The Richness of Mathematics Noticed by Teacher Candidates in a Professional Development School Model

Melissa A. Gallagher, University of Louisiana at Lafayette
Lesley A. King, George Mason University
Jennifer M. Suh, George Mason University
Dori L. Hargrove, Shepherd University

ABSTRACT: This study examined how 16 female teacher candidates in two sections of a math methods course, which implemented high leverage professional development school practices, described the aspects of richness of mathematics (i.e., linking between representations, explanations, mathematical sense-making, multiple procedures or solution methods, patterns and generalizations, and mathematical language) from the *mathematical quality of instruction* (MQI; Hill et al., 2008) when observing a clinical faculty, their peers, and their own instruction. After analyzing written reflections and reflections using the Edthena video tool, we found that the teacher candidates were mainly focused on mathematical language and explanations. They described patterns and generalizations and linking between representations least frequently. Furthermore, their descriptions of mathematical language and explanations centered around instructional practices they could implement in their own classrooms, critical reflections of their teaching, and noticing how the teachers fostered students' use of mathematical language and rich explanations.

This manuscript addresses Essential #4: A shared commitment to innovative and reflective practice by all participants.

The professional development school (PDS) setting affords teacher candidates the opportunity to work with students while simultaneously enrolled in a math methods class. This environment helps to strengthen the connection between theory and practice, as teacher candidates observe a school-based teacher educator (SBTE), design and deliver instruction based on the needs of the students, and engage in meaningful discourse at the conclusion of each lesson. This analysis of instructional practice allows teacher candidates to refine their work and reflect on pedagogical decisions that take place prior to, during, and after instruction. The purpose of this study was to examine the impact of high-leverage clinical practices used as part of a PDS partnership on teacher candidates' understandings of the richness of mathematics. The high-leverage practices implemented were: (a) joint observations and (b) using a research-based observational tool to structure observations. Specifically, the following research questions guided the study:

1. Which aspects of richness of mathematics do teacher candidates notice most and least frequently when observing an SBTE, their peers, and their own instruction?
2. How do teacher candidates describe the aspects of richness of mathematics that they most frequently observed?

Theoretical Framework

We have approached this work from a social constructivist perspective stemming from the work of Vygotsky (1978) in which learning is a social and collaborative endeavor under the guidance of a knowledgeable instructor. In this case, teacher candidates had the support and experience of a SBTE and university teacher educators as they navigated a clinical experience placement, designed and delivered math instruction to students, and reflected upon their own practice and that of their peers and the SBTE. This interactive process, highlighted in reflections and discourse, allowed the teacher candidates, the SBTE, and the university teacher educators to make sense of the mathematical practices evident during instruction. The teacher candidates engaged in multiple rounds of focused observations, one of the pedagogies of practice of PDSs (Yendol-Hoppey & Franco, 2014) using *mathematical quality of instruction* (MQI; Hill et al., 2008) framework. Analyzing the richness of the mathematics using the MQI facilitated the teacher candidates' construction of knowledge and resulted in targeted and specific feedback from the teacher candidates.

Reflective Practice

Through reflective practice, teacher candidates must be metacognitive about a specific aspect of their teaching and then analyze or evaluate that aspect. Dewey viewed reflection as a holistic process of a purposeful search for an answer grounded in experience (Dewey, 1933). Schön (1983) described reflection as a series of steps to make sense of particular details of practice that require analysis in order to draw conjectures about how to improve practice. Further, he wrote that practitioners use past experience to build on the new problem, which provides evidence to conceive a new theory for future action (Schön, 1983). Both Dewey and Schön distinguished between effective reflection and informal reflection where effective reflection involves an analytic approach that requires one to make sense of a situation by analyzing the details and then drawing conclusions about how to improve practice. Both believed that teachers should reflect on their teaching through engaging in ongoing, systematic and disciplined interpretations with the end goal of improving practice (Dewey, 1933; Schön, 1983).

According to Erickson (2011), what teachers notice as evidence from their previous experiences does not always provide the type of information they need to draw relevant conclusions about student learning. Teacher candidates focus on superficial aspects of teaching, overgeneralize student learning, and have an inaccurate sense of the effectiveness of the lesson (van Es & Sherin, 2002; Yayli, 2008). Levin, Hammer, and Coffey (2009) suggest that the way in which teacher candidates structure dilemmas in the classroom determines the effectiveness of their reflection. Thus, teacher candidates need scaffolds to guide what they notice and make sense of those experiences to learn to make informed instructional decisions.

