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EXAMINING PROSPECTIVE TEACHERS' BELIEF AND PEDAGOGICAL CONTENT KNOWLEDGE TOWARDS TEACHING PRACTICE IN MATHEMATICS CLASS: A CASE STUDY

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Abstract

Beliefs and pedagogical content knowledge (PCK) are two factors influencing teaching practice in the classroom. This research aims to describe the beliefs and PCK of the prospective mathematics teachers and the relationship between the two factors on the teaching practices in the mathematics classroom. Participant in this research includes a prospective teacher who has taken a micro teaching subject and has good communication skill. Data were collected through interview and video analysis on the teaching practice in the classroom. The data obtained were coded, simplified, presented, and triangulated for the credibility and concluded. The result of the research shows that the prospective teachers who hold a constructivist belief view mathematics as a dynamic knowledge which evolves and is regarded as the space of creation for humans. Their beliefs on the nature of mathematics support the belief in the teaching-learning process in mathematics classrooms. Furthermore, a good understanding of the prospective teachers have on the components of the PCK has been sufficient, which can be identified in every step of practical activities in the classroom. More elaboration on the relationship between the belief and PCK is presented in this research.

Keywords: Beliefs, Pedagogical Content Knowledge, Teaching Practice

Abstrak

Keyakinan dan *pedagogical content knowledge* (PCK) merupakan dua hal yang mempengaruhi praktik pembelajaran di kelas. Penelitian ini bertujuan untuk mendeskripsikan keyakinan dan PCK mahasiswa calon guru matematika, serta hubungan antara keduanya terhadap praktik pembelajaran matematika di kelas. Partisipan penelitian adalah seorang mahasiswa calon guru yang telah menempuh mata kuliah *micro teaching* dan memiliki kemampuan komunikasi yang baik. Pengambilan data dilakukan dengan teknik wawancara dan analisis video praktik pengajaran di kelas. Data yang diperoleh dilakukan pengkodean, penyederhanan, dipaparkan, ditriangulasi untuk mendapatkan data yang kredibel, kemudian ditarik kesimpulan. Hasil penelitian menunjukkan bahwa mahasiswa calon guru yang memiliki keyakinan konstruktivis memandang matematika sebagai suatu ilmu yang dinamis, berkembang, dan merupakan ruang penciptaan manusia. Keyakinannya terhadap sifat matematika mendukung keyakinannya terhadap pengajaran dan belajar matematika. Selanjutnya pemahaman yang baik dari mahasiswa calon guru terhadap komponen PCK khususnya *knowledge of mathematics*, *knowledge of teaching*, dan *knowledge of students* telah mendukung dalam setiap tahap kegiatan praktik pembelajaran yang dilakukan di kelas. Uraian lebih detail tentang hubungan antarkomponen keyakinan dan PCK dibahas dalam penelitian ini.

Kata kunci: Keyakinan, *Pedagogical Content Knowledge*, Praktik Mengajar

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Several factors are contributing to the inconsistency between what is planned and what is practiced in the teaching and learning process at school. A good understanding of mathematical knowledge can influence teaching instruction in the classroom in a positive way. Teachers need the knowledge to transform the content into the representative form, which is believed to be able to help the students to improve their competence (Shulman, 1986; Muhtadi, *et al*. 2018). Research on teacher's knowledge is based on the pedagogical content knowledge (PCK) introduced by Shulman (1986, 1987). Even though PCK facilitates the instructional practices in the classroom, some teachers who are strong in the content knowledge tend to use traditional teaching method (Mewborn, 2001). This is due to the belief that the traditional method is the most effective method for teaching mathematics to the students. Teachers with this belief will tend to use the same method in their classroom teaching practice. Hence, PCK alone is not sufficient in contributing to an effective classroom teaching practice.

Belief and knowledge are two important factors influencing teaching and learning practice (Fennema & Franke, 1992). Empirically, researchers have found that belief consistently affects teaching practice in the classroom (Pajares, 1992; Stipek, *et al*. 2001; Thompson 1992). Instrumentalist belief tends to be associated with the teaching practices that are more traditional (Stipek, *et al*. 2001). A teacher with an instrumentalist belief is also said to be allowing the students to explore and maintain social context, where mistakes are avoided. On the other hand, researchers have found that there is an inconsistency between the belief and practice in the mathematics classroom (Barkatsas & Malone, 2005; Beswick, 2003; Raymond, 1997). Raymond (1997) assert that the teacher's learning process is much more influenced by his/her knowledge than by his/her belief. Barkatsas & Malone (2005) found that teacher belief is not consistent with her teaching and learning practice. This is due to the classroom situation, experience, and social norms.

Research on the relationship between belief, knowledge, and mathematical learning and teaching practice has been conducted by Wilkins (2008) and Belbase (2012). Wilkins (2008) studied 481 math teachers of primary school to investigate the relationship between belief, content knowledge, and attitude on the inquiry-based teaching practice. Data were collected using a questionnaire and were analyzed using structural equation modeling. The result of the research shows that teachers' beliefs have the most significant influence on the teaching practice, while knowledge has a negative correlation with the beliefs in the classroom teaching and learning practice. Belbase (2012) examined and analyzed beliefs, knowledge, and teaching practice of the math teachers through literature review and drew some pedagogical implications. The result of the research shows that belief influences the teaching practice in the context of a supportive learning environment. The teachers' beliefs influence teachers' development. Teachers' knowledge comprises content knowledge, pedagogical knowledge, technological knowledge, and the combination of this knowledge. Both studies provide an initial picture of the relationship between beliefs and content knowledge in PCK. However, they have not described the specification of the beliefs, PCK, and how the belief aspect and PCK can influence the mathematical teaching practice.

Pedagogical Content Knowledge (PCK)

The initial research on pedagogical content knowledge (PCK) was done by Shulman (1986, 1987), and developed by (An, Kulm, & Wu, 2004; Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Kilic, 2011). Shulman (1986, 1987) stated that PCK is an integration of content knowledge and pedagogical knowledge realized in the method of presentation and content formulation (learning material) which are comprehensible and can facilitate learning. This definition emphasizes on three aspects: 1) content, 2) pedagogy, and 3) student. PCK does not only emphasize on how to utilize a concept to solve a problem but also on a deep understanding of the concept (Ball, *et al*. 2005). Hence, PCK does not focus merely on how to use an algorithm to solve problems, but also on a deep understanding of the concept.

