on whether a student's response fitted Toulmin's Argumentation Pattern definition of warrant; for example, if a warrant response was a description of reasoning and relation between data and claim pertain to CBK, then the student would be scored 2 points, if not, 0 point. If a student gives a backing response, it has to strengthen the warrant by stating the causality or connection of Kinematic-calculus, and if s/he gives a rebuttal response it has to point to the condition under which the claim to knowledge would not hold true in order to receive 2 points. Since there are six elements of arguments, the total possible score for assessing students' arguments was 12. Test-retest reliability coefficient (i.e. the correlation between scores at pretest and posttest) was .76 . To this end, quantitative data included scores from the fourteen items of the CBK (pretest and posttest), whereas qualitative data were gathered from interviews, which were students' responses about their learning of CBK. Analysis of quantitative data involved calculating both descriptive and inferential statistics, testing scores of the outcome variables. Paired-sample t-tests were used to analyse pre-posttests changes in the students' arguments and understanding of CBK. The significant level was set as $p<.05$ during the analyses using the SPSS (version 25). Finally, qualitative data from interviews were analyzed using open coding (Strauss & Corbing, 1998). The researcher was responsible in analyzing the interview responses. The researcher and one of the previously mentioned reviewers of the CBK worksheet read each student's response, code them, and then grouped emerging themes to show the extent to which the students responded to the treatment (arguing to learn). Inter-rater reliability coefficient was .79 showing the comparability between coders (Miles & Huberman, 1994).

Findings and Discussion

The descriptive interpretation of the CBK performances (n = 86) revealed that the frequency and percentage of each score were: 12 points n = 15 (17%), 11 points n = 9 (10%), 10 points n = 6 (10%), 9 points n = 7 (8%), 8 points n = 5 (6%), and so on (see Table 1), and none with zero point. Sixty-nine percent of the students scored 6 or more points on the TAP component in terms of analytic and dialectic arguments, which meant that most students answered most of CBK items correctly.

Table 1 Frequency and percentage scores of students' arguments

	Frequency by (%)											
Score	12	11	10	9	8	7	6	5	4	3	2	1
Students(N = 86)	15 (17%)	9 (10%)	6 (7%)	7 (8%)	5 (6%)	10 (12%)	8 (9%)	9 (10%)	2 (2%)	7 (8%)	4 (5%)	3 (3%)

The analysis of pre-posttests scores related to students' arguments in terms of analytic and dialectic forms of learning calculus-based kinematics (Table 2) revealed significant gain, $t = -.136, p = .001$. This meant that most students demonstrated improved understanding of the content of CBK concepts over the duration of the study. Another point to consider is that there was no prediction that the effect of arguing to learn would lead to high levels of students' understanding of CBK concepts on the pretest and posttest mean scores (12.65, 12.68 respectively, out of twenty possible points). Despite this, the results arising from the data tend to show that the treatment improves slightly students' understanding of CBK concepts and their ability to identify relevant knowledge of these concepts for problem-solving.

Table 2 Pre-posttests scores in students' arguments, understanding and solving of CBK problems

	Arguments: analytic & dialectic		*Understanding content*		*Solving problems inspired by CBK*		*Total Score*	
CBK Test	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Mean (M)	6.57	6.92	12.65	12.68	9.66	10.48	11.67	16.88
Standard Deviation (SD)	1.64	1.56	2.57	2.89	1.83	1.36	2.72	3.74
t	–.136*		–6.71*		–7.38*		–9.89*	
p	<.001		<.001		<.001		<.001	
Cohen's d	.22		.18		.44		1.39	

*(*p<.05)*

The results (Table 2) showed significant gain between pretest and posttest scores related to understanding content of CBK. The paired-sample t-test of the students' performance on all of the posttest scoring items demonstrated significant (p<.05) progress over the pretest performance. The effect sizes (Cohen's d), for students' understanding of CBK were between $.18$ and 1.39. The improvement on the total score was a larger effect size, $\mathrm{d} = 1.39$ and the improvements on solving problems inspired by CBK was also large effect size, $\mathrm{d} = .44$. Given these results, it can be assumed that the arguing to learn (treatment) may have contributed to the students' performances on understanding and solving problems inspired by CBK concepts. These findings add to evidence to support the suggestions of Belland et al. (2011), Ozdem et al. (2017) and Walker and Sampson (2013) that arguing to learn could help promote students' understanding of science concepts.