Teacher Noticing

The elementary classrooms in which teachers work to build students' understanding are complex places characterized by "multidimensionality, simultaneity, and unpredictability" (Doyle, 1977, p. 52). Teachers must manage this complexity by noticing particular events in the classroom (Sherin, Jacobs, & Philipp, 2011). Noticing has been defined as "arranging to alert oneself in the future so as to act freshly rather than automatically out of habit" (Mason, 2011, p. 37). The importance of noticing is described by Mason (2002):

> At the heart of all practice lies noticing: noticing an opportunity to act appropriately. To notice an opportunity to act requires three things: being present and sensitive in the moment, having a reason to act, and having a different act come to mind. (p. 1)

Mason's work has been built upon by many researchers in the math education field. For instance, Jacobs, Lamb, and Philipp (2010) created the teacher noticing framework, in which teachers: (a) attend to, (b) interpret, and (c) respond to student thinking during instruction.

In order for teacher candidates to be successful at noticing, they must learn to attend to their students' thinking and learning. This is not an easy transition as many teacher candidates lack both the observation skills and pedagogical content knowledge necessary to move their reflections from superficial (i.e., hyper-focused on behavior management) to deep (i.e., detecting evidence of student sense-making; Barnhart & van Es, 2015; Stockero, 2014). Many times, novice teachers do not know what to attend to, how to interpret what they see, and they do not know how to respond when confronted with student learning within the complex environment of the classroom (Jacobs et. al, 2010).

Most teacher candidates develop their noticing skills by spending time in classrooms observing SBTEs' instruction early in their educator preparation programs. However, the effectiveness of these observations is unclear (Brophy, 2004). Star, Lynch, and Perova (2011) suppose that teacher candidates may not have developed the skills to take in the vast and complex classroom activities and interactions. Prospective teachers have been found to have less expertise than experienced teachers in attending to and interpreting student understandings (Jacobs et al., 2010).

Knowing the struggles that teacher candidates have when observing and teaching during clinical experiences, teacher educators must find a way to support the development of their noticing skills and deep reflection. The use of video analysis with the benefit of a specific framework has caught the attention of teacher educators and researchers. Findings from prior research are encouraging in providing evidence that structured video analysis benefit the teacher candidate when reflecting on teaching (Barnhart & van Es, 2015; Brunvand & Fishman, 2007; Kleinknecht & Gröschner, 2016; Stockero, 2014). The types of structure or scaffolds vary in these studies, but the results all indicate that the use of video to reflect and analyze teaching that impacted student learning is a valuable tool that teacher candidates use to deepen their reflections about teaching and student learning.

Some of the scaffolds from the research rely on teacher candidates watching and analyzing video of veteran teachers (Brunvand & Fishman, 2007). The teacher candidates use coding frameworks that focus on noticing. Although these frameworks vary, the teacher candidates are taught and use the specific scaffold when viewing case video of veteran teachers and they write a reflective analysis of what they noticed. What is common from the research is that the scaffolding played an important role in what the teacher candidates noticed and helped them focus on student learning (Barnhart & van Es, 2015; Brunvand & Fishman, 2007; Stockero, 2014). These various scaffolds supported the development of teacher candidates' ability to notice and reflect on student learning. The results from these studies indicate that the teacher candidates were able to reflect more deeply about student thinking and did not attend to superficial episodes (i.e., hyper-focused on behavior management) in the classroom (Barnhart & van Es, 2015; Brunvand & Fishman, 2007; Stockero, 2014).

In the study conducted by Barnhart and van Es (2015), after the teacher candidates learned about theories of analytic and responsive teaching practice they used a framework to analyze teaching strategies and student thinking from video of their own instruction. The results revealed that these teacher candidates were able to attend to, interpret, and reflect on teaching and learning in systematic ways. What was different in this study was that after three months from the completion of analyzing their own video, these teacher candidates were still able to demonstrate high levels of sophistication when noticing teaching and learning. This suggests that teacher candidates are able to enact practices learned in a teacher education program.