PCK consists of knowledge of content and student, knowledge of content and teaching, and knowledge of content and curriculum (Ball, *et al*. 2008). However, Hawkins (2012) specifically explained the three components of PCK that one needs to have: (1) knowledge of mathematics (KM), (2) knowledge of teaching (KT), and (3) knowledge of students (KS). Knowledge of mathematics refers to the knowledge and skill of mathematics, which is exclusively used in the teaching practice, such as conceptual knowledge and procedural of the topic of mathematics. Knowledge of teaching is related to the knowledge on how to teach mathematics to students. For example, the knowledge of choosing the examples before starting the lesson, choosing the type of assignment which will help students understand better, the representation that is used, and the questions given during classroom teaching and learning activities. Meanwhile, the knowledge of students refers to the knowledge in understanding students' ideas in solving the problems, diagnosing errors, and making strategies to overcome the errors.

Mathematical content knowledge constitutes a pure knowledge and its organization in the mind of an educator, which includes the ability to explain why one theory needs to be taught, as well as its relation to other theories. Mathematical content knowledge and pedagogical knowledge is one integrated part of the effective mathematical teaching and learning that establishes the concept of mathematics in the student's mind practice (Kahan, Cooper, & Bethea, 2003; Shulman, 1986). Thus, PCK is considered as knowledge of teaching, especially on transforming mathematical knowledge into different kinds of representations and considering obstacles faced by the students, which then enables the comprehension of mathematical content.

Beliefs

Understanding that there might be differences in beliefs is important in developing and ensuring the success of the implementation of mathematics education at schools (Dossey, McCrone, & O'Sullivan, 2006). Beliefs are in the "twilight zone," which means they are between the cognitive domain and the affective domain (Pehkonen & Pietilä, 2003). Belief is the basis for an individual in behaving and in understanding a phenomenon. Belief is a mental condition which is perceived as true and can originate from experiences, either real or imaginative, and influence words and behaviors. Ernest (1989) describes three components of mathematical beliefs: beliefs about the nature of mathematics, teaching, and learning mathematics. Each component of the beliefs consists of three categories of philosophical views, namely: instrumentalist, Platonist, and constructivist. Ernest's views have been adopted and widely used by some scholars (Beswick, 2005, 2012; Buehl & Fives, 2009; Muhtarom, Juniati, & Siswono, 2017a, 2017b, 2018; Muhtarom, *et al*. 2018; Siswono, Kohar, & Hartono, 2017; Siswono, *et al*. 2017; Thompson, 1992). A person who has instrumentalist beliefs sees mathematics as a set of tools consisting of a set of mathematical facts, concepts, rules, and math skills that are not interrelated but useful. Platonist beliefs look at mathematics as being found, not created. It means that mathematics is static, but an integrated field of science where the truth is intertwined by a set of rules, concepts, and theorems. Constructivist view mathematics as dynamic, that is, the space of creation and human discovery that develops continuously where patterns are raised and then filtered into knowledge (Ernest 1989; Thompson, 1992).

There have been many research on the beliefs that focus on students and teacher, but research focusing on prospective teachers are very limited (Bal, 2015; Boz, 2008; Giovanni & Sangcap, 2010). Muhtarom, *et al.* (2017b) assert that from 183 prospective mathematics teacher, most of them do not show consistency beliefs about the nature of mathematics, teaching, and learning mathematics. For example, there are 2.73% prospective teachers who have instrumentalist beliefs about nature of mathematics, but have Platonist beliefs about teaching, and learning mathematics; 7.10% of prospective teachers have Platonist beliefs about nature of mathematics, and teaching, but have constructivist beliefs about learning mathematics. Only 2.19% of prospective teachers consistently have constructivist beliefs about the nature of mathematics, teaching, and mathematics learning.

METHOD

Participants

Before the selection of research subjects, researchers first gave written open questions to 172 prospective mathematics teachers at a private university in Semarang, Indonesia, to get an initial description of his belief in mathematics, teaching and learning mathematics. This data collection is done gradually in each class of the 6th semester. The result is that there are 38 or 22.09% prospective teachers who have instrumentalist beliefs, 10 or 5.81% who consistently have Platonist beliefs and only 3 or 1.74% are consistent with constructivist beliefs, and the rest are not consistent in their beliefs. The research on the consistency of beliefs refers to Beswick (2005).

One of the criteria for choosing a research sample is their willingness to participate and consistently have constructivist beliefs, this is following the current 2013 Curriculum requirements applied in Indonesia in which educators are required to facilitate students to explore information, actively ask questions, and link information is to form a mathematical understanding. Thus, the suitability can be seen between knowledge and beliefs with learning practices conducted in the classroom. A consistent prospective mathematics teacher who has constructivism view was chosen to be the sample of research, who is coded with Fitri (pseudo name). One sample is considered sufficient because the data obtained from all three samples have shown saturation of the data, and there has been sufficient information to replicate the results of research based on repetitive data retrieval (Moleong, 2007). Fitri is a 21-year-old student and Javanese. Fitri was born in Purwokerto Regency, Central Java Province, Indonesia. Before the execution of this research, Fitri did not yet have teaching experience at school.

Instrument and Procedures

Data were collected through semi-structured interviews and observation of classroom teaching practices. Confidential data retrieval was conducted from May to June 2017. The interview on beliefs focused on obtaining data on the beliefs about the nature of mathematics, and beliefs about teaching and learning mathematics, which is adopted from Ernest's (1989) view. The belief in the nature of mathematics is a person's mental state of mathematics as a discipline that is recognized as a truth (Beswick, 2012; Ernest, 1989; Thompson, 1992).