How students demonstrated arguing to learn to support their understanding of CBK problem-solving

The first result from students arguing to learn that seemed to have supported their understanding of CBK problem solving comes from a group of five students responding to the following question designated as (Q3). The problem statement reads:

> An eagle flies in the xy-plane with a velocity vector given by $\vec{V}(t) = (\alpha - \beta t^2)\hat{i} + \gamma t \vec{j}$ with $\alpha = 2.4 m - sec^{-1}$, $\beta = 1.6 m - sec^{-3}$ and $\gamma = 4.0 m - sec^{-2}$. At $t = 0$, the eagle is at the origin. (a) Calculate the position and acceleration of the eagle as functions of time. (b) What is the eagle's altitude (y – coordinate) as it flies over the origin ($x = 0$) for the first time after ($t = 0$).

The principal dynamic cognitive process in each of the following students' arguments concerns how they interpreted the math deeply rooted in the structure of CBK activity. Different bits of their mathematical knowledge are activated and deactivated as they frame and reframe their activity (Bing & Redish, 2009). Below are some extracts from the students' arguments that help to make the point. Two out of five students in the group did not recognize the point of the problem from the beginning. For ease of reference, students are designated as Student 1, Student 2, and so on beside their responses to help the reader follow the sequence of arguments, results and discussion. Here are some of the excerpts from the students' conversations:

> Student 6: I feel that there is no obvious way to derive the equation for $\vec{a}$ without deriving $\vec{r}(t)$
> Student 17: It doesn't say that in the question, does it?
> Student 6: Come on you can read that, it does…
> Student 17: You can read it to us for a start…
> Student 17: Oh, okay…so we need to play a rather sophisticated trick…
> Student 29: …say position is given by $\vec{r}(t) = \int \vec{V}(t)dt = \int(\alpha - (\alpha - \beta t^2)dt\hat{i} + \gamma t\vec{j}...$
> Student 6: Angelo (pseudonym), I don't follow...the last part of the equation, check it…
> Student 17: I'm not following…I understand that $\vec{r}(t) = \int \vec{V}(t)dt = \int(\alpha - ....$, but this $(\alpha - \beta t^2)dt\hat{i} + \gamma t\vec{j}...$ how come?
> Student 29: It is part of the problem statement…let's go back to the question….
> Student 17: Oh, there I see…
> Student 6: But…Angelo, I think the integral part of the equation is this….
> $\vec{r}(t) = \int \vec{V}(t)dt = \int(\alpha - (\alpha - \beta t^2)dt\hat{i} + \int \gamma t\, dt\vec{j}...$, yes this part $\int \gamma t\, dt\vec{j}$ was incomplete…

If students perceive their learning as making sense, or establishing meaning, or gaining understanding then one might reasonably expect when this happens it is something for the students to mark. Notice on one occasion when student 17 asked her peers for clarity "I'm not following…I understand that $\vec{r}(t) = \int \vec{V}(t)dt = \int(\alpha - ....$, but this $(\alpha - \beta t^2)dt\hat{i} + \gamma t\vec{j}...$ how come?" it led her to gain some insight "Oh, there I see." Student 6 must have seen the future use of this piece of mathematical equation ($\int \gamma t\, dt\vec{j}$) that student 29 may have overlooked and makes the assumption that he will have the opportunity to use it when required in problem solving. The arguing to learn continues.

> Student 29: Aha okay…now, $\vec{r}(t) = \int \vec{V}(t)dt = \int(\alpha - (\alpha - \beta t^2)dt\hat{i} + \int \gamma t\, dt\vec{j}...$
> Student 33: The final equation will come to this $\vec{r}(t) = \left(\alpha t - \frac{\beta t^2}{3}\right)\hat{i} + \gamma \frac{t^2}{2}\hat{j}$.
> Student 52: It's now obvious we can use this equation to find position…
> $\vec{r}(t) = \left[\left(\alpha t - \frac{\beta t^2}{3}\right)\hat{i} + \gamma \frac{t^2}{2}\hat{j}\right]$, right?
> Student 6, 29 and 33: Yeah…
> Student 17: Can we not ask sir if it is the correct equation for calculating the position?
> Student 29: That won't be necessary, don't bother him…
> Student 6: Clara, continue with the write-up…
> Student 17: So, $\vec{r}(t) = \left[\left(2.4m - \sec^{-1})t - \frac{1.6 - \sec^{-3}t^3}{3}\right)\hat{i} + (2.0m - \sec^{-2})t^2\hat{j}\right]$
> Student 17: Is that all about the substitution, anything else?
> Student 29, 6 and 52: Yeah, that's the first part of the solution for position we were asked to find.
> Student 33: Next, we must find the acceleration…I think it's just the easiest part of the problem…I can do that…
> Student 17: I see…different from other activities we have completed earlier…like the vector function and things like that.