Kleinknect and Gröschner (2016), conducted a study where the teacher candidates viewed their own video. In this study, the teacher candidates engaged in a structured video feedback cycle using an online tool called V-Reflect. The findings from this study reveal that emphasis on instruction and scaffolding is essential to the teacher candidates' ability to develop the skills necessary to reflect deeply about student learning. The comparison group in this study participated in a class that taught the teacher candidates how to notice. There were positive effects in the comparison groups' responses, which indicated that the course work supported these teacher candidates' ability to notice (from memory), even though they did not have benefit of watching a video of their own teaching. Again, this study suggests that teacher candidates learn to notice through structured course work and are able to transfer that learning when reflecting on their own teaching.

Instructional Practices

For more than 100 years, educational researchers have studied instructional practices, however, they have yet to agree upon a core set of practices that are the most effective at improving student learning. Recently, mathematics educators have begun to close in on a set of practices that have been shown to relate to student achievement (Kelcey, 2015); these practices are known as the MQI. The MQI was first developed as a framework to use when observing teachers and was defined by Hill and colleagues as "a composite of several dimensions that characterize the rigor and richness of the mathematics of the lesson" (2008, p. 431). Hill and colleagues (2008) examined the relationship between mathematical knowledge for teaching and MQI in ten elementary classrooms and found this relationship to be powerful. The MQI has since been validated (Hill et al., 2012) and used as an observation tool (Mitchell & Marin, 2015; White & Rowan, 2014).

Bringing together teacher noticing using video and the MQI, Mitchell and Marin (2015) created a video club wherein four teacher candidates watched video of their own and their peers' instruction and analyzed this using a modified version of the 2008 MQI. Prior to each video club meeting, the researchers chose 20-minute segments of video that showed continuous mathematics instruction characterized by teacher-student interactions. Participants then watched the selected clips, coded using the MQI, and submitted their codes and rationales for their codes to the researchers. These were compiled and when all participants met in the video club, they spent approximately 70 minutes reconciling codes. Following code reconciliation, the teacher candidates whose instruction was featured in the video would reflect on the coding, the coding discussions, and on how they would alter the lesson. Mitchell and Marin found three major changes in the teacher candidates' noticing: an increased focus on pedagogy and mathematical thinking, a shift toward noticing teacher-student interactions, and move away from an evaluative stance. This research suggests that shared observation and analysis of video using a structured protocol, such as the MQI, can foster deeper, more meaningful noticing for teacher candidates.

Our Study

We sought to inquire about what teacher candidates would notice during their observations after using a structured observation tool (i.e., the MQI) in an innovative PDS setting. What distinguishes this model from a traditional clinical experience is that the teacher candidates had the opportunity to: (a) observe the SBTE during math instruction on a daily basis and then participate in a lesson debrief to address questions that arose during the lesson; (b) work as a member of a team to design and teach one small group and one whole group lesson; (c) design an assessment task and work one-on-one with a student; and (d) work with students to design activities for the culminating event on the final day of the math lab experience. The collaborative nature sets this model apart from traditional clinical experience placements, which tend to be limited to a dyad between the cooperating teacher and teacher candidate. Given that teacher candidates often have difficulty noticing the math content of a lesson (Jacobs et al., 2010; Star et al., 2011), we chose to focus on their descriptions of the Richness of Mathematics domain of the MQI, defined as "the depth of the mathematics offered to students" (Learning Mathematics for Teaching, 2014, p. 4).

Methods

This study employed a qualitative design to address the research questions.

Context

Participants were 16 female teacher candidates in two different sections of a mathematics methods course at a large Mid-Atlantic university. Fourteen of the teacher candidates were Caucasian and two of the teacher candidates were Asian, and all were in their second or third semester of their Master's level elementary education preparation program. This is an innovative model in which university coursework was situated in clinical experience at a PDS, which served as the host site for both a summer math methods course and a two-week summer enrichment math lab for 25 rising third through sixth grade students demonstrating potential for giftedness but from backgrounds traditionally under-represented in gifted and talented programs. The purpose of this math lab was to foster gifted potential and provide opportunities for more advanced coursework for elementary students. While we do not have the specific demographic information for the students who participated in the summer math lab, at the school-level 38% of students were Hispanic, 36% White, 12% Asian, 7% African American, and 6% Other.

Also, approximately 40% of the students at the school qualified for free or reduced lunch. Students from this Title I community school were invited to attend the summer enrichment math lab, while the teacher candidates both observed and reflected on the instruction over the two-week timeframe using the MQI.

