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## Examination of Freshmen’s Conceptual Knowledge on Function in the Context of Multiple Representations

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Article Info	Abstract
<p><b>Article History</b></p> <p>Received: 15 March 2019</p> <p>Accepted: 13 June 2019</p>	<p>The aim of this study is to question freshmen’s conceptual knowledge on function. In this context, after the teaching of the function, students’ skills of defining the concept of function, interpreting the definition and multi-representation for the function were examined. In the study, qualitative research approach was adopted and was an example of a case study. The participants of the study constitute 15 students who were educated in the first class of the classroom teaching department of a state university in Turkey. The data of the study was collected in four consecutive stages. Each stage was arranged according to the information obtained from the previous stage. The students were applied to the understanding form (AF), Multi-representation form (MRF), algebraic and graphical representation form (AGRF) and semi-structured interview form (IF) for the function concept developed by the researcher respectively. As a result of the study, it emerged that more than half of the students have difficulty interpreting the definition, although the function was accurately defined. It was found that students were more successful in the schema and list representations than algebraic and graphical representations of function. Moreover, students had difficulty in transforming between algebraic and graphical representations. In general, sources of these difficulties were; lack of adequate understanding of the function definition, not taking into consideration domains of functions, complex structure of the concept of function and negativity of the students’ mathematical backgrounds.</p>
<p><b>Keywords</b></p> <p>Function Multiple representations Mathematics education</p>	

### Introduction

The concept of function is among the important concepts of algebra. It is not only one of the key concepts of mathematics but also it plays a role in the definition of fundamental concepts of analysis, its specific significance, such as limit, derivative and integral. It is on an important position in the development of mathematical thinking and is a concept associated with all branches of mathematics (Polat and Şahiner, 2007). In courses with a strong sequential structure such as mathematics, the winning concepts of new learning depends on the fact that the preliminary concepts have been actualized and having constructed a bond between them (Özdoğan, 2016). In this sense, accurate understanding of all aspects of the function concept will facilitate the understanding of other fundamental concepts associated with the concept of function.

In general, the concept of function is based on a simple logic. The special kind of relation from A to B that each element of the A set, which is one of the two non-empty sets, that belongs to one and only one element of the B set, is called the function defined from A to B (Aydın, Camus and Kaya, 2018). Two criteria are conspicuous in understanding this definition. The first one is "there is not an unpaired element of A set" and the second is "the mapping of any element of A set to just a single element of B". This definition is expressed in mathematical language as follows: "Let f be a relation from A to B. If for all  $x \in A$ , there exists  $y \in B$  such that  $(x, y) \in f$  and if  $(x, y_1) \in f$  and  $(x, y_2) \in f$  then  $y_1 = y_2$ , the relation f is called function from A to B (Öz vd., 2018)."

In order to relation to be described as a function, two conditions must be satisfied at the same time. Therefore, these two criteria can be called the conditions that relation be a function. In this study, the first condition is called "definiteness" and the latter is referred to as the "unique value" condition. The vertical line test, which is used to determine whether a graphically given relation is a function, is an application of unique value condition. In the studies most students often use vertical line testing without resorting to recognition (Özdoğan, 2016) but they have difficulty understanding the logic of the vertical line test (Clement, 2001).

Researches made on functions emphasized that students had difficulty in understanding the concept of function in despite the simplicity of mathematics and the importance of thought (Akkoç, 2006; Clement, 2001; Vinner, 1983; Tall and Bakar, 1992; Vinner and Dreyfus, 1989). The difficulties of the students; in the understanding of the definition (Aydın and Köğçe, 2008; Clement, 2001; Hatisaru and Erbaş, 2013; Tall and Bakar, 1992;), in the development of the appropriate perception or concept image of recognition (Süzer, 2011 Sierpinska, 1992), multiple representations of the function and transitions between representations (Başturk, 2010; Elia and Spyrou, 2006; Hatisaru and Çetinkaya, 2010; Tall and Vinner, 1981; Vinner, 1983; Tall and Bakar, 1991; Sierpinska, 1992; Şandır, 2006; Tall and Bakar, 1992; Yavuz and Hangul, 2014) is possible to collect in four groups as the difficulties experienced.

The concept of function is a rich concept in terms of multiple representations. Wilson (1991) emphasised that students should take advantage of multiple representations to better understand the concept of function. Because learning to use different representations provides a better and comprehensible understanding of the desired concept (Even, 1990). In general, schema, ordered pairs (list), algebraic (equation) and graphical representations are used during the representation of the function examples. In the Turkish mathematical teaching program, the schema, sequential binary, algebraic and graphical representation of the function is emphasized (Taşdan and Çelik, 2015). Using different representations in teaching the concept of function to establish connections between representations help learning. (Taşdan and Çelik, 2015). As Sprinska (1992) emphasized in the use of representations, students should know every representation. In addition, intensity shouldn't be given to a single method. No representation can fully represent the concept of function (Thompson, 1934) to be aware of the limitations of each representation is important in terms of understanding the concept of function (Sierpinska, 1992).