Meanwhile, beliefs about teaching and learning mathematics are a person's mental state on different types of learning approaches, the role of teachers and students in the learning process, activities in motivating students to learn mathematics, and how students learn mathematics. Interviews intended to explore PCK especially related to knowledge of mathematics, knowledge of teaching, and knowledge of students, were conducted in August 2017, when Fitri participated in a three internship program aimed at allowing participants to practice mathematics learning at a state junior high school in Semarang. Interviews on belief and PCK were conducted twice, then triangulation technique was used to obtain credible data. Table 1 illustrates the examples of interview protocols used to explore the beliefs and the PCK of Fitri. All interview protocols are presented so that the readers will have a comprehensive picture of the issues discussed in this research.

Table 1. Questions from the Semi-Structured Interview Protocol

Focus of Research	**Sub Focus of Research**	**Questions**
Beliefs about the nature of mathematics	Definition of Mathematics	Based on your belief, what is mathematics?
	Relationship between mathematics and daily life	What is the relationship between mathematics and daily life? Elaborate!
	The development of Mathematics science	Elaborate your belief on the development of mathematics science!
Belief about the teaching of mathematics	Teaching Approach	Elaborate your belief on the effective approach implemented in teaching mathematics!
	Role of teachers in the instruction	What is the ideal role of the teacher in mathematics instruction!
	Problem-solving	Elaborate your belief on the effective type of exercises/questions to be given to the students! Who should design them, and what is the source? Give your reasons!
	Motivation on learning mathematics	What method do you apply to motivate the students to learn mathematics? Elaborate! What do you do when students think that mathematics is not relevant to their daily life.
Belief about learning mathematics.	The role of the students	How should students learn math? What should students master to learn math?

Focus of Research	Sub Focus of Research	Questions
Knowledge of mathematics	Conceptual Knowledge	What do you know about linear equation system in two variables concept?
	Procedural Knowledge	What are the steps for setting the substitution, elimination, and combined method
Knowledge of teaching	Learning objective and instructional media used	Elaborate on the learning objective that needs to be achieved! What is the design of instructional media to support learning objective?
	The use of representation	What kind of mathematics representation do you use? Describe the advantages of the media
	Math problems	What example of math problems do you use to start the lesson? Wha math problem do you use to understand the material better?
	Questions	When do you give new questions? When do you use questions that ask for clarification from the students?
Knowledge of students	Knowledge of students' ideas in solving the problems.	Describe the students' idea of solving math problems.
	Knowledge of students' errors	With the aforementioned ideas, Is the student making an error? Elaborate!
	Knowledge in solving students' errors	How do you improve your students' errors?

Two teaching-learning sessions with duration 2 x 40 minute was recorded to obtain data on classroom teaching practices. Fitri teaches in class VIIIA SMP Negeri 6 Semarang, Central Java, Indonesia with the material of linear equation system in two variables. After the series of learning activities have been completed, the interview was conducted to describe Fitri's reflection on what she has stated in the previous interviews so that a relationship between beliefs, PCK, and teaching practices can be made.

Data Analysis

Video recordings and interviews are transcribed, then read and verified again by Fitri to ensure the accuracy of the data. It is then coded and analyzed in an interesting pattern to describe the beliefs and PCKs owned by Fitri. The response patterns associated with beliefs interviews are categorized into instrumentalist, Platonist, and constructivist beliefs for dimensions of mathematical properties and teaching and learning of mathematics. Meanwhile, the pattern of PCK responses describes Fitri's knowledge to teach linear equation system in two variables material by considering the errors made by the students. Overall, the recorded video was played and viewed together with Fitri to explore classroom learning practices.

Data analysis is done through the process of (1) data reduction, which refers to the activity of the election process, centralization of attention, simplification, abstraction, and transformation of raw data. (2) Data presentation, which constitutes organizing data of encoding results through the pairing of first and second data, then subsequently compared to obtaining credible data, and (3) conclusions drawing from data collected and conclusions verification about beliefs, PCK and learning practices undertaken (Moleong, 2007). To demonstrate the relationship between beliefs components, PCK and mathematics learning practices, qualitative data were analyzed using the QSR NVivo 11 software on cluster analysis features (Bazeley, 2007; Bazeley & Jackson, 2013; Muhtarom, Murtianto, & Sutrisno, 2017). Nvivo software is the most effective tool in analyzing qualitative data because it provides completed tools and ideal in analyzing qualitative data (Bazeley, 2007; Bazeley & Jackson, 2013; Hamrouni & Akkari, 2012). Thus, Pearson correlation will be obtained to illustrate the relationship between the components in this research.

RESULT AND DISCUSSION

Fitri's Beliefs about Nature of Mathematics, Teaching and Learning of Mathematics

Fitri tends to consistently have constructivist beliefs about the nature of mathematics, teaching and learning mathematics. Fitri beliefs that mathematics is a dynamic science and mathematical knowledge develops as humans always strive to do continuous research of new problems existing in life has influenced her beliefs in teaching mathematics and learning mathematics. This is in line with the opinions of some experts (Amirali & Halai, 2010; Ernest, 1989; Grigutsch, Raatz, & Törner, 1998; Thompson, 1992; Viholainen, Asikainen, & Hirvonen, 2014). Mathematics, as a dynamic field of human creation, continues to evolve according to discovery patterns and the results remain open to revision (Ernest, 1989). Mathematics is seen as an active construction process (Grigutsch, *et al.* 1998; Shahrill, *et al*. 2018). Ongoing research on problems in life is based on experience and observation of the regularity of the phenomena of social context. Thus, mathematics is always changing and never static (Buehl & Fives, 2009). The relevance of this relationship can be seen when Fitri sees that a suitable approach to be applied in mathematics teaching is those which is student-centered where learning begins with giving the students problems (Barkatsas & Malone, 2005; Ahamad, *et al*. 2018), helping students understand problems, discussing and finding solutions (Felbrich, Kaiser, & Schmotz, 2014; Stipek, *et al*. 2001; Prahmana & Suwasti, 2014), and presenting it. This approach makes learner as a focus rather than the mathematical content (Beswick, 2012).