The MQI

We used the 4-point version of the MQI (Learning Mathematics for Teaching, 2014). This version of the MQI has four broad domains with individual items under each domain. For the purposes of this study, we are focused on the Richness of Mathematics domain, which contains six individual items and an overall richness item: (1) linking between representations, (2) explanations, (3) mathematical sense-making, (4) multiple procedures or solution methods, (5) patterns and generalizations, and (6) mathematical language. The MQI provides a detailed description of each of these teaching practices, including examples of what constitutes each practice. For instance, the MQI states "For Linking Between Representations to be scored above a Not Present: At least one representation must be visually present; [and] The explicit linking between the two representations must be communicated out loud" (Learning Mathematics for Teaching, 2014, p. 5). The MQI also includes a detailed rubric for rating each item as *not present, low, mid,* or *high* in an observation.

The Math Methods Course

Before beginning observations at the camp, the university teacher educators introduced the teacher candidates to the MQI. Together they watched, analyzed, and discussed video of teachers' instruction in relation to the different components of the MQI. During the first week of the summer camp, the teacher candidates spent the mornings observing the SBTE's instruction on algebraic reasoning. After the students left each day, university teacher educators facilitated a debrief with the SBTE and the teacher candidates and then the teacher candidates wrote individual reflections on the SBTE's instruction. During the second week of the summer camp, the teacher candidates took turns teaching a lesson to the students. They planned and taught these lessons in groups of three to four, and when they were not teaching, they were observing their peers' teaching. Later the same day, the teacher candidates wrote reflections on their peers' instruction. In addition to observations of others' instruction, each teacher candidate also planned and taught one lesson with a group. Immediately following these lessons, the whole class gathered together and the university teacher educator led a debrief of the lesson with the class, facilitating student-led questions and reflections. After the debrief, teacher candidates were asked to review the video of their instruction on Edthena and leave reflective comments throughout. Edthena is an online collaborative video repository. Although the two sections of the methods courses were not combined, the university teacher educators planned together and both groups observed the same lessons taught by the same SBTE, although with different groups of students.

Data Sources

The data sources used to explore the research questions were teacher candidates' written reflections of their observations of the SBTE, written reflections of their observations of their peers, and their written reflections in Edthena. The teacher candidates wrote reflections for every observation of the SBTE and of their peers. On two occasions they were asked to focus specifically on the Richness of Mathematics codes of the MQI: once when they were observing the SBTE and once when they were observing their peers. These were the reflections used in this study. Their Edthena comments on their own instruction were also included as data sources. For these reflections, the teacher candidates were not directed to use any aspect of the MQI. Also, because the teacher candidates taught their lessons in small groups (i.e., three to four teacher candidates worked together to plan and teach one lesson to a class of approximately 12 students), the comments left on the videos were visible to the other teacher candidates with whom they had taught. Although all three data sources were teacher candidates' written reflections, they were written at different times and with different foci. Additionally, the reflections on their own instruction were different than the other forms of reflection because they were semi-public (public to their teaching group) comments on videos.

Data Analysis

All three data sources were coded using a priori coding (Saldaña, 2013) in order to answer the first research question. As we were interested in the aspects of richness of mathematics which the teacher candidates noticed most frequently, as well as how they described these aspects, we coded all data sources for references to these: linking between representations, explanations, mathematical sense-making, multiple procedures or solution methods, patterns and generalizations, and mathematical language, using the definitions established in the MQI. We used Dedoose 7 (SocioCultural Research Consultants, 2017), an online qualitative data analysis platform, to code the data. The first two authors each independently coded the same 12 written reflections using these Richness of Mathematics codes, then we came together to discuss, reach agreement, and establish coding decisions, such as the decision to code discrete ideas independently even if they were adjacent and described the same aspect of richness of mathematics. Once agreement was reached, we divided up the remainder of the reflections and coded these independently.