According to Clement (2001), teachers should determine the level of understanding the concept of function of the students, give more time to the meaning and definition of the concept of function, use typical examples as well as non-typical examples and function and to ensure that the definition is functionally used by discussing invalid examples with students. Polat and Şahiner (2007) determined that when the course content, course plan and method prepared by taking into consideration, common errors in the function are largely resolved. This study was designed as a product of this idea. At the end of the function instruction in accordance with the suggestions for teaching the function in the related literature, it was tried to find out whether the students have conceptual knowledge level about function concept. The most well-known theoretical framework used to determine the level of conceptual knowledge is the classification that was revealed by Benjamin Bloom and named as "Bloom taxonomy". This classification has been revised for the purpose of eliminating deficiencies and making it more modern (Anderson and Krathwohl, 2001). With the revised new taxonomy, the opportunity to evaluate learning or objectives not only in terms of information but also in terms of process (Bekdemir and Selim, 2008). According to this taxonomy, there is a significant indication of the level of conceptual information (arising from the level of knowledge), to be able to explain/interpret, to give examples, and to be able to convert into another form (Demirel, 2017). In this context, the skills of the freshmen in the department of elementary school teachers to define, interpret, sample and represent the function in different ways were examined and the answers to the following research questions were sought:

1. How do students define the concept of function?
2. How do students interpret the concept of function?
3. What kind of examples do students give for the function?
4. How is the ability of students to function in different representations?
5. How is the ability of students to transform between algebraic and graphical representations?

## **Method**

Qualitative research approach has been adopted in the study. The study was designed as a case study of qualitative research patterns. The case study is an in-depth representation and examination of a limited system (Merriam, 2013).

## **Participants**

The research group of the study consists of students who were studying in the first class of the department of elementary school teachers at a public university in the Eastern Anatolia region of Turkey. The classroom consisted of 17 students. As two students did not want to participate in the study, the data collection process of

the study was conducted with 15 voluntary students. The teaching of the function subject was conducted to the students under the Basic Mathematics course in the curriculum of the department of elementary school teachers. Students come across with the concept of function for the first time in the first year of high school. When students' mathematics curriculum at that period was examined (MEB, 2013), it was seen that target acquisitions were involved such as "to explain concept of function, to do the graphic representation of the function, to draw graphs of exponential functions and to explain injective and surjective functions". The function, as an interest in this study, is treated as "a relation that maps each element of a set (domain) to one and only one element of another set (codomain). The function is described as a machine that produces for some input values ( $x$ ), to the output values ( $f(x)$ ) in the framework of a specific rule. In this context, the  $f(x)$  in table or rule is given for an  $x$  value, and  $f(1)$ ,  $f(2)$ ,  $f(a)$ ,  $f(2x)$ ,  $f(x+1)$ , etc. values are found. The domain and image sets are shown on the graph of the function. In a graph of a function, the line that is plotted parallel to the y-axis from each point where the function is defined on the x-axis intersects the graph of the function in exactly one point (vertical line test). After the definition is given verbally in the program, it is visually described by the cluster mapping schema and examples of the function's schema, ordered pairs (list), algebraic, and graphical representations (Gokbaş and Erdogan, 2016; Taşdan and Çelik, 2015). Türkelli (2016) determined that the ninth-grade high school students were more successful in the schema and list representations. In this program he also participated in the criticism of the previous program (Akkoç, 2006), he had opinion that there was more emphasis on the diagram and the list representation of the function.

Semi-structured interview was conducted with six students who volunteered among 15 students. The results obtained from the study were shared and the students' opinions were taken. In the study, the names of S1, S2..., S15 were used instead of the actual names of the students.

### Function Instruction

In the function instruction, the studies on the function of the researchers had been examined. A list of common mistakes and recommendations for effective teaching were compiled. The list was considered during the instruction. In this list; To stand on the points where students have difficulty, to benefit from multiple representations, to be equally involved in each representation, to be aware of the limitations of representations, not to start with a direct definition of function, to present real-life examples, describing, interpreting and expressing the function definition in different ways, using as many different relation examples as possible, making a definition-oriented assessment of function, emphasizing the relationship definition with rules and discussions with students in all events were prominent. The instruction of the function was completed in six weeks, two hours a week.

The teaching of function was started by giving examples of real-life. Examples given were discussed in the classroom environment. One of the examples given for the concept of function was "child-mother relationship". The relationship between children in a group and the women of children with their mothers was defined as function. The children's set was named domain and the set of women including the children's mothers was named as codomain. The function rule also stated that every child should have a mother and that a child should have only one mother. As a result of this relationship, it was stated that a set of mothers, where each child is paired, is called a set of images. It was emphasized that children in a set of definitions can be both in a finite number or an infinite number. The sample given in the classroom environment was discussed and the examples of functions offered by the students were evaluated in terms of function conditions.