Despite the fact her group has just had the experience of understanding how to derive, interpret and compute the CBK problem; Clara (student17) does not identify this as part of the purpose of understanding the CBK concepts. To her, it is rather about practising routine skills.

> Student 33: We can say the acceleration is given by $\vec{a} = \frac{d}{dt}\left((\alpha - \beta t^2)\hat{i} + \gamma t\, j\right)$, is this correct guys?
> Student 29: …wait a minute, look what happens when we do this…we get a derivative equal to

acceleration… $-2\beta t\hat{i}+\gamma\hat{j}$
Student 6: There was nothing wrong with that.
Student 52, 6, and 17: Yeah, Brian was going to come to that…it wasn't all that more difficult than the first two derivations.

In what followed, another group of six students was invited to respond to question (b) of CBK (Q3). These extracts of students' conversations are interesting because they reveal students' separation of maths-in-physics classroom practice from their understanding of CBK problems.

Student 4: What is the eagle's altitude at the origin?
Student 19: Isn't it what we need to find?
Student 4:…I mean in real life…what's the eagle's altitude at the origin?
Student 32: Most probably the reference point before the eagle gears up into the air.
Student 85: We can only find it by solving for the time at $r_x(t)=0$.
Student 47: Do you think $r_x(t)=0$ is going to be of any use to us?...i don't understand…
Student 85: What don't you understand there?
Student 32: But the eagle's altitude is given as $(y-\text{coordinate})$ …
Student 4: I got what Solomon is saying, yes…even though it was given as $(y-\text{coordinate})$, the problem says 'as it flies over the origin ($x=0$) for the first time after $t=0$.
Student 85: That's what you would expect him to understand, except that he assumes I have no purpose of stating that $r_x(t)=0$.

The experiences of arguing to learn seem to have been guiding the students to do structured thinking by detecting that certain claims made to knowledge by their peers need to be backed up. It is useful for students to collect a variety of evidence-based claims and personal opinions from which to decide what to believe or do (Walker & Sampson, 2013). The arguing to learn continues.

Student 4: Can I solve it?
Students 85, 19 and 30: Go ahead.
Student 4: Alright, $\alpha t=-\frac{\beta t^3}{3}=0,\ \alpha t=\frac{\beta t^3}{3}...$
Student 19: And that will lead to $\alpha t=\frac{\beta t^3}{3}$, which gives us $t=\sqrt{\frac{3\alpha}{\beta}}=2.12s$, right?
Student 85, 30 and 4: Yeah, all in order.
Student 32: …then we can say with confidence that the eagle is at $x=0$ (the origin), 2.12 seconds, right?
Student 4: Yes…after $t=0$.
Student 47: But what does $r_y(t)$ equal to?
Student 30: That should be $r_x(t)=2.0m-sec^{-2}(t)$, not so?
Student 32: No, Amanda is right, we should be dealing with $r_y(t)$ and not $r_x(t)$ …we dealt with it already.
Student 47 and 4 : Guys let's move ahead, we have little time left… we know that $r_y(t)=\gamma\frac{t^2}{2}$.
Student 85: Yeah…since we now know the time is 2.12s, the eagle's altitude $r_y(t)$ at that time is, $r_y(2.12)=\gamma\frac{t^2}{2}=2.0m-sec^{-2}(2.12)^2=...$
Student 19: And that gives 9m.
Student 4: It turns out that the eagle's altitude is 9m.
Student 47: Yeah, as it flies over the origin $x=0$ for the first time after $t=0$.