Fitri believes that students should be given non-routine questions to complete in the learning activities, this is in line with Muhtarom, *et al.* opinion (2018), and students are also allowed to make their problems and solutions independently (Siswono, *et al.* 2017). To facilitate the achievement of the learning goal of mathematics, the teacher acts as a facilitator in the learning activities. This means teachers should facilitate inquiry, allowing students to develop solutions to their problems, and enabling students to play an active role in learning activities. Facilitator can also mean applying classroom activities to help students create new mathematical concepts, as well as to encourage reasoning, creativity and information gathering, learning occurs when there is social interaction involving collaborative dialogue with other students as well as teachers (Stipek, *et al.* 2001), co-operativeconstructivist view (Barkatsas & Malone, 2005), and learner-focused content (Beswick, 2012).

Fitri believes that students need to be given the freedom to construct their knowledge. The teacher only gives direction for further students to dig further understanding. Thus, when students learn mathematics, they need to be able to construct a mathematical concept, develop it, and solve it appropriately. This is in line with the opinion of Ernest (1989) which states that learning is an active understanding construction, perhaps even problem posing and problem-solving. The learning objective constitutes acquiring skills in reasoning and building new things (Felbrich, *et al*. 2014). Beswick (2005, 2012) views mathematical learning as an autonomous exploration of self-interest. Thus, the things students should master in learning mathematics is to construct mathematical concepts, develop them and solve the math problems appropriately through practicing solving varied problems from various sources of references, making summaries, and constructing those mathematical concepts (Ernest, 1989; Thompson, 1992).

Fitri’s beliefs that mathematics develops by the problems of daily life, mathematics provides solutions which can be used to solve problems in mathematics itself and everyday life, can be identified Fitri's from the way she started the lesson where she gave students problems to solve. The problem should be made as relatable as possible to the contextual and real problem so that it is close to the students' daily life where mathematics can be used to provide solutions (Muhtarom, *et al.* 2017b; Siswono, *et al.* 2017). Through this method, Fitri has shown her way of motivating students by pointing out the relevance of mathematics to everyday life. The inspirational figures of mathematics also form mathematics based on observations and patterns that exist in life. This is in line with Stipek, *et al*. (2001) which suggest the importance of motivating students during learning activities.

This description has explained the consistency of Fitri's beliefs about how to teach mathematics and how students learn in the classroom as described by some researchers (Boz, 2008; Ernest, 1989; Thompson, 1992). Fitri's belief that mathematics is a dynamic field of human creation, which continues to evolve according to the pattern of discovery has influenced her decision-making to use student-centered learning approach that is more learners-focused than content-focused. This approach suggests that students should practice solving varied problems from various reference sources, making summaries, and constructing concepts and procedures while solving math problems. The conclusion is Fitri's belief that mathematics is a dynamic science, evolves, and is a space of human creation, has supported her belief in her teaching practice.

The above description is supported by the positive relationship between each component of belief using software QSR NVivo 11. The correlation coefficient used in this cluster analysis is the Pearson correlation coefficient. Obviously, Figure 1 shows that there is a strong relationship between beliefs about nature of mathematics and beliefs about teaching mathematics, which is indicated by a correlation coefficient of .651, a fairly strong relationship between beliefs about nature of mathematics and beliefs about learning mathematics, which is indicated by a correlation coefficient of .569, and a strong relationship between beliefs about nature of mathematics teaching and beliefs about learning mathematics, which are indicated by a correlation coefficient of 0.741.

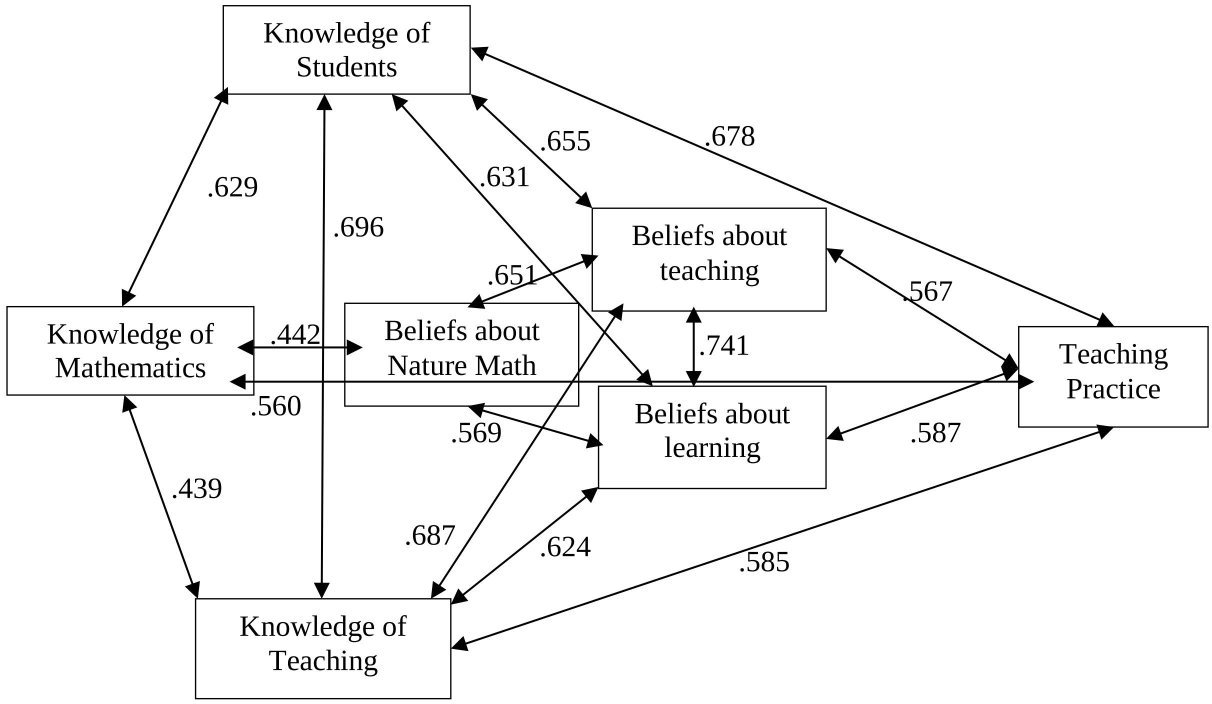


Figure 1. The relation between Beliefs, PCK and Teaching Practice

Fitri's Pedagogical Content Knowledge (PCK)