After evaluating the function examples of the students, the formal definition of function was presented. The concept of relation in this definition was reminded again. Students were asked to make explanations of what they understood from this definition. At the end of the discussions, formal definition was explained to students. Later, examples for each representation functions or not function were evaluated with the students. It was paid attention not to weigh any representation, to evaluate an equal number of samples for each representation. Definition-oriented arguments were used in the evaluation of the samples. It was stated that the vertical correct test used for evaluating examples of graphical representations is an application of the single value condition in the function definition.

After the instruction of interpretation of the function definition, the function in multiple representations and the evaluation of the examples of non-function, function types (fixed function, unit function, equal function, one-to-one function, injective function, permutation function, single/double function, number of functions) were educated by giving equal weight to multiple representations in a discussion environment. In the graphical representation of the injective function, the relationship between horizontal line test and injective function

definition were highlighted. In the teaching of the “graph of a function”, drawing of the graphs of the first and second-degree functions were investigated. Then, using the graph of the function of the  $y = x^2$ , by using the discovery approach,  $y = x^2 \pm k$ ,  $y = (x \pm k)^2$  and  $y = x^2 + bx + c$  graphs of the functions were examined. The examples were plotted on the board at first and then the relationship of functions with the  $y = x^2$  function was shown by means of dynamic graphic drawing programs. The instruction was ended with the teaching of operation in functions, inverse function and composition of functions.

### Data Collection Tools

The data of the study was collected in four consecutive stages according to the results obtained from each section. The students were respectively applied to the Understanding Form for the function concept (UF), Multi-representation form (MRF), Algebraic and Graphical Representation Form (AGRF) and Semi-Structured Interview Form (SSIF). UF focuses on the function definitions, insights and examples of students. With the MRF, the students were tried to expose their ability to make examples on multiple representations of the function and to determine whether the relations presented in multiple representations were functions. AGRF focuses exclusively on algebraic and graphic representations of relations. In the final stage, semi-structured interviews were conducted with the students in order to evaluate the results obtained. After the completion of the teaching of the function, the data were applied one week apart. The opinions and approval of an expert in mathematics education was taken during the development of data collection tools. Information about data collection tools is provided in Table 1.

Table 1. Information about the data collection tools used in the study

Forms	Questions contained in the form
Understanding Form for Function (UF)	<p><i>Describe the concept of function.</i></p> <p><i>Describe what you understand from the function description with your own sentences.</i></p> <p><i>Please give an example of function.</i></p>
Multiple Representation Form (MRF)	<p><i>Use the Venn schema, list method, algebraic expression, and graphical representations to give an example of function.</i></p> <p><i>Please indicate whether the relations represented in different structures are functions.</i></p> <p>The form includes two relation examples for schema, list, algebraic, and graphic representation, respectively. Relations in schema, list and graphic form do not satisfy definiteness and unique value conditions, whereas relations in algebraic representation are in violation of both conditions.</p> <p>Algebraic examples are <math>y^2 = x</math> and <math> y  = x + 1</math> relations from real numbers to real numbers.</p> <p>Graphical examples are <math>y^2 = x + 1</math> relation from <math>[-1, \infty)</math> to real numbers and <math>y = x + 1</math> from real numbers to real numbers relation plotted graph in the range of <math>[-1, \infty)</math>.</p>
Algebraic and Graphic Representation Form (AGRF)	<p><i>Indicate whether algebraic and graphically presented relations are function.</i></p> <p>In this form, there are both algebraic and graphical representations for three relations. Algebraic representations of examples are presented below.</p> $f(x) = 1, R \rightarrow R,$ $x^2 + y^2 = 1, R \rightarrow R$ $f(x) = \sqrt{x + 1}, R \rightarrow R$
Interview Form (IF)	<p><i>In the study, it was determined that students had difficulty in algebraic and graphical representations while there was no difficulty in determining whether giving samples about the schema and list representation of the function and whether the given samples are function or not. What do you think about the cause of this situation?</i></p>

### Data Analysis

In the analysis of the data, both descriptive analysis and content analysis were used. Content analysis was performed of the function descriptions, interpretations and function examples of the students. Descriptive analysis was used to analyze the data obtained in evaluating the performance of students for multiple representations. The students' function definitions, interpretations and examples of the students were resolved with content analysis and the examples given by the students in multiple representations were resolved with the

help of descriptive analysis, depending on whether they were mathematically valid or invalid. In the formation of categories, the opinion of an expert teacher was taken who was a doctor in mathematics education. The categories were finalized by compromising with the expert teacher. In the study the students' expressions that allowed the formation of categories was presented without modification on. In this way, it was aimed to ensure the validity and reliability of the study

## Results and Discussion

### Students' Definitions, Comments and Examples of Functions

In the first phase of the study, the general skills of the function concept of students were tried to uncover. Students were asked to define the concept of function, explain what they understood from the definition by their own sentences and give them an example of function. The data obtained from the students is summarized in Table 2.