Students' arguments thus suggest they were framing their activity as math consistency. For example, when student 17 asked student 29 to clarify certain part of the equation he was busy deriving, he gestures to the different paths as he points out to Student17 that what she is asking is given in the problem statement. He was more willing to negotiate meaning with her when he says "let's go back to the problem". Perhaps his intention was to use available data traceable in the problem statement to convince his peer. His data is a familiar mantra (though he omits mentioning how the equation he was deriving $\vec{r}(t) = \int \vec{V}(t)dt = \int (\alpha - (\alpha - \beta t^2)dt\hat{i} + \int \gamma t\, dt \vec{j} ...$ is only valid for the context they were considering). Even so his warrant suggests he was framing his use of maths-in-physics in a different way. His oversight (or mistake) in dealing carefully with the last section of the equation was only realised when his peer (student 6) alerted him. Moreover, he responds to reframing request made by student 6 and repeats his equation as he remains in invoking authority. At any rate, his response shows he may have overlooked the missing part of the equation. While student 47 thinks there should be different amounts of information they need to address their own problem (Q3b), his peer (student 4) provided him a justification for the assumption, student 85 had made. As the arguing to learn progresses there were several instances in which students were able to identify both valid arguments and invalid arguments of their peers. It appeared that students were very accomplished in the practice of arguing to learn, they knew how to behave and thrive in this area. They were aware of the goals of the practice, and shared expectations which enabled them to work towards the goals.

Benefits of using argumentation instruction

The interview responses were explored to determine the benefits (if any) of using argumentation instruction on students learning of CBK concepts. The driving question is expressed in italics, students are designated using the foregoing pattern beside their responses, and the author's elaborations are in normal font.

> Interviewer: How have your experiences in the arguing-to-learn-based lectures improved your understanding of CBK concepts and problem-solving skills?
>
> Student 11: It was helpful in doing some activities, but not all. In some activities, I feel you need to concentrate and not let people distract your thinking process, especially with calculus…
>
> Student 38: Me…I feel that it was useful and practical for me. There are little difficulties of following the procedures…like in some activities we completed you find that other students' voices were minimal, some students tend to dominant the flow.
>
> Student 19: When engaging in this CBK activity, arguing to learn for me was fruitful. It helps us to provide reasons, even when we don't agree with other people's opinions…we must provide justifications to convince them about our own views on any side of issue…that for me is good.
>
> Student 52: I enjoyed it. We students could apply these concepts of warrant, backing and rebuttal into our learning of other subjects…like chemistry, biology and maths. In terms of solving problems, I must say…it leads me to make good decisions.
>
> Student 6: Before, I do not like solving problems involving calculus, now…that is changing. In our group, Angelo was very helpful. He was good in keeping ideas flowing. Some of the activities we completed together…I only understood them because of the way we were learning them.
>
> Student 85: It was helpful…the practices were sort of mental activity that helped to sharpen our skills…what else can I say, oh yes…one more thing interesting was engaging my fellow students, I learned a lot doing so.

Responses such as these led the author to believe that for the students in this study calculus-based kinematics (CBK) lessons belonged to a practice which was quite different from their own ways of learning. Student 38 shared a concern about the domineering attitudes of some of her peers during arguing to learn. She describes this as procedural difficulties. In response to this finding, this is due, to some extent, to the limited time frame involved in completing some of the CBK activities, with most activities lasting a few minutes to complete. While this poses particular pedagogical challenges insofar as facilitating arguing to learn is concerned, other researchers, who have examined the ways students participate in argumentation, have suggested that argumentation practice (or protocol – terms and conditions) must be thoroughly explained to students before exposing them to its practice (Erduran et al., 2004; Walker & Sampson, 2013). Nonetheless, data from this study also indicate that success in CBK has as much to do with learning with special discourse rules and interpreting symbolic notations in a clever way. And because of the fragmented nature of some students experience about the instructional approach they did not seek any coherence within the use of maths-in-physics and thus when it is clearly lacking it fails to interrupt their flow of activity and so there was no reason to reflect.