Fitri has conceptual knowledge and procedural knowledge related to linear equation system in two variables. Conceptual knowledge is demonstrated through understanding the definitions of linear equation system in two variables, the concept of elimination methods and the concept of substitution methods. This can be seen when Fitri understands the concept of linear equation system in two variables that it consists of two linear equations, with every variable to the first power. The concept of elimination is where one variable is being eliminated, while the substitution concept is to replace one variable with another variable by the given equation. Procedural knowledge is identified from an understanding of the steps to solve math problems using methods of elimination or substitution. The understanding of the steps of the elimination method is indicated from the knowledge on determining which variable should be omitted. When the coefficient is different, it must be equalized first by multiplying it with the opposite (look for the smallest multiplier) of the coefficient to be omitted. The second equation should go through the same steps before it can be solved. Understanding the step of the substitution method is indicated from her ability in determining the equation where x or y will be changed into $x = \ldots$ (x in y) or $y = \cdots$ (y in x), then substituting the equation into one of the equations. If the first equation is changed, then it must be substituted to the second equation and vice versa. Next, after the value of x or y is obtained, the next step is to substitute them into one of the equations.

Fitri was able to understand the ideas of students in solving the given problem very well. This is shown from her fluency in describing each answer given by the student and was able to identify the errors made by the students in completing the worksheet or the questions made by the students themselves. Fitri further explains that misconceptions of the student in solving the student worksheet occur in the concept of counting operation, for example the negative number is subtracted by the negative number $(-6-(-3)=-9)$, the negative number is subtracting the positive number $(-12-(48)=60)$, and another error on the substitution concept when the student cannot change a linear equation of two variables into the variable x in y. While the error procedure in performing algebra operations such as errors in multiplication, addition, subtraction which affects the results. To solve the student's misconception, Fitri gave an example of the problem, emphasized the error, and emphasized the concept of 'sakubeta' on the same variable. If the sign was the same, then it must be subtracted. In contrast, if the sign was different, then it should be added.

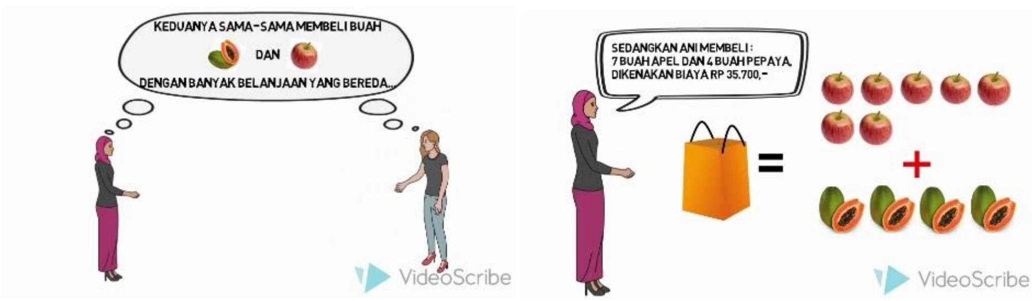
Fitri's understanding of the knowledge of mathematics (KM) has supported her decision in planning classroom activities (KT). For example, Fitri prefers examples at the beginning of the lesson using contextual story problems, while the question to deepen the understanding of the material was done by giving a story problem that has a different level of difficulty with the ones previously used when starting the lesson. Fitri designed the learning activities using the image and symbol representation. The image representation is used to clarify the concept of the given problem so that it is completely contextual to the students. The table representation is used to assist the student in modeling the given problem so that the student can have a clear illustration. Fitri asks new questions when students have understood the lesson and can use linear equation system in two variables method of completion, whereas clarifying questions are given when students have already presented their work and invited other students to respond. A teacher should be able to ask appropriate, meaningful questions to understand students' thinking processes (Muzaini, Juniati, & Siswono, 2019; Turnuklu & Yesildere, 2007). They also need to have the ability to create solutions to overcome students’ learning difficulties.

This result is supported by Shulman (1986), who states that understanding of PCK is a strong foundation for effective teacher education. A deep understanding of mathematics and students' knowledge was not enough to teach mathematics (Black, 2008; Turnuklu & Yesildere, 2007). Knowledge of mathematical pedagogy was necessary. Furthermore, teachers should be able to understand students 'difficulties, understand the reasons behind it, and be able to ask appropriate and meaningful questions to understand their thinking processes. Even Kilic (2011) confirms that the content and student knowledge is the determinant of a teacher's success in mastering the class and answering students’ questions. The description is supported by cluster analysis with QSR NVivo 11 software, presented in Figure 1. For example, there is a strong relationship between knowledge of mathematics and knowledge of students, which is indicated by a correlation coefficient of 0.629. There is a strong relationship between knowledge of students and teaching practices, which is indicated by a correlation coefficient of 0.678, and the relationship is quite strong between knowledge of teaching and teaching practices, with a correlation coefficient of 0.585.

The Relationship between Beliefs and PCK toward Teaching Practice

Fitri's instructions are constructivist-oriented, which reflect what she believes. Fitri believes that mathematics is a dynamic branch of science. In line with the beliefs about teaching that was mentioned before, the appropriate approach for learning mathematics is student-oriented or student-centered where learning activities start with the provision of contextual problems, teachers help students discuss and find solutions to problems before presenting them. This belief is implemented in the practice of learning through question and answer method and discussion method. Instructional media, which contains contextual illustrations and representations of images and symbols, is an indicator that she has applied what she has planned and what she believes. This can be seen from the teaching practice she underwent. Fitri used image representation and symbols. The image representation is used to clarify the concept of the given problem so that it is completely contextual for the students, while the symbol representation by using certain variables is used to facilitate problem-solving. The following is the record of Fitri's instructions.

PPM.K.06 : Now sit in a group. Please set the chair facing each other (handing out student worksheets) pay attention and watch the video I am showing you. Then please fill in the exercises in your worksheet by the information from the video. Pay attention! (Playing the videos associated with linear equation system in two variables, students are focusing and listening). Please work on and discuss with your friends in your group (Teacher walks around and guides the discussion).