Table 2. Function descriptions of students, comments and examples of functions they offer

Function description (f)	Function definition Interpretation (f)	Example (f)
Mapping all elements of domain to only one element of codomain (8)	Mapping (6)	The algebraic notation that domain and codomain sets were not identified (10)
Mapping each element in domain (4)	Definition-compatible Comments (3)	Example of a specified function with schema representation (4)
Single mapping from domain to codomain (1)	Linking to a rule (1)	Example of the function specified by the list method (1)
	domain and codomain set (2)	
Mapping one element of domain to one element of codomain (1)	No idle element (1)	
Injective match from domain to codomain (1)	Machine (1)	
	No answer (1)	

In Table 2, it was determined that eight students could accurately identify the function. This means that approximately half of the students could conduct at least the level of knowledge about the function concept. Examples of these students include the statements of S10 and S2.

*S10: The function is called to match only one element in the codomain of each element in the domain set. In order to be a function, there will be no idle element in the domain and only one image.*

*S2: Function is a relation in which there is no idle element left in the domain and all the elements are mapped to only one elements of codomain. For example: Every kid has to go to a house to be protected from the cold, and if he doesn't, he freezes. So, it cannot be a function. This is how each element in the domain must match only one of the codomains.*

Seven students could not be able to accurately identify the function. Four of these students defined "mapping each element in the domain" they did not state that this mapping was only one. As an example for these students, the function definitions of S13 and S4 are as follows.

*S13: The function is to map each element in the domain to the image set.*

*S4:  $f$ , when going from  $A$  to  $B$ , each element of  $A$  (with no idle element) matches any element of  $B$ , this connection is called a function.*

In the earlier research, students were asked to define the concept of function. In some researches, students never touched the formal recognition (Hatusaru and Erbaş, 2013; Tall and Bakar, 1991; Türkelli, 2016), in some studies, very few students could make a satisfactory formal description (Clement, 2001; Ural, 2012; Vinner, 1983). Polat and Şahiner (2007) found that 40% of the students in the department of elementary school teachers study had a misconception that the function have to be injective and surjective.

The student S15, defined the function “ $f(x)$  relation that matches only one of the defined image sets from A to B is called a function”. This definition has been evaluated in the “single mapping from domain to codomain” category. S8 made “an elements in domain corresponds to an element in codomain and function is an expression defined from A to B” definitions and tis definition provided “Mapping one element of domain to one element of codomain” category. S14 expressed “If the images for all elements of the real numbers are different from each other, we can call it a function”. S14 specified the difference of each element in the domain as a function rule. It is possible to say that the concept of injective function is dominant within the function definition of this student. Students were asked to interpret the concept of function in the way they understood. Students interpret this definition mostly as a mapping from the domain to the codomain. The following are sample student expressions that allow this category to occur.

S5: *It is the elements in a set matches a correspond and a value in another set.*

S4: *I think function is the thing with the entrants and subtracts.*

The three students correctly interpreted the definition using the features found in the formal definition. The statements of S6 and S9 from these students are presented as examples.

S6: *There are two sets called domain and codomain. For example, if we say child as the domain, the codomain will be the mother. There will be no idle element in the domain. Every child has to have a mother and only a mother.*

S9: *There will be no idle elements in the domain, and there will be only one image of the elements in the codomain.*

The function comments of six students were comments that were incompatible with the function definition, such as linking to a rule, domain and codomain, machine, no idle element. They could not explain their relationship with the function concept. Examples of students' comments are presented below.

S1: *Binding an unknown to a rule with mathematical operations.*

S12: *Function get any  $f(x)$  set. The domain is a set of values for a cluster that is  $f(x)$ .*

S8:  *$f$ , is a relation between A and B and which satisfy each element in the domain so that there is no idle element in the domain.*

S11: *The function is a machine. You put the boards in the machine and want to have a table and other stuff. It is something like that.*

In the previously conducted researches, students also had the concept of function; any mapping (Ural, 2012; Vinner and Dryfus, 1989), mapping rule (Vinner, 1983), dependency relationship (Türkelli, 2016; Vinner and Dryfus, 1989), formula, equation or equality (Tall and Bakar, 1991; Türkelli, 2016; Vinner, 1983; Vinner and Dryfus, 1989), machine (Clement, 2001) matched the statements like that. Doruk and Kaplan (2018) found that the comments of the function definition of primary mathematics teachers were in the form of conceptual comprehension, misunderstanding and concept complexity. Similarly, in this study, there were interpretations that are not compatible with the definition. Students were asked to provide a function example in order to reveal the function sample in their mind. Many of the students emerged that they offer formulas that do not specify a domain and codomain (Fig. 1). Examples of valid functions are for schema and list representation (Figure 2-3). Below are examples of each category.

$$f(x) = ax + b \Rightarrow \underline{2x + 1}$$

Figure 1. S10's Domain and Codomain Unspecified Algebraic Representation

$$X = \{1, 2, 3, 4\} \quad X \rightarrow y \quad f = \{(1,1) (2,3) (3,5) (4,0)\}$$

$$y = \{1, 3, 5, 2, 0, 6\}$$

Figure 2. Example of the Function Specified by the List Representation of S11

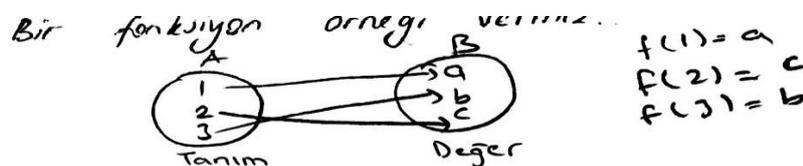


Figure 3. Example of the Function Specified by the Schema Representation of S3.