For the second research question, we pulled all the coded excerpts of text related to the two most frequent richness of mathematics codes. Then we engaged in a round of initial coding (Saldaña, 2013), whereby each researcher independently coded the excerpts related to each of the two most frequent richness of mathematics codes in order to determine what the

Table 1. Frequency of Aspects of Richnesss of Mathematics

	Reflections on SBTE	*Reflections on Peers*	*Reflection on Self*	*Totals*
Explanations	37	35	64	136
Linking between representations	10	15	10	35
Mathematical language	85	43	25	153
Mathematical sense-making	22	22	30	74
Multiple procedures or solution methods	8	26	25	59
Patterns and generalizations	5	2	10	17
Totals	167	143	164	474

participants were noticing within these categories. Next, we came together to discuss our codes and determine agreed upon codes for each segment. Third, initial codes were categorized and refined using axial coding (Saldaña, 2013) so that within each hypothetical code, the dominant and less important codes were selected (Boeije, 2010).

Findings

The findings from the first research question indicated that in all their reflections, the teacher candidates most frequently noticed mathematical language (32.3%) and explanations (29.5%; Table 1). In their reflections on the SBTE, they reflected on mathematical language (51.2%). The second most often described MQI item in their observations of the SBTE was explanations (22.1%). In their reflections on their peers, they described their peers' mathematical language (28.9%) and explanations (26.8%) in nearly equal amounts. When reflecting on their own instruction in Edthena, the teacher candidates most often reflected on their explanations (39.4%). They reflected nearly equally on their mathematical sense-making (17.6%), mathematical language (15.9%), and multiple procedures or solution methods (14.7%). The least discussed aspects were patterns and generalizations (3.5%) and linking between representations (7.3%). This is possibly because of the emphases of these lessons; the lesson by the SBTE was primarily about building knowledge of math terms, and the lesson by their peers was about explaining a new method for solving algebraic equations. As such, although the teacher candidates had opportunities to comment on other aspects of the richness of mathematics that were incorporated into these lessons, they did so with much less frequency. It is also possible that patterns and generalizations is a more complex mathematical practice and thus more difficult for the teacher candidates to notice (Gallagher & Suh, under review).

With regard to the second research question, we examined how the teacher candidates described mathematical language and explanations. They typically described mathematical language in terms of: (a) a teacher defining/reviewing math terms; (b) a teacher's level of fluency with mathematical language; (c) a teacher's use of a word wall; and (d) a teacher's encouragement of students' fluent use of mathematical language (Table 2). For instance, in her reflection on her peers' instruction, Molly[1] noticed how they "started the lesson off by reviewing the terms they would be using throughout the lesson." The teacher candidates also tended to notice the teacher's fluent use of mathematical language and reflected on how they could have improved their own fluency. In Samantha's observation of her peers, she noticed how another teacher candidate modeled fluent math language for her students. "When Charlotte asked a student how to solve a problem and they told her to subtract a

[1] All names are pseudonyms.

Table 2. Frequency of Descriptions of Mathematical Language and Explanations

	Reflections on SBTE	*Reflections on Peers*	*Reflection on Self*	*Totals*
Mathematical Language				
Defining/reviewing math terms	29	18	7	54
Teachers' fluency with mathematical language	15	13	14	42
Use of a word wall	19	2	1	22
Students' use of mathematical language	22	9	1	32
Explanations				
Clarity/thoroughness of explanation	12	5	12	29
Explanation of mathematical symbols or notation	2	3	5	10
Explanation of steps for a procedure	9	8	3	20
Students' explanations	0	6	27	33
Explanation of why a procedure worked	7	8	4	19
Totals	115	72	74	261

Note. Where text was coded for multiple MQI indicators, the secondary code was only applied to the MQI code primarily described by the text. Thus, the total number of descriptors for each MQI indicator may not match the total number of codes for that indicator in Table 1.

quantity from both sides, she responded by saying 'Yes! We could use inverse operations!' and she wrote the term on the board." In their reflections on their own instruction, the teacher candidates were more likely to note instances where they could have improved their mathematical language. For instance, Kristen said, "I think we all made the mistake of forgetting to remind students of the term variable throughout the entire lesson."

Another aspect of mathematical language that the teacher candidates noticed was the SBTE's use of a math word wall.

> He then moved into his lesson by starting with a word wall. The word wall included a bunch of new and old vocabulary words that he asked to students to raise their hands and ask questions if they could not remember what they meant. Some kids did not know what prime and composite numbers were. I thought this was a great idea because the students were able to refresh themselves on the mathematical language they were to learn. (Kim)

The teacher candidates also noticed when the SBTE or the other teacher candidates who were teaching a lesson encouraged students to use mathematical language fluently. Victoria noticed, "As the children began to ask questions in an attempt to guess the number, [the SBTE] pushed them to think about the types of questions they were asking. He asked them to use their vocabulary words and challenged them to use different strategies to get at the number." These descriptions of mathematical language as not just using math vocabulary, but encouraging a fluent use of language in the classroom by using a word wall and modeling for students, show a deep understanding of how teachers can support elementary students' math language.