PPM.K.17 : Look at this, here is the linear equation system in two variables table. So the image of the apple and papaya can be presented in a tabular form. This table is used to make it easier for you to create a mathematical model from linear equation system in two variables. The third question, please make the linear equation system in two variables model.

Student : $5x + 5y = 30{,}000$; and $7x + 4y = 35{,}700$; specify the solution for $5x + 10y = \ldots$?

PPM.K.18 : I will eliminate y variable, how do you do that?

Student : Through the elimination method, by equating the coefficient of y variable.

PPM.K.19 : Look at this, the y coefficient in equation one is 5, and the y coefficient in equation two is 4. What is the value of x obtained?

Fitri has been able to connect the representations with fundamental ideas and other representations to make students better understand the material being taught (Ball, *et al*. 2008). It should be noted, however, that even though the prospective teachers have used some representations, students are only using symbol representations in solving the math problem. This is possible because the emphasis of conceptual understanding appears to be very limited compared to a procedural understanding of problem-solving. In line with this, Black (2008) has explained that procedural knowledge is very dominant in the practices of classroom instruction. The fact that students' tendency to use symbolic representations related to mathematical problem-solving procedures including in solving the problem of linear equations system of two variables is influenced by the teachers' habits while teaching in the classroom (Nizaruddin, Muhtarom, & Murtianto, 2017). Furthermore, Fitri's content and student knowledge have influenced the organization of her instructional decisions, class explanations, guiding the students, and the way they conduct mathematical discussions.

Fitri's knowledge and beliefs are implemented in her learning practice. Fitri knows that a suitable example to start the instruction is a contextual story problem (see PPM.K.06). She also understands that it will be more effective to give the problem with a different level of difficulty when aiming at deepening the understanding on the content. This is in line with what Fitri believes that teachers should provide non-routine questions for students to complete, as long as learning is made by teachers and students. The following is a quote from Fitri's learning practice.

PPM.K.23 : If you still feel confused, please ask? Now try to do the problem in this worksheet (Distribute the second worksheet). Please work in groups.

Student : (Students discuss problem-solving, and the teacher walks around guiding the students) In a housing complex residents, there is Mr. Hasyim, Mr. Ikhsan, and Mr. Alwi plans to create a gazebo pole is made of adobe. Mr. Hisham gazebo pole shape is shown in Fig. (1). Mr. Ikhsan has a shape like a gazebo pole Fig. (2). While Mr. Alwi wanted to make a different gazebo pole shape with the shape of the gazebo Mr. Hasyim and Mr. Ikhsan (see Fig. (3)). Determine how high the gazebo pole belongs to Mr. Alwi!

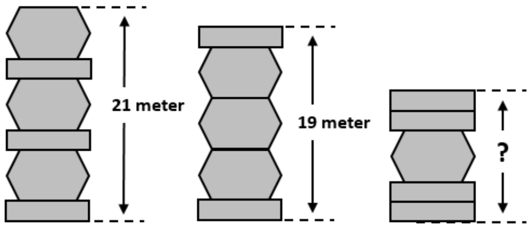


Fig. (1) Fig. (2) Fig. (3)

In accordance with her belief that knowledge of mathematics continues to evolve and is the product of human findings, the student's roles are to construct an active understanding through problem posing and solving (Ernest, 1989), building understanding through reasoning (Felbrich, *et al*. 2014), and building knowledge their own mathematics as facilitated by their teacher (Beswick, 2012). This can be seen from the classroom instruction practiced by Fitri. In her instruction, the use of media is only to motivate students by relating mathematics with everyday life. Fitri does not explain the subject matter (linear equation system in two variables), when the student discusses to solve the problem, she helps the students who were having difficulty solve the problem by giving direction, then the student solve it based on their understanding. Although the understanding of the elimination, substitution, and combination method are still limited; the teacher acted as a facilitator in helping students construct mathematical concepts so that they can solve math problems appropriately. Students were trained to solve problems with different level of difficulty, and are also trained to solve the problems they make themselves. This is consistent with what Fitri has been believed that the teacher acts as a facilitator while the student is an active knowledge constructor.

Focus on the interaction between students and teachers can also be identified in Fitri's classroom teaching practice. During the discussion, teacher-student interactions happened and were established one-on-one with the students in each group. During the presentation, there was an interaction between the teacher and the whole student. The dialogue conducted by Fitri tends to use questions to ask for clarification of student comments or answers. The high frequency indicates that Fitri will approach students to seek information through a series of questions leading to the discussion. Fitri believes that through the provision of questions, students will be actively engaged in the learning process. Through the discussion, students were also trained to be able to construct mathematical concepts, practice solving various problems, and develop the problems. Fitri also used questions as a way to assess the experience and knowledge of her students. In other words, the more the feedback gained from the students, the better the teachers understand the weaknesses and continue to improve them. This fact is consistent with the knowledge of when to ask questions to students and Fitri's belief that the students' involvement in collaborative learning is always beneficial to the learning process. The following is a quote from Fitri's learning practice.

PPM.K.10 : What do you understand about this equation?
Student : Model of linear equation system in two variables, so 5x + 5y = 30,000 and so on
PPM.K.11 : And then?
Student : And then we need to define the solution. Bu I am confused about the question, does it means that the price of 1 apple should be multiplied by this (5 apples) and the same with papaya?
PPM.K.14 : What word indicates what?
Student : The one that asks. So this is the given information only, then we should make an example and then create an equation of a mathematical model, and then solved using the combined method, is that correct?
PPM.K.18 : How to remove y variable?
Student : Eliminate it by equating the coefficient of y variable.
PPM.K.24 : (guiding the discussion) what is the first step to do it?
Students : The information that is known and asked, and then makes an example.

Turnuklu & Yesildere (2007) explain that teachers should be able to ask appropriate, meaningful questions to understand students' thinking processes and should also have the ability to create solutions to student's learning difficulties. The teacher's role is to diagnose student errors and point out student difficulties, guide and facilitate students, rather than providing answers and explanations. Teachers should also recognize students' needs in understanding the lesson. Fitri has done these in the implementation of mathematics learning practice. It is clear that Fitri has used question and answer methods and discussion methods during classroom learning activities. The application of the use of this method of learning is supported by her knowledge of learning and media objectives, the use of examples and tasks is supported by the use of representation, and ideas on when to ask students questions about clarification and when to ask new questions which was done when students have understood the material. This is supported by cluster analysis using QSR NVivo 11 software presented in Figure 1. It is clear that the correlation coefficient between each component of belief and PCK component to the teaching practice is obtained. Overall belief correlates with PCK so as to support the practice of mathematics learning in the classroom.