**Students' Skills on Multiple Representations of Functions**

In the first part of the study, it was concluded that the students had the wrong conceptions, such as the function must have only the definiteness and the unique value condition, and that they had difficulty in giving examples in accordance with the multiple representations. It was also found that students did not specify a domain and codomain in function examples. In the second part of the study, in order to obtain more detailed information on these difficulties, students were asked to question the skills of the multi-representation of the function. In this respect, it was tried to find out the students' exemplary and evaluation skills for multiple representations of the function.

First, it was questioned the students skills of giving function examples in multiple representations. Students were asked to give an example of the function's schema, list, algebraic (equation) and graphical representation. Students were specifically asked to specify domain and codomains in the function examples. The characteristics of the samples given by the students are presented in Table 4.

Table 4. Characteristics of the function examples that students provide in multiple representations

	Schema	List	Algebraic	Graphics
Valid example	14	13	5	4
Invalid example	1	1	8	9
No response		1	2	2

14 students gave valid examples with schema method. Similarly, 13 students did not have difficulty finding appropriate function examples with the list method. However, most of the students had difficulty in giving an example of functions suitable for graphic and algebraic form. Only four students gave graphical function and five students gave function examples in algebraic form. Students in these two representations had difficulty in specifying domain and codomain. In Figure 5, the valid and invalid sample pairs given by the students for the schema, list, algebraic and graphic representation of the function are presented respectively.

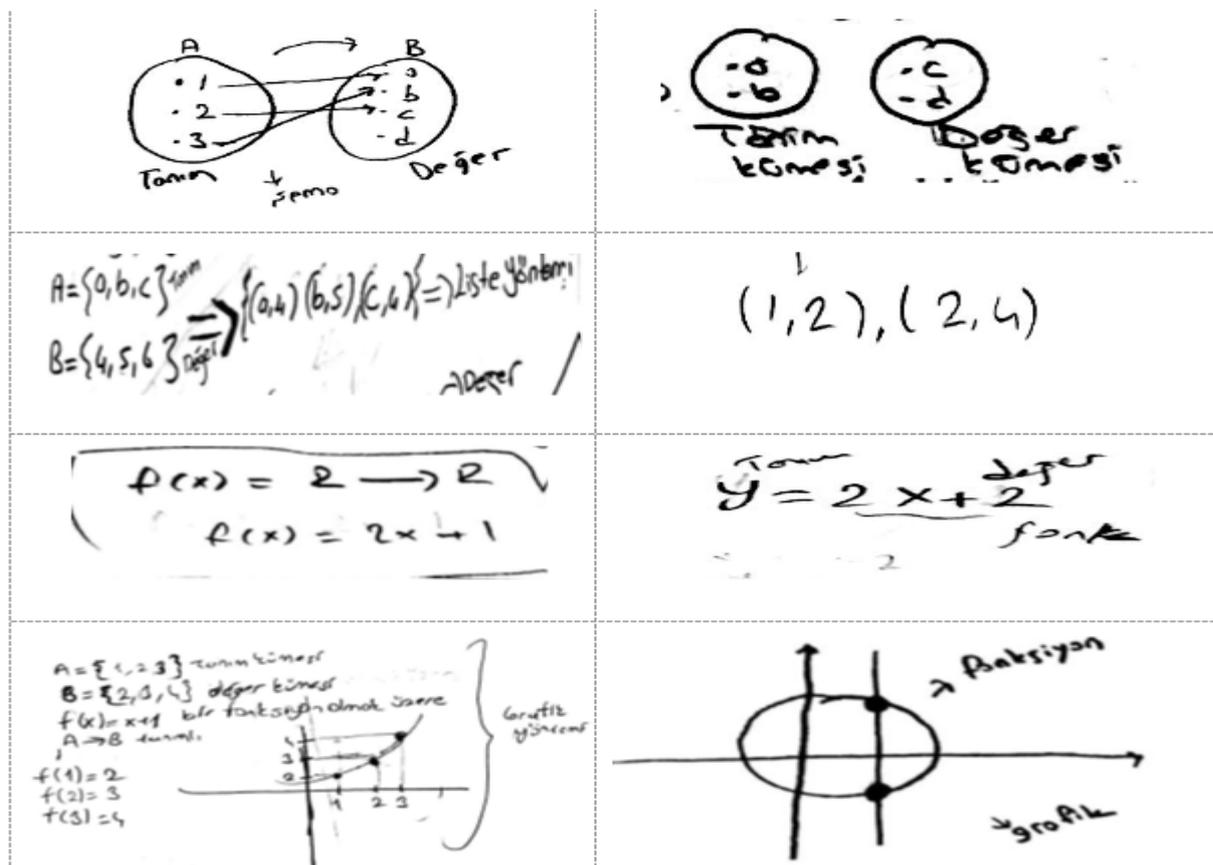


Figure 5. Examples of valid and invalid functions that students provide for schema, list, algebraic and graphic representation respectively