When examining how teacher candidates described explanations, we found five main themes: (a) clarity/thoroughness of explanation; (b) explanation of mathematical symbols or notation; (c) explanation of steps for a procedure; (d) students' explanations; and (e) explanation of why a procedure worked. The teacher candidates often noticed strengths and weaknesses in all of these areas. When describing the clarity or thoroughness of an explanation, the teacher candidates often noticed the SBTE's use of examples, commented on the success of their peers' explanations, or reflected on how their own explanations could have been better. For instance, Amelia said, "Our conclusion should have been more in depth and direct to provide closure for the students." Kim commented on the clarity of her peers' instruction saying, "I would argue that they could have spent a little more time explaining what an inverse operation is and show examples."

The teacher candidates often noticed the explicit explanation of the meaning behind mathematical symbols or notation. Amelia wrote, "'I am glad that Kristen wrote the plus sign and the minus sign above the total sum and total difference columns because it brought more meaning than just having the words." The description of explanations in the MQI focuses on why a procedure works, why a solution method is appropriate, or why an answer is true; however, the teacher candidates often reflected on the use of explanations of the steps for a procedure, reflecting when these steps were done well or noting the lack of connection to conceptual understanding. Mariela reflected on her peers' instruction, "The students could understand it when it was explained to them step by step but on their own it seemed like it was kind of swimming around and not on solid foundation." Mariela's focus in many of her reflections seemed to be around the need for students to know the steps to solving an equation. Lisa, on the other hand, was more focused on the importance of building conceptual understanding. She reflected on the SBTE's explanation of steps, saying, "'Although [he] took time to discuss the elements of an equation, when it came time to solve, his speech to the students was more of a discussion of procedure than real explanations."

The focus of the Richness of Mathematics domain of the MQI is teachers' instruction, and therefore the focus of the teacher candidates' reflections on explanations should have all centered around the teachers, however, the most common theme of the teacher candidates' description of explanations had to do with student explanations. Although we feel that this does not precisely fit within Richness of Mathematics, we nevertheless present it here, as the teacher candidates chose to include it in their Richness reflections. In writing about students' explanations, the teacher candidates often reflected on how the teachers elicited explanations from students. For instance, Joan reflected on her peers' instruction, "As students provided answers he would consistently ask them, 'How did you solve that?' or 'Explain to me what you did there.'" In a reflection on her group's instruction, Molly said,

> I like that Alyssa asked Student M why she multiplied the number of animals times 2 to really get her to think about why she was setting her problem up that way. Having her think more metacognitively about the problem helped her to get a better understanding of why the problem was set up that way. Alyssa did a nice job of prompting her to think about why she was setting up the problem that way, and by having her explain herself I think helped her grasp the concept better. (Molly, Edthena comment)

When describing why a procedure works, the teacher candidates were either noticing how the teachers had done this well or how they could have done it better. Sophia noticed that the SBTE "provided some explanations regarding why a solution makes sense, such as showing how a number that is even would also be divisible by two during a mystery number guessing game, but only in response to student confusion." Joan reflected critically on her own instruction,

> I wish I had instead prompted the group to have a discussion about what they were working on and to come up with some ideas about why the strategy they

were learning was a great way to solve two-variable equation problems.

These descriptions of explanations by the teacher candidates are, for the most part, aligned with the MQI's description of explanations. Furthermore, their frequent discussion of procedural and conceptual instruction shows an awareness of these two different foci within mathematics and, for the most part, a valuing of teaching the concepts which underlie the procedures.

Discussion

The findings of this study suggest that utilizing the MQI with teacher candidates can provide a structure for observations that supports meaningful noticing and reflection on a variety of aspects of mathematics instruction. Teacher candidates, who have limited experience in a classroom setting, were able to focus their observations by concentrating on specific criteria. This fine-grain analysis may not have been evident otherwise; had we not utilized the MQI, some teacher candidates may not have been aware of exactly what to notice during an observation and may have instead made only broad comments about the instruction. The MQI framework appears to have assisted teacher candidates with reflecting on instruction in meaningful ways based on the quality of comments regarding math content and math pedagogy. Furthermore, the candidates transferred their knowledge of MQI when reflecting on the Edthena video, which did not require them to make explicit connections to the MQI. Similar to the work of Kleinknect and Gröschner (2016), these findings suggest that if candidates learn to notice with a framework, they will transfer it to other situations.