The above description indicates that this research is consistent with previous research. However, it provides the additional belief that understanding beliefs is an important step in understanding learning practices (Ernest, 1989; Pajares 1992; Stipek, *et al.* 2001, 2001; Thompson 1992). The results of this research are also consistent with the literature research which highlights the positive relationship between content knowledge and teaching practices (Belbase, 2012; Fennema & Franke 1992). As previously described, knowledge of mathematics on linear equation system in two variables material, knowledge of ideas and errors made by students, and knowledge to overcome student errors, have supported prospective teachers in designing learning activities, and then apply them in the classroom.

CONCLUSION

The findings of this research indicate the important role of beliefs and PCK as a factor affecting the learning and teaching practice, as shown by Fitri's experience. The results of this research are consistent with the literature research which highlights the positive relationship between belief and knowledge of teaching practices. Beliefs correlate with PCK and teaching practices as a system. The results of this research can be used as an input for the curriculum of Mathematics Teacher Education program to integrate the philosophy of the belief and to strengthen students' PCK capability which will result in the integration between cognitive ability and mathematical teaching knowledge. It is also important to understand that prospective teachers need to be allowed to have a sustainable learning experience. This is useful for giving them a chance to reflect on their beliefs and PCK so they can improve in the future. The results of this research are hoped to be able to provide a diversity of relationships between beliefs and PCK on learning practices conducted by prospective mathematics teachers.

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REFERENCES

Ahamad, S.N.S.H., Li, H.C., Shahrill, M., & Prahmana, R.C.I. (2018). Implementation of problem-based learning in geometry lessons. *Journal of Physics: Conference Series, 943*(1), 012008. https://doi.org/10.1088/1742-6596/943/1/012008.

Amirali, M., & Halai, A. (2010). Teachers' knowledge about the nature of mathematics: A Survey of Secondary school teachers in Karachi, Pakistan. *Bulletin of Education and Research, 32*(2), 45-61.

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the US. *Journal of Mathematics Teacher Education, 7*(2), 145–172. https://doi.org/10.1023/B:JMTE.0000021943.35739.1c.

Bal, A.P. (2015). Examination of the mathematical problem-solving beliefs and success levels of primary school teacher candidates through the variables of mathematical success and gender. *Educational Sciences: Theory and Practice, 15*(5), 1373–1390. https://doi.org/10.12738/estp.2015.5.2573.

Ball, D.L., Hill, H.C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*(1), 14-46.

Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407. https://doi.org/10.1177/0022487108324554.

Barkatsas, A.T., & Malone, J. (2005). A typology of mathematics teachers' beliefs about teaching and learning mathematics and instructional practices. *Mathematics Education Research Journal, 17*(2), 69–90. https://doi.org/10.1007/BF03217416.

Bazeley, P. & Jackson, K. (2013). *Qualitative data analysis with NVivo*. Los Angeles: Sage Publications.

Bazeley, P. (2007). *Qualitative data analysis with Nvivo qualitative project book*. Thousand Oaks: Sage Publications.

Belbase, S. (2012). *Teacher belief, knowledge, and practice: A trichotomy of mathematics teacher education.* Retrieved from https://files.eric.ed.gov/fulltext/ED530017.pdf.

Beswick, K. (2003). Accounting for the contextual nature of teachers' beliefs in considering their relationship to practice. In Mathematics education research innovation, networking, opportunity: *Proceedings of the 26th Annual Conference of the Mathematics Education Research Group of Australasia* (vol. 1, pp. 152–159). Geelong: Pymble N.S.W. MERGA Inc.

Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal, 17*(2), 39–68. https://doi.org/10.1007/BF03217415.

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics, 79*(1), 127–147. https://doi.org/10.1007/s10649-011-9333-2.

Black, J.W. (2008). Content knowledge and pedagogical content knowledge of algebra teachers and changes in both types of knowledge as a result of professional development. *Proceedings of the 5th Annual TEAM-Math Partnership Conference Pre-Session* (pp. 30–40).

Boz, N. (2008). Turkish pre-service mathematics teachers' beliefs about nature of mathematics teaching. *Australian Journal of Teacher Education, 33*(5), 66-80. https://doi.org/10.14221/ajte.2008v33n5.5.

Buehl, M.M., & Fives, H. (2009). Exploring teachers' beliefs about teaching knowledge: Where does it come from? Does it change? *The Journal of Experimental Education, 77*(4), 367–408. https://doi.org/10.3200/JEXE.77.4.367-408.

Dossey, J. A., McCrone, S. S., & O'Sullivan, C. (2006). Problem solving in the PISA and TIMSS 2003 assessments. *Technical Report NCES 2007-049*. Washington DC: US Department of Education.

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. *Science and Technology Education, 35*, 99-101.

Felbrich A., Kaiser G., Schmotz C. (2014) The Cultural Dimension of Beliefs: An Investigation of Future Primary Teachers' Epistemological Beliefs Concerning the Nature of Mathematics in 15

Countries. In: Blömeke S., Hsieh FJ., Kaiser G., Schmidt W. (eds) *International Perspectives on Teacher Knowledge, Beliefs and Opportunities to Learn*. *Advances in Mathematics Education* (pp. 209-229). Dordrecht: Springer. https://doi.org/10.1007/978-94-007-6437-8_10

Fennema, E., & Franke, M. (1992). Teachers knowledge and its impact. In Grouws D. A. (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan Publishing Company.

Giovanni, P., & Sangcap, A. (2010). Mathematics-related beliefs of Filipino college students: Factors affecting mathematics and problem solving performance. *Procedia Social and Behavioral Sciences*, *8*, 465–475. https://doi.org/10.1016/j.sbspro.2010.12.064.