Examples of relation were presented in different representations of the misconceptions identified to the students. The students were presented with a schema, a list, algebraic and graphical forms, and the relation of the function that did not provide the definiteness and unique-value conditions. Students were asked to reach a reasoned decision about whether these relations were function or not. A total of 12 relation examples were presented to students, two of which represent each representation. Examples presented in algebraic form ( $y^2 = x$  and  $|y| = x + 1$  relations defined on the real numbers) were evaluated in both categories because they violated both conditions. In the graphical form,  $y = x + 1$ , which does not meet the definiteness condition,  $y^2 = x + 1$  graphs which do not meet the unique value condition are used. In table 5, information about the skills of the students is presented.

Table 5. The right decisions given by the students in determining whether the relation of the same characteristics represented in different structures

Relation Properties	Schema	List	Algebraic	Graphics
No definiteness condition	12	8	5-5	5
No unique-value condition	9	13	5-5	9

In Table 5, it was determined that the students were successful in the schema representation for the non-function relation which did not provide the definiteness condition (12) and that they demonstrated an above average success in the list representation (8). Most of the students were unsuccessful in evaluating algebraic and graphically represented relations. Most of the students were successful in the schema, list and graphical representation in determining the non-function relation that did not provide the unique value condition. In algebraic and graphical form, the students did not have the same success in detecting non-function relation. In the graphical representation, most of the students were successful in determining that the  $y^2 = x + 1$  relation was not only a violation of the unique value condition. Students decided with the help of vertical line testing. This showed that the vertical line test is popular among students.

When the average achievements were evaluated by taking into account both types of relations, it was found out that students were more successful in schema (average = 10.5) and list representation (average = 10.5), than algebraic (average = 5) and graphical representation (average = 7). With the effect of the vertical line test's popularity, it was seen that the students were more successful in the algebraic representation than graphical representation. When the findings were evaluated, it was found that even if they had the same characteristics, most of the students were successful in evaluating their functions with schema and list methods but failed to make evaluations in their algebraic and graphical representations. Examples of different decisions given in Table 6 are presented.

Table 6. Examples of decisions made by students for different representations of the same characteristics

Multiple representations (Unsatisfied condition)	Sample Student Statements
Schema (Unique value)	S3: It's function. Because the domain has no idle element and is also combined with the codomain. S10: It is not a function. Because an element in a domain must match only one member of the codomain.
List (Unique value)	S6: It is not a function. An element in the domain matched two distinct elements from the codomain. A child can't have two mothers. S2: It's function. Because mothers match with their children.
Algebraic (Definiteness-Unique value)	S8: It's function. It can get different values for each Y and x values. S11: It is not a function. Because Y does not provide the for negative values of X. S6: It is not a function. $x = 1$ has two images as both $-1$ and $1$ .
Graphics (Definiteness)	S4: It's function. Because a value in the definition has not been matched twice. S11: Is not the function, because the domain has an idle element.

When Table 6 was examined, it was revealed that students misused the function definition when making wrong decisions. Even if the justifications were true, it was observed that the students had difficulty in applying these reasons to the samples. As it was seen in Table 6, it was determined that the students decided on the wrong decisions by taking into consideration only one of the conditions of being function. In this sense, it is possible to say that there are deficiencies in the idea that students should provide both conditions in the function definition for a relation to be a function. From the examples they offered to the wrong decisions of the students, it can be said that they are keeping any of the conditions of function more prominent. For example, S3 made a false decision by referencing the definiteness condition of being a function for the relation shown by the schema, in

which a unique-valued condition is violated. Similarly, S4 made the wrong decision by accepting a reference to the unique value condition of being a function for the relation represented by the graph, in which the terms of the definiteness were violated.

A source of the wrong decisions given by the students is the misinterpretation of the analogies used for function definition. The deficiencies in understanding the "child-mother relationship" analogy for the function definition was effective in making the students wrong decisions. Another thought that was effective when students' misjudgment is that the unique-value condition was confused with the injective function. This situation emerged both in the interpretation of the function definition of the students and in the process of evaluating the relation. Some students used horizontal lines instead of using vertical lines to determine whether the relations are function or not.

In earlier studies, it was determined that Turkish students were more successful in the schema and list representation (Akkoç, 2006; Türkelli, 2016). The researchers stated that in the Turkish curriculum, they were given weight to the schema and the list method associated with it (Akkoç, 2006; Türkelli 2016; Yavuz and Başturk, 2011). In the studies conducted in connection with this situation, it was determined that schema and list representations were evoked definitional properties in the students' minds, and other representations evoked examples (Taşdan ve Çelik, 2015).