Regarding clinical relevance, we believe the teacher candidates found the opportunity to see teaching in action at the same time they were enrolled in the math methods course to be powerful. The setting afforded them the chance to take what they were learning in their course and implement it with students in a more real-time scenario than is possible in a traditional program in which teacher candidates complete coursework before working with students during their internships. The real-world/real-time aspect of this setting created a unique opportunity for teacher candidates to learn about and develop their skills in math content and math pedagogy concurrently.

In terms of limitations, the dataset could have been more robust. For each our of 16 participants, we had one reflection after observing the SBTE, one reflection after observing peers, and one reflection after observing their own teaching on video. If we were to conduct this study again, our data could include multiple observations of the SBTE, peers, and the teacher candidates themselves, which would offer more than simply a single snapshot of what the teacher candidates were thinking during the lessons. In a similar vein, the summer enrichment math lab took place over a two-week period, providing only a small window of time during which the teacher candidates observed and reflected on mathematics instruction. Had the experience taken place over a longer stretch of time, both our analysis and our findings may have differed because we may have been able to explore patterns of changes in the reflections of the teacher candidates. Finally, in terms of the teacher candidates' lessons, the feedback they received came mainly from one another; in other words, novices provided feedback to other novices. If we were to conduct this study again, we could ask the SBTE to provide feedback to the teacher candidates and have each teacher candidate reflect upon this feedback. This has the potential to not only add depth and dimension to the teacher candidates' reflections, but also to their instruction during subsequent lessons.

Future research is needed and we would like to see additional opportunities for teacher candidates to work in a comparable setting while enrolled in math methods courses so as to explore whether similar findings would result with different teacher candidates and SBTEs. Another possibility for future research would be to revisit the teacher candidates once they have a year of teaching experience and ask them to reflect on the same video of their own teaching a second time and compare what they observed about their teaching as a teacher candidate before and after they have classroom teaching experience. Lastly, a comparison of reflections written by teacher candidates who were utilizing the MQI framework and those who were not could possibly reveal a difference in both the overall quality of their reflections and also the particular aspects of the instruction that were highlighted by each group.

Conclusions

Regardless of years of experience, teachers require dedicated time for professional learning in order to understand new concepts and skills, get support and feedback while trying new approaches, and integrate them into their practice (Zeichner & Conklin, 2008). The PDS model supports this professional learning as teacher candidates have the opportunity to both observe the instructional practice of an experienced math educator and work collaboratively with peers in a supportive environment. This PDS experience allowed teacher candidates to make the connection between the theoretical knowledge acquired in a math methods class and the practical application of said knowledge while under the guidance of university teacher educators.

Utilizing an observation tool such as MQI can help teachers become reflective and responsive practitioners through fine-grain analysis of their instruction. In this case, teacher candidates were able to think deeply about the richness of the mathematics being presented to students through observation of instruction provided by the SBTE, their peers, and themselves. This differs from a traditional field experience in which teacher candidates are often isolated in classrooms with one cooperating teacher, which limits any opportunity the teacher candidate might have to debrief about what he/she has observed. The model presented here encourages collaborative work and meaningful discourse that takes advantage of the shared experience wherein teacher candidates and SBTEs work together to analyze the mathematics instruction taking place. SUP

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Dr. Gallagher is an Assistant Professor. Her research interests include mathematics teachers' knowledge, beliefs, and practices. In particular, she studies instructional practices that support the mathematics achievement of English Learners.

Lesley A. King is a doctoral candidate. Her research interests include instructional coaching, teacher pedagogy, and teacher content knowledge, as well as the connection between mathematics and literacy in the elementary classroom.

Dr. Suh is an Associate Professor. Her research focuses on developing teachers' pedagogical mathematics knowledge, building children's mathematical meaning through math modeling, and promoting equitable access to 21st century skills.

Dr. Hargrove is an Assistant Professor. Her research focuses on professional development of teachers, selecting and implementing high cognitive demand tasks, and elementary teachers' mathematical content and pedagogical content knowledge.