Grigutsch, S., Raatz, U., & Törner, G. (1998). Einstellungen gegenüber mathematik bei mathematiklehrern. *Journal Für Mathematik-Didaktik*, *19*(1), 3–45. https://doi.org/10.1007/BF03338859.

Hamrouni, A.D., & Akkari, I. 2012. The entrepreneurial failure: exploring links between the main causes of failure and the company life cycle qualitative analysis using NVivo 7 software. *International Journal of Business and Social Science*, *3*(4), 189-205.

Hawkins, W.J. (2012). An investıgatıon of primary teachers' pedagogical content knowledge when teaching measurement to years three and four. *Proceedings of the 12th International Congress on Mathematical Education*. Seoul: Seoul National University.

Kahan, J.A., Cooper, D.A., & Bethea, K.A. (2003). The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of prospective teachers. *Journal of Mathematics Teacher Education*, *6*(3), 223–252. https://doi.org/10.1023/A:1025175812582.

Kilic, H. (2011). The nature of pre-service teachers' pedagogical content knowledge. *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education–CERME7* (pp. 2690–2696). Rzeszów: University of Rzeszów.

Mewborn, D. (2001). Teachers content knowledge, teacher education, and their effects on the preparation of elementary teachers in the United States. *Mathematics Teacher Education and Development*, *3*(1), 28–36.

Moleong, L.J. (2007). *Qualitative Research Method* [in Bahasa]. Bandung: Remaja Rosdakarya.

Muhtadi, D., Wahyudin, Kartasasmita, B.G., & Prahmana, R.C.I. (2018). The Integration of technology in teaching mathematics. *Journal of Physics: Conference Series*, *943*(1), 012020. https://doi.org/10.1088/1742-6596/943/1/012020.

Muhtarom, M., Juniati, D., Siswono, T.Y.E., & Rahmatika, I. (2018). Teachers' and students' beliefs in mathematics at State Senior High School 5 Semarang. *Jurnal Riset Pendidikan Matematika*, *5*(1), 64-72. https://doi.org/10.21831/jrpm.v5i1.18734.

Muhtarom, Murtianto, Y.H., & Sutrisno. (2017). Thinking process of students with high-mathematics ability (A study on QSR NVivo 11-assisted data analysis). *International Journal of Applied Engineering Research*, *12*(17), 6934-6940.

Muhtarom., Juniati, D., & Siswono, T.Y.E. (2017a). Exploring beliefs in a problem-solving process of prospective teachers with high mathematical ability. *Global Journal of Engineering Education*, *19*(2), 130-136.

Muhtarom., Juniati, D., & Siswono, T.Y.E. (2017b). *Consistency and inconsistency of prospective teachers' beliefs in mathematics, teaching, learning and problem solving*. AIP Conference Proceedings (Vol. 1868, p. 050014). College Park: AIP Publishing.

Muhtarom., Juniati, D., & Siswono, T.Y.E. (2018). Exploring prospective teachers' beliefs about nature of mathematics. *Journal of Engineering and Applied Sciences, 13*(10), 3547-3554.

Muzaini, M., Juniati, D., & Siswono, T. Y. E. (2019). Exploration of student's quantitative reasoning in solving mathematical problem: case study of field-dependent cognitive style. *Journal of Physics: Conference Series*, *1157*(1), 032093. https://doi.org/10.1088/1742-6596/1157/3/032093.

Nizaruddin, Muhtarom, & Murtianto, Y.H. (2017). Exploring of multi mathematical representation capability in problem solving on senior high school students. *Problems of Education in the 21st Century*, *75*(6), 591-598.

Pajares, M.F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, *62*(3), 307–332. https://doi.org/10.3102/00346543062003307.

Pehkonen, E., & Pietilä, A. (2003). On relationships between beliefs and knowledge in mathematics education. *Proceedings of the Third Conference of the European Society for Research in Mathematics Education*. Bellaria: University of Pisa and ERME.

Prahmana, R.C.I., & Suwasti, P. (2014). Local instruction theory on division in Mathematics GASING: The case of rural area's student in Indonesia. *Journal on Mathematics Education, 5*(1), 17-26. https://doi.org/10.22342/jme.5.1.1445.17-26.

Raymond, A.M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching. *Journal for Research in Mathematics Education*, *28*(5), 550–576. https://doi.org/10.2307/749691.

Shahrill, M., Putri, R.I.I., Zulkardi, & Prahmana, R.C.I. (2018). Processes involved in solving mathematical problems. *AIP Conference Proceedings*, *1952*(1), 020019. https://doi.org/10.1063/1.5031981.

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14. https://doi.org/10.3102/0013189X015002004.

Shulman, L.S. (1987). Knowledge and teaching: Foundations for the new reform. *Harvard Educational Review*, *57*(1), 1-22. https://doi.org/10.17763/haer.57.1.j463w79r56455411.

Siswono, T.Y.E., Kohar, A.W., & Hartono, S. (2017). Secondary teachers' mathematics-related beliefs and knowledge about mathematical problem-solving. *Journal of Physics: Conference Series*, *812*(1), 012046. https://doi.org/10.1088/1742-6596/812/1/012046.

Siswono, T.Y.E., Kohar, A.W., Rosyidi, A.H., & Hartono, S. (2017). Primary school teachers' beliefs and knowledge about mathematical problem-solving and their performance on problem-solving task. *World Transactions on Engineering and Technology Education*, *15*(2), 126–131.

Stipek, D.J., Givvin, K.B., Salmon, J. M., & MacGyvers, V.L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education*, *17*(2), 213–226. https://doi.org/10.1016/S0742-051X(00)00052-4.

Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). Reston: National Council of Teachers of Mathematics.

Turnuklu, E.B., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal*, *1*, 1-13.

Viholainen, A., Asikainen, M., & Hirvonen, P.E. (2014). Mathematics prospective teachers' epistemological beliefs about the nature of mathematics and the goals of mathematics teaching and learning in the beginning of their studies. *Eurasia Journal of Mathematics, Science & Technology Education*, *10*(2), 159–171. https://doi.org/10.12973/eurasia.2014.1028a.

Wilkins, J.L. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education*, *11*(2), 139–164. https://doi.org/10.1007/s10857-007-9068-2.