### Students' Transformation Skills between Algebraic and Graphical Representations

The data in Table 5 revealed that students had more difficulty in algebraic and graphic representations of functions. The third part of the study was conducted to uncover the ability of students to transform between these two representations. Both algebraic and graphical representations of the three relations, which are on real numbers, are presented to the students. Students were asked to determine whether these relations are functional and to justify their decisions. In table 7, information about the different decisions given for the same relation with the correct answers is presented.

Table 7. Decisions made by students for algebraic and graphic representation of the same relation

Relations	Algebraic representation		Graphical representation		Different decision
	Function	Not function	Function	Not function	
First relation ( $f(x) = 1, R \rightarrow R$ )	13*	2	11*	4	6
Second relation ( $x^2 + y^2 = 1, R \rightarrow R$ )	3	12*	2	13*	5
Third relation ( $f(x) = \sqrt{x+1}, R \rightarrow R$ )	4	11*	6	9*	7

\* Correct answers

According to Table 7, it was revealed that most of the students were successful in determining whether the relations offered to them were function or not. Nevertheless, it was determined that some students made different decisions to different representations of the same relations. In the general of the study, it was revealed that 12 students made different decisions for algebraic and graphical representation of the same relation. This showed that most of the students were low on the ability to transform between algebraic representation and graphic representation. Six students for the first relation, five for the second and seven for the third relation made different decisions to the different representations of the same relation. In order to understand the cause of this difficulty, it was focused on the students who had this kind of contradiction. Six students made different decisions for algebraic and graphic representation of the constant function. When examining the reasons that students used to make these decisions, it emerged that four students misinterpreted the unique value condition of the function. A student could not have been able to justify the wrong decision, and a student stated that there were no values that would be equal to the constant number, so that there wasn't any idle element in domain. This situation revealed that the students misinterpreted the conditions of function. The following are examples of the decisions given by the algebraic and graphic representation of the constant function of S5.

S5 (Algebraic representation): It's function. Specifies a constant function. Specifies a function when  $x$  returns a value and if  $Y$  can find the corresponding.

S5(Graphical representation): It is not a function. Because an element in the domain cannot be equal to the two elements in the codomain.

Five students have made different decisions for the algebraic and graphical representation of the unit circle. It was noteworthy that the students misinterpreted the conditions of function. Three students did not find a reason for their wrong decision. Following are examples of the decisions given by the algebraic and graphic representation of unit circle of S9.

*S9 (Algebraic representation): It's function. It doesn't matter if the equation passes through the square. Because he says the function is from real numbers to real numbers.*

*S9 (Graphical representation): It is not a function. Because when we download one line, it intersects two different points.*

Seven students made different decisions for algebraic and graphical forms of the third relation, in which the definiteness condition is not met, a student made the wrong decision for both cases. In making wrong decisions; it was effective that the idea of the fact that the three students would have an image of each element in the domain, that the two students needed to be the image of an element, and that the two students did not take into account the domain. One student did not find a reason for his wrong decision. The following are the reasons for the algebraic and graphical representation of S3, S8 and S7, who used different reasons.

*S3 (Algebraic representation): It is not a function. Because square-rooted expressions do not indicate function.*

*S3 (Graphical representation): It's function because it's cut at one point.*

*S8 (Algebraic representation): It's function.  $y$  is defined for each  $x$ .*

*S8 (Graphical representation): It is not a function. It is defined in  $R$ , the roots cannot be negative.*

*S7 (Algebraic representation):  $[-1, \infty]$  is also defined. So, it is a function.*

*S7 (Graphical representation): It is not function because square-rooted expressions cannot be a function.*

This result is consistent with the study results where students had low ability to transform between representations (Baştürk, 2010; Şandır, 2006; Tall and Bakar, 1992; Yavuz and Hangul, 2014). Difficulties students had in evaluating functions; students used the concept injective function for the concept of function (Bayazit, 2008; Doruk and Kaplan, 2018; Polat and Sahiner, 2007; Vinner, 1983), the function was seen only as a mapping (Ural, 2012), not paying attention to the domain and codomain in function (Hatsaru ve erbaş, 2013; Özüdoğru, 2016), seeing the function just as a rule (Dreyfus, 1989) and the use of samples without mathematical reference in evaluation (Özüdoğru, 2016; Tall and Bakar, 1992) can be associated with the difficulties previously identified. Many of the students in the study of Tall and Bakar (1992) did not accept the function of constant function in algebraic form and accepted the graphical form. Most of the students accepted the circle as a function. Accordingly, the students in the study can be said to be more successful in the fixed function and circle.