> Student 20: Unfortunately, I noticed no difference in my understanding of CBK concepts. I do not like to involve myself in debate activities, whenever I hear the word argument; I have a feeling of dislike.
>
> Student 45: I compete with other students for completion of most of the activities. In some cases I lost concentration, or made mistakes that I could have avoided if I were to solve the problems on my own.
>
> Student 19: Yes, yes…it has improved my understanding…I cannot speak for everyone. But from what I have noticed in my group, we were all excited whenever we set to solve problems involving calculus. One of the tricks that I think other students are failing to realize is the rules to apply.
>
> Student 34: Right from my high school days, calculus has always being inspiring aspect of mathematics that I enjoy. It is vast, complex and interesting. My father was a mathematics teacher, so in that belief I started early liking maths and sciences. So the instruction has brought more insight to my learning.
>
> Student 4: Like detectives hunting for clues and building ideas from them…that is how I will describe my experience of arguing to learn CBK. So I find it helpful.
>
> Student 1: Yes, it has offered me the opportunity to my learning of CBK and my understanding of all the physics activities we did have changed. Before arguing to learn was introduced, I was attending calculus tutorial sessions, and now I have realized what have been missing in my failed attempts to understand calculus.
>
> Student 66: To some extent, it was helpful. One thing I dislike about it is time consuming. It takes time for most of us to agree on a particular idea, even when the evidence is overwhelming. They still want to argue more and more, that to me is time wasting.
>
> Student 86: I used to be afraid whenever I am called upon to answer questions during calculus problem solving; I just think my fear has reduced. I can say it is beneficial to me and to my group because we all learn from one another…and help each other to identify mistakes.

Although calculus can be sophisticated, once its constructs are understood, students can go beyond the techniques and the symbolic manipulations to see the subject matter for what it is. Integrating argumentation experience with CBK was to encourage better and thorough understanding. Hence, through the activity of arguing to learn, the students supported their understanding of CBK problem-solving, which had significant teaching-learning benefits.

Conclusion and suggestions

This study empirically has explored students' arguments and understanding of calculus-based kinematics. In the study, argumentation in the form of arguing to learn has shown how students exert various pushes and pulls on each other as they try to negotiate a common understanding. The analysis of how students demonstrated arguing to learn to support their understanding of CBK problem-solving is necessary to discriminate how they frame their maths-in-physics activity. The evidence presented has shown that each student frames his/her CBK activity differently and hence tries to apply a different type of warrant to judge the validity of their claims. This has important implications for teachers as well as researchers in the area of this study. For example, in a conventional lecture, information on presenting CBK tends to only flow from the lecturer (educator) to the students. But engaging students in arguing to learn could be helpful to improve students' understanding of CBK concepts. And referring back to the theoretical framework which based on Toulmin's (2003) Argumentation Framework, arguing to learn is the practice which is goal directed rather than the person who enters the practice, the person is subjectified by the practice and is motivated or led by his expectations regarding needs to be fulfilled. Excerpts from the interview showed that students felt that the modalities associated with arguing to learn lead to waste of time. This point to treatment fidelity and begs for the question, was the treatment effective in promoting the desired outcomes for all the students who received it? In a way, the analysis of students' interactions with the treatment shows that it made a relative difference in supporting their understanding of CBK. There are other argumentation patterns that can be applied for classroom-based research (Erduran &

Jimenez-Aleixandre, 2012; Evagorou & Osborne, 2013; Ghebru & Ogunniyi, 2017; Iwuanyanwu, 2019a). Future studies can further investigate this dimension.

Limitations

Ideally, it would have been more fruitful to conduct this study among all the undergraduate physics students to better their understanding of CBK, but due to institutional constraints, access to students in other undergraduate physics levels was not feasible at the time of this study. The second year group included in the study was more readily accessible, hence the use of a convenience sampling. And, whilst a probability sampling technique would have been preferred, the convenience sample was the only sampling technique that allowed the researcher to gather useful data and information that would not have been possible using other sampling techniques, which require formal access to other undergraduate students. In addition, the question of whether treatment (arguing to learn) was effective in helping students to better understand CBK is treated with caution given the design of the study. On this point, since the study adopted one-group pre-posttest design which has no control or comparison group to address other external influences, thus, the study cannot speak to how much of the improvement in students' understanding of CBK are solely due to the treatment (arguing to learn). Besides, it neither generalise its findings beyond the sampling frame nor the design. However, the basic point to consider here is that knowledge gained across time (pre-test and posttest) is representative of the population from which the sample was drawn. In conclusion, the limitations pointed out should be considered in the design of future studies.

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