### The Reasons of Difficulties in Algebraic and Graphical Representation According to Students

It emerged that students had difficulty in determining whether the relations presented in algebraic and graphical form are function, while the relations presented by the scheme and list method were successful. It was determined that this difficulty was profound enough to make different decisions about whether the algebraic and graphical representation of the same relation was function. It was determined that the ability of students to transform between algebraic representation and graphical representation was quite low. The thoughts of the students were wondered about the cause of this difficulty. The results of the study were shared, and interviews were conducted with six students who volunteered to express their ideas.

As a result of the negotiations with the students, the difficulties were cause of the deficiencies of the mathematical backgrounds of the students, the defects in the past teaching method, schema and list representation are just related to function concept and these representation are suitable for the simulation (mother-child relationship) made about the concept of function, the fact that algebraic and graphic representations are more complex, function covers different subjects and skills. When they compared algebraic and graphic representations, they stated that algebraic representation is harder than graphic representation as vertical line test cannot apply in algebraic representation. Below are the students' opinions on this subject.

*S1: My teacher, the questions you asked me about functions was the part I couldn't do was about showing it as an equation. And the reason why I couldn't do it was not to study hard enough where I was lacking. But other than that, the graphic, list and schema are simpler to show, it can be determined by drawing with the pen, but I could not do algebraic because it requires more algebraic processing. There seems to be a distinct difficulty in algebraic expressions. I couldn't do because of that.*

*S2: I think the reason for this is that teachers who had been teaching lessons for a long time with rote-based logic. To be able to do the schema and list method of the functions and not be able to do the graphic and equation stems is because of the basis of the mathematics that were not done in time. I have no trouble identifying functions, but I know why my friends can't.*

*S3 Since the equation and graphic are more complex than the schema and list method, students are experiencing difficulties in determining whether such relations are functional.*

*S4: My teacher, I've been able to do questions about the list and the schema method. Because these two methods were directly related to the subject of the function, I could understand it, but I think I couldn't do it because the graphics and the equation covered some of the other math issues. So, we can call it a fundamental lack.*

*S5: I think make the topic of mathematics in the story allows the student to find a permanent solution on the subject. Of course, such as the schema and the list that can be displayed visually, keep the perception of the students high and provides benefits in the retrieval of desired return. For example, the mother-children story in the schema method. As the equation is thought of as complex structures It's difficult to comment on and build a solution in the mind. I think that this challenge is the inability to make a story for these methods.*

*S6: I think we can do it because the list and schema method are somewhat simplified and memorized. But the equation and graphics are a little more difficult for mathematical processing and technical wants, we can't. I think it's a bit of a lack of basis.*

## Conclusion

The study revealed that students could often define the concept of function but had difficulty in interpreting the concept of function. When the reasons given by the students during the study and the reasons given to them in evaluating the relations given to them, it was seen that there were some cognitive difficulties that prevent the fact that students could not have conceptual knowledge about the definition of function. These difficulties are listed below.

- Failure to consider functions as common.
- Seeing only one condition adequate for function.
- The function was considered only as a mapping, regardless of the conditions of the functions.
- Usage of non-mathematical referenced warrants and prototype examples.
- Perceiving unique-value condition as a definition of injective function.
- Applying vertical line used to test the unique value condition of function as horizontal due to the perception of injective function.
- Focus only on rule or shape of relations, regardless of domain and codomain of functions.
- Misinterpretation of analogies used for function definition.

The difficulties in understanding the conditions of function and the misinterpretation of the analogies used for the function have been identified for the first time. As a result of the negotiations with the students, the difficulties were cause of the deficiencies of the mathematical foundations of the students, the defects in the past teaching method, the schema and list representations are only relevant to the function subject and the analogy (mother-child relationship) made about the concept of function, the fact that algebraic and graphic representations are more complex and they stated that they were caused by the fact that they covered more different subjects and skills.

When the students' decisions on different representations of the same relation have been examined, it emerged that most of the students were contradicted. Most of the students had difficulty in transforming between algebraic and graphical representations. In the study, some positive results were obtained when compared to the difficulties detected in the literature and the results of the function teaching. It emerged that students had achieved more success in understanding of formal function definition, performing definition-based evaluation without resorting to prototype examples, evaluation of relations presented multiple representations.

## Recommendations

For the effective function instruction, it is important to conceptually understand the definition of function by the students. Accordingly, the meaning of the two conditions in the definition of function, underlying mathematical ideas and that two conditions are indispensable for the function and other concepts and differences should be

emphasized. The simulations made for the function definition should be carefully selected and the relationship with the function concept should be explained in detail. In the evaluation of the course, the examples that students can use conceptually to define the function definition can be discussed. Discussions about the conditions which the non-functional examples might function are useful.

In order to overcome the difficulties of the students, studies can be made about multiple representations of pupils and their transformations on each other. Considering that this difficulty can be caused by the deficiencies of mathematical basis, studies can be done about the students' skills of interpreting equations and reading graphics. By taking into consideration the results obtained from the studies, the necessary study can be done to improve the students' skills. In this way, the difficulties and negative judgments of students in algebraic and graphical representation can be overcome. It is deducing that this type of work will be beneficial for the conceptual learning of the function.

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